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Experimental detection of modal interaction in the non-linear vibration of a hinged–hinged beam

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Abstract

The geometrically non-linear vibration of an aluminium beam hinged at both ends is investigated experimentally. The beam is excited transversely with a harmonic excitation and the amplitudes of the first and higher harmonics are analyzed at different points in order to detect the modes involved in the motions. It is demonstrated that internal resonances occur between the first and higher order modes, as well as between the second and higher order modes.

1. Introduction

Generally, if a beam vibrates with amplitudes of the order of its thickness under the action of a harmonic excitation, the response is periodic but not harmonic, and therefore its autospectrum includes the frequency of excitation and some of its harmonics [1,2]. In geometrically non-linear systems the natural frequencies and the mode shapes of vibration may change with the vibration amplitude [3–6], simply because the stiffness of the system is not constant. The variation of the natural frequencies makes it likely that some of them become commensurable, that is, related by an equation of the form $m_1\omega_1 + m_2\omega_2 + \cdots + m_n\omega_n = 0$, where m_i are integers. When this happens, and due to the coupling caused by the non-linearity, energy can be interchanged between the different modes of vibration.

As a result of this *internal resonance* phenomenon [2], the structure responds in two or more coupled modes, and it could be said that the structure's motion is defined by *non-linear mode shapes* [7], which change during the period of vibration. The effects of the coupled response have

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been demonstrated in doubly clamped and doubly hinged beams numerically [5,6], and in the experimental analyses of clamped–clamped beams [8]. Interesting experimental analysis of cantilever beams [9,10], and numerical and experimental analyses of hinged–hinged beams have been carried out [11–13]. However, it seems that the occurrence of modal interaction in hinged–hinged beams has not been definitely demonstrated experimentally, namely through an analysis of the mode shapes.

In this work, the geometrically non-linear vibrations of a hinged–hinged aluminium beam are investigated experimentally. The main goals are to verify how important the non-fundamental harmonics can be, where fundamental indicates the frequency of excitation; to search for internal resonances; and to assess the variation of the natural frequencies and mode shapes with the vibration amplitude. Moderate vibration amplitudes are considered.

2. Experimental procedure

The dimensions of the aluminium alloy beam analyzed are $560 \times 20 \times 3.3\,\mathrm{mm}$. To simulate the hinged–hinged boundary conditions, the set-up shown in Fig. 1 was implemented. Twenty-five measuring points were marked on the beam, as represented in Fig. 2. The beam was both screwed and glued at its ends to two axes, which were linked to rigid steel blocks by ball bearings. The bearings are not lubricated or sealed and were carefully chosen and aligned, in order to reduce the resistance to rotation. An electromagnetic exciter was connected to the beam by a drive rod and a force transducer. The signal sent to the exciter was generated by an analyzer, constituted by software, a PC and a front-end. The accelerations were measured using one accelerometer with 2 g. The total mass of the beam between supports is approximately 100 g.

In order to verify that the conditions implemented approach the ideal hinged–hinged boundaries, the experimental linear natural frequencies were compared with the numerical frequencies. The latter were computed following Timoshenko's model and the following properties for the aluminium alloy: Young's modulus $E = 7 \times 10^{10}\,\mathrm{Pa}$, the Poisson coefficient, $v = 0.33$, mass density $\rho = 2778\,\mathrm{Kg/m3}$, and shear modulus of elasticity $G = \lambda E/(2(1+v))$, where the shear correction factor is $\lambda = 5/6$. These frequencies are shown in Table 1, where one can see that the agreement is quite reasonable for the first and third natural frequencies, and not so good for the others. The differences encountered can be explained by the fact that the boundaries are

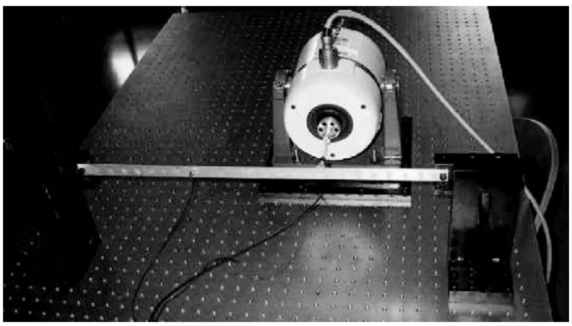

Fig. 1. Experimental set-up.

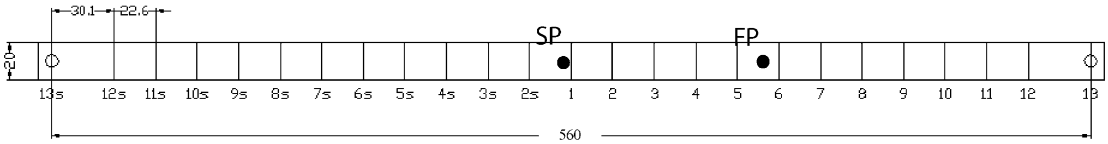

Fig. 2. Beam dimensions (in mm), points of measurement and of excitation.

Table 1
Natural frequencies ω (Hz)

Mode	1	2	3	4	5
ω_e, experimental	23.1	80.8	202	329	520
ω_n, numerical	23.9510	95.787	215.46	383.38	600.62
Relative error $(\omega_e - \omega_n)/\omega_n$	−3.5%	−15.6%	−6.0%	−14.2%	−13.4%

not exactly hinged and particularly by the influence of the mass of the transducers, which should be larger for higher frequencies due to inertia effects.

In the non-linear studies, the beam was excited transversely with sine excitations at frequencies close to its first and second natural frequencies. At each excitation amplitude, the frequency of excitation was slowly changed. Measurements were carried out when the force was applied at a point 8 mm to the left of point 6 and 2 mm to the left of point 1 (Fig. 2). With the first point, designated by **FP**, the first two modes were analyzed, and with the second point, **SP**, the first mode. The deviations of points **FP** and **SP** with relation to the equally spaced lines marked on the beam are due to mounting restrictions.

Some difficulties experienced during the analyses should be noted. First, although a light-weight accelerometer was used, the linear natural frequencies were not independent of its position on the beam. Second, the exciting system slightly restricted the deflection of the point of the beam connected to the vibrator. As a result of this, when exciting at point **FP**, which is not close to the beam's center, the first measured mode of vibration is not really symmetric and the second mode is not exactly antisymmetric. Also as a result, the exciting system itself might have introduced some harmonic distortion [14]. Although the beam is quite thin, a medium size exciter was used, so that its stroke is large enough to apply a sinusoidal excitation when the beam suffers large displacements. In order to reduce the effects of the harmonic distortion of the exciting system, only moderately large vibration amplitudes are investigated.

It was decided to always use a force transducer, because it was necessary to adjust the gain of the amplifying system, in order to keep the amplitude of excitation constant for different frequencies and to verify if the exciting signal was sinusoidal. Unfortunately, for some excitation frequencies and amplitudes, the signal coming from the force transducer was not sinusoidal. This type of signal has been found by other authors [14], and in this case appears to be a consequence of the beam's dynamic response, which affects the force transducer. In fact, the signal applied to the beam has a form close to that of a sine wave, as was demonstrated by fixing a very-light-weight accelerometer (1 g) to the force transducer. An accurate measurement of the amplitude of the force applied became somewhat difficult at certain frequencies, and, when given, the values in the next sections should be regarded as approximated values.

The unwanted effects referred to in the foregoing were minimized as far as possible, and the tests were often repeated in order to confirm the validity of the data obtained.

3. Non-linear vibrations

3.1. First non-linear mode

First, the situation where the excitation is implemented in point **FP**, at frequencies close to the first natural frequency, will be considered. In this case, three amplitudes of excitation were applied: 0.1, 0.3 and 0.45 N. Estimates of the amplitudes of the first harmonic (represented by W_1) at point 1, divided by the beam thickness h, are shown in Fig. 3. It is seen that the resonance curves bend towards the right as the amplitude of vibration increases. An approximation to the mode shape was defined by measuring the displacement amplitude of the first harmonic at half of the points marked on the beam. The normalized shapes obtained are shown in Fig. 4. Numerical work predicts that the mode shape of hinged–hinged beams does not change with the vibration amplitude [3–5] and the experimental analysis carried out here at moderate vibration amplitudes confirms this view.

The autospectrum of the motion of point 10s, when the excitation is harmonic, with 25.25 Hz frequency and 100 mN amplitude (Fig. 5) shows that that the eighth harmonic is excited. Fig. 6 shows its deflection, which is similar to the third mode.[1] This is most probably due to a modal interaction between the first and third modes, since the eighth harmonic has a frequency of 202 Hz, which is the linear natural frequency of the third mode.

The shapes of the eighth harmonic were displayed for other excitation amplitudes, at excitation frequencies close to the resonance frequency. For 0.3 N and 0.45, the frequency chosen was 26 Hz, and probably because the eight harmonic of 26 occurs above the third natural frequency, no connection was found between the third mode and the eight harmonic in these cases.

The detection of even harmonics, such as the eighth, which also occurred in Ref. [3], is not expected in a cubic system excited by an harmonic force. Their appearance may be due to quadratic non-linearities associated with the longitudinal inertia, or/and to perturbations introduced by the exciting system [14,15].

At 26 Hz, either with 0.3 or 0.45 N (the latest example can, again, be seen on Fig. 5), the second and third harmonics are very important. The deflection of the second harmonic did not allow conclusions to be drawn about which modes influence it. But, as clearly shown in Fig. 7, the deflection of the third harmonic is very much influenced by the second mode of vibration. This excitation of the second mode occurs because the frequency of the third harmonic is very close to the second natural frequency (Table 1). Therefore, some energy is transferred from the first to the second mode of vibration.

Moreover, the relative importance of the third harmonic and second mode clearly increase with the amplitude of vibration. This is demonstrated by the autospectra of the accelerations, at point 65 due to excitations with different amplitudes, which are shown in Fig. 8. It is clear that whilst the first harmonic increased 1.22 times, the third increased 5.05.

[1] Amplitudes at points 4s–2s were multiplied by minus one.

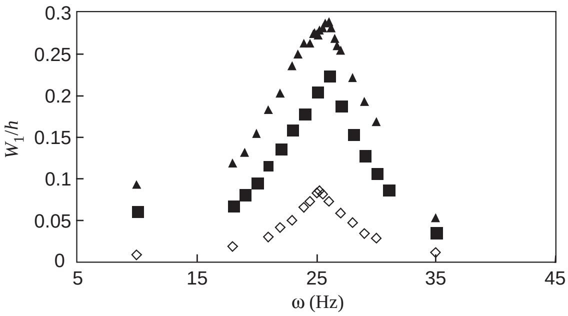

Fig. 3. Resonance curves at point 1, first harmonic, excitations applied at point **FP** with amplitudes: ◇, 100 mN; ■, 300 mN and ▲, 450 mN.

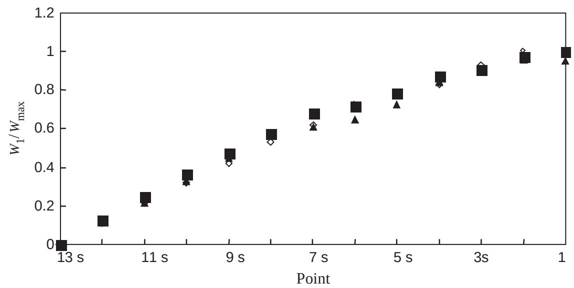

Fig. 4. Normalized deformation of first harmonic, excitations applied at point **FP** with amplitudes: ◇, 0.1 N; ■, 0.3 N and ▲, 0.45 N.

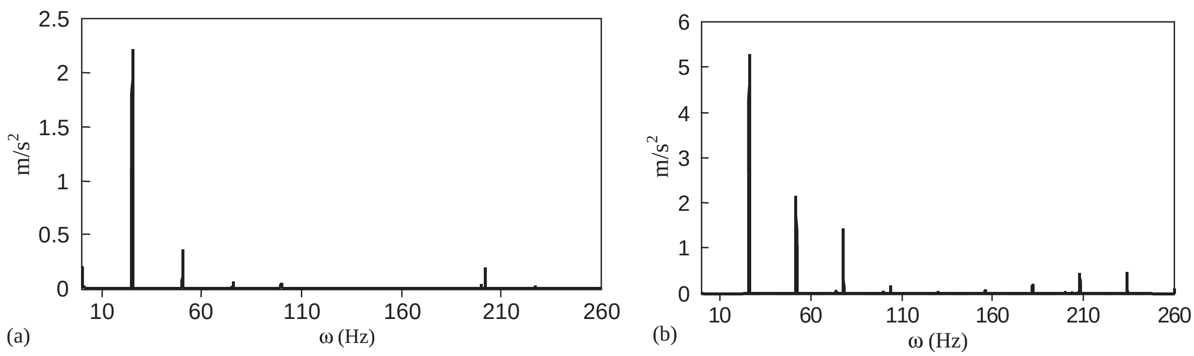

Fig. 5. Autospectra of acceleration at point 10 s, due to excitations of (a) 0.1 N, 25.25 Hz and (b) 0.45 N, 26 Hz.

Another set of trials were carried out with the exciter connected to point **SP**. In these experiments all the settings of the amplification system were constant during each frequency sweep. This meant that the force applied actually changed during the frequency sweep and, therefore the force amplitude in Newton will not be given. Each excitation shall instead be identified by the corresponding setting in volts.

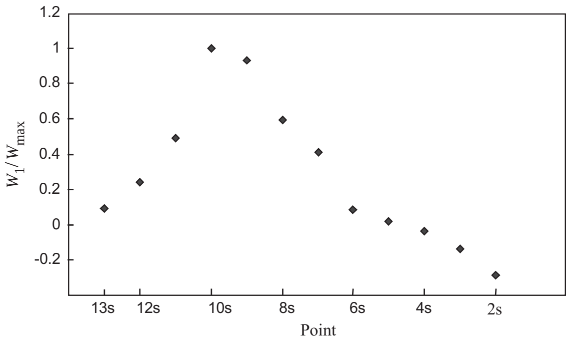

Fig. 6. Normalized deformation of eighth harmonic, excitation applied at point **FP** with amplitude 100 mN. Excitation at 25.25 Hz. Only half the beam is shown.

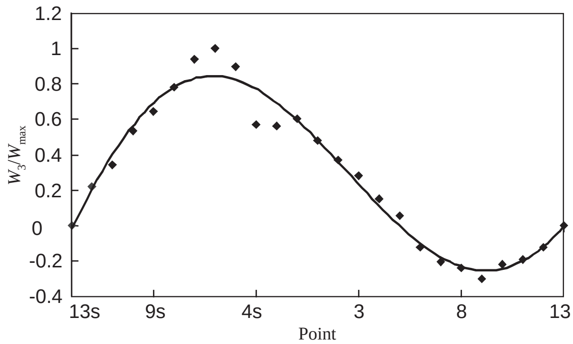

Fig. 7. Normalized deformation of third harmonic, excitation applied at point **FP** with amplitude 0.45 N (◆ measurements, - fitted curve).

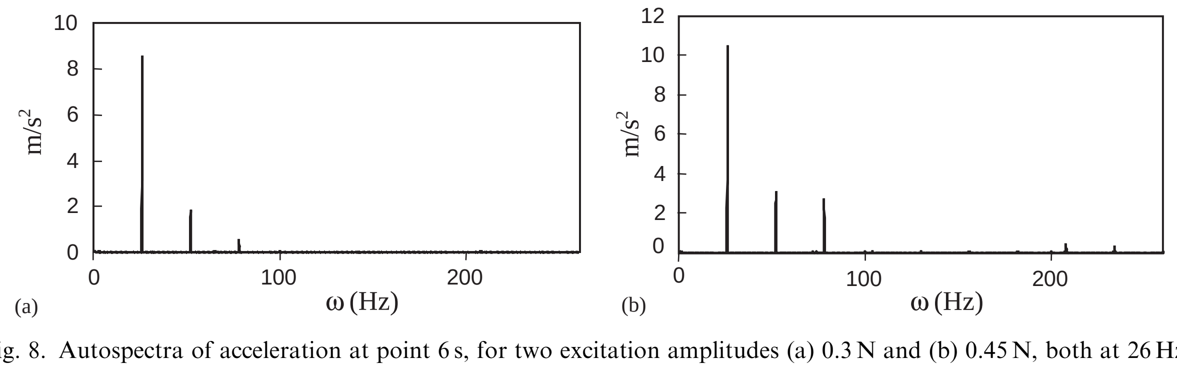

Fig. 8. Autospectra of acceleration at point 6 s, for two excitation amplitudes (a) 0.3 N and (b) 0.45 N, both at 26 Hz.

In many cases the first harmonic is by far the more important; however some very interesting phenomena were again found. Looking for example at Fig. 9, a case can be seen where higher harmonics have a determining weight on the response. In the case shown, an excitation of the seventh harmonic occurs around 22 Hz, i.e., when the frequency of the seventh harmonic is

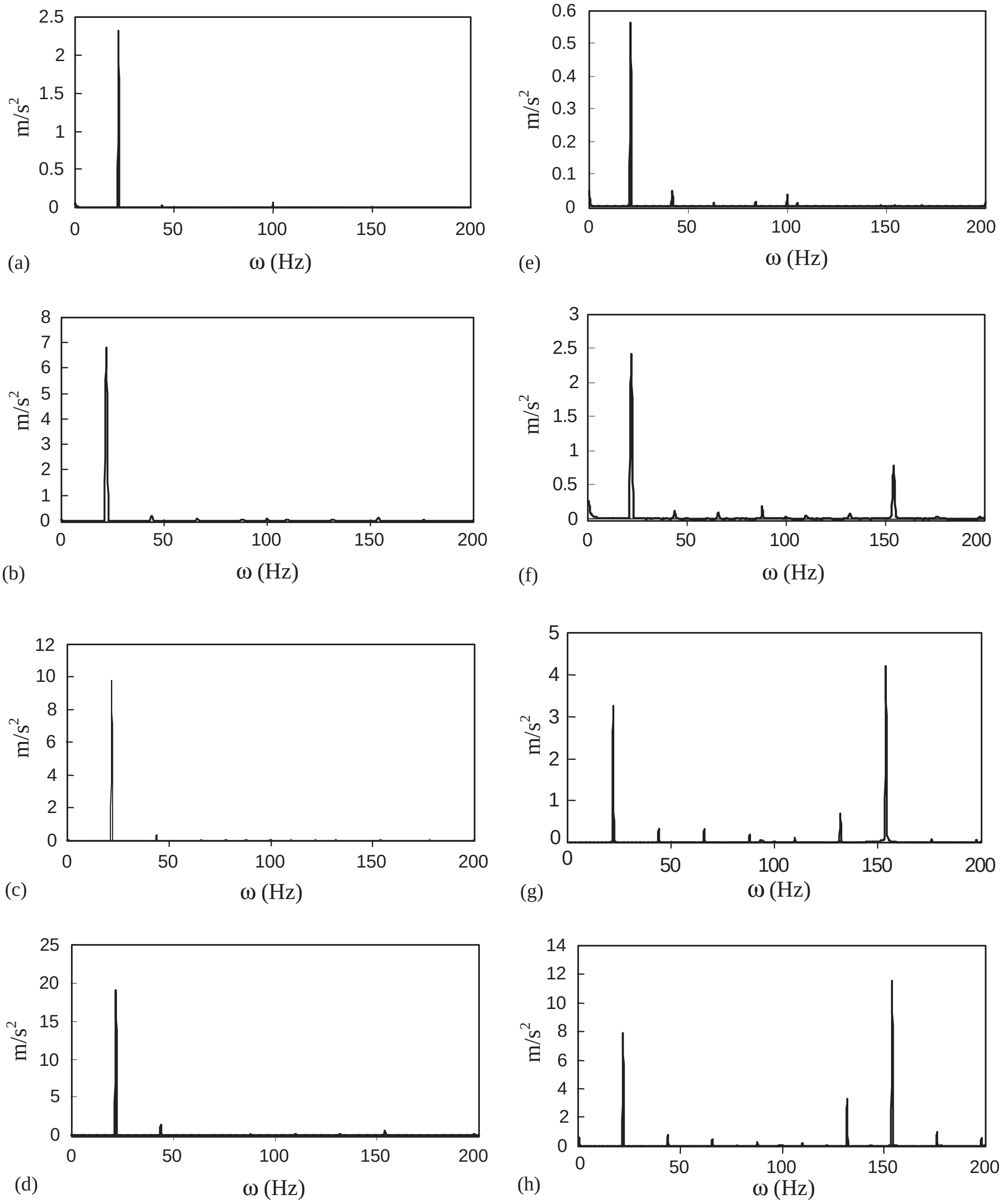

Fig. 9. Autospectra for two excitation amplitudes. Point 1: (a) 0.25 V; (b) 0.5 V; (c) 1 V; (d) 2 V. Point 10: (e) 0.25 V; (f) 0.5 V; (g) 1 V; (h) 2 V.

154 Hz. It was verified that the importance of this harmonic increases as the fundamental frequency approaches 22 Hz and decreases afterwards. The seventh harmonic excitation is already visible when the tension in the amplifying system is set to medium values (0.5 and 1 V), but it is

very small at 0.25 V and its importance markedly increases with the amplitude of vibration, as a result of the non-linear effects. At 1 and 2 V, point 10, the seventh harmonic amplitude is larger than the one of the fundamental harmonic.

Analyzing the deflection shapes in Fig. 10, it can be seen that the excitation of the seventh harmonic is linked with the second mode, whose linear natural frequency is close to 80 Hz; that is, around half that of the seventh harmonic. Thus, it is possible that the second mode is excited due to a "two to seven" internal resonance.

3.2. Second non-linear mode

To study the second non-linear mode, the excitation was applied at point **FP**. Fig. 11 shows some resonance curves measured in point 6, and it is clear that the resonance frequency increases with the vibration amplitude. The jump phenomenon is viewed best as seen in Fig. 12 where the forward and backward sweeps are superimposed. Again, the experimental detection of the hardening spring effect confirms our expectations from numerical analysis [3–6]. In some measurements the fundamental linear frequency changed, shifting the corresponding curves. This is most probably due to an environmental effect, and these trials are not discussed here.

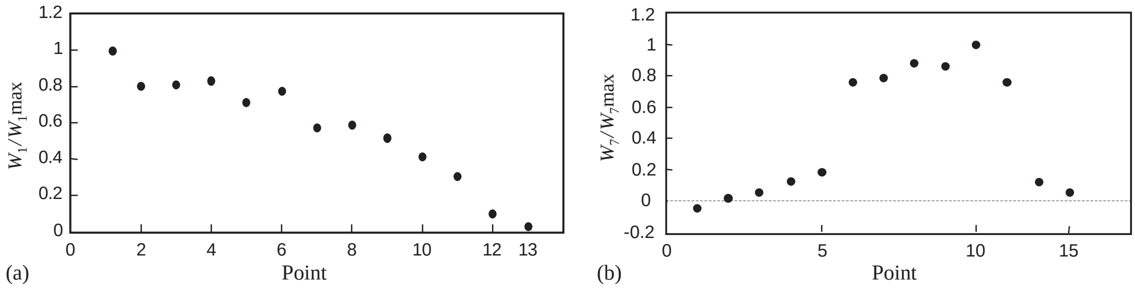

Fig. 10. Normalized deformation of (a) 1st and (b) 7th harmonic, 22 Hz excitation applied at point **SP**.

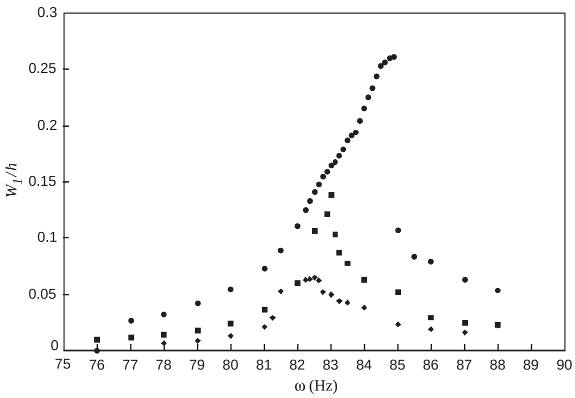

Fig. 11. Resonance curves at point 8, 1° harmonic, excitations applied at point **FP** with amplitudes: ● 150 mN, ■ 300 mN and ◆ 800 mN.

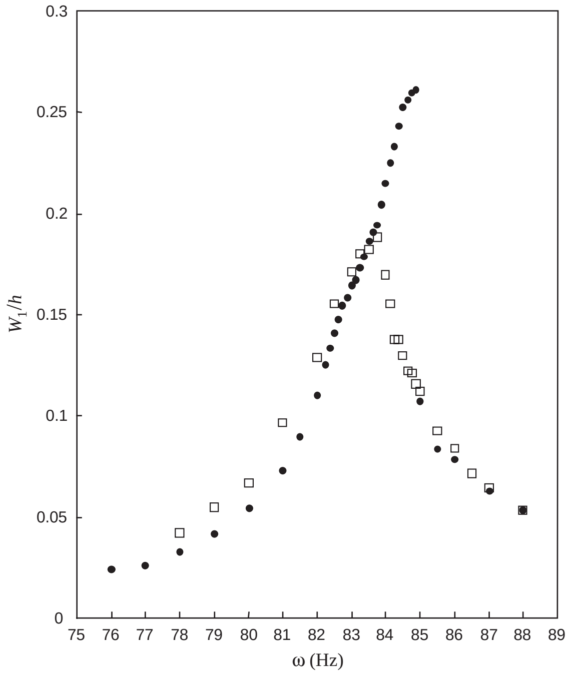

Fig. 12. Resonance curves at point 8, first harmonic, excitations applied at point **FP** with amplitude: 0.8 N. ● Forward and □ backward sweeps.

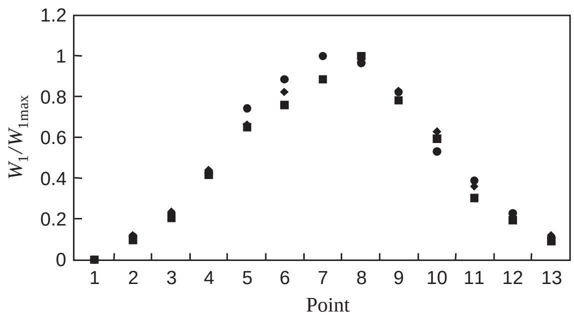

Fig. 13. Normalized deformation of half-beam, first harmonic, excitations applied at point **FP** with amplitudes: ♦ 300 mN (80.75 Hz), ■ 450 mN (81,125 Hz) and ● 600 mN (81,375 Hz).

An approximation to the mode shape was defined by measuring the displacement amplitude of the harmonics. The shapes obtained were normalized by dividing them by the maximum amplitude. The results are shown in Fig. 13. Apparently, the second mode shape of this hinged–hinged beam does not change with the vibration amplitude, or at least the variations are not large enough to supplant the uncertainties inherent in these analyses.

Some higher harmonics were also present in the responses, as shown in Fig. 14 where the autospectra of the acceleration at point 3 are displayed for different excitation frequencies. In particular the third harmonic is almost as important as the first, when the excitation is at 78 Hz.

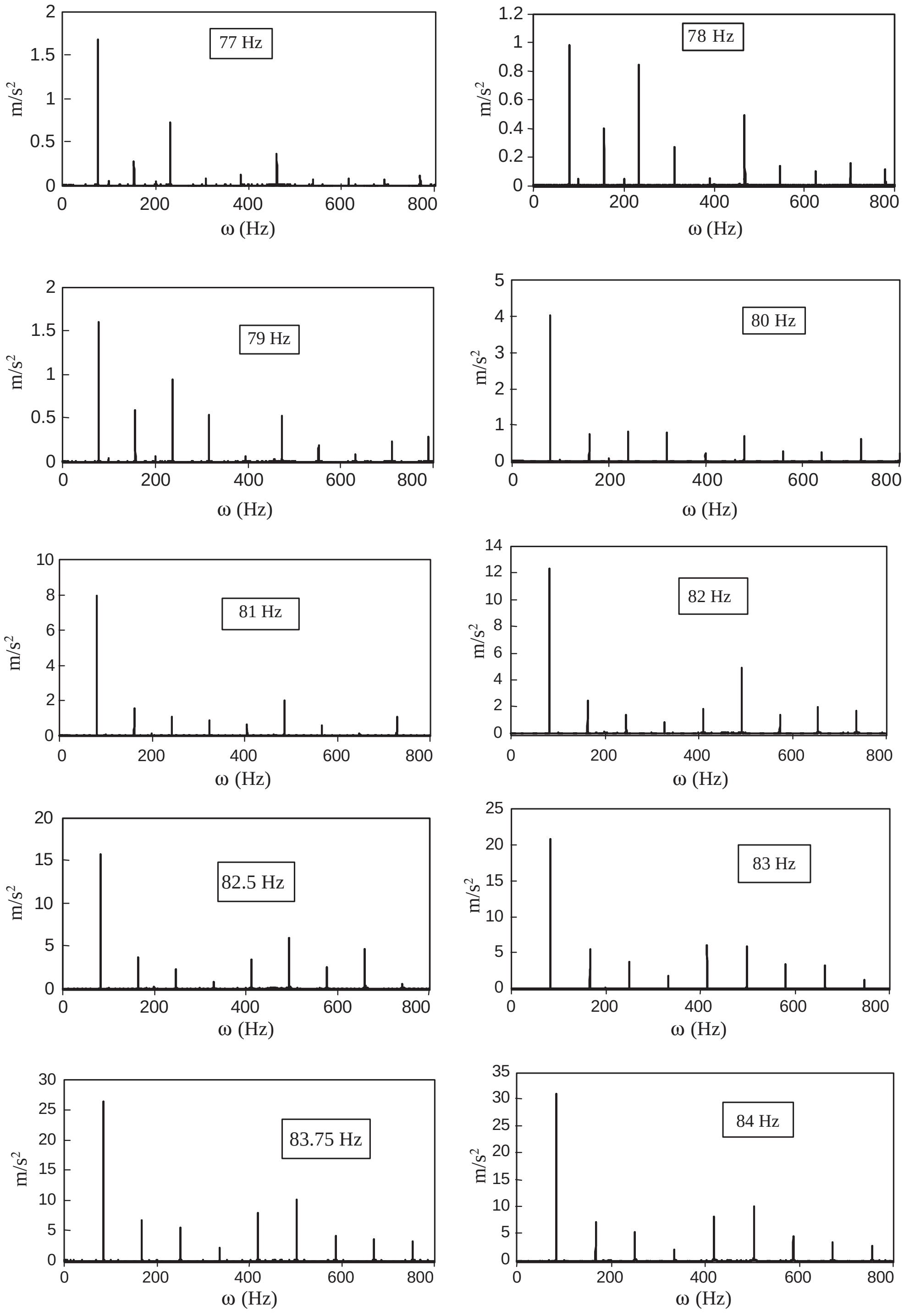

Fig. 14. Autospectra at point 3 for excitation with 0.8 N.

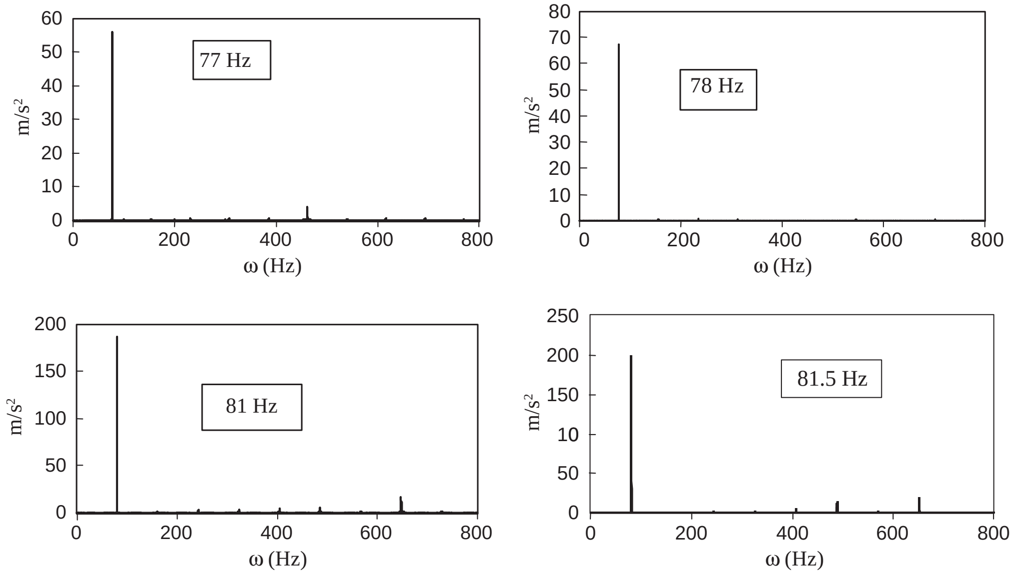

Fig. 15. Autospectra at point 6 s for excitation with 0.8 N.

The higher harmonics are very visible at point 3, because the second mode, which is the one directly excited, has a small amplitude at this point. In fact, looking at Fig. 15, it can be seen that much larger amplitudes are attained by the first harmonic at point 6 s, where the second mode is prominent, and that here the relative importance of higher modes and modes not directly excited is disguised. They do exist, however.

Another phenomenon due to the non-linearity, and which was often experienced near the resonance frequencies, is the jump between two limit cycles [2], which was reflected in vibrations with different amplitudes and different spectra, but with the same fundamental frequency. Particularly the limit cycles with larger displacement amplitudes have a small domain of attraction, and therefore it is quite difficult to define them experimentally.

4. Conclusions

A hinged–hinged beam was excited transversely by sinusoidal forces of moderately large amplitude, in order to provoke non-linear oscillations. The autospectra at several points and the deflection of the first and higher harmonics were analyzed. The first two non-linear modes were investigated.

It was confirmed that modal interaction occurs in hinged–hinged beams vibrating with moderately large displacements; that is that due to non-linear coupling, higher order modes can be excited by harmonics of the frequency of the excitation. This results in a more severe change of the non-linear mode during the vibration period. Moreover, the fact that a higher order shape is present leads to higher slopes in the deformed beam and, therefore, can lead to larger stresses.

In several cases, for the moderately large amplitudes investigated, the directly excited mode is clearly the most important. Phenomena encountered before by other authors, such as the variation of the resonance frequency with the vibration amplitude and the jump between different limit cycles at certain frequencies, were also found here.

Acknowledgements

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