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Dynamic behaviour of an anisotropic liquid-saturated porous medium in frequency domain

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Abstract

A general solution to the field equations of an anisotropic liquid-saturated porous medium has been obtained, in the transformed form, using the Fourier transform. Assuming the disturbances to be harmonically time dependent, the transformed solution is obtained in the frequency domain. An application of a time-harmonic concentrated force acting at some interior point of an infinite medium has been considered to show the utility of the solution obtained. The transformed solutions are inverted numerically, using a numerical inversion technique to invert the Fourier transform. The results in the form of stress components have been obtained numerically and discussed graphically for a particular model. The results of the corresponding problem in isotropic liquid-saturated porous medium can be derived as a special case.

1. Introduction

The liquid-saturated porous solids are often present on and below the surface of the earth. The liquid-saturated porous medium is of great importance in the fields of earth sciences and engineering. Recently, de Boer [1] presented a comprehensive review of the porous media theory. Biot [2,3] developed a systematic theory of wave propagation in liquid-saturated porous solids, and provides a potentially powerful tool for studying the behaviour of many kinds of porous media. The classical poroelastic model of Biot has been widely used by various authors. Deresiewicz and Skalak [4] derived the conditions sufficient for uniqueness of solution of the field

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equations of Biot's theory of liquid-filled porous media. Burridge and Vargas [5] gave the fundamental solution in dynamic poroelasticity theory given by Biot. Altay and Dokmeci [6] proved the uniqueness of the solution of Biot poroelasticity equations.

Many researchers have discussed the different types of problems related to liquid-saturated porous medium using different theories, e.g., Armero and Callari [7], Fellah and Depollier [8], Reddy and Tajjuddin [9], Schanz and Cheng [10], etc. The various problems of the deformation of the liquid-saturated porous medium have been discussed by many authors in many different ways, e.g., Paul [11,12], Pal [13], Philippacopolous [14], Sharma [15], Sharma and Gogna [16], Cederbaum [17], Kumar et al. [18,19], etc.

There are reasonable grounds for the assumption that anisotropy may exist in the continents. Anisotropy has significant effects on the characteristics of various phenomena occurring in an earthquakes, e.g., wave propagation. Therefore, many investigators have studied the problems related to anisotropic liquid-saturated porous medium. Kazi-Aoual et al. [20] discussed the Green's function for transversely isotropic poroelastic medium. Sharma and Gogna [21] have studied the wave propagation in anisotropic liquid-saturated porous solids. The wave propagation theory in anisotropic periodically layered fluid-saturated porous media has been discussed by Sun et al. [22]. Carcione [23] has discussed the plane wave theory and numerical simulation for wave propagation in anisotropic liquid-saturated porous medium. Propagation of plane waves in transversely isotropic fluid-saturated porous medium was studied by Wang and Zhang [24]. Sharma [25], considering a model involving a layer of transversely isotropic liquid-saturated porous medium, discussed the dispersion in oceanic crust, which may help in identifying an earthquake preparation region.

The solution of the dynamic problem in frequency domain is simpler than in the time domain. As the time variable is missing in the frequency-domain formulation, the dynamic problem become the static-like problem. Also, the solution in the frequency domain is important, whenever we have to discuss the wave propagation phenomena. Therefore, many researchers have dealt with the dynamical problems in frequency domain, e.g., Sato [26], Dominguez [27], Rajapakse and Senjuntichai [28], etc.

The determination of the state of stress in the materials of the earth due to the presence of certain sources is of great importance. The field of geomechanics, dealing with the various phenomena occurring in an earthquake, deals with the problem of dynamic behaviour of an earth material due to the presence of certain sources. Here, in this investigation, the dynamic behaviour of a transversely isotropic liquid-saturated porous medium due to a time-harmonic concentrated point force has been discussed, in frequency domain, by assuming the disturbances to be harmonically time dependent.

2. Basic equations

Following Biot [2,3], the equations of motion for the liquid-saturated porous medium in the absence of body forces, without dissipation are given as

$$\sigma_{ij,j} = \frac{\partial^2}{\partial t^2}(\rho_{11} u_i + \rho_{12} U_i), \tag{1}$$

$$\sigma_{,i} = \frac{\partial^2}{\partial t^2}(\rho_{12}u_i + \rho_{22}U_i) \quad (i = x, y, z), \tag{2}$$

where σ_{ij} are the stress components in the solid, $\sigma = -\beta p$ is the stress in the fluid (p is the pressure in the fluid and β is the porosity); u_i, U_i $(i = x, y, z)$ are the components of the displacement vectors in the solid and liquid parts, respectively, of the porous medium; ρ_{11}, ρ_{12} and ρ_{22} are the dynamical coefficients and are related to the mass densities of the solid ρ_s and fluid ρ_f as

$$\rho_{11} + \rho_{12} = (1-\beta)\rho_s, \quad \rho_{12} + \rho_{22} = \beta\rho_f, \tag{3}$$

so that the mass density of the bulk material is

$$\rho = \rho_{11} + 2\rho_{12} + \rho_{22} = \rho_s + \beta(\rho_f - \rho_s). \tag{4}$$

The stress–strain relations for the transversely isotropic liquid-saturated porous solid with symmetry about the z-axis are given by Biot [29] as

$$\begin{aligned}
\sigma_{xx} &= 2Ne_{xx} + A(e_{xx} + e_{yy}) + Fe_{zz} + M\varepsilon,\\
\sigma_{yy} &= 2Ne_{yy} + A(e_{xx} + e_{yy}) + Fe_{zz} + M\varepsilon,\\
\sigma_{zz} &= Ce_{zz} + F(e_{xx} + e_{yy}) + Q\varepsilon,\\
\sigma_{yz} &= Le_{yz}, \quad \sigma_{xz} = Le_{xz}, \quad \sigma_{xy} = Ne_{xy},\\
\sigma &= M(e_{xx} + e_{yy}) + Qe_{zz} + R\varepsilon,
\end{aligned} \tag{5}$$

where

$$e_{ij} = \begin{cases} \dfrac{\partial u_i}{\partial x_j}, & i = j,\\[2mm] \dfrac{\partial u_i}{\partial x_j} + \dfrac{\partial u_j}{\partial x_i}, & i \neq j, \end{cases} \tag{6}$$

$$\varepsilon = \frac{\partial U}{\partial x} + \frac{\partial V}{\partial y} + \frac{\partial W}{\partial z}, \tag{7}$$

A, N, F, M, C, Q, L and R are the elastic constants for transversely isotropic liquid-saturated porous solid. These elastic constants can be reduced to that of isotropic liquid-saturated porous solid through relations

$$F = A, \quad M = Q, \quad L = N, \quad C = A + 2N. \tag{8}$$

3. Formulation of the problem

A transversely isotropic liquid-saturated porous medium with symmetry about z-axis, taken vertically downwards, has been considered. The problem considered is two-dimensional plane strain, i.e., the field components along the y direction are zero and others are independent of y-co-ordinate. An application of a time-harmonic concentrated point force acting at some point, taken as origin, in the interior of an infinite medium along vertical direction, is considered, i.e., the force is applied at the origin of the co-ordinate system along z-axis.

4. Solution of the problem

We are assuming the disturbances to be harmonically time dependent ($e^{i\omega t}$), and since the problem considered is two-dimensional plane strain, so we take

$$\vec{u} = (u, 0, w), \qquad \vec{U} = (U, 0, W) \tag{9}$$

and

$$u(x,z,t) = u(x,z)e^{i\omega t}, \qquad w(x,z,t) = w(x,z)e^{i\omega t},$$
$$U(x,z,t) = U(x,z)e^{i\omega t}, \qquad W(x,z,t) = W(x,z)e^{i\omega t}, \tag{10}$$

where (u, w) and (U, W) represent the displacement components in the solid and liquid parts of the porous aggregate, respectively; and 'ω' is the angular frequency. Now, using the non-dimensional variables and coefficients defined as

$$u' = \frac{u}{h}, \quad w' = \frac{w}{h}, \quad U' = \frac{U}{h}, \quad W' = \frac{W}{h},$$
$$x' = \frac{x}{h}, \quad z' = \frac{z}{h}, \quad t' = \frac{t}{t^*}, \quad \omega' = \omega t^*,$$
$$\sigma'_{zz} = \frac{\sigma_{zz}}{C}, \quad \sigma'_{zx} = \frac{\sigma_{zx}}{C}, \quad \sigma' = \frac{\sigma}{C} \tag{11}$$

and

$$a^2 = \frac{Q}{C}, \quad b^2 = \frac{R}{C}, \quad c^2 = \frac{A+2N}{C},$$
$$d^2 = \frac{L}{C}, \quad e^2 = \frac{F+L}{C}, \quad f^2 = \frac{M}{C},$$
$$R_{11} = \frac{\rho_{11}}{\rho}, \quad R_{12} = \frac{\rho_{12}}{\rho}, \quad R_{22} = \frac{\rho_{22}}{\rho}, \tag{12}$$

where

$$t^* = h\sqrt{\frac{\rho}{C}}, \tag{13}$$

and 'h' has the dimension of length, and then applying the Fourier transformation, Sneddon [30], with respect to 'x' defined as

$$\{\hat{u}(q,z), \hat{w}(q,z), \hat{U}(q,z), \hat{W}(q,z)\} = \int_{-\infty}^{\infty} \{u(x,z), w(x,z), U(x,z), W(x,z)\}e^{iqx}\,dx \tag{14}$$

on the reduced non-dimensional form of equations. We obtain a system of four ordinary differential equations, which is written in the matrix differential equation form (after suppressing the primes) as

$$A_1\ddot{V} + B_1\dot{V} + C_1 = 0, \tag{15}$$

where dot represents the differentiation with respect to z,

$$V = [\hat{u}, \hat{w}, \hat{U}, \hat{W}]^{T}, \ {}^{T} \text{ stands for transpose,}$$

$$A_1 = \begin{bmatrix} d^2 & 0 & 0 & 0 \\ 0 & 1 & 0 & a^2 \\ 0 & 0 & 0 & 0 \\ 0 & a^2 & 0 & b^2 \end{bmatrix}, \qquad B_1 = \begin{bmatrix} 0 & -\mathrm{i}qe^2 & 0 & -\mathrm{i}qf^2 \\ -\mathrm{i}qe^2 & 0 & -\mathrm{i}qa^2 & 0 \\ 0 & -\mathrm{i}qa^2 & 0 & -\mathrm{i}qb^2 \\ -\mathrm{i}qf^2 & 0 & -\mathrm{i}qb^2 & 0 \end{bmatrix},$$

$$C_1 = \begin{bmatrix} -(q^2c^2 + H_{11}) & 0 & -(q^2f^2 + H_{12}) & 0 \\ 0 & -(q^2d^2 + H_{11}) & 0 & -H_{12} \\ -(q^2f^2 + H_{12}) & 0 & -(q^2b^2 + H_{22}) & 0 \\ 0 & -H_{12} & 0 & -H_{22} \end{bmatrix} \tag{16}$$

and

$$H_{11} = -R_{11}\omega^2, \quad H_{12} = -R_{12}\omega^2, \quad H_{22} = -R_{22}\omega^2. \tag{17}$$

To solve the system of equations (15), we assume

$$V(q,z) = X(q)\mathrm{e}^{mz}, \tag{18}$$

which gives us the characteristic equation

$$\det(m^2A_1 + mB_1 + C_1) = 0, \tag{19}$$

i.e.,

$$T_0m^6 + T_1m^4 + T_2m^2 + T_3 = 0, \tag{20}$$

where

$$\begin{aligned}
&T_0 = -d^2H_{22}X, \quad T_1 = T_{11} + T_{12}q^2, \\
&T_2 = T_{21} + T_{22}q^2 + T_{23}q^4, \quad T_3 = T_{31} + T_{32}q^2 + T_{33}q^4 + T_{34}q^6, \\
&T_{11} = ZX + d^2H_{22}Y, \quad T_{12} = (XX_1' + b^2X_1 + f^2X_2)H_{22}, \\
&T_{21} = -(Y + d^2H_{22})Z, \\
&T_{22} = -\big[(X + X')H_{11}H_{22} + 2(a^2e^2 - f^2 - a^2c^2 + e^2f^2)H_{12}H_{22} \\
&\qquad + 2b^2d^2Z + (X_1 + c^2)H_{22}^2 - 2(b^2e^2 - a^2f^2)H_{12}^2\big], \\
&T_{23} = -\big[b^2X_1 + c^2X + d^2X' + f^2X_2\big]H_{22}, \\
&T_{31} = Z^2, \quad T_{32} = (Y' + d^2H_{22})Z, \\
&T_{33} = X'Z + d^2H_{22}Y', \quad T_{34} = d^2H_{22}X'
\end{aligned}$$

and

$$\begin{aligned}
&X = b^2 - a^4, \quad X' = b^2c^2 - f^4, \\
&X_1 = d^4 - e^4, \quad X_1' = c^2 + d^2, \\
&X_2 = 2a^2e^2 - f^2, \quad Z = H_{11}H_{22} - H_{12}^2, \\
&Y = b^2H_{11} + H_{22} - 2H_{12}a^2, \quad Y' = b^2H_{11} + c^2H_{22} - 2H_{12}f^2.
\end{aligned} \tag{21}$$

Using Cardan's method, Eq. (20) is reduced to

$$z^3 + 3Hz + G = 0, \tag{22}$$

where

$$\begin{aligned} &z = m^2 + \frac{T_1}{3T_0}, \\ &H = \tfrac{1}{3}(T_2' - \tfrac{1}{3}T_1'^2), \quad G = T_3' - \tfrac{1}{3}T_1'T_2' + \tfrac{2}{27}T_1'^3, \\ &T_1' = \frac{T_1}{T_0}, \quad T_2' = \frac{T_2}{T_0}, \quad T_3' = \frac{T_3}{T_0}. \end{aligned} \tag{23}$$

The roots of Eq. (22) are given by

$$z = r + s, \tag{24}$$

where r and s are given by

$$r^3 = \frac{-G + \sqrt{G^2 + 4H^3}}{2}, \quad s^3 = \frac{-G - \sqrt{G^2 + 4H^3}}{2},$$

satisfying

$$rs = -H.$$

Hence, roots of Eq. (20) are given by

$$m_n^2 = z_n - \frac{T_1}{3T_0}, \quad n = 1, 2, 3. \tag{25}$$

The eigenvectors $X(q)$ associated to different eigenvalues $\pm m_1$, $\pm m_2$ and $\pm m_3$ are obtained as

$$\begin{aligned} &X_i^{\mathrm{T}} = [-k_i', l_i', -m_i', n_i'], \quad \text{for } m = m_i, \\ &X_{i+3}^{\mathrm{T}} = [-k_i', l_i', -m_i', n_i'], \quad \text{for } m = -m_i, \ i = 1, 2, 3, \end{aligned} \tag{26}$$

where

$$\begin{aligned} &n_i' = n_{i1}n_{i2} - n_{i3}n_{i4}, \quad m_i' = n_{i1}m_{i1} - n_{i3}m_{i2}, \\ &l_i' = \frac{1}{n_{i1}}[n_{i4}m_i' - m_{i2}n_i'], \quad k_i' = \frac{1}{k_{i4}}[k_{i1}l_i' - k_{i2}m_i' + k_{i3}n_i'], \\ &k_{i1} = a^2m_i^2 - H_{12}, \quad k_{i2} = -\mathrm{i}qm_ib^2, \\ &k_{i3} = b^2m_i^2 - H_{22}, \quad k_{i4} = -\mathrm{i}qm_if^2, \\ &m_{i1} = \mathrm{i}qm_i\{f^2(f^2q^2 + H_{12}) + b^2(d^2m_i^2 - c^2q^2 - H_{11})\}, \\ &m_{i2} = f^2(a^2m_i^2 - H_{12}) - e^2(b^2m_i^2 - H_{22}), \\ &n_{i1} = f^2(m_i^2 - d^2q^2 - H_{11}) - e^2(a^2m_i^2 - H_{12}), \\ &n_{i2} = (f^2q^2 + H_{12})^2 + (b^2q^2 + H_{22})(d^2m_i^2 - c^2q^2 - H_{11}), \\ &n_{i3} = \mathrm{i}qm_i\{e^2(f^2q^2 + H_{12}) + a^2(d^2m_i^2 - c^2q^2 - H_{11})\}, \\ &n_{i4} = -\mathrm{i}qm_i(a^2f^2 - b^2e^2), \quad i = 1, 2, 3. \end{aligned} \tag{27}$$

Thus, a general solution of Eq. (15) in the transformed form, is obtained as

$$V(q,z)=\sum_{i=1}^{3}\{B_iX_i(q)e^{m_iz}+B_{i+3}X_{i+3}(q)e^{-m_iz}\}, \tag{28}$$

where B_i $(i=1,2,3,4,5,6)$ are arbitrary constants to be determined from the boundary conditions. Using Eq. (10), a general solution for the two-dimensional plane strain problem of a transversely liquid-saturated porous medium in the frequency domain, is obtained as

$$V(q,z,\omega)=\sum_{i=1}^{3}\{B_iX_i(q)e^{m_iz}+B_{i+3}X_{i+3}(q)e^{-m_iz}\}e^{i\omega t}. \tag{29}$$

5. Application

Let us consider a time-harmonic concentrated normal point force of magnitude F_0 is acting at the origin along the z direction in an infinite transversely isotropic liquid-saturated porous medium. The appropriate boundary conditions in the present case at $z=0$ are given as

$$\begin{aligned}
&u(x,0^+,\omega)-u(x,0^-,\omega)=0,\quad w(x,0^+,\omega)-w(x,0^-,\omega)=0,\\
&W(x,0^+,\omega)-W(x,0^-,\omega)=0,\quad \sigma_{zz}(x,0^+,\omega)-\sigma_{zz}(x,0^-,\omega)=-F_0\delta(x)e^{i\omega t},\\
&\sigma_{xz}(x,0^+,\omega)-\sigma_{xz}(x,0^-,\omega)=0,\quad \sigma(x,0^+,\omega)-\sigma(x,0^-,\omega)=F_0\delta(x)e^{i\omega t}.
\end{aligned} \tag{30}$$

Using Eqs. (11) and (14), these boundary conditions (30) can be written in the non-dimensional transformed form (after suppressing the primes) as

$$\begin{aligned}
&\hat{u}(q,0^+,\omega)-\hat{u}(q,0^-,\omega)=0,\quad \hat{w}(q,0^+,\omega)-\hat{w}(q,0^-,\omega)=0,\\
&\hat{W}(q,0^+,\omega)-\hat{W}(q,0^-,\omega)=0,\quad \hat{\sigma}_{zz}(q,0^+,\omega)-\hat{\sigma}_{zz}(q,0^-,\omega)=-F_0e^{i\omega t},\\
&\hat{\sigma}_{xz}(q,0^+,\omega)-\hat{\sigma}_{xz}(q,0^-,\omega)=0,\quad \hat{\sigma}(q,0^+,\omega)-\hat{\sigma}(q,0^-,\omega)=F_0e^{i\omega t}.
\end{aligned} \tag{31}$$

Now, from Eqs. (5)–(7), after reducing them to non-dimensional transformed form by using Eqs. (11), (12) and (14), and with the help of Eq. (29), we obtain the displacement and stress components in the transformed form (after suppressing the primes) as

$$\begin{aligned}
&\hat{u}(q,z,\omega)=-\sum_{i=1}^{3}B_{i+j}k_i'e^{-\gamma zm_i}\cdot e^{i\omega t},\quad \hat{w}(q,z,\omega)=\sum_{i=1}^{3}B_{i+j}l_i'e^{-\gamma zm_i}\cdot e^{i\omega t},\\
&\hat{U}(q,z,\omega)=-\sum_{i=1}^{3}B_{i+j}m_i'e^{-\gamma zm_i}\cdot e^{i\omega t},\quad \hat{W}(q,z,\omega)=\sum_{i=1}^{3}B_{i+j}n_i'e^{-\gamma zm_i}\cdot e^{i\omega t},\\
&\hat{\sigma}_{zz}(q,z,\omega)=\sum_{i=1}^{3}B_{i+j}P_{i+j}e^{-\gamma zm_i}\cdot e^{i\omega t},\quad \hat{\sigma}_{xz}(q,z,\omega)=\sum_{i=1}^{3}B_{i+j}R_{i+j}e^{-\gamma zm_i}\cdot e^{i\omega t},\\
&\hat{\sigma}(q,z,\omega)=\sum_{i=1}^{3}B_{i+j}H_{i+j}e^{-\gamma zm_i}\cdot e^{i\omega t},
\end{aligned} \tag{32}$$

where

$$\gamma = \mathrm{sign}(z) \quad \text{and} \quad \begin{cases} j = 0 & \text{if } z < 0, \\ j = 3 & \text{if } z > 0 \end{cases} \tag{33}$$

and

$$\begin{aligned} &P_i = l'_i m_i + \mathrm{i}q(e^2 - d^2)k'_i + a^2 n'_i m_i + \mathrm{i}qa^2 m'_i, \\ &P_{i+3} = -l'_i m_i + \mathrm{i}q(e^2 - d^2)k'_i - a^2 n'_i m_i + \mathrm{i}qa^2 m'_i, \\ &R_i = -(k'_i m_i + \mathrm{i}ql'_i)d^2, \quad R_{i+3} = (k'_i m_i - \mathrm{i}ql'_i)d^2, \\ &H_i = a^2 l'_i m_i + \mathrm{i}qf^2 k'_i + b^2 n'_i m_i + \mathrm{i}qb^2 m'_i, \\ &H_{i+3} = -a^2 l'_i m_i + \mathrm{i}qf^2 k'_i - b^2 n'_i m_i + \mathrm{i}qb^2 m'_i, \quad i = 1, 2, 3. \end{aligned} \tag{34}$$

Substituting the transformed displacements and stresses given by Eq. (32) in the transformed boundary conditions (31), we obtain a system of six equations in six unknowns B_1, B_2, B_3, B_4, B_5 and B_6, which on solving gives

$$\begin{aligned} &B_4 = B_1 = -s_1 \frac{(t_3 + r_3)s_2 - (t_2 + r_2)s_3}{\Delta} F_0, \\ &B_5 = B_2 = s_1 \frac{(t_3 + r_3)s_1 - (t_1 + r_1)s_3}{\Delta} F_0, \\ &B_6 = B_3 = -s_1 \frac{(t_2 + r_2)s_1 - (t_1 + r_1)s_2}{\Delta} F_0, \end{aligned} \tag{35}$$

where

$$\begin{aligned} &\Delta = (t_2 s_1 - t_1 s_2)(r_3 s_1 - r_1 s_3) - (r_2 s_1 - r_1 s_2)(t_3 s_1 - t_1 s_3), \\ &r_n = P_{n+3} - P_n, \quad s_n = R_{n+3} - R_n, \quad t_n = H_{n+3} - H_n, \quad n = 1, 2, 3. \end{aligned} \tag{36}$$

Thus, expressions (32) give the displacement and stress components with the help of Eqs. (35) and (36) in the transformed form, in frequency domain, for an infinite transversely isotropic fluid-saturated porous medium due to a time-harmonic concentrated normal point force acting along the z-axis. These, displacement and stress components in the transformed form, on inversion enable us to give the displacement and stress components in the physical form. To invert the Fourier transforms in the transformed form expressions, we make use of a numerical inversion technique to get the results in the physical form, numerically.

6. Special cases

(i) With the help of relations (8), we obtain the solution of the problem for isotropic liquid-saturated porous medium. The resulting solution of the problem agree well with the one obtained by Kumar et al. [18] with suitable change in notations as

$$\begin{aligned} &a^2 = f^2 = b^2, \quad b^2 = d^2, \quad d^2 = a^2 \\ &e^2 = 1 - a^2, \quad \omega^2 = -p^2, \end{aligned} \tag{37}$$

and taking the dissipation factor as zero.

(ii) Further, in the limiting case of β, Q, $R \to 0$, and taking $A = \lambda$ and $N = \mu$, we obtain the solution of the problem in classical elastic medium, which is known as Lamb's problem.

7. Inversion of the transforms

The transformed solutions are functions of depth variable 'z', the Fourier transform parameter q and the frequency ω, and hence are of the form $\hat{f}(q,z,\omega)$. To get the function $f(x,z,\omega)$ in the physical form, we invert the Fourier transform by using

$$f(x,z,\omega) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \mathrm{e}^{-\mathrm{i}qx}\hat{f}(q,z,\omega)\,\mathrm{d}q = \frac{1}{\pi}\int_{0}^{\infty} (\cos(qx)f_e - \mathrm{i}\sin(qx)f_o)\,\mathrm{d}q, \tag{38}$$

where f_e and f_o are even and odd parts of the function $\hat{f}(q,z,\omega)$, respectively. Thus, expression (38) gives us the function $f(x,z,\omega)$. Now, we have to evaluate the integral in Eq. (38) and the method for evaluating the integral is described by Press et al. [31], which involves the use of Romberg's integration with adaptive step size. This, also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

8. Numerical results and discussion

Numerical calculations are made by considering a particular model of a transversely isotropic liquid-saturated porous medium. In the model considered, the elastic constants for the transversely isotropic liquid-saturated porous medium are chosen as

$$A = 4.43 \times 10^{10}\ \mathrm{dyn/cm^2}, \quad F = fA,$$

$$Q = 0.743 \times 10^{10}\ \mathrm{dyn/cm^2}, \quad M = mQ,$$

$$N = 2.765 \times 10^{10}\ \mathrm{dyn/cm^2}, \quad L = nN,$$

$$R = 0.326 \times 10^{10}\ \mathrm{dyn/cm^2}, \quad C = F + 2L.$$

For $f = m = n = 1.0$, these constants represent the elastic constants of an isotropic kerosene-saturated sandstone, Fatt [32]. The dynamical coefficients are given as

$$\rho_{11} = 1.926\ \mathrm{g/cm^3}, \quad \rho_{12} = -0.00214\ \mathrm{g/cm^3}, \quad \rho_{22} = 0.21534\ \mathrm{g/cm^3},$$

Also, we have considered

$$F_0' = \frac{F_0}{C} = 1.0 \quad \text{and} \quad t = 1.0.$$

The anisotropy of the medium is represented by a set of values of the parameters, f, m, n. Here, we take

$$f = 1.5, \quad m = 1.2, \quad n = 2.0,$$

but, one can take some other set of values, also.

The stress components, representing the dynamical behaviour, are calculated on the plane $z = 1.0$ against non-dimensional distance 'x' for three different values of non-dimensional frequency 'ω', i.e., $\omega = 1.0$, 2.0 and 5.0, taking the medium to be transversely isotropic liquid-saturated porous solid. The stress distribution curves are shown in Figs. 1–3. Further, the stress distribution curves of transversely isotropic as well as isotropic liquid-saturated porous medium are plotted for one fixed value of non-dimensional frequency 'ω', i.e., $\omega = 2.0$, in Figs. 4–6, depicting the effect of transverse isotropy.

From Figs. 1–3, it is observed that the maximum (absolute) stresses occur corresponding to minimum frequency ($\omega = 1.0$), i.e., the impact of a time-harmonic impulsive force is large for small frequencies. Further, it is observed that the impact of the applied force goes on decreasing with increase in frequency. It is clear from Figs. 1–3, that the stress components are approaching towards zero value with reference to frequency 'ω'. But, this approach towards zero value is not uniform for all the cases. This is due to the fact that the medium considered is liquid-saturated porous, which is a two-phase medium involving an elastic solid matrix with pores saturated with fluid. The disturbances travelling through these different constituents of the medium suffer sudden changes, resulting in an inconsistent/non-uniform approach towards zero value. Otherwise, when

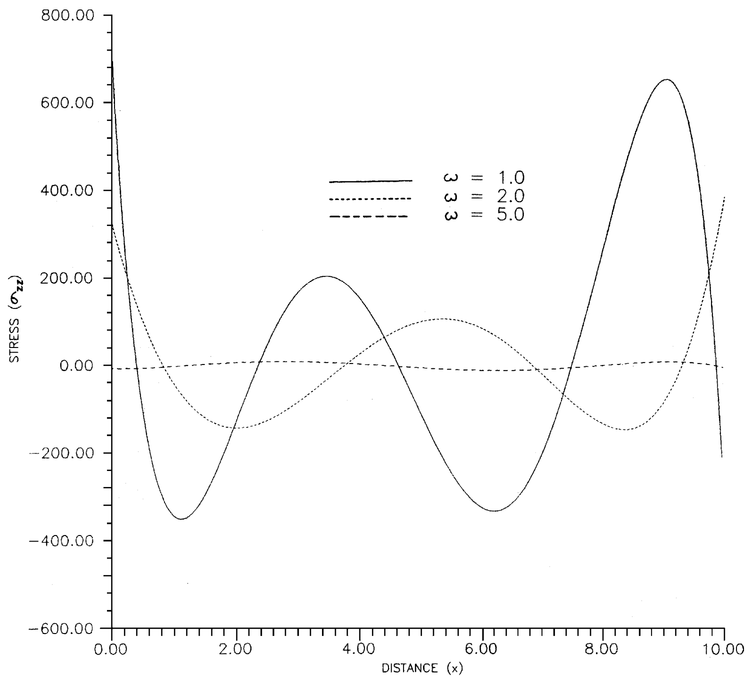

Fig. 1. Normal stress distribution in the solid part of the transversely isotropic porous aggregate at the level $z = 1.0$.

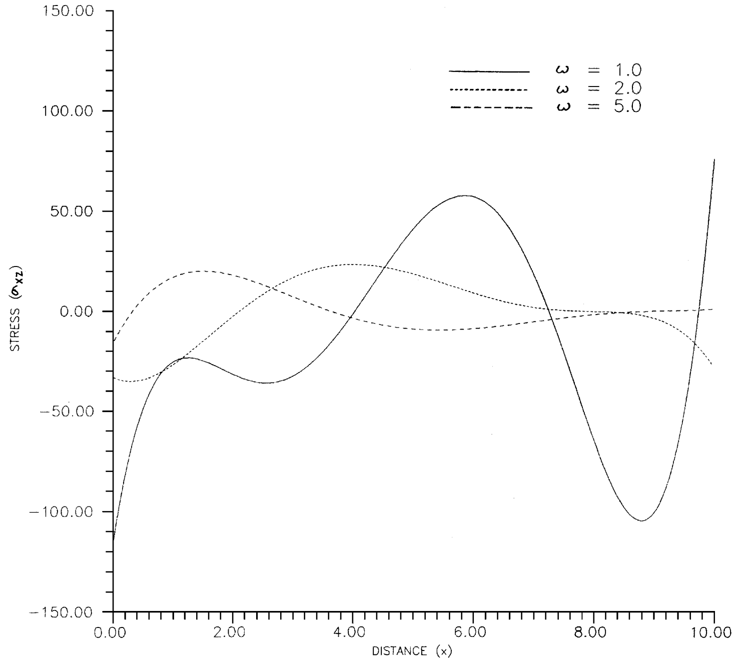

Fig. 2. Tangential stress distribution in the solid part of the transversely isotropic porous aggregate at the level $z = 1.0$.

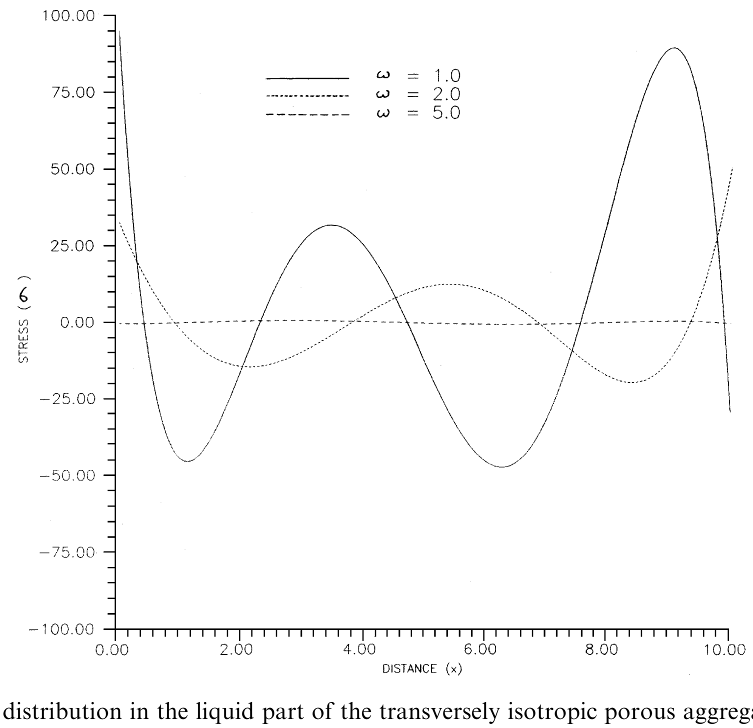

Fig. 3. Normal stress distribution in the liquid part of the transversely isotropic porous aggregate at the level $z = 1.0$.

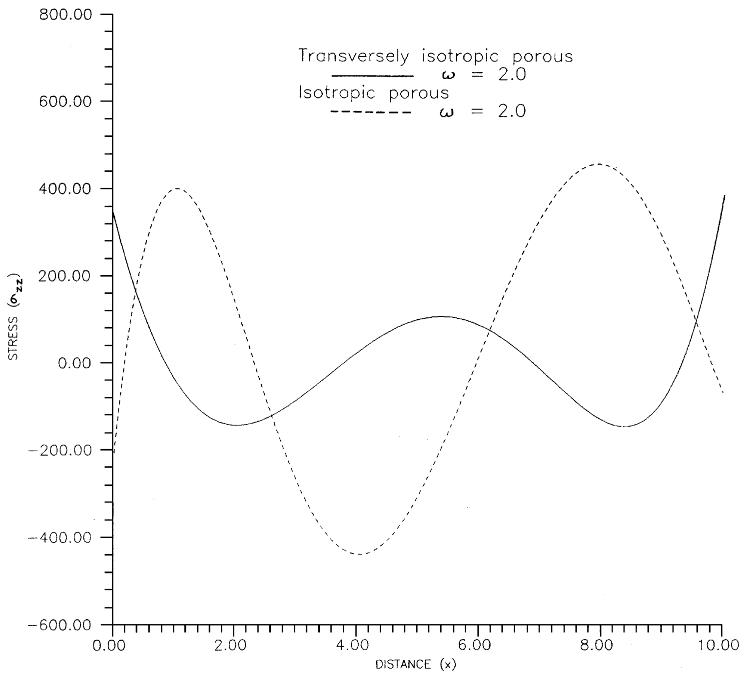

Fig. 4. Normal stress distribution in the solid part of the porous aggregate at the level $z = 1.0$.

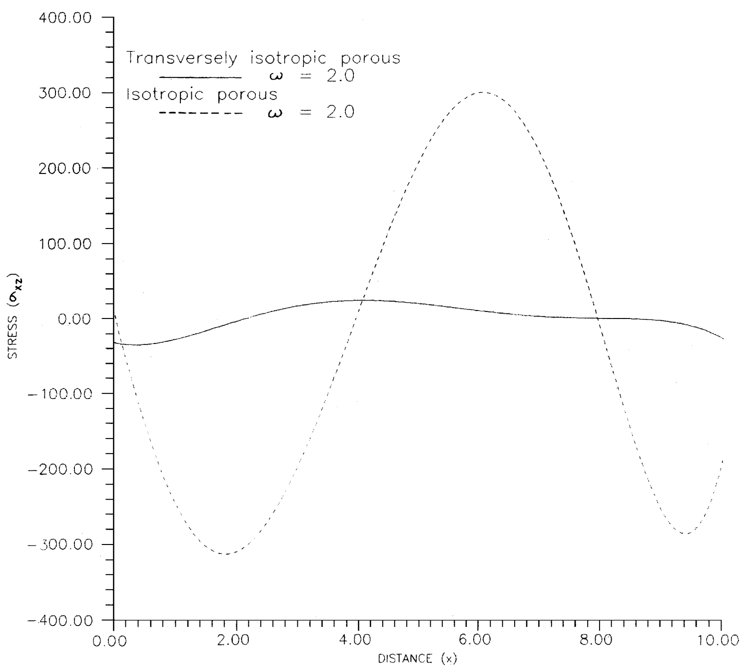

Fig. 5. Tangential stress distribution in the solid part of the porous aggregate at the level $z = 1.0$.

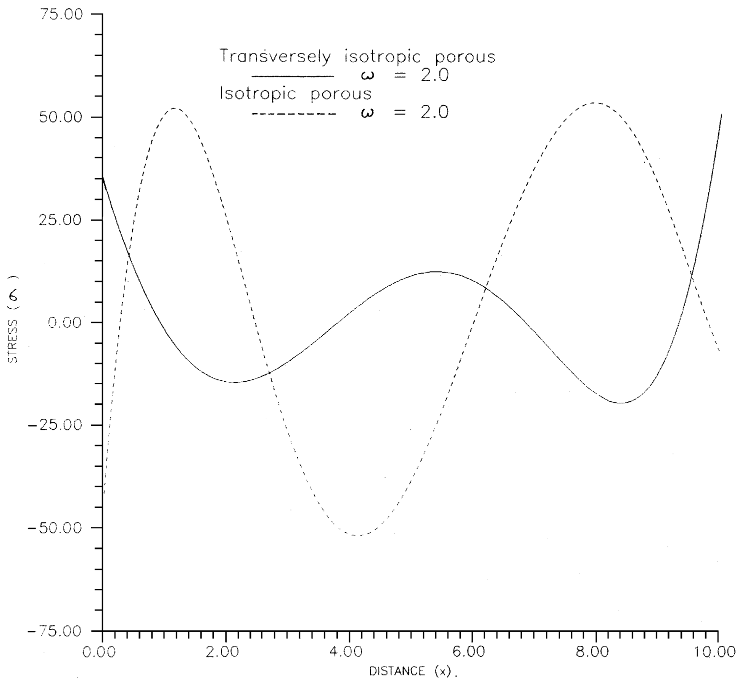

Fig. 6. Normal stress distribution in the liquid part of the porous aggregate at the level $z = 1.0$.

it passes through either solid or liquid, it shows a uniform change. It is observed that the magnitude of displacement and stress components increase or decrease along an oscillatory path with the increase in distance 'x'. It is revealed that with the increase in distance each distribution curve follow an oscillatory pattern.

Figs. 4–6 show the change in distribution curves of isotropic liquid-saturated porous medium due to transverse isotropy of the porous medium. That means, the transverse isotropy of the porous medium affects the disturbances produced due to time-harmonic impulsive force. It is observed that the effect is mainly on the magnitude of stresses. This effect is in the form of decrease in the range of values of the distribution curves. This decrease in the range of values is also not uniform due to the complexity of the medium considered. It is also observed that the transverse isotropy of the liquid-saturated porous medium disturb the oscillatory pattern of the distribution curves of isotropic liquid-saturated porous medium. It is revealed that the effect of transverse isotropy of the porous medium is mainly quantitative in nature.

The trend of curves exhibits the properties of liquid-saturated porous medium. The disturbances produced in the liquid-saturated porous medium are affected by anisotropy of the medium, which in turn will effect the various phenomena, e.g., wave propagation. The results of this problem are very useful in the two-dimensional treatment of the dynamic response due to impulsive harmonic sources of the poroelastic solids, which has various application in the fields of geomechanics and engineering.

Appendix. Nomenclature

A, N, F, M, C, Q, L, R	elastic constants for transversely isotropic liquid-saturated porous medium
ρ_{11}, ρ_{12}, ρ_{22}	dynamical coefficients
ρ_s, ρ_f, ρ	mass densities of the solid, fluid, bulk material
β	porosity
ω	angular frequency
t	time variable
x, y, z	cartesian co-ordinates
u_i, U_i	components of displacements in the solid and liquid parts of the porous aggregate along x_i direction
$\vec{u}$, $\vec{U}$	displacement vectors in the solid and liquid parts of the porous aggregate
u, w	tangential and normal components of the displacement in the solid part
U, W	tangential and normal components of the displacement in the liquid part
p	pressure in the fluid
σ_{ij}	components of stress in the solid part
σ	stress in the fluid part
e_{ij}	components of strain in the solid part
e	dilatation in the solid part
ε	dilatation in the liquid part
h	a quantity having the dimension of length

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