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Journal of Sound and Vibration 276 (2004) 475–490
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Experimental validation of a quasi-steady theory for the flow through the glottis

C.E. Vilain[a], X. Pelorson[a,*], C. Fraysse[b], M. Deverge[b], A. Hirschberg[b], J. Willems[b]

[a] *INPG, Universite Stendhal, UMR CNRS Q5009, Institut de la Communication Parlée, 46 avenue F. Viallet, Grenoble Cedex 01 F-38031, France*

[b] *Fluid Dynamics Laboratory, Technical University of Eindhoven, Postbus 513, 5600 MB Eindhoven, The Netherlands*

Received 20 January 2003; accepted 28 July 2003

Abstract

In this paper a theoretical description of the flow through the glottis based on a quasi-steady boundary layer theory is presented. The Thwaites method is used to solve the von Kármán equations within the boundary layers. In practice this makes the theory much easier to use compared to Pohlhausen's polynomial approximations. This theoretical description is evaluated on the basis of systematic comparison with experimental data obtained under steady flow or unsteady (oscillating) flow without and with moving vocal folds. Results tend to show that the theory reasonably explains the measured data except when unsteady or viscous terms become predominant. This happens particularly during the collision of the vocal folds.

1. Introduction

Voiced sound production, or phonation, is produced by a modulation of the flow while the vocal folds are self-oscillating. Typically, the glottis, the narrow passage between the vocal folds, forms a converging channel when the vocal folds are separating and a diverging channel when the vocal folds are approaching. It can be shown that the self-sustained oscillation of the vocal folds is essentially driven by the difference between the hydrodynamic forces exerted on the folds during the opening and the closing phase of the glottis [1].

While simple theories using ad hoc assumptions such as imposing a flow separation at a fixed point within the glottis (e.g., Refs. [2–4]) can be justified when the glottis forms a converging

*Corresponding author. Fax: +33-4-7682-5946.
E-mail address: pelorson@icp.inpg.fr (X. Pelorson).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2003.07.035

channel, they remain very crude during the closure of the vocal folds when the glottis forms a diverging channel.

An empirical prediction was proposed by Liljencrants [5] based on numerical simulations of the glottal flow. The flow separation is simply predicted to occur at the point where the glottal diameter exceeds the minimum glottal diameter by a fixed amount (10% or 20%) Lous et al. [6] have successfully used this criterion coupled to a two-mass model of the vocal folds.

Although attractive in its simplicity, such a criterion has the drawback of being purely geometrical and thus insensitive to the flow conditions.

Based on steady flow measurements on mechanical replicas of the vocal folds (e.g., Ref. [7]) or using numerical simulations (e.g., Ref. [8]) Scherer et al. have proposed empirical correction coefficients to explain the departures from inviscid predictions. Recently, Zhang et al. [9] have used empirical discharge coefficients coupled with a quasi-steady flow approximation in order to describe the modulation of the flow induced by an oscillating in vitro model of the glottis. The present goal is to predict theoretically these empirical coefficients.

In previous works, attempts to describe the flow more accurately were made using a third order Pohlhausen method [10–12]. This theory was tested against in vitro experiments under steady or unsteady flow conditions. In particular, the unsteady flow conditions were obtained by using impulsively started flows generated by opening a mechanical valve. Although comparable, these flow conditions were quite different from those expected during phonation. Further, it was found that the theoretical solutions were difficult to obtain numerically due to the non-linearity of the flow equations [13,14].

Therefore in this paper another theoretical prediction is proposed for the flow within the glottis based on the Thwaites' method [13–15]. After a definition of the basic assumptions, this method will be described. The experimental validation will then be presented based on measurements on rigid replicas of the vocal folds. The use of rigid replicas allows for an accurate control of the geometry of the glottis and hence for quantitative measurements. Steady flow conditions are first considered. Unsteady flow measurements will then be presented using two different set-ups. In the first one, the unsteadiness is imposed by a siren while the replicas are fixed. In the second set-up, one vocal fold replica is forced to oscillate using a motor. This latter case allows thus to evaluate the effects of moving boundaries on the flow.

2. Theory

2.1. Assumptions

In the following the direction of the flow will be considered to follow the x dimension. The transverse dimension is denoted by y. In Table 1 are presented some typical values for the physical quantities of importance for this study. These quantities should be understood as average values observed for a male speaker during normal (conversational) speech. In order to quantify some necessary assumptions a dimensionless analysis can be carried out on the basis of these typical values.

The Reynolds number is defined as $Re_h = v_g h_g / \nu$, where h_g is the minimum glottal aperture (henceforth called the glottal height), v_g is the velocity at this point and ν the kinematic viscosity

Table 1
Typical values for relevant physical quantities characterizing the glottal flow (from Ref. [16])

Quantities		Notations	Typical values
Geometrical	Vocal folds transversal length	L_g	14 mm
	Mean glottal height	h_g	1 mm
	Vocal folds longitudinal length	L	6 mm
Physical	Phonation pressure[a]	P_{sub}	100–1000 Pa
	Mean glottal flow velocity	v_g	15–40 m/s
	Fundamental frequency of oscillation	f	80–200 Hz
	Density of the air	P_0	1.2 kg/m^3
	Sound celerity	c_0	350 m/s
	Kinematic viscosity of the air	v	1.5×10^{-5} m^2/s

[a] Pressures are given relative to atmospheric pressure.

coefficient. The Reynolds number is usually used as a measure for the importance of the inertial flow effects with respect to the viscous ones. For the purpose of this study another dimensionless number, based on the Reynolds number will be used

$$\frac{Re_h h_g}{L} = \frac{v_g h_g^2}{vL},$$

where L is the length (along the x-axis) of the glottis.

The boundary layer theory assumes that all viscous effects can be confined within a thin region of the flow near the walls. If it is assumed that inertial terms and viscous terms are of the same magnitude in this thin region [13] one gets an estimation of the boundary layer thickness, δ:

$$\delta = \sqrt{\frac{vL}{v_g}}.$$

Therefore,

$$\left(\frac{h_g}{\delta}\right)^2 = \frac{v_g h_g}{v}\,\frac{h_g}{L} = Re_h\,\frac{h_g}{L}.$$

In other words, the dimensionless number $Re_h h_g/L$ provides information about the thickness of the boundary layer and thus about the validity of the boundary layer concept. Using the values of Table 1, one has typically $Re_h h_g/L = 500$ which justifies the assumption of thin boundary layers.

The Strouhal number is defined by $Sr_L = \frac{fL}{v_g}$, where f is the fundamental frequency of oscillation and v_g the flow velocity at h_g. The Strouhal number is often referred as a dimensionless frequency but can also be understood as a measure of the importance of the inertial effects with respect to the convective ones. More precisely, consider the mean volume flow due to walls movement ($\frac{dh_g}{dt} L L_g$) and the mean volume flow through the glottis ($v_g h_g L_g$). Since, in the first approximation, one has $f = 1/h_g \; dh_g/dt$, it can be easily shown that the Strouhal number can be interpreted as a measure of the relative importance of the flow induced by wall motion with respect to the flow induced by the pressure difference across the glottis. Typical Strouhal numbers for phonation, obtained from Table 1, are of order of 10^{-2}. This justifies not only the use of a quasi-steady flow theory but also to neglect the effects of moving walls on the flow. However, this analysis clearly fails when the glottis closes since h_g tends toward 0 and f cannot be estimated by $1/h_g \, \mathrm{d}h_g/\mathrm{d}t$ anymore.

The squared value of the Mach number, $M = v_g/c_0$, where c_0 is the speed of sound, is a measure for the importance of the compressibility effects [14]. Typical Mach numbers for phonation are of order of 10^{-1}. Compressibility effects can, in first approximation, be neglected.

At first sight it can seem surprising to describe a sound source, such as the glottis, by means of incompressible theory since sound is by definition a compressible phenomenon. The key point is that, as long as the acoustical wavelength, λ remains much larger than the flow region, one can assume a locally incompressible flow. A measure for this is the Helmholtz number $He = L/\lambda$. Using values of Table 1, one gets $He = 10^{-3}$, which means that the glottis can be considered as a compact source.

Finally, as a last assumption, it is considered that since the width of the glottis, L_g is much larger than the glottal height, h_g a two-dimensional flow can be assumed.

2.2. Prandtl equations

The dimensionless analysis shows that a two-dimensional boundary layer theory can be used to describe the glottal flow. Thus, the flow through the glottis is divided into two regions: the bulk flow, considered as one dimensional and inviscid and the two-dimensional boundary layer flow, where all viscous effects are confined. Let $u_e(x)$ be the longitudinal velocity in the bulk flow. Upstream of the separation point, the momentum conservation equation reduces, for the bulk flow, to

$$u_e \frac{\partial u_e}{\partial x} = -\frac{1}{\rho}\frac{\partial p}{\partial x}, \qquad \frac{\partial p}{\partial y} = 0. \tag{1}$$

The first equation is often called the Euler equation. Its solution is the Bernoulli equation

$$p(x) + \tfrac{1}{2}\rho u_e^2(x) = \text{constant}. \tag{2}$$

Within the boundary layer, let $u(x,y)$ and $v(x,y)$ be the longitudinal and transversal components of the velocity. The momentum conservation equation reduces to [13]

$$u\frac{\partial u}{\partial x} + v\frac{\partial v}{\partial y} = -\frac{1}{\rho}\frac{\partial p}{\partial x} + \nu\left(\frac{\partial}{\partial y}\right)^2 u, \qquad \frac{\partial p}{\partial y} = 0. \tag{3}$$

If one adds the local mass conservation equation

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0,$$

to the latter system, one gets the so-called Prandtl equations.

The local mass conservation equation can be integrated over the height $h(x)$ of the channel. Considering that $h(x)/Lg \ll 1$, one can neglect the lateral boundary layers. This yields the relation

$$U_g = \text{constant}, \text{ where } U_g = L_g \int_0^{h(x)} u(x,y)\,\mathrm{d}y \text{ is the volume flow velocity.}$$

Using the definition of the displacement thickness for a symmetric flow

$$\delta_1(x) = \frac{1}{2}\int_0^{h(x)} \left(1 - \frac{u(x,y)}{u_e(x)}\right)\mathrm{d}y,$$

the mass conservation equation can be rewritten as

$$U_g = L_g(h(x) - 2\delta_1(x))u_e(x). \tag{4}$$

Downstream of the point where the flow separates from the walls, a free jet is formed. The vorticity is convected in the jet and a turbulent mixing regime sets-in. Thus the boundary layer theory does not apply anymore downstream of the flow separation point. It is here assumed that the pressure within the jet is constant and equals the supraglottal pressure, P_{supra}. Let u_j be the velocity within the jet. Neglecting the inlet velocity, $u_e(0)$ with respect to the jet velocity one obtains

$$u_j = \sqrt{\frac{2(P_{sub} - P_{supra})}{\rho}}.$$

The volume flow velocity is thus given by $U_g = u_j L_g(h_s - 2\delta_{1,s})$.

The main theoretical difficulty consists in finding the values of h_s and $\delta_{1,s}$ at the flow separation point.

2.3. A theory based on Thwaites' method

A way to solve the Prandtl equations is to integrate them along a cross-section of the channel. This yields the van Kármán equation [13]

$$u_e^2(x)\frac{\mathrm{d}\theta(x)}{\mathrm{d}x} + (2+H)\theta(x)u_e(x)\frac{\mathrm{d}u_e(x)}{\mathrm{d}x} = \frac{\tau_0(x)}{\rho}, \tag{5}$$

where $u_e(x)$ is the local velocity at the outer edge of the boundary layer and where θ is the momentum thickness, defined by

$$\theta(x) = \frac{1}{2}\int_0^{h} \frac{u(x,y)}{u_e(x)}\left(1 - \frac{u(x,y)}{u_e(x)}\right)\mathrm{d}y,$$

H is a shape factor defined by

$$H = \frac{\delta_1}{\theta},$$

and τ_0 is the shearing stress at the wall

$$\tau_0(x) = \rho\nu \left.\frac{\partial u(x,y)}{\partial y}\right|_{y=0}.$$

Among the different methods to solve the von Kármán equation, the semi-empirical method developed by Thwaites [13–16] will be developed. This method consists in a rewriting of the von Kármán equation (5) using the following shape factors:

$$\lambda(x) = \frac{\theta^2(x)}{\nu}\frac{\mathrm{d}u_e(x)}{\mathrm{d}x},$$
$$S(x) = \frac{\tau_0(x)\theta(x)}{\rho\nu u_e(x)}$$
$$F(x) = 2\frac{u_e(x)\theta(x)}{\nu}\frac{\mathrm{d}\theta(x)}{\mathrm{d}x}. \quad \text{(6a–c)}$$

The von Kármán equation can be rewritten as follows [15]:

$$F = 2[S - \lambda(2 + H)]. \quad (7)$$

In practice, one assumes that F is given by a linear relationship with λ:

$$F(\lambda) = 0.45 - 6.0\lambda.$$

By integrating this expression along x, one gets the Thwaites' equation

$$\theta^2(x)u_e^6(x) - \theta^2(0)u_e^6(0) = 0.45\nu\int_0^x u_e^5(x)\,\mathrm{d}x. \quad (8)$$

Together with the Bernoulli equation (2) and the integral mass conservation equation (4), Thwaites' equation (8) provides a set of three equations which fully characterize the flow upstream of the separation point. These equations are summarized as follows:

$$\theta^2(x)u_e^6(x) - \theta^2(0)u_e^6(0) = 0.45\nu\int_0^x u_e^5(x)\,\mathrm{d}x,$$
$$L_g[h(x) - 2H(\lambda)\theta(x)]u_e(x) = U_g,$$
$$\mathrm{p}(x) + \tfrac{1}{2}\rho u_e^2(x) = P_{supra} + \tfrac{1}{2}\rho u_j^2 = P_{sub}. \quad \text{(9a–c)}$$

Eq. (9c) is obtained using the Bernoulli equation for a steady incompressible frictionless flow. It is implicitly assumed that downstream of the flow separation point the pressure P_{supra} is uniform.

Flow separation is predicted to occur when the parameter λ—defined in (6a)—reaches a critical value, λ_s [13]. Based on empirical data, the original Thwaites' method uses a critical value $\lambda_s = -0.09$. Pelorson et al. [10] used an analytical value for λ_s obtained by assuming a third order polynomial law for the velocity within the boundary layer. This critical value $\lambda_s = -0.0992$ will be used in the following.

In practice, the numerical solution of Eqs. (9) is carried out in the following way. The volume flow velocity U_g is guessed and the two equations (9a,9b) are solved step by step until the flow separation point is reached. Then, for a given transglottal pressure ($P_{sub} - P_{supra}$), u_j can be calculated using Eq. (9c). This allows to get a new estimation for the volume flow velocity, U_g.

This process is repeated until a stable value for U_g is found. Once U_g, h_s and $\delta_{1,s}$ are known the pressure distribution within the whole glottal channel can be calculated using Eq. (9c).

3. Experimental validation

In order to validate the theory, different glottal configurations were used as presented in Fig. 1. All glottal configurations have a length, L of 20 mm and a width, L_g of 30 mm. Considering the typical dimensions presented in Table 1, this corresponds to a glottis scaled-up by a factor 3. At a given Reynolds and Strouhal numbers this implies that the oscillation frequency of the mechanical replicas is a factor 9 lower than those expected within the (human) glottis. The experimental glottis are made of metal (brass or aluminum).

Two mechanical replicas, SC1 and SC2 represent a uniform glottis with a rounded entrance and a sharp edged outlet. The radius of curvature of the inlet of replica SC1 was 10 mm while SC2 has a much smaller radius of curvature: 2 mm. The last mechanical replica, RC approximates the glottis as the channel formed between two half cylinders.

The pressure inside the mechanical replicas could be measured using pressure taps located at the throat of the channel. Pressure taps have a diameter of 0.4 mm. All experiments were performed in a $5 \times 5 \times 3$ m^3 room.

3.1. Steady flow measurements

3.1.1. Set-up

The experimental set-up used in this section is described in details by Hofmans et al. [12]. It consists of a very large pressure reservoir (10^3 m^3) connected to the glottal replicas via a 0.7 long cylindrical pipe with a diameter of 3 cm. The steady pressure upstream the replicas, P_{supra} and within the replicas, P_g were measured using Betz water manometer with an accuracy of 0.5 Pa.

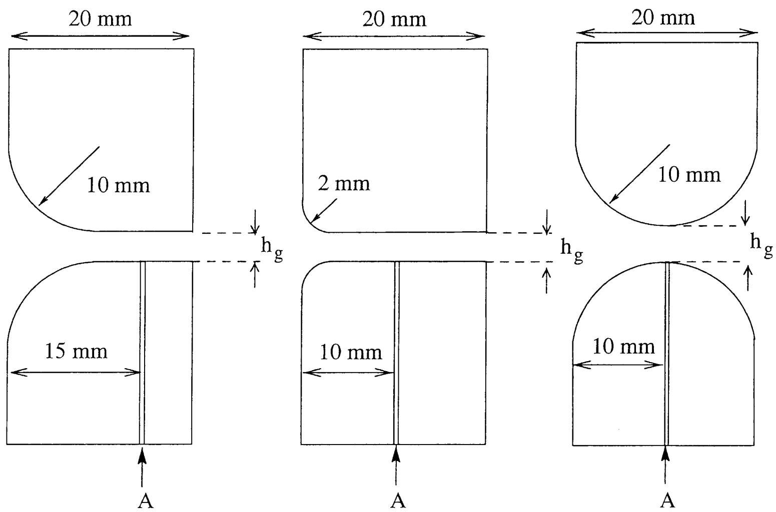

Fig. 1. Geometry of the vocal folds replicas. From left to right: straight channel 1 (SC1), straight channel 2 (SC2) and rounded channel (RC).

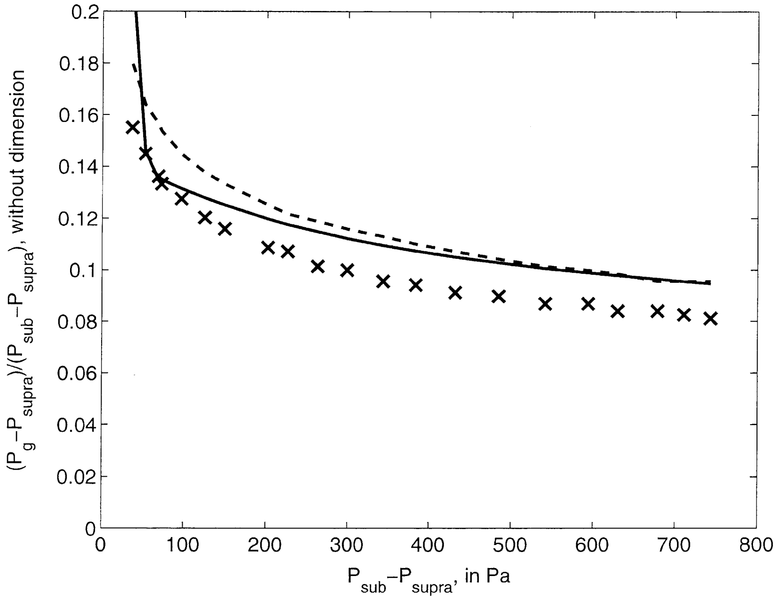

Fig. 2. Uniform glottal geometry (SC1) with $h_g = 0.99$ mm: comparison between experimental and theoretical pressure ratio $(P_g - P_{supra})/(P_{sub} - P_{supra})$ for different transglottal pressure $P_{sub} - P_{supra}$. Crosses, experimental values; solid line, theoretical values computed by Thwaites equations; dashed line, theoretical values computed by Pohlhausen equations.

3.1.2. Results

In Fig. 2 results are presented concerning the uniform glottal geometry SC1 with a glottal height h_g of 0.99 mm. In such a case it is theoretically assumed that flow separation occurs at the sharp edged exit, $x = L$ of the glottal replica. The agreement between the measured pressure difference $P_g - P_{supra}$, at $x = 3L/4$ and the theoretical predictions is good (within 15%). Similar comparison for $P_g - P_{supra}$, at $x = L/2$ in the rounded replica, RC is presented in Fig. 3. As for the previous replica, the channel height h_g is 0.99 mm.

Although the agreement between the experimental data and the theory is less satisfying than for the uniform geometry case, the discrepancies still remain within 30%. In Figs. 2 and 3 are also presented the theoretical predictions obtained using the Pohlhausen theory as described by Hofmans [11]. Since the predictions are very close, this tends to prove that the present theory is a valid alternative to the computationally expensive Pohlhausen theory.

Lastly, it must be noted that the predicted glottal pressures are always higher than the measured ones. Hofmans [11] explained this systematic discrepancy as the possible effect of a pressure recovery in the flow downstream of the separation point.

3.2. Unsteady flow conditions

3.2.1. Rigid vocal folds

3.2.1.1. Set-up. The experimental set-up is presented in Fig. 4. The air supply is provided by a ventilator (ventola 613380, Aug. Laukhuff Orgelteile) connected to a siren. This allows one to

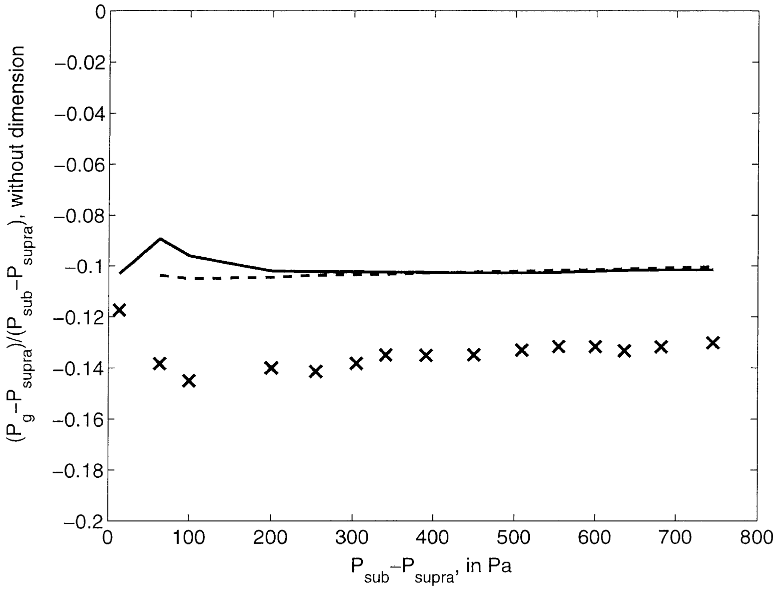

Fig. 3. Rounded glottal geometry (RC) with $h_g = 0.99$ mm: comparison between experimental and theoretical pressure ratio as a function of the transglottal pressure, $P_{sub} - P_{supra}$. Crosses, experimental values; solid line, theoretical values computed by Thwaites equations; dashed line, theoretical values computed by Pohlhausen equations.

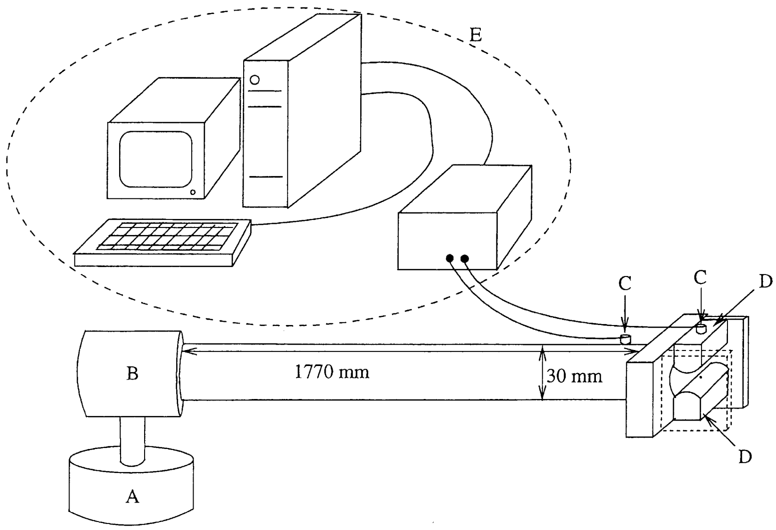

Fig. 4. Global view of the experimental set-up. A, ventilator; B, siren; C, pressure transducers; D, vocal folds replicas; E, acquisition system.

generate oscillating flows with a fundamental frequency ranging from a few Hertz up to a few hundred Hertz. The siren is connected to the glottal replica via a 1.77 m long pipe with a diameter of 30 mm. The pressure upstream of the replica, $P_{sub} - P_{supra}$ and within the replica, $P_g - P_{supra}$

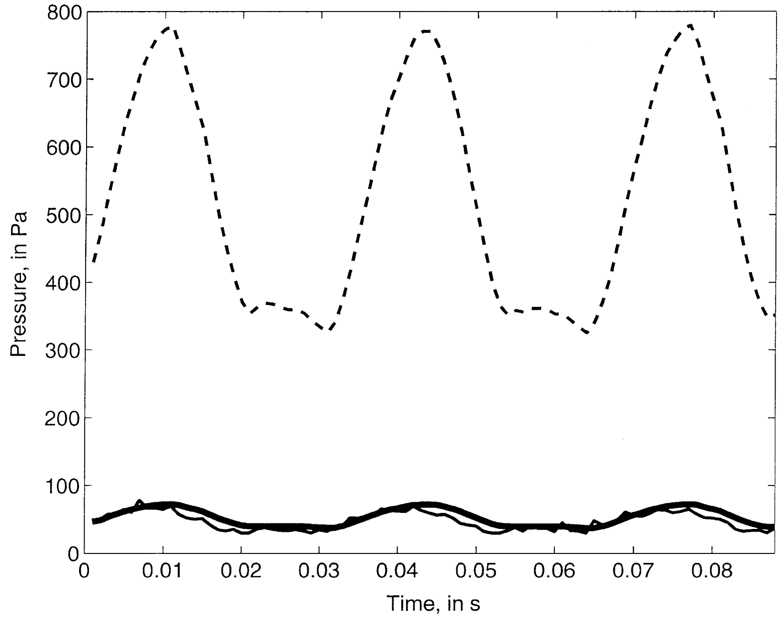

Fig. 5. Straight channel glottal geometry (SC1) with $h_g = 1$ mm. Comparison between the measurements and the theoretical predictions for $Re_h h_g / L = 100$ and $Sr_L = 0.03$. Dashed line, experimental values of transglottal pressures ($P_{sub} - P_{supra}$); thinner solid line, experimental values of ($P_g - P_{supra}$); thicker solid line; theoretical values of ($P_g - P_{supra}$) computed with Thwaites equations.

are measured using Kulite XCS-093 pressure transducer and sampled at 10 kHz. The Kulite transducers were calibrated by means of a Betz water manometer with an accuracy of 0.5 Pa.

3.2.1.2. Results. In Fig. 5 is presented an example of measurement of the pressure difference $P_g - P_{supra}$ performed on the uniform glottal replica SC1. The glottal height h_g is 1 mm and the fundamental frequency of the pulsated flow is 30 Hz. This corresponds thus to a Strouhal number $Sr_L = 0.02$ and a dimensionless number $Re_h h_g / L = 100$. The agreement between the experimental data and the theory appears to be within the same order of magnitude as for the steady flow case (within 20%).

A more systematic comparison is presented for the case of the rounded replica RC in Fig. 6. In this case the glottal height was $h_g = 2$ mm and the fundamental frequency of the oscillating flow was varying from 38 up to 540 Hz. This corresponds to Strouhal number varying from $Sr_L = 0.03$ up to 0.4 and to a dimensionless number $Re_h h_g / L = 400$.

As one could have expected the adequacy of the quasi-steady theory decreases as the unsteadiness of the flow increases. For a fundamental frequency of 38 Hz, the agreement between the theory and the experimental data is comparable to the one observed during the steady flow measurements (within 30%). Up to 239 Hz the order of magnitude of the glottal pressure appears to be reasonably predicted by the theory. The major discrepancy occurs due the presence of a phase shift between the theoretical predictions and the experimental data. The case of a 540 Hz pulsated flow gave the worst results, however the flow conditions imposed during these

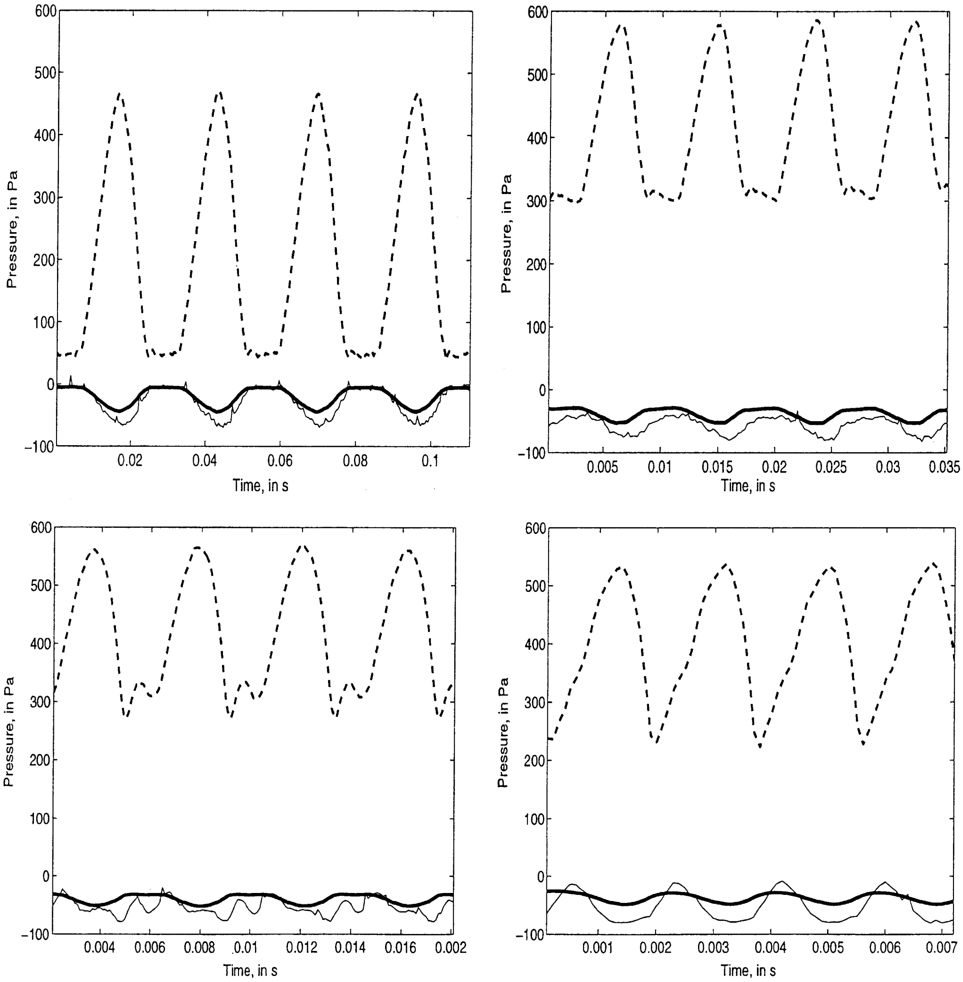

Fig. 6. Rounded glottal channel (RC) with $h_g = 2$ mm. Comparison between the measurements and the theoretical predictions for a fixed dimensionless number $Re_h h_g / L = 400$ and various fundamental frequencies. Top left, $f_0 = 38$ Hz; top right, $f_0 = 102$ Hz, bottom left, $f_0 = 239$ Hz; bottom right, $f_0 = 540$ Hz. Dashed lines, experimental values of transglottal pressures $(P_{sub} - P_{supra})$; thinner solid line; experimental values of $(P_g - P_{supra})$; thicker solid line, theoretical values of $(P_g - P_{supra})$ computed with Thwaites equations.

measurements are extreme and are not expected to be relevant for normal phonation. Indeed, as explained in Section 2.1, this in vitro condition would correspond, in real life, to a fundamental frequency of order of 5 kHz.

3.2.2. *Oscillating vocal folds*

So far, only rigid non-moving vocal folds replicas were considered. In order to evaluate the unsteadiness generated by the motion of the vocal folds walls, another set-up including an oscillating replica is developed.

3.2.2.1. Set-up. The experimental set-up depicted in Fig. 7 is inspired by the work of Mongeau et al. [17] and Barney et al. [18]. A constant flow alimentation from an 8 bar air supply was provided using a sonic valve to a large reservoir (0.5 m^3). The reservoir was filled with acoustical foam in order to prevent acoustical resonances and was connected to the glottal replica using a 300 mm long, 30 mm diameter, pipe. It appeared that, under these conditions, the reservoir pressure fluctuations were only of order of 10% of $P_{sub} - P_{supra}$. One mechanical vocal fold replica was kept fixed while the other one was forced to move using an eccentric motor type Maxon RE40. Stable oscillations were obtained up to 35 Hz. At constant Reynolds and Strouhal numbers this would correspond to a fundamental frequency of 300 Hz for a human glottis. Although the minimum glottal height could be adjusted, all results presented here correspond to oscillations were the vocal folds never collide together, in other words, the glottal replica is never closing completely. An optical sensor (OPTEK OPB700) allows for the measurement of the instantaneous position of the moving fold and thus for the measurement of $h_g(t)$. As in Section 2.2.1 the pressure upstream of the replica and within the replica (on the rigid vocal fold) were measured using Kulite XCS-093 pressure transducers and sampled at 10 kHz.

3.2.2.2. Results. An example of result for the uniform replica SC2 is presented in Fig. 8. The fundamental frequency imposed on one fold is 35 Hz. This corresponds to a Strouhal number averaged over a period, $Sr_L = 0.02$ and a dimensionless number averaged over a period, $Re_h h_g / L = 50$. The agreement between the theoretical prediction and the experimental data appears to be very good again (within 20%).

In Fig. 9 is presented an example of results for the rounded geometry RC. In this case, the fundamental frequency was chosen as 35 Hz and the minimum glottal height was 0.35 mm.

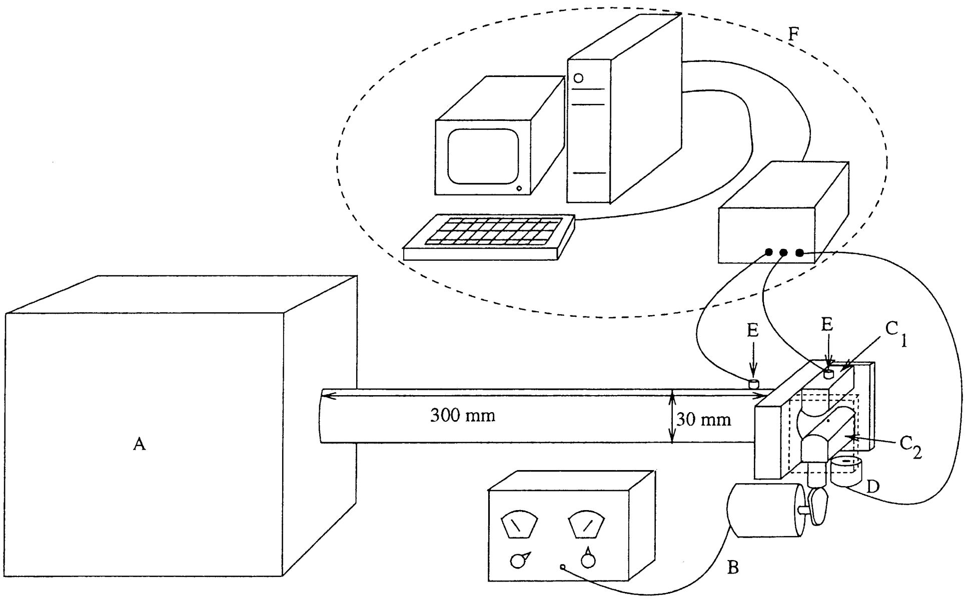

Fig. 7. Experimental set-up including moving vocal fold replica. A, Pressure reservoir; B, eccentric motor; C, vocal fold replicas: fixed replica (C1) and moving replica (C2), D, optical sensor; E, pressure transducers; F, acquisition system.

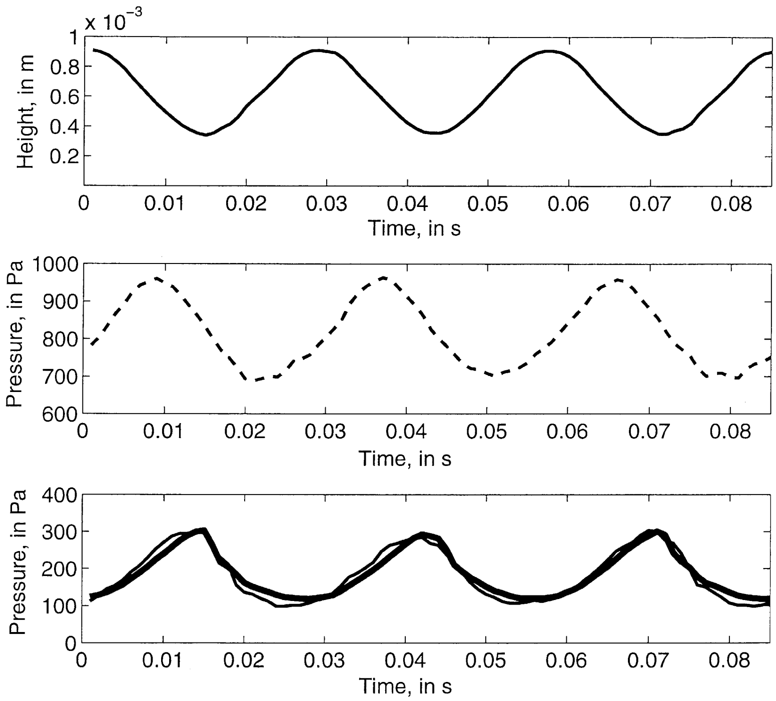

Fig. 8. Straight channel (SC1) of variable glottal height. Top, glottal height h_g; middle, transglottal pressure $P_{sub} - P_{supra}$; bottom, measured glottal pressure, $P_g - P_{supra}$ (thinner solid line) versus theoretical predictions (thicker solid line).

Average characteristics of the flow are thus a Strouhal number averaged over a period, $Sr_L = 0.02$ and a dimensionless number $Re_h h_g / L$ averaged over a period, $Re_h h_g / L = 80$. While the measured pressure difference $P_g - P_{sub}$ agrees within 30% with the theoretical predictions, a significant phase shift in the oscillating component of the signal can be observed.

In Fig. 10, another example is presented using the same rounded geometry, RC and a fundamental frequency of 35 Hz but with a much smaller minimum glottal height: 0.08 mm. In such a case the mechanical glottis is almost completely closing. While a 30% agreement can be observed between the theory and the measured data for most of one period of oscillation, large discrepancies appear when $h_g(t)$ becomes smaller than 0.2 mm. During this period of time, typical Strouhal numbers are of order of 0.2 and dimensionless numbers $Re_h h_g / L$ are of order of 5. These values are in contradiction not only with the quasi-steady assumption but also with the assumption of thin boundary layers (see Section 2.1). It is therefore not surprising that such large theoretical errors are observed.

4. Conclusion

As an alternative to simple ad hoc [5] or empirical [8,9] descriptions, a theoretical description of the flow through the glottis has been proposed. This theory is based on Thwaites' method to solve

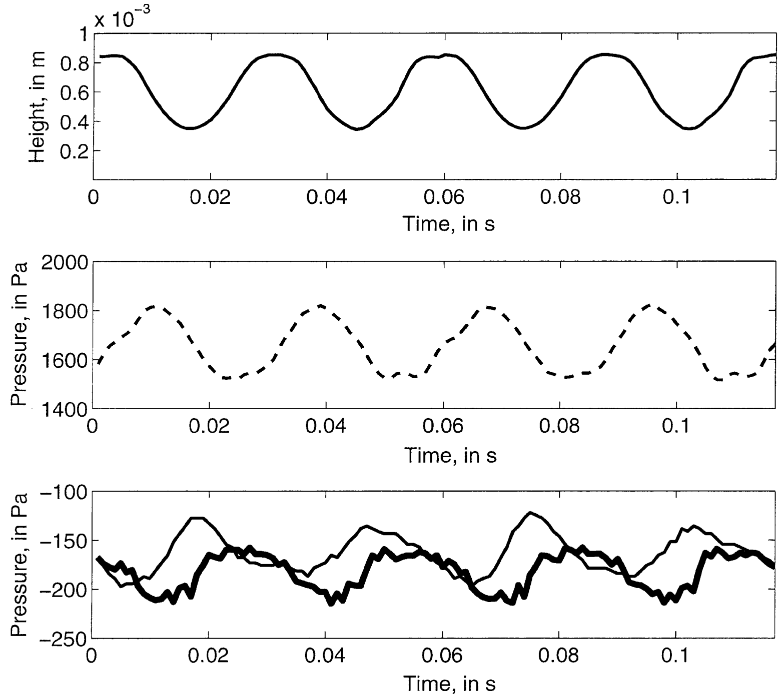

Fig. 9. Rounded channel (RC) of variable glottal height. Top, glottal height, h_g; on middle, transglottal pressure, $P_{sub} - P_{supra}$; bottom, measured glottal pressure (thinner solid line) versus theoretical predictions (thicker solid line).

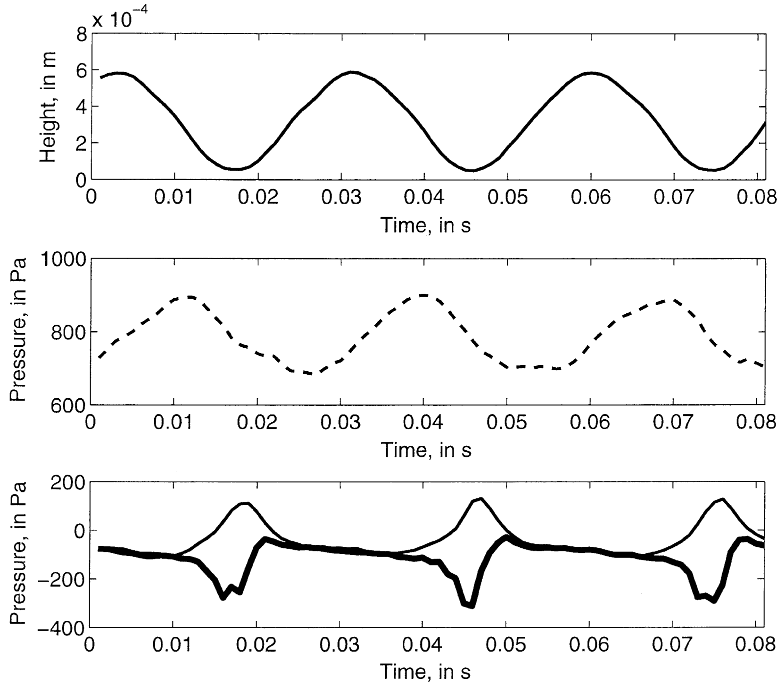

Fig. 10. Rounded channel (RC) of variable glottal height. Top, glottal height, h_g; middle, transglottal pressure; $P_{sub} - P_{supra}$; bottom, measured glottal pressure, $P_g - P_{supra}$ (thinner solid line) versus theoretical predictions (thicker solid line).

the Prandtl equations. Compared with the Pohlhausen one described by Hofmans [11], the Thwaites' method appears to be much simpler to implement numerically.

Under steady flow conditions the agreement between this theory and the measurements appears to be satisfactory since discrepancies for the glottal pressure remained within 30% at the most. In term of volume flow velocity this would correspond to an agreement within 15%.

Under unsteady flow conditions, it was found that same order of agreement could be observed. The theory fails for Strouhal numbers of the order of $SR = O(0.1)$. In such a case the quasi-steady assumption does not stand any longer. However, such Strouhal numbers are not expected during normal voicing but could occur during singing of for some pathological voices.

Another, more problematic, limitation of the theory occurs during the collision of the vocal folds. In such a case, a double limit is faced. The Strouhal number is not a measure for the unsteadiness of the flow anymore and viscous effects are obviously dominant when the glottis is almost closed. Alternative theories to predict the flow during the closure of the glottis are then obviously needed.

Acknowledgements

Part of this work was supported by Grant EURODOC from the Rhone-Alpes region (SAFIR number 99009385), by Grant VAN GOGH from the French Ministry of National Education, Research and Technology (number 01381XE) and by a grant from CNRS (number DRI-EUR 99/NWO/NLD). The authors also wish to thank H. Koolmes and F. Van Uittert from TUe for their help in the design of the experimental set-up.

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