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Active control of vibration using a fuzzy control method

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Abstract

Active control of vibration has been the subject of a lot of research in recent years. Adaptive linear filtering techniques have been extensively used for the active control of vibration. A popular adaptive filtering algorithm is the filtered-X LMS algorithm because of its simplicity and its relatively low computational load. However, the filtered-X LMS algorithm is limited to linear control problems using feedforward control techniques. A fuzzy logic system based control structure and adaptive algorithm suitable for driving non-linear feedforward active vibration control systems is presented in this paper. This kind of non-linear fuzzy control method is used in two kinds of active vibration control problems, as follows: (1) A fuzzy logic system is used to approximate a non-linear sensor path function and to suppress the primary disturbance that is a non-linear function of the reference signal. (2) A fuzzy logic system is used in active control of vibration with non-linear piezoelectric actuators. The numerical simulation results show the effectiveness of this adaptive fuzzy control method.

1. Introduction

In many industrial and defense applications, noise and vibration are important problems. Active control of sound and vibration has been the subject of a lot of research in recent years, and examples of applications are now numerous [1].

The active control of vibration using feedforward control techniques has been the topic of much recent research. Adaptive linear filtering techniques have been extensively used for the active control of sound and vibration, and many of today's implementations of active control use these techniques. The most common form of adaptive algorithm/architecture combination is a

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transversal filter-based controller (FIR or IIR filter), adapted using a gradient descent-type algorithm. A popular adaptive filtering algorithm, because of its simplicity and its relatively low computational load, is the multi-channel filtered-X LMS algorithm (or the multi-error LMS algorithm) [2]. This algorithm is a steepest descent algorithm that uses an instantaneous estimate of the gradient of the cost function (i.e., the mean squared error).

Such a system is limited to linear control problems. In other words, the control input signal, as well as the associated measured error signal used in the adaptation process, must be a linear function of the reference signal used by the adaptive filter to derive the control signal. The linear digital controller described so far (using FIR filters or IIR filters) may not perform well in cases where non-linearities in an active vibration control system are found.

Few practical implementations of non-linear active controllers have been realized. There are, however, several instances where a non-linear adaptive control scheme may be proved superior to a linear one [3,4]. Non-linear active controllers may be required in the case where the actuators used in active vibration control systems exhibit non-linear characteristics because the most common source of non-linearity in the field of active control of vibration is the actuator. For example, the surface-mounted piezoelectric actuator for actively controlling structural vibration exhibits non-linear harmonic response characteristics. Other examples where the structures to be controlled exhibit a non-linear behavior are the vibration behavior of plates exhibiting buckling, or when a tonal reference signal is provided to the control system, but the actual primary disturbance to be controlled comprises both the tone and harmonics.

What is needed in the above instances is some non-linear adaptive control algorithm other than the linear control method. The use of a non-linear controller can improve the control performance on a system with a non-linear behavior. A multi-layer perception neural-network based control structure was previously introduced as a non-linear active controller [3–5] with a training algorithm based on an extended back propagation scheme. One possible architectural candidate for a non-linear adaptive control method is the fuzzy logic system based controller.

Recently, fuzzy controllers have been developed which perform well in non-linear and dynamic control systems [6–8]. In this paper, a non-linear fuzzy control method is used in two different kinds of active vibration control problems. The first considers a non-linear sensor path where a fuzzy logic system is used to approximate the above non-linear sensor path function, and a feedforward control method using a fuzzy logic based control system is considered with the aim of deriving an architecture/algorithm combination. A training algorithm is derived for this purpose. The numerical simulation results of the active vibration control system for the composite beam show that the fuzzy control method achieves better results than the conventional filtered-X LMS algorithm.

In the second, the fuzzy control method is used in active vibration control with a non-linear piezoelectric actuator. The numerical simulation results show the effectiveness of the fuzzy adaptive control method.

2. Fuzzy control method

There are four principal elements in a fuzzy logic system: fuzzifier, fuzzy rule base, fuzzy inference engine and defuzzifier. We consider a multi-input and single-output (MISO) fuzzy logic system that constructs the fuzzy controller. The lth MISO IF–THEN rule is defined as follows:

$$Rule^l: \text{ IF } x_1 \text{ is } A_1^l \text{ and } x_2 \text{ is } A_2^l \ldots \text{ and } x_n \text{ is } A_n^l \text{ THEN } y \text{ is } B^l, \tag{1}$$

where $Rule^l$ denotes the lth fuzzy rule, $l = 1, 2, \ldots, M$ is the number of rules in the rule base. $\mathbf{x} = [x_1, \ldots, x_n]^T \in U = U_1 \times \cdots \times U_n$, $U_i \in R, y \in R$, where $\mathbf{x}$ and y represent the input and output variable of the fuzzy logic system. A_i^l is the fuzzy set of the ith input variable in the lth rule. B^l is the membership function of the output variable.

The fuzzy logic system with singleton fuzzifier, product-inference rule, center average defuzzifier, and Gaussian membership function is defined as follows [9]:

$$u(k) = \frac{\sum_{i=1}^{M} \omega^i y^i}{\sum_{i=1}^{M} \omega^i}, \tag{2}$$

where $u(k)$ is the output of the fuzzy logic system, k is time step, $y^i \in R$ is any point at which $\mu_{B^i}(y^i)$ achieves its maximum value, $\mu_{B^i}(y^i) = 1$. $\mu_{A_j^i}(x_j)$ is the Gaussian membership function of the input variable x_j, $\mu_{A_j^i}(x_j) = \exp[-((x_j - \bar{x}_j^i)/\sigma_j^i)^2]$, where x_j^{-i} and σ_j^i are the corresponding centers and standard deviations of the Gaussian membership functions, $\omega^i = \prod_{j=1}^{N} \mu_{A_j^i}(x_j)$. So, the output $u(k)$ is a function of adjustable parameter set $\Theta(k)$, $\Theta(k) = \{y^i(k), \bar{x}_j^i(k), \sigma_j^i(k)\}$.

In the active vibration control using the fuzzy control method, the above input variable x_j of the fuzzy controller is the reference signal. Details are shown in the numerical simulation examples in the next section.

One of the most important advantages of a fuzzy logic system is that the fuzzy logic system has the capability to approximate non-linear mappings.

Universal Approximation Theorem [10]: For any given real continuous function f on the compact set $U \subset R^n$ and arbitrary $\varepsilon > 0$, there exists a fuzzy logic system f^* in the form of (2) such that $\sup_{x \in U} |f^*(x) - f(x)| < \varepsilon$.

The fuzzy adaptive filter, which is constructed from a set of changeable fuzzy IF–THEN rules and was first proposed by Wang [11], has drawn a great deal of attention because of its universal approximation ability in non-linear problems [12,13]. But few applications of adaptive fuzzy filters to active control of vibration have been realized.

The fuzzy filter can be viewed as a mapping from the multi-input into the single-output. These fuzzy rules come either from human experts or by matching input–output pairs through an adaptation procedure. The constructed filter in Eq. (2), based on fuzzy rules, can be used as the initial filter for the adaptation procedure. Therefore, the parameters of the membership functions in the fuzzy rules will change during the adaptation procedure.

The fuzzy adaptive filter is able to adapt to minimize some criterion function. To adjust the parameter set $\Theta(k)$ of the membership functions that characterize the fuzzy concepts in the IF–THEN rules, we use the adaptive algorithm based on least mean squares.

Because our aim is to reduce the structure's vibrational amplitude, we use the error signal $e_u(k)$, which is the vibration response of the cantilever beam in the next section, to construct the criterion function $J(k)$ as follows:

$$J(k) = \frac{1}{2} e_u(k)^2. \tag{3}$$

So the parameter set $\Theta(k) = \{y^i(k),\ \bar{x}_j^i(k),\ \sigma_j^i(k)\}$ can adapt as follows:

$$\Theta(k+1) = \Theta(k) - \alpha \frac{\partial J(k)}{\partial \Theta(k)}, \tag{4}$$

where α is a small positive constant ($\alpha = 0.05$ in numerical simulations in the next section) and the partial differential of each parameter in the criterion function $J(k)$ is as follows:

$$\frac{\partial J(k)}{\partial y^i} = -\frac{e_u(k)\omega^i}{\sum_{i=1}^{M} \omega^i}, \tag{5}$$

$$\frac{\partial J(k)}{\partial x_j^i} = -\frac{2e_u(k)(x_j - \bar{x}_j^i)\omega^i(y^i(k+1) - y^i(k))}{(\sigma_j^i)^2 \sum_{i=1}^{M} \omega^i}. \tag{6}$$

$$\frac{\partial J(k)}{\partial \sigma_j^i} = -\frac{2e_u(k)(x_j - x_j^{-i})^2\omega^i(y^i(k+1) - y^i(k))}{(\sigma_j^i)^3 \sum_{i=1}^{M} \omega^i} \tag{7}$$

The ability of a fuzzy logic system to be "trained" to perform some desired task using the gradient descent-based algorithm has been well documented. It would seem therefore likely that such an architecture/algorithm combination could be employed to perform the previously mentioned non-linear active control of vibration tasks, where the fuzzy logic system would be used to derive an output signal which would "cancel" the unwanted primary disturbance.

The application of fuzzy set theory to control problems has been the focus of numerous studies. The motivation is often that the fuzzy set theory provides an alternative to the traditional modelling and design of control systems when system knowledge and dynamic models in the traditional sense are uncertain and time varying. Although achieving many practical successes, fuzzy control has not been viewed as rigorous due to the lack of formal synthesis techniques that would guarantee the basic requirements for the control systems, such as global stability.

In general, because the LMS algorithm does not guarantee convergence to globally optimal parameters, we need good initial parameters for its convergence for the adaptive fuzzy logic system. Also, since minimizing $J(k)$ can be viewed as matching the input and outputs, the fuzzy adaptive filter combines both linguistic and numerical information in its design. In this case, the initial parameters of the fuzzy adaptive filter can be constructed based on linguistic rules from human experts. Therefore, the adaptation procedure of the fuzzy adaptive filter could have a quick convergence, provided that there are sufficient linguistic rules. Also, the fuzzy logic system has an inherent robustness because it uses linguistic information.

The universal approximation theorem is the theoretical basis of the active control of vibration using fuzzy control method described in this paper. We, therefore, would like to apply this non-linear fuzzy filter to two kinds of active control of vibration problems:

(1) The fuzzy logic system is used to approximate the non-linear sensor path function. This kind of non-linear fuzzy controller is used to suppress a primary disturbance that is a non-linear function of the reference signal.
(2) The fuzzy logic system is used for active control of vibration with non-linear piezoelectric actuators.

Two typical structures of feedforward active control of vibration systems having non-linear sensor path and non-linear piezoelectric actuator are portrayed in Fig. 1 and Fig. 2, respectively.

As with the linear LMS filter based systems, implementation of a gradient descent algorithm in a fuzzy feedforward active control system is not straightforward. Referring to Fig. 1, the reason for the complication is a (system dependent) cancellation path transfer function between the control signal input to the system and the associated error measurement output. This transfer function incorporates the frequency response characteristics of the control actuator and the error sensor, as well as the response characteristics of the structural system which separates them, including delays due to the finite distance between source and sensor. The existence of this cancellation path transfer function must be taken into account. For the linear structure, the cancellation path transfer function can be modelled using the LMS algorithm for a linear identification before the control stage. There are numerous examples of this aspect [14]. In the above fuzzy control method, the fuzzy logic system is only used to simulate the non-linear sensor path, and the cancellation transfer function is constructed for the LMS algorithm. The input signal of the fuzzy logic system is the reference signal from the non-linear sensor path.

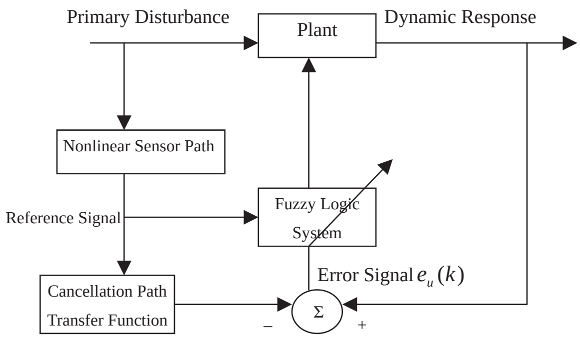


Fig. 1. Block diagram of the non-linear fuzzy control method for the active control of vibration having non-linear sensor path.

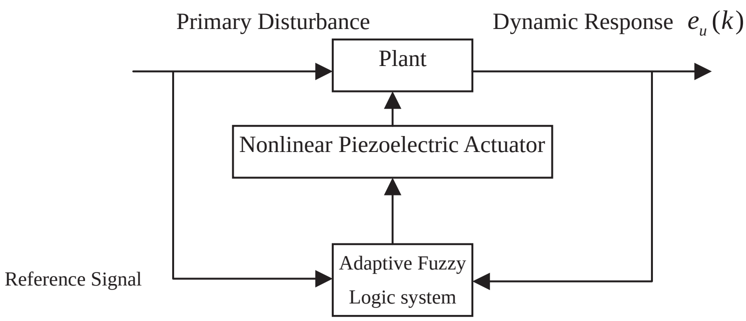


Fig. 2. Block diagram of the non-linear adaptive fuzzy control method for the active control of vibration having non-linear piezoelectric actuator.

In the second active control of vibration having non-linear piezoelectric actuators, the fuzzy logic system uses very little numerical data to realize the active vibration control. There is an almost negligible plant cancellation transfer function and the electronic components to consider.

Most control methods have required mathematical information about the plants, sensors and actuators. However, the mathematical information is subject to inaccuracies due to truncation and the finite-length digits of microprocessors in real-time implementations. The adaptive fuzzy control method gives an effective way to control the plant.

3. Numerical Simulation

3.1. Active control of vibration having non-linear sensor path

In this section, we illustrate some numerical simulation results to demonstrate the effectiveness of the above fuzzy control method. First, consider an active feedforward vibration control problem of a graphite/epoxy composite cantilever beam whose stacking sequence is $[(0^\circ/90^\circ)_2]$. Fig. 3 presents the flow diagram of the fuzzy control for the composite beam. All material properties and dimensional parameters are given in Table 1.

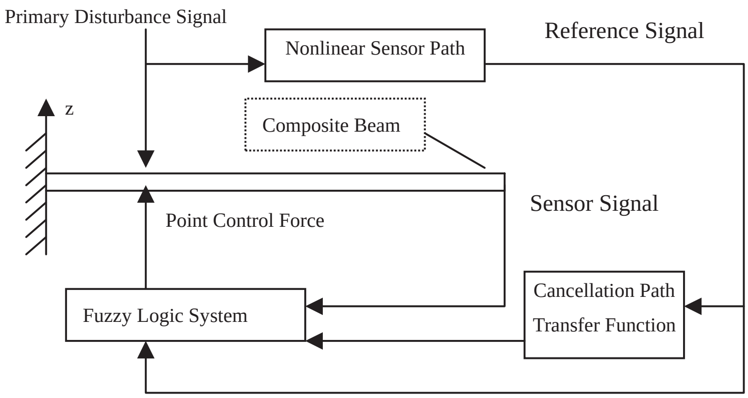


Fig. 3. Flow diagram of the fuzzy control for the composite beam.

Table 1
Material properties and dimensional parameters of the composite beam

Material property		Dimensional parameter	
E_1, GPa	140	Length, mm	80
E_2, E_3, GPa	9	Width, mm	5
G_{12}, G_{13}, GPa	5.6	Height, mm	2
G_{23}, GPa	2		
v	0.28		
ρ, kg/m^3	1400		

The finite element model of the composite beam is shown in Fig. 4. It is divided into eight elements. In the examples, the primary disturbance and control force act on the 24th node in the Z direction. The primary disturbance and control signal are both supplied to the beam by point force.

The fuzzy system is used to approximate the non-linear sensor path. In the next three examples, the input signal x_j of the fuzzy system is the reference signal and the output signal is the "identified" opposite phase primary disturbance signal. Before performing the active control, 500 bits of data are employed to "train" the fuzzy logic system, and the training signal $J(k)$ is constructed by the dynamic response of the free end of the composite beam. There are two Gaussian membership functions in each input, and therefore, there are four fuzzy rules in the adaptive fuzzy logic system.

3.1.1. Example one

The first case to be considered is for a linear problem. The primary disturbance and reference signal are all 50 Hz signals. The corresponding result is shown in Figs. 5(a) and (b). From the results it is demonstrated that the non-linear fuzzy controller can work perfectly for a simple linear case.

3.1.2. Example two

Consider the primary disturbance to be sinusoidal at 50 and 100 Hz, as in Eq. (8), but a non-linear sensor path that results in only a tonal reference signal of 50 Hz is provided to the control system, as in Eq. (9), and the actual primary disturbance to be controlled comprises both the tone and harmonic. The fuzzy logic system is used to approximate the non-linear sensor path. Its input signal is $r(t)$, and it derives a control signal to cancel the primary disturbance.

$$x(t) = \sin(2\pi 50t) + \sin(2\pi 100t) \quad \text{(N)}, \tag{8}$$

$$r(t) = \sin(2\pi 50t). \tag{9}$$

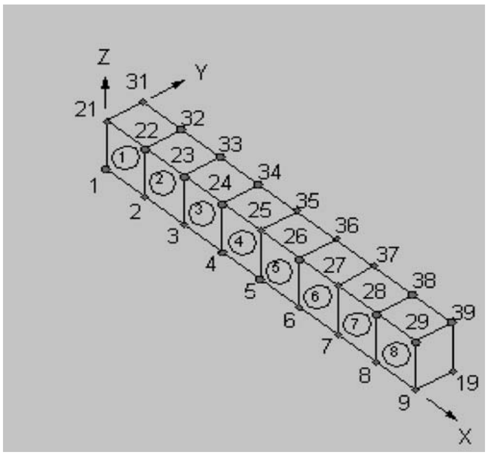
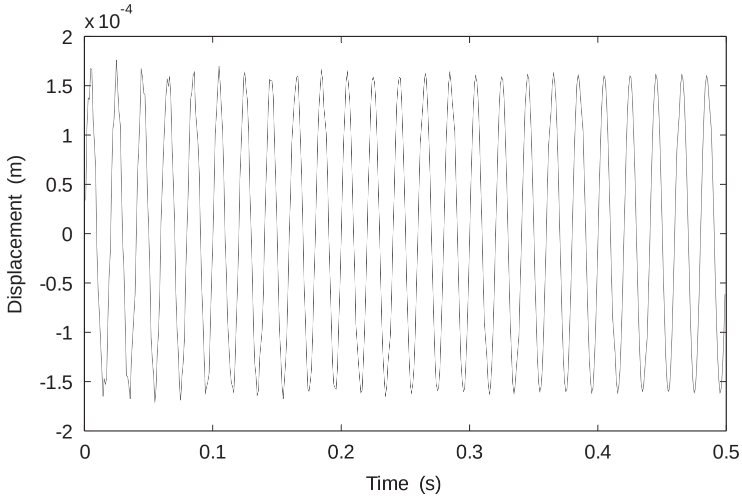
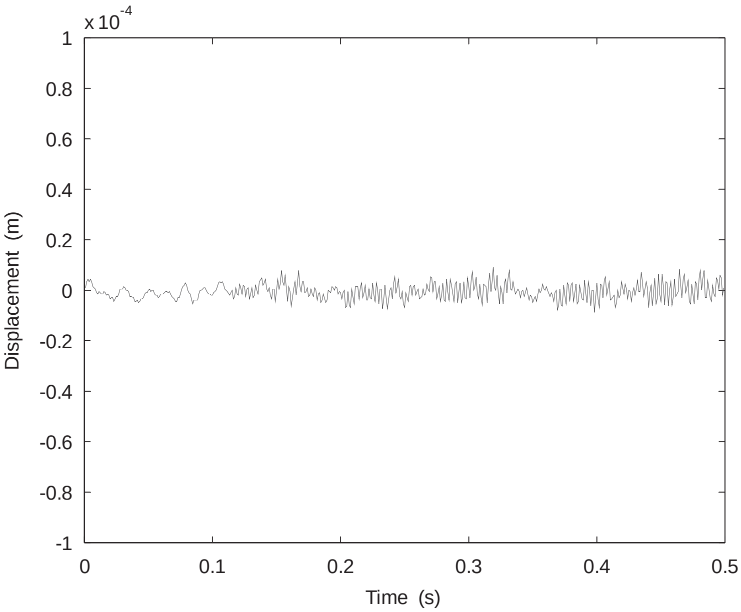


Fig. 4. Finite element model of the composite beam.



(a) Without control



(b) With control

Fig. 5. Example one: The dynamic response of the free end of the composite beam without and with control.

Fig. 6 presents the dynamic response of the end of composite beam without control. Figs. 7(a) and (b) present the dynamic response of the end of the composite beam with the linear filter-X LMS control method and the non-linear fuzzy control method, respectively.

It can be observed that the filtered-X LMS controller significantly reduces the 50 Hz tonal disturbance, but the dynamic response of the non-linearity of the harmonic of the 50 Hz tone at

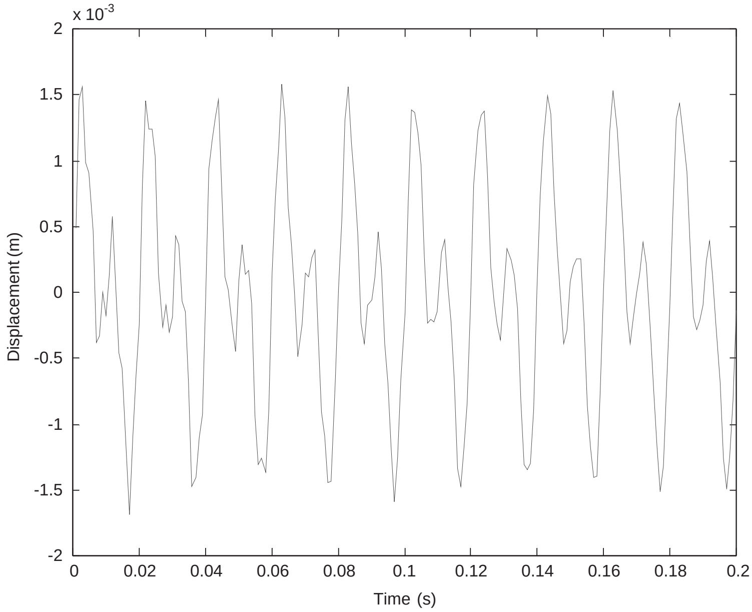


Fig. 6. Time history of the free end displacement of the composite beam without control.

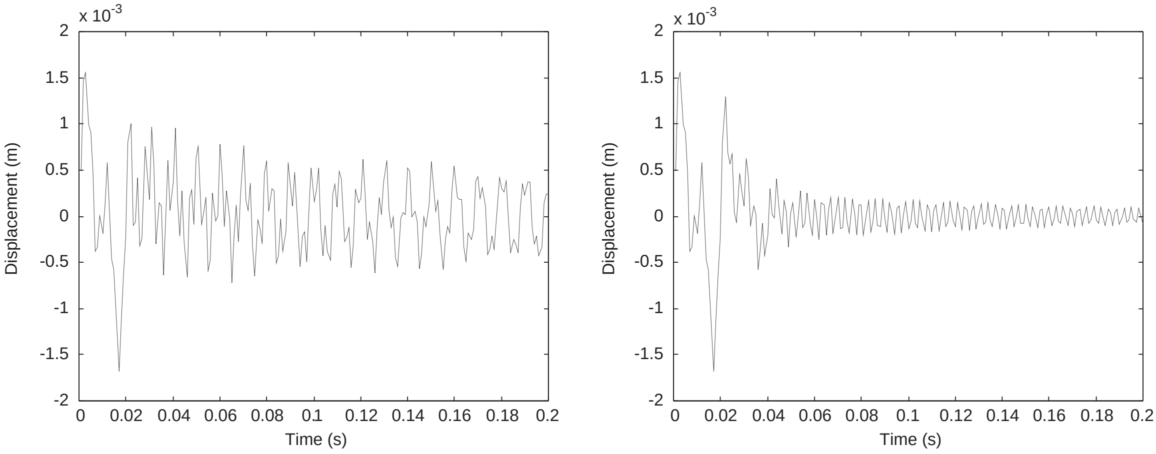


Fig. 7. Example two: Time history of the free end displacement of the composite beam with control.

100 Hz obviously exists. From the simulation results shown in Fig. 7(b), it can easily be determined that not only the tonal disturbance of 50 Hz but also the harmonic disturbance of 100 Hz are all suppressed effectively by the fuzzy control method.

3.1.3. Example three

Consider the primary disturbance $x(t)$ is given as follows

$$x(t) = \sin(2\pi 50t) + \sin(2\pi 100t) \quad (\mathrm{N}), \tag{10}$$

provided that the dynamic characteristic of the unknown two-order non-linear sensor path [15] is

$$d(k) = f(x(k),\ x(k-1)) = \frac{\sin(x(k))x(k-1)}{1+[x(k-1)]^2}, \tag{11}$$

where $x(k)$ is the primary disturbance signal, $f(\cdot)$ is the non-linear sensor path function, and $d(k)$ is the distorted reference signal.

Because the non-linear sensor path function is unknown, the fuzzy logic system is used to approximate this non-linear function to control the primary disturbance. Here its input signal is $d(k)$.

The distorted reference signal caused by the non-linear sensor path is shown in Fig. 8.

Fig. 9 presents the dynamic response of the free end of the composite beam with control. From the simulation results, it can easily be determined that the fuzzy control method can effectively

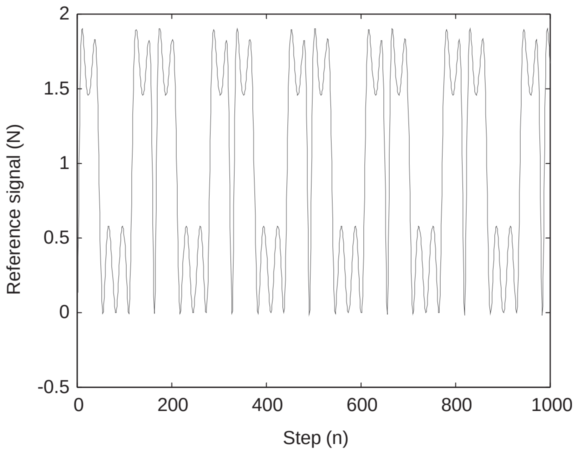


Fig. 8. The distorted reference signal.

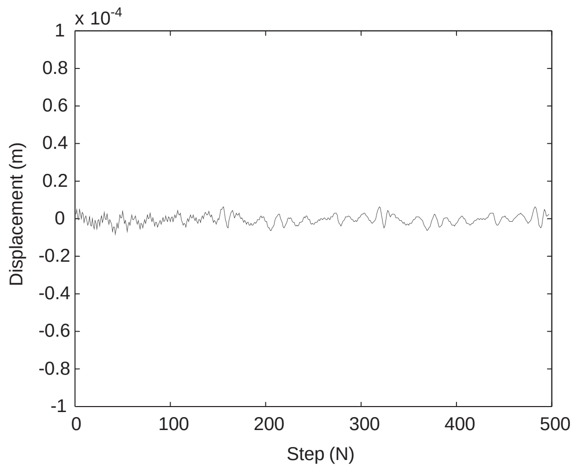


Fig. 9. Example three: The dynamic response of the free end of the composite beam with control.

control the structural vibration response, though the reference signal is a non-linear function of the primary disturbance signal. It is impossible for the filtered-X LMS control method to have perfect control effectiveness due to the non-linear sensor path.

3.2. Active control of vibration having non-linear piezoelectric actuators

In this section, we consider a numerical simulation of active control vibration of a cantilever beam with non-linear piezoelectric actuators using the adaptive fuzzy control method. It is schematically presented in Fig. 10. The input signal x_j of the adaptive fuzzy logic system is the reference signal, its order n is equal to 7, and each input sample x_j has seven linguistic terms, which are initially equally distributed in input signal range [−10,10]. In addition, the initial standard deviations are set to be 0.1.

All material properties and dimensional parameters of the cantilever beam and piezoelectric actuators are given in Tables 2 and 3.

The primary disturbance is supplied to the beam by point force located 65 mm from the end of the beam as follows:

$$F = 10\sin(2\pi 50t) + 10\sin(2\pi 100t) \quad \text{(N)}. \tag{12}$$

For the linear piezoelectric actuator, the pure bending moment is given as follows:

$$M = b_a d_{31a} E_a y_t V, \tag{13}$$

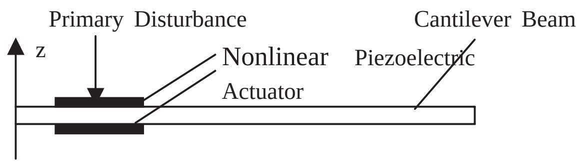


Fig. 10. Sketch diagram of adaptive fuzzy control for the cantilever beam with non-linear piezoelectric actuators.

Table 2
Material properties and dimensional parameters of the cantilever beam

Material	Density	E	v	Length	Width	Height	Boundary
Fe	7800 kg/m^3	200E9 N/m^2	0.3	80 mm	5 mm	2 mm	fixed–free

Table 3
Material properties and dimensional parameters of the piezoelectric actuator

Material	Density	Height	Length	Width	Piezoelectric constant (d_{31})
PZT	7600 kg/m^3	0.5 mm	10 mm	5 mm	190E–12 m/v
E	v	Location			
63E9 N/m^2	0.3	Located 60 mm from the end of the beam			

where b_a is the width of the piezoelectric actuator, d_{31a} is a piezoelectric constant, E_a is Young's modulus of piezoelectric actuator, y_i is the width of an actuator and beam, V is the applied voltage.

It is difficult to construct a general quantitative model for the non-linear piezoelectric actuator due to its complex characteristics. The non-linear characteristics of piezoelectric actuators can be described by a non-linear curve equation. For the sake of numerical simulation, we assume the input voltage–output bending moment relation of non-linear piezoelectric actuator is given as follows:

$$M = b_a d_{31a} E_a y_t (0.05V^2 + 0.9V + 0.05). \tag{14}$$

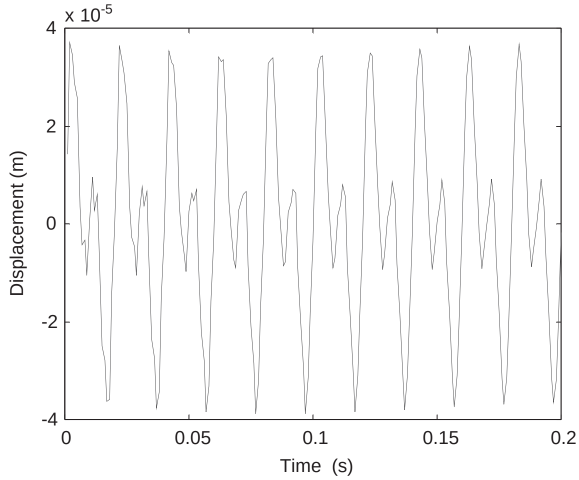
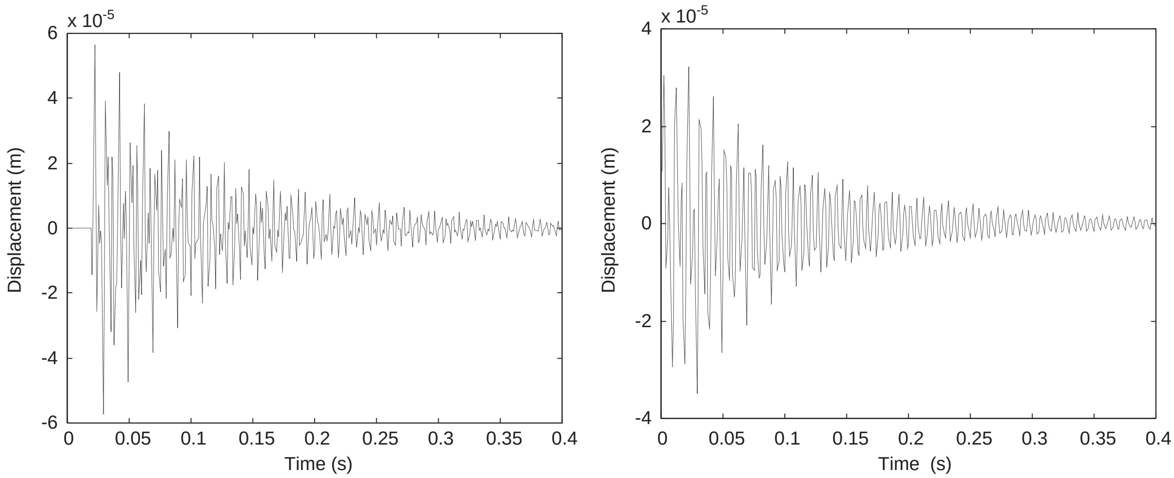


Fig. 11. Time history of the free end displacement of the beam without control.



(a) With the filtered-X LMS control method

(b) With the adaptive fuzzy control method

Fig. 12. Time history of the free end displacement of the beam with control having the non-linear piezoelectric actuators.

The extent of non-linearity of the piezoelectric actuator in Eq. (14) is small. In fact, the piezoelectric actuator's non-linearity is a very complex problem [16,17].

Fig. 11 presents the dynamic response of the end of the beam without control. Figs. 12(a) and (b) present the dynamic response of the end of the beam with the linear filter-X LMS control method and the non-linear adaptive fuzzy control method, respectively. The results showed that the two control methods all have excellent effectiveness.

In addition, we must point out that the filtered-X LMS control method and the non-linear adaptive fuzzy control method all have excellent control effectiveness because the non-linear extent of piezoelectric actuators in Eq. (14) is small. When the non-linear extent of piezoelectric actuators is high, the fuzzy adaptive control method will achieve better result than the linear filtered-X LMS method.

4. Conclusion

In this paper considering non-linear sensor paths and non-linear piezoelectric actuators, we have presented two kinds of active vibration control systems using a fuzzy logic system. Numerical simulation results showed the effectiveness of this feedforward active control of vibration systems.

Acknowledgements

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