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Crack identification in double-cracked beams using wavelet analysis

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Abstract

A method for crack identification in double-cracked beams based on wavelet analysis is presented. The fundamental vibration mode of a double-cracked cantilever beam is analyzed using continuous wavelet transform and both the location and depth of the cracks are estimated. The location of the cracks is determined by the sudden changes in the spatial variation of the transformed response. To estimate the relative depth of the cracks, an intensity factor is established which relates the size of the cracks to the coefficients of the wavelet transform. It is shown that the intensity factor follows definite trends and therefore can be used as an indicator for crack size. The proposed technique is validated both analytically and experimentally in case of a double-cracked cantilever beam having cracks of varying depth at different positions. In the light of the obtained results, the advantages and limitations of the method are presented and discussed.

1. Introduction

The existence of fatigue cracks in structural members subjected to repeated loading presents a serious threat to the integrity of structures. For this reason, many studies have been carried out in the last few decades in an attempt to find methods for non-destructive crack detection in structures. As a result, a variety of analytical, numerical and experimental investigations for detecting cracks now exists. A review of the state of the art of vibration based methods for testing cracked structures is presented in Ref. [1].

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In the analytical models of cracked beams, a crack is usually treated as a local change of flexibility in the vicinity of crack location. In a pioneering work of Dimarogonas [2], the crack was modelled as a massless rotational spring and its equivalent stiffness was computed as a function of crack depth using fracture mechanics methods. This model has been validated by subsequent research work and has been successfully applied to investigate the link of the reduction in natural frequencies to the characteristics of the crack. This motivated the dynamic analysis of cracked structures and the use of natural frequency shifts for detecting crack size and location.

Anifantis et al. [3] found that one could determine the size and position of large cracks if the changes in two natural frequencies were known. Using a similar approach Masoud et al. [4] investigated the vibrational characteristics of a fixed–fixed beam with a symmetric crack. Rizos et al. [5] suggested a method for using measured amplitudes of a cracked cantilever beam vibrating at one if its natural modes to identify crack location and depth. Narkis [6] simulated the crack by an equivalent spring connecting the two segments of a cracked beam and developed a closed-form solution, which he applied to the inverse problem of localization of cracks in the basis of natural frequency measurements. A variational approach to the problem of cracked beams has been used by Chondros et al. [7]. They developed a continuous vibration theory of cracked Bernoulli–Euler beams and they reported experimental results for the variation of the fundamental frequency of simply supported cracked beams.

All the above studies were concentrated on the analysis of the effect of a single crack on the dynamics of simple structures such as shafts and beams. Ostachowicz and Krawczuck [8] have investigated a continuous model of a beam with cracks, which were modelled by rotational springs and calculated by the dynamic behaviour, in the case of two cracks. Shen and Pierre [9] considered the same problem in case of symmetric cracks. Surcase et al. [10] analyzed the problem of double-cracked beams using both analytical models and FEM. They studied both analytically and experimentally the changes in natural frequencies for different crack locations and sizes. The dynamic behaviour of a double-cracked beam and a rotor with two cracks were investigated by Ruotolo et al. [11] and Sekhar [12], respectively. The natural frequencies of a beam with an arbitrary number of cracks was studied by Shifrin and Ruotolo [13]. They proposed a method for evaluating natural frequencies of such a beam that requires calculation of a $(n+2)$ determinant instead of a $(4n+4)$ usually needed. Recently, Khiem and Lien [14] proposed a more simplified method for evaluating the natural frequencies of beams with an arbitrary number of cracks. Their method is based on the use of the rotational spring model of cracks and leads to determinant calculation of a 4×4 matrix.

All the above-mentioned studies, however, concentrated on finding changes in dynamic behaviour when the damage is known. If the structure is cracked in more than one position, the problem of crack identification becomes quite complex. In case of a double-cracked beam, one has to deal with four unknown parameters, i.e., the locations and depths of the two cracks. Therefore, apart from natural frequency changes, the use of an additional defect information carrier for crack appearance becomes important. In searching for an additional defect information carrier for crack appearance, the use of the mechanical impedance was introduced by the authors [15]. It was shown that using mechanical impedance measurements the position of a crack can be determined. Once the position is fixed, natural frequency changes can be used to estimate crack size. The method, however, lacks accuracy for small cracks.

In the light of the above discussion, it is obvious that the literature on crack detection in beams has been so far dominated by studies based on methods that utilize natural frequency changes. Having in mind that small cracks have a minor effect on natural frequencies, more sensitive methods capable of detecting small changes become important. Recently, methods based on wavelet analysis are emerging and become a promising damage detection tool [16,17]. The advantage of wavelet analysis is that it breaks down a signal in a series of functions (wavelets) and allows the identification of local features from the scale and position of wavelets. The signal, for example, can be the displacement response over a region of interest for a structure. Using the wavelet transform, the local features in a spatially distributed structural response signal can be identified with a desired resolution. Deng and Wang [18] applied the discrete wavelet transform to locate a crack along the length of a beam. Quek et al. [19] also used wavelet analysis for crack identification in beams under both simply supported and fixed–fixed boundary conditions. No attempt, however, was made to estimate the size of the crack.

Hong et al. [20] used the Lipschitz exponent for the detection of singularities in beam modal data. The Mexican hat wavelet was used and the damage extent has been related to different values of the exponent. The correlation, however, of the damage extent to the Lipschitz exponent is sensitive to both sampling distance and noise resulting in limited accuracy of the prediction.

In the present work, a method for crack identification in double-cracked beams based on wavelet analysis is presented. The fundamental vibration mode of a cantilever beam having two transverse surface cracks is wavelet transformed and both the location and relative depth of the cracks are estimated. For this purpose, a symmlet wavelet having four vanishing moments is utilized as the analyzing wavelet. The position of the cracks is estimated by the variation of the transformed spatial response due to the high-resolution property of the wavelet transform. To estimate the depth of the cracks an intensity factor is established, which relates the depth of the cracks to the coefficients of the wavelet transform. It is shown that the intensity factor obeys a polynomial law and therefore can be used for accurate prediction of relative crack depths. The feasibility of the proposed approach is demonstrated both analytically and experimentally on double-cracked cantilever beams with cracks of varying depths at different locations. In view of the obtained results, the advantages and limitations of the method are presented and discussed.

2. Continuous wavelet transform as damage detection tool

The wavelet transform is a mapping from a function $f(x)$ depending on time or space to a function $W[f, \psi(s,b)]$ depending on a dilation parameter s and a temporal or spatial translation parameter b. It is defined in Ref. [21] as

$$W[f, \psi(s,b)] = \frac{1}{\sqrt{s}} \int_{-\infty}^{+\infty} f(x)\psi^{*}\left(\frac{x-b}{s}\right) \mathrm{d}x, \tag{1}$$

where $(*)$ denotes the complex conjugate. The function $\psi(x)$ is called the analyzing wavelet. It has to fulfil the admissibility condition which for most cases reduces to the zero mean condition. The zero mean condition implies that the wavelet changes its sign at least once along the real line and this gives the wave property to the wavelet, i.e., oscillation. The effective length (support) of the wavelet is finite and this explains the term wavelet, or small wave. The parameter, s-called scale, controls the width of the wavelet. A high value of s corresponds to big wavelets, so that low-frequency components can be looked through, while a low value of s corresponds to small wavelets suitable for the analysis of high-frequency components. Both big and small wavelets can be shifted using parameter b to cover the range over which function f is defined (or the signal is sampled). In other words, a multiscale and well localized analysis of the function is possible looking at the function's interesting features through a wavelet window. An important concept of the wavelet transform-based analysis is that of vanishing moments. If a wavelet has N vanishing moments it is blind to polynomials up to order $N-1$. This is expressed mathematically as

$$\int_{-\infty}^{+\infty} x^n \psi(x)\,\mathrm{d}x = 0 \quad \text{for } n = 0, 1, \ldots, N-1. \tag{2}$$

A way to obtain a wavelet with N vanishing moments is to take the Nth derivative of a smooth function $\phi(x)$, called the scaling function [22]. In this case the wavelet transform reads

$$\begin{aligned} \frac{1}{\sqrt{s}}\int_{-\infty}^{+\infty} f(x)\psi\left(\frac{x-b}{s}\right)\mathrm{d}x &= \frac{(-s)^N}{\sqrt{s}}\int_{-\infty}^{+\infty} f(x)\left(\frac{\mathrm{d}^N}{\mathrm{d}x^N}\phi\left(\frac{x-b}{s}\right)\right)\mathrm{d}x \\ &= \frac{(-s)^N}{\sqrt{s}}\int_{-\infty}^{+\infty}\left(\frac{\mathrm{d}^N}{\mathrm{d}x^N}f(x)\right)\phi\left(\frac{x-b}{s}\right)\mathrm{d}x. \end{aligned} \tag{3}$$

In other words, the wavelet transform of $f(x)$ with a wavelet having N vanishing moments is a smoothed version of the Nth derivative of $f(x)$ at various scales. This is an extremely useful property, because it provides the means to look into different rates of change of the signal in a controlled manner by the choice of a suitable wavelet. In addition, this can be done separately for every scale of interest, isolating coarse from fine features. Wavelet theory provides a way to estimate the local smoothness of functions, which can be characterized by Hoelder exponents. A function $f(x)$ has a local Hoelder exponent h at x_o if, and only if, a polynomial $P(x)$ of order $n<h$ and a constant C exists, so that

$$|f(x) - P(x)| \leqslant C|x - x_o|^h. \tag{4}$$

The polynomial $P(x)$ is associated with the Taylor expansion of $f(x)$ at x_o. By examining the decay of wavelet maxima coefficients as s tends to zero, it can be proved [23] that, for isolated singularities, the wavelet maxima obey an exponential law with an exponent equal to the Hoelder exponent, i.e.,

$$|W[f, \psi(s,x)]| \leqslant C s^{h+1/2}. \tag{5}$$

At a more practical level, the points of sharp variations of a signal cause local maxima at a fixed scale of the wavelet transform modulus. The decay of the modulus maxima along a modulus maxima line is characterized by the Hoelder exponent. By rewriting Eq. (5) in logarithmic form

$$\log_2|W[f,\psi]| = \log_2 C + (h + 1/2)\log_2 s, \tag{6}$$

the constants h and C can be estimated by linear interpolation so that the square error is minimized. It is important at this point to clarify the role of each constant in Eq. (6). The Hoelder exponent h accurately describes the type of a singularity. A Dirac delta function, for example, is Hoelder -1 at $x=0$. Generally speaking, one degree of smoothness is lost with differentiation. Consequently, the first derivative of Dirac delta function is Hoelder -2 and the step function is

Hoelder 0. Assume next that all singularities in a signal are of the same type, say for convenience they resemble a step function. In this case, all singularities are characterized by the same exponent and only constant C changes. Each singularity might be characterized from its relative magnitude which is described by constant C. Therefore, constant C can be considered as an intensity factor. In an actual application of Eq. (6), the slope h and constant C should be determined and the size of the damage can be estimated. This procedure will be illustrated in the subsequent section with numerical examples.

Finally, the choice of the analyzing wavelet is left to be discussed. In practice, wavelets with higher number of vanishing moments give higher coefficients and more stable performance. On the other hand, one should bear in mind that the effective support of a wavelet is increased with the number of vanishing moments. Therefore, a compromise between number of vanishing moments and adequate localization should be accomplished. After some experimentation a symmlet wavelet having four vanishing moments has been chosen and used as analyzing wavelet throughout the present work.

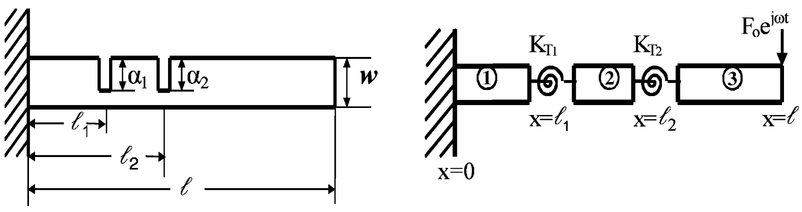
3. Wavelet analysis of double-cracked beam

3.1. Vibration model of cracked cantilever beam

To illustrate the feasibility of applying the wavelet transform to double-cracked beams, numerical simulations on double-cracked cantilever beams were performed. The objective of the investigation is twofold:

- To demonstrate the feasibility of wavelet transform to detect cracks by capturing local perturbations in the transformed structural response.
- To investigate the relation between wavelet transform coefficients and crack extent described in terms of the intensity factor.

A cantilever beam of length ℓ, of uniform rectangular cross-section $w \times w$ having two transverse surface cracks located at positions ℓ_1 and ℓ_2 is considered as shown in Fig. 1(a). The cracks are assumed to be open and have uniform depths α_1 and α_2, respectively. Due to the localized crack effect, the double-cracked beam can be simulated by three segments connected by two massless springs [5,6] (Fig. 1(b)). For general loading, a local flexibility matrix relates displacements and forces. In the analysis, since only bending vibrations of thin beams



(a) (b)

Fig. 1. (a) Cantilever beam under study, (b) double-cracked cantilever beam model.

are considered, the rotational spring constant is assumed to be dominant in the local flexibility matrix [3].

The bending constant K_T in the vicinity of the cracked section is given by

$$K_T = \frac{1}{c} \quad \text{with } c = 5.346\frac{w}{EI}J\left(\frac{\alpha}{w}\right), \tag{7}$$

where α is the depth of the crack, E is the modulus of elasticity of the beam, I is the moment of inertia of the beam cross-section and $J(\alpha/w)$ is the dimensional local compliance function, given by

$$\begin{aligned} J(\alpha/w) = {} & 1.8624(\alpha/w)^2 - 3.95(\alpha/w)^3 + 16.37(\alpha/w)^4 - 37.226(\alpha/w)^5 \\ & + 76.81(\alpha/w)^6 - 126.9(\alpha/w)^7 + 172(\alpha/w)^8 \\ & - 43.97(\alpha/w)^9 + 66.56(\alpha/w)^{10}. \end{aligned} \tag{8}$$

The displacement in each part of the beam is

$$\begin{aligned} &\text{part 1:} \quad n_1(x) = c_1 \sin K_B x + c_2 \cos K_B x + c_3 \sinh K_B x + c_4 \cosh K_B x, \\ &\text{part 2:} \quad n_2(x) = c_5 \sin K_B x + c_6 \cos K_B x + c_7 \sinh K_B x + c_8 \cosh K_B x, \\ &\text{part 3:} \quad n_3(x) = c_9 \sin K_B x + c_{10} \cos K_B x + c_{11} \sinh K_B x + c_{12} \cosh K_B x \end{aligned} \tag{9}$$

with $K_B^4 = \omega^2 \rho A \ell^4/(EI)$, where A is the cross-sectional area, ω is the vibration angular velocity, ρ is the material density and c_i, $i = 1, 2, \ldots, 12$ are constants to be determined from the boundary conditions.

Assuming that a driving force $F = F_o e^{j\omega t}$ is acting at the free end of the beam, the boundary conditions at both ends are

$$\begin{aligned} &\text{at } x = 0: \quad n_1(0) = 0, \quad n_1'(0) = 0, \\ &\text{at } x = \ell: \quad M_3(\ell) = 0, \quad F_3(\ell) = F_o, \end{aligned} \tag{10}$$

where prime denotes derivative with respect to x. For each connection between the two segments, conditions can be introduced which impose continuity of displacement, bending moment and shear. Moreover, an additional condition imposes equilibrium between transmitted bending moment and rotation of the spring representing the crack.

Consequently, the boundary conditions at the crack positions can be expressed as follows:

$$\begin{aligned} \text{at } x = \ell_1: \quad & n_1(\ell_1) = n_2(\ell_1),\ M_1(\ell_1) = M_2(\ell_1),\ F_1(\ell_1) = F_2(\ell_1), \\ & -EI\frac{\partial^2}{\partial x^2}n_1(\ell_1) = K_{T_1}\left[\frac{\partial}{\partial x}n_1(\ell_1) - \frac{\partial}{\partial x}n_2(\ell_1)\right], \end{aligned} \tag{11}$$

$$\begin{aligned} \text{at } x = \ell_2: \quad & n_2(\ell_2) = n_3(\ell_2),\ M_2(\ell_2) = M_3(\ell_2),\ F_2(\ell_2) = F_3(\ell_2), \\ & -EI\frac{\partial^2}{\partial x^2}n_2(\ell_2) = K_{T_2}\left[\frac{\partial}{\partial x}n_2(\ell_2) - \frac{\partial}{\partial x}n_3(\ell_2)\right]. \end{aligned} \tag{12}$$

The resulting characteristic equation for the above-described system can be solved numerically and both the natural frequencies and mode shapes of the beam can be obtained.

For numerical simulations a plexiglas beam of total length 300 mm and rectangular cross-section $20 \times 20\ \text{mm}^2$ is considered. Two cracks, located at $x = 60$ and 120 mm from clamped end, are introduced. Based on the theoretical model presented above, the fundamental vibration mode of the beam was calculated for different crack depths. Since small to medium cracks are considered as the most interesting cases, the depth of the crack located at $x = 60$ mm was fixed at 10% while the depth of the crack at $x = 120$ mm was varied from 5% to 40%. These five cases are numbered as in Table 1. The simulated fundamental vibration mode for case (c) is shown in Fig. 2.

3.2. Determination of crack location

To determine the location of the cracks, the calculated displacement response of the double-cracked cantilever beam considered was wavelet transformed. The displacement data obtained follow a sampling distance of 1 mm resulting in a number of 301 points available. All data are normalized so that a direct comparison between different cases is possible. The wavelet analysis is performed using the algorithm supplied in Matlab's wavelet toolbox [24]. The continuous wavelet transform is preferred instead of the discrete version, as the redundancy of information it provides is useful for analysis purpose. The wavelet transform is implemented for scales 1–25 with the sym4 wavelet as the analyzing wavelet.

Table 1
The five cases with cracks of varying depths at different locations

Case	$x = 60$ mm	$x = 120$ mm
(a)	10%	5%
(b)	10%	10%
(c)	10%	20%
(d)	10%	30%
(e)	10%	40%

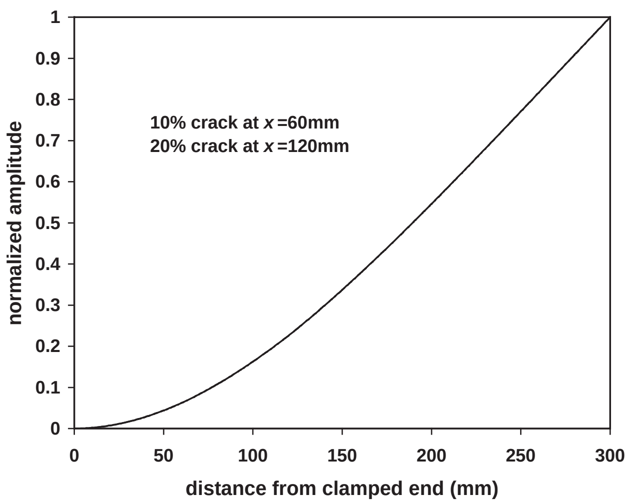


Fig. 2. Calculated fundamental vibration mode of the cracked cantilever beam.

Fig. 3 presents the results of the analysis for all five cases shown in Table 1 for a single scale $s = 5$. The wavelet transform coefficients exhibit high values at $x = 60$ and 120 mm providing evidence of crack existence at these positions. Moreover, the relative magnitudes of the wavelet transform coefficients give an indication about the relative size of the damage. In case (c), for example, a small crack located at $x = 60$ mm is followed by a deeper crack at $x = 120$ mm.

To be certain about the presence of the crack, however, one has to examine in detail the behaviour of the wavelet maxima at these points as the scale increases. Fig. 4 presents a three-dimensional plot of the wavelet transform coefficients for case (c) for scales 1–25. It can be seen that the absolute value of the wavelet maxima decreases in a regular manner with decreasing scale. This kind of behaviour is typical for singular points of a response signal generated by cracks. The absence of coefficients of significant value at any other locations away from the cracks is characteristic. This is attributed to the fact that the analyzed data stem from theoretical calculations of the response and hence contain no noise or measurement errors. In real experiments, however, noise is expected to corrupt the response data. It is known [21] that the wavelet transform coefficients behave in a completely different manner if they are generated by noise disturbances. They characterized by negative Hoelder exponents and consequently, the wavelet modulus maxima increase with decreasing scale. This fact provides a way of discriminating singular points generated by noise. This issue will be examined in detail in a subsequent section.

The results of this section show that the location of cracks in a multi-cracked beam can be accurately determined by applying continuous wavelet transform to displacement response data.

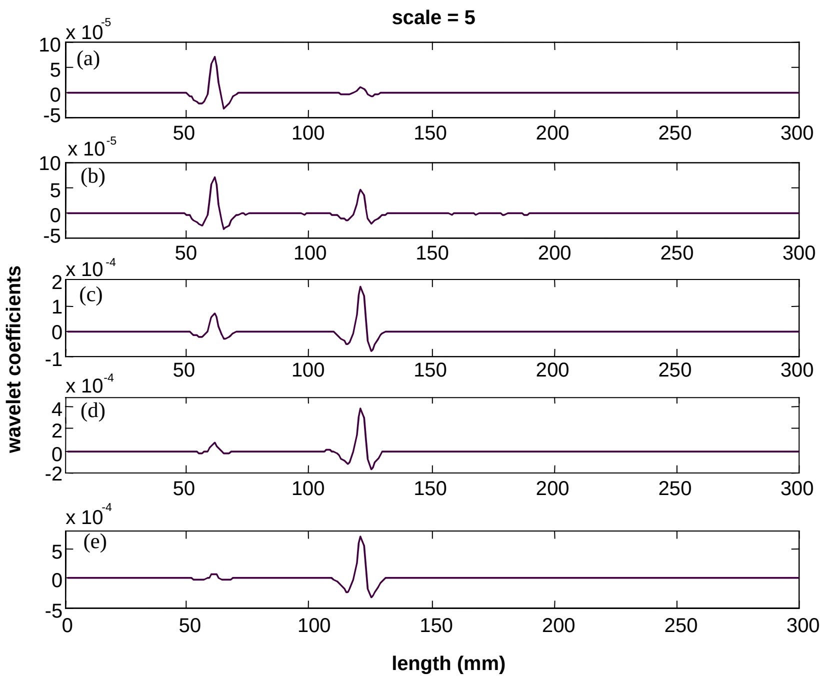


Fig. 3. Wavelet coefficients plots for all cases of Table 1 for scale $s = 5$.

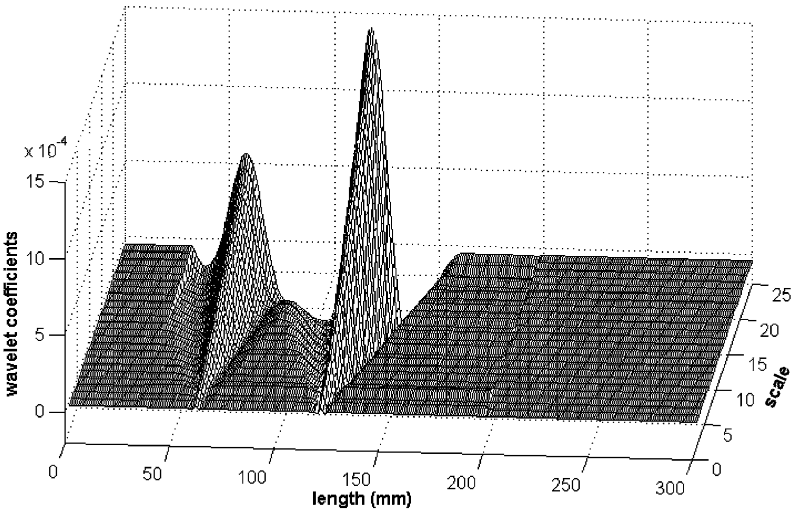


Fig. 4. Three-dimensional plot of wavelet transform showing the trend of wavelet modulus maxima at crack locations (10% crack at $x = 60$ mm, 20% crack at $x = 120$ mm).

3.3. Estimation of crack size

As already mentioned, Eq. (6) can be used to relate crack extent to the coefficients of the wavelet transform. For that purpose, one has to examine the modulus maxima lines of the wavelet transform. Case (c) is considered again.

Fig. 5 shows the wavelet maxima lines versus scale at crack site $x = 120$ mm for varying crack depths from 5% to 40%. The wavelet coefficients are parallel lines of the same slope, or in other words of the same Hoelder exponent. The Hoelder exponent for all cases has a constant value equal to 1. This means that the mode function is exactly one time differentiable at the location of the crack and, as explained above, a wavelet of two vanishing moments would suffice. The constant value of the Hoelder exponent implies singularities of the same type caused by the same physical cause, which in our case is the existence of a crack.

From Eq. (6) it follows that for the fixed exponent h, only the constant C changes. Hence, each parallel line can be discriminated by its constant C. It can be seen that constant C increases with increasing damage extent. In this sense, constant C can be considered as an intensity factor which relates the extent of the crack to the coefficients of the wavelet transform.

Fig. 6 presents the intensity factor C versus crack depth. It can be seen that the intensity factor follows a second order polynomial law with crack depth. Therefore, Fig. 6 can be used for predicting the relative crack depth for a given intensity factor.

An important point to be clarified is the dependence of the wavelet coefficients on the location of the crack. This will be illustrated by means of Fig. 7. In this figure the wavelet modulus maxima coefficients are plotted versus the scale in the case of two cracks of the same relative depth 10% located at different positions namely, $x = 60$ and 120 mm from clamped end. It can be seen that, even though the cracks are of the same depth, the coefficients at $x = 60$ mm appear to have higher values compared to the corresponding coefficients at $x = 120$ mm. More specifically, the intensity

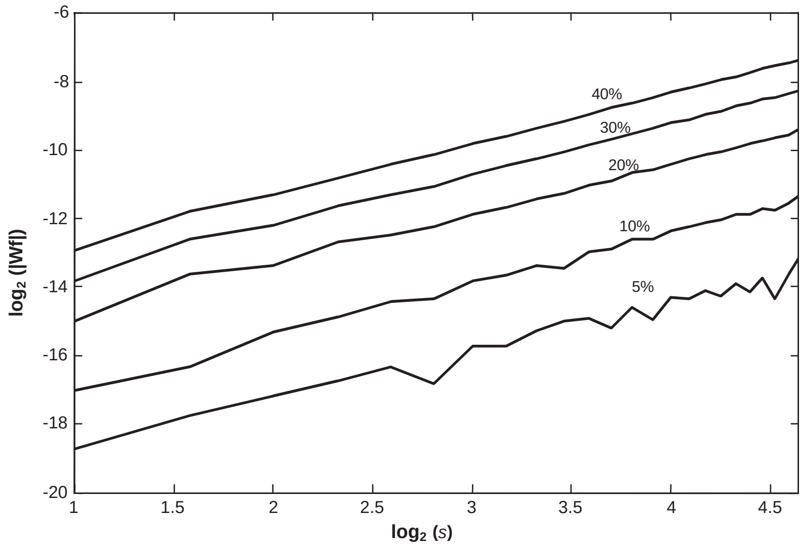


Fig. 5. Wavelet modulus maxima versus scale for varying crack depth. Crack located at $x = 120$ mm from clamped end.

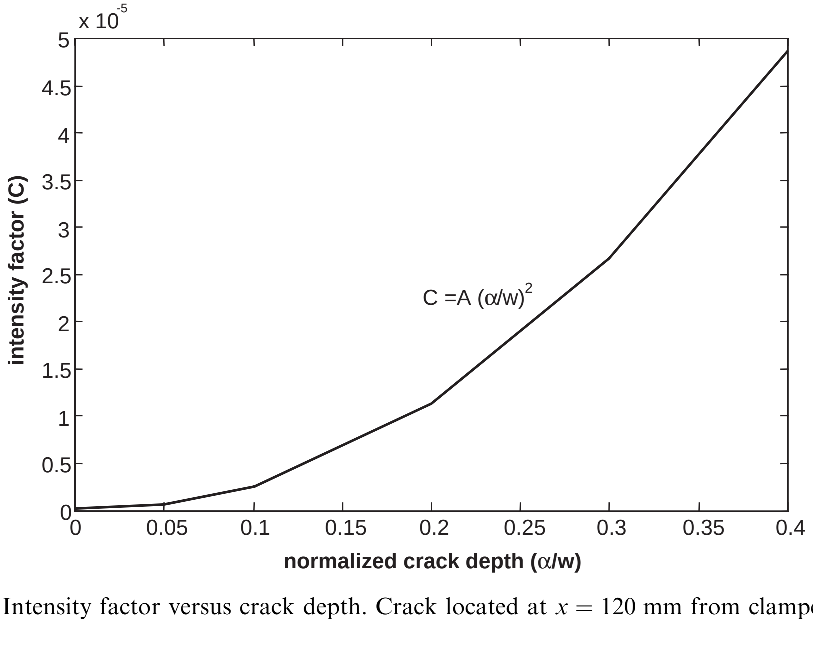


Fig. 6. Intensity factor versus crack depth. Crack located at $x = 120$ mm from clamped end.

factor at $x = 60$ mm is $C = 4.67 \times 10^{-6}$, i.e., 1.65 time higher than the intensity factor $C = 2.82 \times 10^{-6}$ at $x = 120$ mm. This is due to the fact that the slope of the vibration mode analyzed is different at the two locations. The slope biases the wavelet transform coefficients and therefore a correction is necessary if cracks at different locations are to be compared.

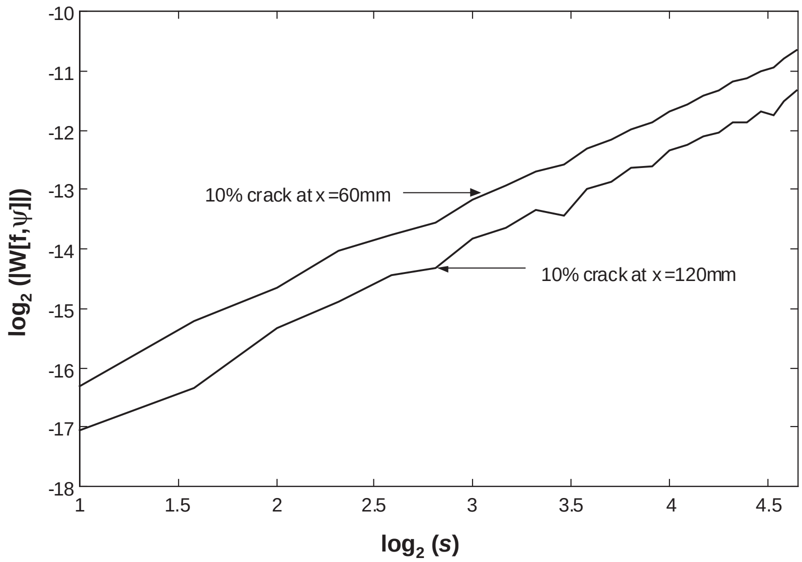


Fig. 7. Comparison of the intensity factors of two identical cracks located at different positions from the clamped end.

4. Experimental investigation

To validate the analytical results of the wavelet analysis, an experiment on a plexiglas beam has been performed.

A 30 cm long plexiglas beam of rectangular cross-section $20 \times 20\ \mathrm{mm}^2$ was clamped to a vibrating table. Two cracks of relative depths 20% and 30% were introduced at $x = 60$ and 120 mm from the clamped end, respectively. An electromagnetic vibrator by Link and two 4375 B&K accelerometers were used. Harmonic excitation was used via a 2110 B&K analyzer and the fundamental mode of vibration was investigated.

The vibration amplitude was measured with a sampling distance of 10 mm, which was the effective diameter of the accelerometer used, so that a total number of 30 measuring points were obtained. Mode shape was measured by using two calibrated accelerometers mounted on the beam. One accelerometer was kept at the clamped end as the reference input, while the second accelerometer was moved along the beam to measure the mode amplitude. The accelerometer had a mass of 2.4 g, which was small compared with the 4.6 g/cm mass per unit length of the beam. A plot of the measured fundamental mode shape of the beam is shown in Fig. 8.

Because of the sparse sampling, the wavelet transform if implemented directly would detect many points of sample data as singularities. Therefore, to smooth the transition from one point to another an over-sampling procedure was necessary. For that purpose, a cubic spline interpolation was used which resulted in a total number of 301 points available.

Fig. 9 presents the results of the wavelet analysis of the experimental data. The analysis was carried out for scales 1–25 in accordance to the procedure followed in the case of the theoretical data. It can be seen that the results are not smooth as in the case of the simulated response. The wavelet coefficients exhibit significant values in more than two points along the beam. It is

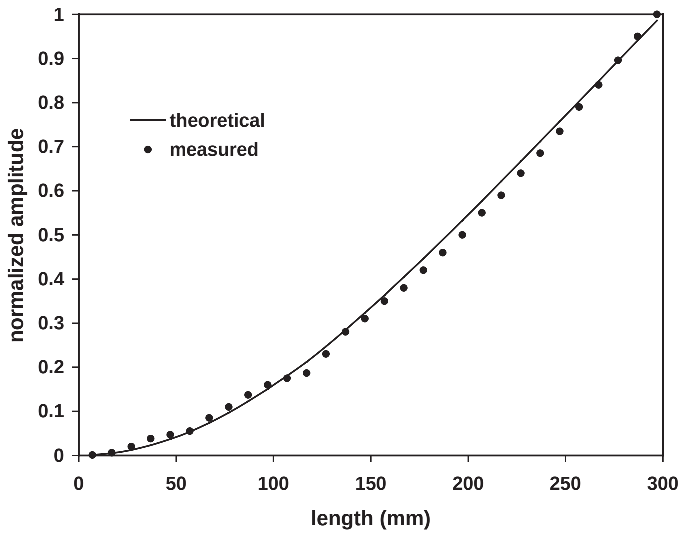


Fig. 8. Comparison between calculated (—) and measured (●●●) fundamental vibration mode of the double-cracked cantilever beam (20% crack located at $x = 60$ mm and 30% crack located at $x = 120$ mm from clamped end).

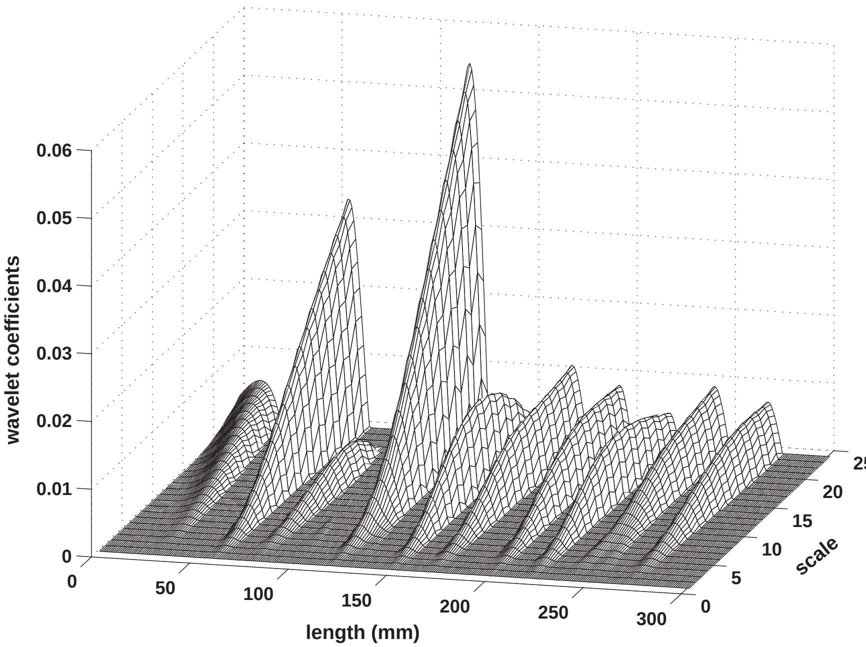


Fig. 9. Three-dimensional wavelet transform of the experimental mode shape of the double-cracked cantilever beam.

obvious that measuring errors and noise corrupt the response data. The behaviour, however, of the wavelet coefficients caused by errors is completely different from those caused by cracks. A careful observation reveals that the coefficients do not increase regularly with scale. This observation provides a way to discriminate real singular points caused by the presence of a crack from those caused by errors. On the other hand, at $x = 60$ and 120 mm, where the cracks are located, the coefficients exhibit higher values compared to all other points and more important, they decrease in a very regular manner with scale. Therefore, at $x = 60$ and 120 mm singularities caused by cracks are present.

To estimate the size of the cracks, the wavelet maxima for different scales were calculated at crack locations. The resulting values of the Hoelder exponent and intensity factor are $h = 1.17$, $C = 1.85 \times 10^{-5}$ for the crack at $x = 60$ mm and $h = 1.21$, $C = 1.1 \times 10^{-5}$ for the crack at $x =$ 120 mm, respectively.

The experimental Hoelder exponents are 15% higher compared to the theoretical value. This means that the singularities generated by the cracks appear to be smoother as a result of the inadequate sampling density. Using the calculated results of the constants h and C, the prediction scheme based on the relation of the intensity factor C to the relative depth (Fig. 6) yields crack depths of approximately 30% and 40% instead of the actual depths 20% and 30%. This is expected since the prediction scheme is based on the exact Hoelder exponent equal to $h = 1.00$. As the exponent increases the intensity factor also increases resulting in a larger singularity, i.e., a larger crack.

The experimental results show that a key issue in applying wavelet analysis based on the distributed signals is the spatial resolution and accuracy required in making measurements. In this case, the spatial resolution was determined by the particular accelerometer used, which allowed a total number of only 30 measuring points. Therefore, new measuring techniques able to pick up the perturbations caused by the presence of a crack are needed. The application of a laser vibrometer, for example, would enhance the efficiency of the method providing non-contact measurements of the vibration mode with the desired accuracy [25].

5. Conclusions

A method for crack detection in double-cracked beam structures based on wavelet analysis has been presented. The viability of the method has been demonstrated by analyzing the fundamental vibration mode of a double-cracked cantilever beam using a symmlet.

The location of the cracks was determined by the sudden changes in the spatial response of the transformed signal at the site of the crack. Such local changes usually are not obvious from the response data, they are, however, discernible as singularities when using wavelet analysis due to its high-resolution properties. For the estimation of the relative depth of the cracks an intensity factor was established. It relates the size of the cracks to the corresponding wavelet coefficients. It was shown that the intensity factor changes with crack depth according to a second order polynomial law and therefore, can be used as an indicator for crack extent.

The numerical results were confirmed by the application of wavelet analysis to actual experimental data of a double-cracked cantilever beam. Using the noisy experimental data, the location of the cracks were accurately determined by making use of the different behaviour of the wavelet coefficients. The estimation, however, of the crack depths based on experimental data provided difficulties. Measurement errors result in higher values of the intensity factor which in turn leads to overestimation of crack depths. Hence, a reliable estimation of crack depths becomes questionable in case of small cracks. It seems that a key issue for an effective application of wavelet analysis for crack identification is the spatial resolution and the accuracy of the response data. In that vein, new techniques, like laser scanning vibrometers, allowing non-contacting accurate measurement of mode shapes would enhance the potential of the proposed method.

In conclusion, the presented results provide a foundation for using wavelet analysis as efficient crack detection tool. The advantage of using wavelet analysis is that local features in a displacement response signal can be identified with a desired resolution, provided that the response signal to be analyzed picks up the perturbations caused by the presence of the cracks. In that vein, a key issue is the spatial resolution and the accuracy of the measurements. Furthermore, the sensitivity of the analysis to random noise need also to be investigated. The results of the present work refer to a cantilever beam but can be easily extended to include more complex structures and boundary conditions. Work is already under way in this direction and will be the subject of a future publication.

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