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Frequency selection method for FRF-based model updating

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Abstract

In this paper, a new frequency selection method for efficient FRF-based model updating is proposed. Using the proposed method, the frequency points used for updating can be selected in an automatic way such that the selected frequencies can carry as much information as possible with a limited number of frequencies. In order to demonstrate the effectiveness of the proposed method, a numerical example of truss structure is used. Different sets of frequency points for updating are compared in terms of numerical stability as well as updated results. Finally, the proposed method is extended to deal with rotor–bearing systems.

1. Introduction

In many engineering problems, mathematical model accuracy has been an essential part for design and analysis. As a way of improving dynamic models, model updating has been widely used for correcting the mathematical models from experimental data.

For model updating techniques, either identified modal parameters such as eigenvalues and eigenvectors or measured frequency response functions (FRFs) are widely used as reference data [1,2]. In this work, the method using measured FRFs is mainly discussed because it has the following advantages: (1) Errors from modal parameter extraction can be eliminated because modal parameter identifications are not required; (2) The amount of data available is much larger than that of modal parameters [3,4].

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For model updating using FRFs, a large number of frequency points in measured FRFs can be used as reference data. However, in practice, a limited number of frequencies among a large number of candidate reference data are used for practical reasons. Furthermore, some frequencies in a large number of candidate frequencies are believed to be redundant for updating parameters [2]. Therefore, the selection of frequency points has been one of the important issues in model updating using FRFs [3–7]. The best policy is to select a limited number of frequency points such that the selected frequencies can carry as much information as possible. However, it should be noted that a sufficient number of updating frequencies are required to be selected for an over-determined set of equations.

Until now, there seems to be still controversial results on the optimum frequency points for updating. Some researchers recommended that frequency points around resonances should be avoided [3,6]. Recently, D'Ambrogio and Fregolent proposed the use of the information density matrix to select the optimum frequencies [4]. However, the selected frequencies are close to resonances, which may contradict conclusions by other researchers. In addition, not so many papers have dealt with the automatic selection of frequency points for updating with minimum engineering judgment. In this study, a new automatic frequency selection method is proposed such that row vectors in sensitivity matrix associated with selected frequencies are not only sensitive to parameter changes but also more linearly independent of one another. Thus selected frequencies have advantages because they are apt to be more informative and computationally efficient.

For the demonstration of the effectiveness of the proposed new method, a truss structure is taken as a numerical simulation model. In the numerical simulation, numerical aspects such as stability and convergence are studied according to different cases of selected frequency points for updating. Finally, the method is extended to deal with the case of rotor–bearing systems. Updating of rotor–bearing systems has been considered to be more difficult than ordinary stationary structures because of their non-self-adjoint problems [8,9].

2. Model updating using FRF

The model updating using FRFs as reference data has some advantages: (1) errors due to modal parameter extraction can be avoided; (2) a large number of data are available for updating although it does not mean that information available is proportionally increased according to the increase of the number of data.

In this study, sensitivity of frequency response matrix (FRM), $[H]$, with respect to an updating parameter, ϕ, is used for model updating using FRFs, as [2]

$$\frac{\partial [H]}{\partial \phi} = -[H]\frac{\partial [Z]}{\partial \phi}[H]. \tag{1}$$

Here, dynamic stiffness matrix, $[Z]$, can be written as

$$[Z(\omega)] = [-\omega^2[M] + \mathrm{j}\omega[C] + [K]] \text{ and } [H] = [Z]^{-1}, \tag{2}$$

where $[M]$, $[C]$, $[K]$, ω and j are mass, damping, stiffness matrix, frequency in rad/s and imaginary unit, respectively. From Eq. (1), updating with respect to an updating parameter, ϕ, can be formulated, for a frequency point, ω_k, with response and excitation co-ordinates, i and j, respectively, as

$$H_{ij}^{e}(\omega_k) - H_{ij}^{a}(\omega_k) = -\{H_i^a(\omega_k)\}^{\mathrm{T}} \frac{\partial[Z(\omega_k)]}{\partial\phi}\{H_j^a(\omega_k)\}\Delta\phi$$
$$\text{or}\quad \varepsilon_k = S_k\Delta\phi, \tag{3}$$

where $\{H_i^a(\omega_k)\}^{\mathrm{T}}$ and $\{H_j^a(\omega_k)\}$ are ith row and jth column vectors of analytical FRM, $[H(\omega_k)]$. Here, superscripts, e and a, represent experimental and analytical FRF, respectively. Note that updating needs iterative process due to the first order approximation of sensitivity with respect to an updating parameter. Using Eq. (3), model updating problem can be written in a simple matrix form as

$$[S]\{\Delta\phi\} = \{\varepsilon\}. \tag{4}$$

Here, the sizes of sensitivity matrix, $[S]$, updating parameter vector, $\{\Delta\phi\}$, and output residual vector, $\{\varepsilon\}$, are $N_f \times N_p$, $N_p \times 1$ and $N_f \times 1$, respectively. The size of N_f depends on the number of frequency points used as reference data and the number of measured co-ordinates whereas N_p is the number of parameters to be updated. The updating problem can be easily turned into over-determined set of equations using more frequency points than updating parameters. Then, the least-square solution for Eq. (4) can be given by [2]

$$\{\Delta\phi\} = [S]^{+}\{\varepsilon\} \quad \text{where } [S]^{+} = [S^{\mathrm{T}}S]^{-1}S^{\mathrm{T}}. \tag{5}$$

Here, superscript '+' represents the pseudo-inverse. The parameters are updated iteratively using Eq. (5) as

$$\{\phi_{new}\} = \{\phi_{old}\} + \{\Delta\phi\}, \tag{6}$$

until the residual, $\{\varepsilon\}$, becomes sufficiently small. Here, the sensitivity matrix, $[S]$, may have complex value whereas the parameters to be updated have real value from the physical point of view. To solve this, the equation is partitioned into [2]

$$[S_r]\{\Delta\phi\} = \begin{bmatrix} \mathrm{Real}([S]) \\ \mathrm{Imag}([S]) \end{bmatrix}\{\Delta\phi\} = \begin{Bmatrix} \mathrm{Real}(\{\varepsilon\}) \\ \mathrm{Imag}(\{\varepsilon\}) \end{Bmatrix}, \tag{7}$$

where Real(.) and Imag(.) represent the real and imaginary component of matrix $[S]$ or $\{\varepsilon\}$. Then, $[S_r]$ becomes $2N_f \times N_p$ matrix with real component.

3. Frequency point selection method

The sensitivity matrix, $[S]$, can be conveniently written in either column or row vectors as

$$[S] = [\{X_1\},\{X_2\}\cdot\cdot\{X_{N_p}\}] \quad \text{or} \quad [S] = \begin{bmatrix} \{Y_1\}^{\mathrm{T}} \\ \{Y_2\}^{\mathrm{T}} \\ \vdots \\ \{Y_{N_f}\}^{\mathrm{T}} \end{bmatrix}, \tag{8}$$

where $\{X_i\}$, $i = 1,2,\ldots,N_p$, are column vectors in sensitivity matrix whereas $\{Y_i\}^{\mathrm{T}}$, $i = 1,2,\ldots,N_f$, are the row vectors. The column vectors are associated with parameters to be updated whereas the row vectors are associated with selected frequencies as reference data for updating. Since the pseudo-inverse of sensitive matrix, $[S]$, is involved in parameter updating at each iteration, the sensitivity matrix $[S]$ should be well conditioned in order to get stable updated parameters during iterations. If the matrix, $[S]$, is ill conditioned, the updated results can be easily affected by the measurement noise in residual vector, $\{\varepsilon\}$. The treatment of ill-conditioned matrix and multi-collinearity is extensively discussed in Refs. [10–13].

In order for sensitivity matrix to be well conditioned, both column and row vectors in the sensitivity matrix are needed to be not nearly dependent one another. Here, the linear dependency among the column vectors can be more critical than that of row vectors because the number of row vectors is larger than that of column vectors for an over-determined set of equations. However, since the column vectors are associated with parameters to be updated, the choice of column vectors may be not as easy as the selection of frequency points. So, in this study, selection method of frequency points related with row vectors is mainly discussed.

There can be a lot of different choices for row vectors in sensitivity matrix of FRF-based updating because they are associated with selected frequencies. The choice of frequency points is important because model updating using too many frequencies is not efficient from a computational point of view. On the other hand, wrong selection of frequency points may result in ill conditioning or not enough information for updating parameters. In this study, optimal frequency points for updating are sought such that the row vectors in sensitivity matrix associated with selected frequencies can be more linearly independent one another. For this purpose, the linear dependency of row vectors in sensitivity matrix is investigated to select the optimum frequency points for updating.

3.1. Evaluation of vector dependency

In order to match analytical and experimental eigenvectors, modal assurance criterion (MAC), which can measure the consistency of two vectors, has been widely used [14,15]. In this paper, a concept similar to the MAC is used in order to investigate the linear dependency between row vectors or column vectors in sensitivity matrix. In this section, the concept of MAC is briefly summarized for understanding of the method employed in this paper.

To compare the two vectors directly, say $\{\psi_i\}$ and $\{\psi_k\}$, the scale factor should be multiplied to the vector $\{\psi_k\}$ as

$$SF = \frac{\{\psi_k\}^*\{\psi_i\}}{\{\psi_k\}^*\{\psi_k\}}. \tag{9}$$

Here, superscript * represent the complex conjugate transpose. Then, the two vectors can be easily compared and the consistency can be evaluated from

$$\xi = \{\psi_i\}^*(\{\psi_i\} - SF\{\psi_k\}). \tag{10}$$

The value of ξ in Eq. (10) can be normalized as

$$\frac{\xi}{\{\psi_i\}^*\{\psi_i\}} = 1 - \frac{\left|\{\psi_i\}^*\{\psi_k\}\right|^2}{(\{\psi_i\}^*\{\psi_i\})(\{\psi_k\}^*\{\psi_k\})}, \tag{11}$$

where the value of $\xi/(\{\psi_i\}^*\{\psi_i\})$ becomes between 0 and 1 according to the consistency of two vectors, $\{\psi_i\}$ and $\{\psi_k\}$. For example, if the two vectors are orthogonal, the value becomes one. On the other hand, if the two vectors are linearly dependent, then the value becomes zero. The Eq. (11) can be expressed alternatively as

$$MAC_{ik} = 1 - \frac{\xi}{\{\psi_i\}^*\{\psi_i\}} = \frac{\left|\{\psi_i\}^*\{\psi_k\}\right|^2}{(\{\psi_i\}^*\{\psi_i\})(\{\psi_k\}^*\{\psi_k\})}. \tag{12}$$

Then, the value of MAC_{ik} becomes one when the two vectors are linearly dependent. On the other hand the value becomes zero when the two vectors are linearly independent, i.e., orthogonal. Therefore, the MAC, which has been used for matching experimental and analytical eigenvectors, can be effectively used for measuring dependency between vectors. In this paper, for the sake of convenience, the square root value of MAC is used in order to measure the linear dependency among either row or column vectors of sensitivity matrix.

3.2. Measurement dependency index

In order to measure the linear dependency of row vectors in sensitivity matrix, the concept of the so-called measurement dependency index (MDI) is used in this study. Similarly to the MAC, the dependency between row vectors, say $\{Y_i\}^{\mathrm{T}}$ and $\{Y_k\}^{\mathrm{T}}$, in the sensitivity matrix can be measured from

$$MDI_{ik} = \frac{\left|\{Y_i\}^*\{Y_k\}\right|}{sqrt\left((\{Y_i\}^*\{Y_i\})(\{Y_k\}^*\{Y_k\})\right)}, \tag{13}$$

where *sqrt*(.) means square root. Here, the total number of row vectors in sensitivity matrix is assumed to be N. Then, MDI forms $N \times N$ matrix with elements between 0 and 1 according to the degree of linear dependency among row vectors of sensitivity matrix. Since the value of MDI_{ik} indicates the linear dependency between the two ith and kth row vectors, the value close to 1 means that corresponding two row vectors in the sensitivity matrix have similar effects and one of the two rows can be discarded.

In this study, for the selection of frequency points for updating, the MDI is used such that the row vectors associated with selected frequencies can be more linearly independent. The selection of frequencies can be simply implemented by following procedures. Firstly, a large number of candidate frequency points for updating are grouped into smaller number of frequency sets such that the row vectors in the same group are apt to be linearly dependent. This classification can be implemented by setting a threshold value in MDI matrix. Here, the row vectors in the same group can be considered to have similar information with one another. Then, only a few frequency points in each group can be selected as reference data for updating and the other frequencies in the same group can be discarded. For a representative frequency point in a group, the frequency point with highest sensitivity is selected for updating in this study. For the selection of most sensitive frequencies, the sensitiveness of each row vector in the group is compared using the value of $\left|\{Y_i\}^*\{Y_i\}\right|$ associated with ith row vector in the classified group. Thus, N_f frequencies for updating are selected from the total N candidate frequencies.

The threshold value for MDI_{ik} is determined from engineering judgment. If the threshold value is set to be close to 0, then less frequency points are selected for updating. However, the row vectors associated with selected frequencies tend to be more linearly independent. If the threshold value is close to 1, then the number of selected frequencies can be increased accordingly. However, the row vectors in sensitivity matrix are likely to be more linearly dependent one another. As a rule of thumb, the threshold value is recommended to be set such that the number of selected frequencies becomes at least 2 times the number of updating parameters in order to ensure an over-determined set of equations. Here, it should be noted that there should be sufficient equations that are not nearly dependent in order to update parameters concerned. After selection of optimum frequency points, adding more equations may be useful to improve the results in terms of averaging out noise effects. However, it should be noted that there is saturation point where adding more frequencies gives no further improvement only resulting in computational burden [16].

The optimum frequencies for updating may vary according to iterations since the sensitivity matrix can be changed according to updated parameters at each iteration. However, the structure of the sensitivity matrix may not vary significantly during iterations if the initial analytical parameters are not very different from their exact values. Then, initially selected optimum frequencies can be effectively used throughout the iterations.

The proposed method has advantages because the information embedded in a limited number of frequencies can be maximized. Thus selected frequencies may result in a better conditioning in sensitivity matrix because the row vectors are apt to be linearly independent one another. Furthermore, optimum frequency points for updating can be selected in an automatic way with minimum engineering judgment.

3.3. Parameter dependency index

A similar concept to MDI can be used for the investigation of the linear dependency among column vectors in sensitivity matrix, which are associated with updating parameters. This can be simply realized by the so-called parameter dependency index (PDI), which uses column vectors, say $\{X_i\}$ and $\{X_k\}$, instead of row vectors in Eq. (13). However, it should be noted that the PDI deals with its complex values in a different way from MDI. This feature can be easily understood from Eq. (7) in which real and imaginary parts of the sensitivity matrix are dealt with separately. By considering this feature, PDI can be simply obtained from the absolute value of d_{ik} and the cosine value of angle, θ, as

$$PDI_{ik} = |d_{ik}||\cos(\theta)|, \tag{14}$$

where d_{ik} and θ are defined by

$$d_{ik} = \frac{\{X_i\}^*\{X_k\}}{sqrt\,((\{X_i\}^*\{X_i\})(\{X_k\}^*\{X_k\}))} = |d_{ik}|e^{j\theta}. \tag{15}$$

Here, j means the imaginary number. The PDI value close to 1 indicates that two column vectors are nearly linearly dependent each other. On the other hand, the PDI value close to zero indicates that two column vectors are almost linearly independent. Note that the selection of frequency points can affect the linear dependency among column vectors. Therefore, the optimum frequency points for updating do not necessarily result in better conditioning of sensitivity matrix with low PDI values. Therefore, PDI values are recommended to be checked to ensure the stable parameter updating after optimum frequency points are sought.

If a sensitivity matrix is ill conditioned due to the linearly dependent column vectors associated with updating parameters, the number of updating parameters is recommended to be reduced or regularization method should be used. The treatment of ill-conditioned problem is beyond the scope of this paper and needs further research. However, on the other hand, the concept of PDI can be used for the selection of updating parameters for better-conditioned sensitivity matrix. For example, the parameters, which have PDI value close to 1, are not recommended to be updated simultaneously for stable convergence.

4. Numerical examples

4.1. Truss structure

To demonstrate the practical applicability of the proposed method, an analytical finite element (FE) model of a plane truss structure with 36 elements, which is shown in Fig. 1, is considered for updating model. The FE model has 3 degrees of freedom at each node, which results in total 90 degrees of freedom. For simulations, the following parameters are used for the initial analytical FE model: modulus of elasticity, $E = 0.75 \times 10^{11}\ \mathrm{N/m^2}$; second moment of area of each member, $I = 0.0756\ \mathrm{m^4}$; cross-sectional area of each member, $Ar = 0.004\ \mathrm{m^2}$. In this study, without loss of generality, the stiffness modelling errors in the analytical model are considered. For reference data for updating, the same FE model with 10% lower moduli of elasticity ($E = 0.675 \times 10^{11}\ \mathrm{N/m^2}$) at 2nd, 4th, 6th, 10th, 11th and 12th members is used in order to generate simulated experimental FRFs. Thus, the updating problem becomes adjusting moduli of elasticity of 12 members in analytical FE model to minimize the difference between reference (experimental) and analytical FRFs.

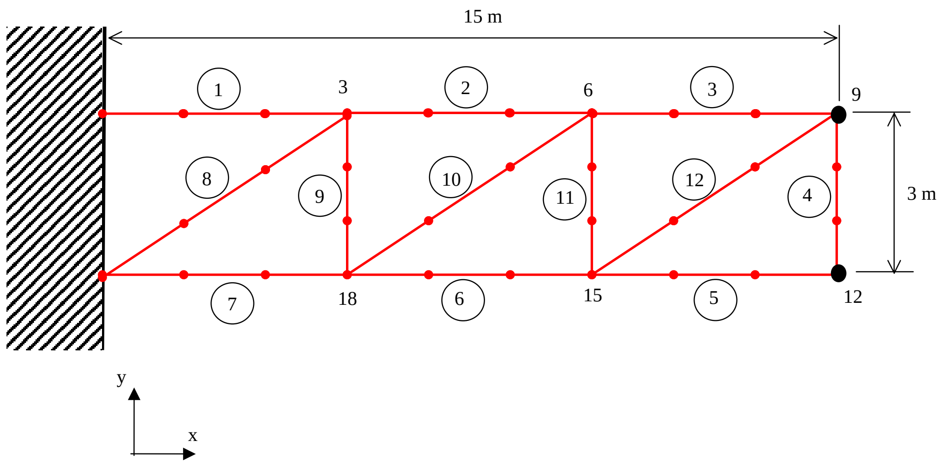

Fig. 1. Truss structure: ● measured nodal point; • non-measured nodal point.

Note that there is always incompatibility in the size of analytical FE and measured experimental models to be compared. Due to limited number of measurement sensors and exciters, the measured d.o.f.s are likely to be far less than that of analytical model. In addition, rotational degrees of freedom in analytical FE model are difficult to be measured in practice. For the simulation of co-ordinate incompleteness, 4 FRFs of translational degrees of freedom at nodes 9 and 12 with excitation of node 12 in the Y direction are assumed to be available as reference data throughout simulations. In reference FRF data, 1% random noise is added for the simulation of measurement errors.

Fig. 2 shows typical analytical and experimental FRFs, which are generated from the FE model seen in Fig. 1. In this study, 500 frequencies of each FRF with frequency resolution of 1 Hz are considered as candidate reference frequencies for updating. Note that there is considerable discrepancy between analytical and experimental FRFs near resonances as seen in Fig. 2. The use of sensitivity defined by Eq. (3) near resonance frequencies may result in poor convergence due to non-linearities and irregularities [6]. Furthermore, the FRFs close to resonances are sensitive to damping and subject to various errors. Since there is no guarantee that the selected frequencies are not coincided with resonances, engineering judgment is needed to avoid the selection of frequencies near resonances. In this study, frequencies close to resonances are excluded from the candidate frequencies prior to selecting optimum frequencies for updating.

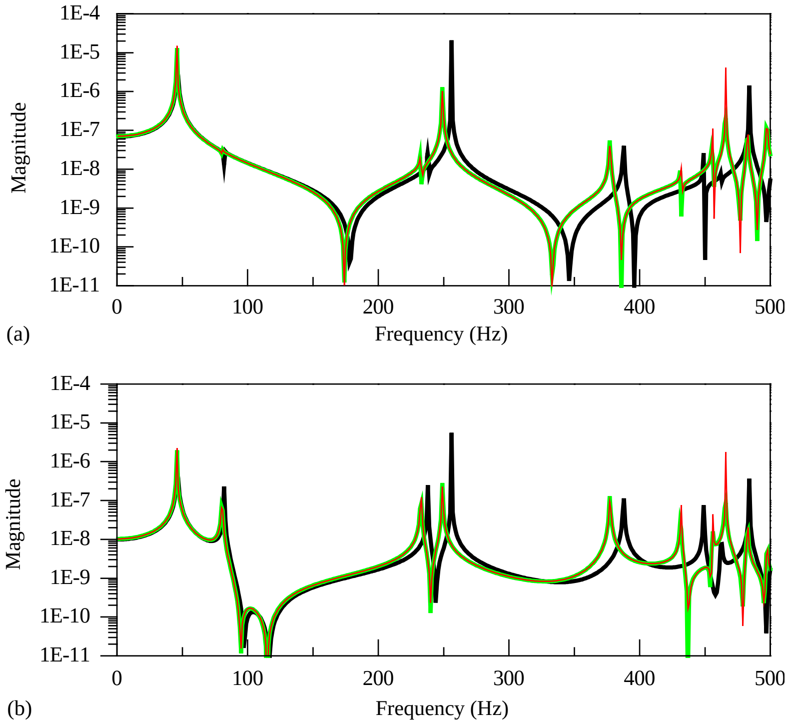

Fig. 2. Magnitude plots of typical FRFs: (a) $H_{12y12y}(\omega)$; (b) $H_{12x12y}(\omega)$; ▬ initial analytical FRF; —— simulated experimental FRF; ▬ updated FRF using 32frequency points by proposed method.

This may result in loss of some information due to the elimination of frequencies near resonances. However, the information embedded can remain almost the same if other frequencies, which have similar effect, are selected instead of excluded frequencies near resonance. In the numerical example of the truss structure, frequency points close to resonances, such as, 40–50, 75–85, 210–270, 360–400 and 420–500, are excluded from the total candidate frequencies. The other frequencies from each FRF are evaluated according to the MDI values in order to select the optimum reference frequencies. In this study, the reference frequencies for updating are selected from each FRF such that the selected frequencies in each FRF can have MDI values less than 0.8. Further reduction of frequency data among FRFs is not considered in this study in order to ensure an over-determined set of equations for updating 12 parameters. Case 1 in Table 1 shows 32 frequencies, which are selected by the proposed method. Note that the number of selected frequencies can differ according to the FRFs measured at different measurement co-ordinates. For example, the FRF measured in the X direction of node 12, H_{12x12y}, has more number of selected frequency points than the other FRFs concerned. The fact indicates that the method can be used for selection of measurement co-ordinates for model updating, where

Table 1
Comparison of various frequency sets for updating of truss structure

Case 1 (32 frequency points selected by proposed method)	
H_{9y12y}	51, 271, 314, 343, 401 414 (Hz)
H_{9x12y}	51, 86, 126, 209, 271, 401, 418 (Hz)
H_{12y12y}	51, 115, 204, 271, 359, 401 (Hz)
H_{12x12y}	51, 63, 73, 87, 98, 117, 167, 271, 315, 340, 359, 401, 413 (Hz)
Condition number of $[S]^T[S]$: 1.16E+7 (1.19E+4)[a]	
Case 2 (32 frequency points avoiding resonances)	
H_{9y12y}	5, 51, 100, 150, 200, 271, 320, 401 (Hz)
H_{9x12y}	5, 51, 100, 150, 200, 271, 320, 401 (Hz)
H_{12y12y}	5, 51, 100, 150, 200, 271, 320, 401 (Hz)
H_{12x12y}	5, 51, 100, 150, 200, 271, 320, 401 (Hz)
Condition number of $[S]^T[S]$: 2.93E+7 (3.92E+004)[a]	
Case 3 (2000 frequency points)	
H_{9y12y}	1, 2, 3…499, 500 (Hz)
H_{9x12y}	1, 2, 3…499, 500 (Hz)
H_{12y12y}	1, 2, 3…499, 500 (Hz)
H_{12x12y}	1, 2, 3…499, 500 (Hz)
Condition number of $[S]^T[S]$: 3.65E+012 (3.49E+003)[a]	
Case 4 (32 equally spaced frequency points)	
H_{9y12y}	60, 120, 180, 240, 300, 360, 420, 480 (Hz)
H_{9x12y}	60, 120, 180, 240, 300, 360, 420, 480 (Hz)
H_{12y12y}	60, 120, 180, 240, 300, 360, 420, 480 (Hz)
H_{12x12y}	60, 120, 180, 240, 300, 360, 420, 480 (Hz)
Condition number of $[S]^T[S]$: 3.14E+006 (8.24E+003)[a]	

[a] Condition numbers in ( )where weightings on all information are used.

the number of measurement sensors is limited. The co-ordinates, which have more independent equations for updating (or optimal frequency points for updating), are recommended to be selected for the location of measurement sensors.

For comparison, different sets of frequencies shown in Table 1 are considered for reference data of updating: case 1 shows 32 frequencies, which are selected by the proposed method; case 2 in Table 1 shows almost equally spaced 32 frequencies avoiding resonance region; In case 3, all frequencies available from experimental FRFs are used for reference data; In case 4, 32 frequencies, where some of the frequencies are selected near resonances, are used.

Model updating using Eq. (5) is likely to be ill conditioned if the least-square solutions are dominated by a few equations with coefficient of great magnitude [16,17]. Therefore, all information is recommended to be weighted such that sensitivity matrix can be balanced [17]. For this purpose, both ith row vector of sensitivity matrix and corresponding output residual in Eq. (4) are divided by $\sqrt{\{Y_i\}^*\{Y_i\}}$, $i = 1, 2, \ldots, N_f$. Here, $\{Y_i\}^{\mathrm{T}}$is ith row vector of sensitivity matrix. Then, each equation associated with selected frequencies can have equal contribution to least-square solutions. The condition numbers of $[S]^{\mathrm{T}}[S]$ in each case shown in Table 1 indicate the significant increases in numerical stability by using such weightings. The condition number is based on singular value decomposition (SVD), which is defined by the ratio of the largest to smallest singular value of the matrix. The low number in condition number of $[S]^{\mathrm{T}}[S]$ means the better-conditioned sensitivity matrix [2].

Fig. 3 shows convergence plots of updated parameters for each case of frequency sets. Here, weightings on all information are used in order to increase the numerical stability of updating. In both cases, 1 and 2, in Table 1, 32 frequency points, which seems to be sufficient in number for over-determined equations, is used as reference data for updating 12 parameters. However, it should be noted that actual information might not be sufficient for updating parameters if a lot of

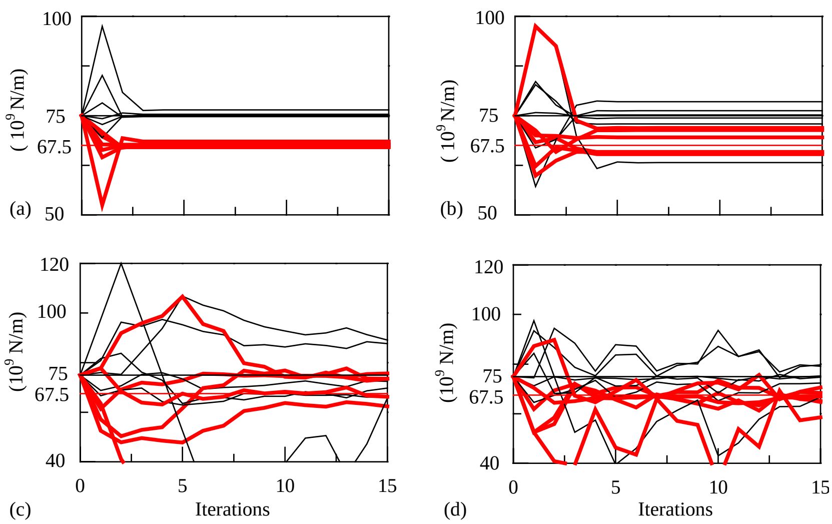

Fig. 3. Convergence plots of updated parameters of the truss structure: (a) case 1; (b) case 2; (c) case3; (d) case4; modulus of elasticity of; — 1st, 3rd, 5th, 7th, 8th and 9th members; ▬ modulus of elasticity of 2nd, 4th, 6th, 10th, 11th and 12th members.

row vectors in sensitivity matrix are almost linearly dependent one another. If information is not sufficient for updating parameters concerned, updated parameters are likely to be deviated from exact values in presence of measurement noise as seen in Fig. 3(b). However, updated results using 32 frequencies of case 1 are close to the exact parameters because selected frequencies are more informative even though the same number of frequency points is used.

Note that FRF sensitivity in Eq. (3) may have problems of severe non-linearity and irregularity especially when it is derived near resonances. Fig. 3(c) shows the updated results using all 2000 frequencies available. Here, parameters do not converge due to the use of frequencies near resonances in spite of using all information available. The similar results can be observed in Fig. 3(d) where some of frequencies in case 4 are selected near resonances.

4.2. A rigid rotor–bearing system

In this section, the proposed method is extended to the case of rotor–bearing systems. Because of the gyroscopic effect in a rotor system, the system becomes non-self-adjoint unlike ordinary stationary structures. So the treatment of model updating may be different accordingly [8,9]. For modal testing of rotors, different approaches have been used [18–22]. In this study, for the updating of a rotor–bearing system, the so-called directional frequency response functions (dFRFs), which have been developed by Lee et al. [20–22], are used as reference data for better physical understanding of rotors and straightforwardness of updating. In this paper, model updating of rotor–bearing systems is formulated in terms of complex co-ordinate. Then, the frequency selection method is applied to select optimum frequencies. For simplicity, but without loss of generality, a rigid rotor–bearing system shown in Fig. 4 is taken as a numerical example.

4.2.1. Equation of motion of rotor–bearing systems

The equation of motion for a rotor–bearing system can be written as

$$[M]\{\ddot{q}\} + ([G] + [C])\{\dot{q}\} + [K]\{q\} = \{f\}, \tag{16}$$

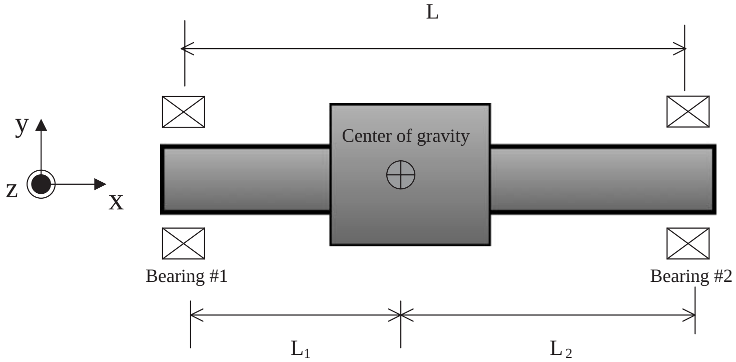

Fig. 4. Co-ordinate system of a rigid rotor–bearing system.

where $\{q\}$ and $\{f\}$ are the co-ordinate vector and force vector, respectively. If the real co-ordinates of a rigid rotor bearing system are defined by the bearing co-ordinates in Fig. 4 as

$$\{q\} = \begin{Bmatrix} y_1 \\ y_2 \\ z_1 \\ z_2 \end{Bmatrix} \text{ and } \{f\} = \begin{Bmatrix} f_{y1} \\ f_{y2} \\ f_{z1} \\ f_{z2} \end{Bmatrix} \tag{17}$$

then, the mass, gyroscopic, damping and stiffness matrix in a rigid rotor–bearing system can be written as [21]

$$[M] = \begin{bmatrix} ml_2 & ml_1 & 0 & 0 \\ -i_t & i_t & 0 & 0 \\ 0 & 0 & ml_2 & ml_1 \\ 0 & 0 & -i_t & i_t \end{bmatrix}, \quad [G] = \Omega \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & -i_p & i_p \\ 0 & 0 & 0 & 0 \\ i_p & -i_p & 0 & 0 \end{bmatrix},$$

$$[C] = \begin{bmatrix} c_{yy1} & c_{yy2} & c_{yz1} & c_{yz2} \\ -c_{yy1}l_1 & c_{yy2}l_2 & -c_{yz1}l_1 & c_{yz2}l_2 \\ c_{zy1} & c_{zy2} & c_{zz1} & c_{zz2} \\ -c_{zy1}l_1 & c_{zy2}l_2 & -c_{zz1}l_1 & c_{zz2}l_2 \end{bmatrix}, \quad [K] = \begin{bmatrix} k_{yy1} & k_{yy2} & k_{yz1} & k_{yz2} \\ -k_{yy1}l_1 & k_{yy2}l_2 & -k_{yz1}l_1 & k_{yz2}l_2 \\ k_{zy1} & k_{zy2} & k_{zz1} & k_{zz2} \\ -k_{zy1}l_1 & k_{zy2}l_2 & -k_{zz1}l_1 & k_{zz2}l_2 \end{bmatrix}, \tag{18}$$

where m, Ω, c_{ijk} and k_{ijk}, $i = y, z$, $j = y, z$, $k = 1,2$ are total mass, rotational speed, damping and stiffness of y, z direction in each bearing component, respectively, and the i_p and i_t are defined as $i_p = J_p/L^2$, $i_t = J_t/L^2$. Here J_p and J_t are polar and transverse mass moments of inertia of the rigid rotor. The dimensionless bearing location, l_i, $i = 1,2$, is defined as L_i/L where L_i, $i = 1,2$, are related with bearing locations from the center of gravity of the rotor and L is the total length of the distance as shown in Fig. 4.

The real co-ordinates defined by Eq. (17) can be transformed into complex co-ordinate vector, $\{p_c\}$ and complex force vector, $\{g_c\}$, using transformation matrix, $[T]$ as [21,23]

$$\{p_c\} = \begin{Bmatrix} p_1 \\ p_2 \\ \bar{p}_1 \\ \bar{p}_2 \end{Bmatrix} = \begin{Bmatrix} y_1 + jz_1 \\ y_2 + jz_2 \\ y_1 - jz_1 \\ y_2 - jz_2 \end{Bmatrix} = [T]^{-1}\{q\},$$

$$\{g_c\} = \begin{Bmatrix} g_1 \\ g_2 \\ \bar{g}_1 \\ \bar{g}_2 \end{Bmatrix} = \begin{Bmatrix} f_{y1} + jf_{z1} \\ f_{y2} + jf_{z2} \\ f_{y1} - jf_{z1} \\ f_{y2} - jf_{z2} \end{Bmatrix} = [T]^{-1}\{f\}, \tag{19}$$

where j is imaginary unit and transformation matrix can be written as

$$[T] = \frac{1}{2}\begin{bmatrix} I & I \\ -\mathrm{j}I & \mathrm{j}I \end{bmatrix}. \tag{20}$$

Here, $[I]$ is an 2×2 identity matrix. Then, the equation of motion in complex form can be written as [21,23]

$$[M_c]\{\ddot{p}_c\} + [C_c]\{\dot{p}_c\} + [K_c]\{p_c\} = \{g_c\}, \tag{21}$$

where $[M_c] = [T]^{-1}[M][T]$, $[C_c] = [T]^{-1}([G] + [C])[T]$, $[K_c] = [T]^{-1}[K][T]$. Similarly to Eq. (2), the dFRFs of rigid rotor–bearing system, $[H_c(\omega)]$, which are defined by complex input and output, can be written as

$$\begin{Bmatrix} P_1(\omega) \\ P_2(\omega) \\ \hat{P}_1(\omega) \\ \hat{P}_2(\omega) \end{Bmatrix} = [H_c(\omega)] \begin{Bmatrix} G_1(\omega) \\ G_2(\omega) \\ \hat{G}_1(\omega) \\ \hat{G}_2(\omega) \end{Bmatrix},$$

$$[H_c(\omega)] = \begin{bmatrix} H_{p_1g_1}(\omega) & H_{p_1g_2}(\omega) & H_{p_1\hat{g}_1}(\omega) & H_{p_1\hat{g}_2}(\omega) \\ H_{p_2g_1}(\omega) & H_{p_2g_2}(\omega) & H_{p_2\hat{g}_1}(\omega) & H_{p_2\hat{g}_2}(\omega) \\ H_{\hat{p}_1g_1}(\omega) & H_{\hat{p}_1g_2}(\omega) & H_{\hat{p}_1\hat{g}_1}(\omega) & H_{\hat{p}_1\hat{g}_2}(\omega) \\ H_{\hat{p}_2g_1}(\omega) & H_{\hat{p}_2g_2}(\omega) & H_{\hat{p}_2\hat{g}_1}(\omega) & H_{\hat{p}_2\hat{g}_2}(\omega) \end{bmatrix}. \tag{22}$$

where P_i, $\hat{P}_i$, G_i and $\hat{G}_i$ are Fourier transform of p_i, $\bar{p}_i$, g_i and $\bar{g}_i$, $i = 1,2$. Here, the so-called normal dFRFs, $H_{p_ig_k}(\omega) = \bar{H}_{\hat{p}_i\hat{g}_k}(-\omega)$, $i,k = 1,2$, have different frequency characteristics according to positive and negative frequencies such that the gyroscopic effect can be easily understood [21,22]. On the other hand, the so-called reverse dFRFs, $H_{p_i\hat{g}_k}(\omega) = \bar{H}_{\hat{p}_ig_k}(-\omega)$, $i,k = 1,2$, have been known as diagnostic tools for anisotropy in bearing parameter since the slight deviation in parameters from isotropic condition of bearing can be effectively identified [21,22]. These features are also useful in updating of parameters in rotor–bearing systems. The details of complex modal testing of rotors are beyond the scope of this paper. Readers may refer to Refs. [20–24] for details.

4.2.2. *Model updating of rotor–bearing system*

In this section, the updating of a rigid rotor–bearing system shown in Fig. 4 is formulated. Here, it is assumed that only bearing co-ordinate #1 is excited and measured such that only normal dFRF, $H^e_{p_1g_1}(\omega)$, and reverse dFRF, $H^e_{p_1\hat{g}_1}(\omega)$, are available from experimental data.

Using measured normal dFRF, $H^e_{p_1g_1}(\omega)$, and analytical dFRFs, model updating with respect to a parameter, ϕ, can be formulated using Eq. (3) as

$$S_1 \Delta\phi = \varepsilon_1, \tag{23}$$

where

$$S_1 = -\{H^a_{c_1}(\omega_k)\}^{\mathrm{T}} \frac{\partial[Z_c(\omega_k)]}{\partial\phi} \{H^a_{c_1}(\omega_k)\} \quad \text{and} \quad \varepsilon_1 = H^e_{p_1g_1}(\omega_k) - H^a_{p_1g_1}(\omega_k). \tag{24}$$

Here, the analytical dFRFs, $\{H^a_{c_1}(\omega_k)\}^{\mathrm{T}}$ and $\{H^a_{c_1}(\omega_k)\}$, are the first row and column vectors of $[H_c(\omega_k)]$ in Eq. (22), respectively. In many cases, the derivatives of analytical dynamic stiffness in complex formulation with respect to a parameter, $\partial[Z_c(\omega_k)]/\partial\phi$, may not be available directly. The simple method to solve this problem is to calculate the derivative of analytical dynamic stiffness in real formulation, $(\partial[Z_r(\omega_k)])/\partial\phi$, and convert it into complex form using transformation matrix defined in Eq. (20) as

$$\frac{\partial[Z_c(\omega_k)]}{\partial\phi} = [T]^{-1}\frac{\partial[Z_r(\omega_k)]}{\partial\phi}[T]. \tag{25}$$

Similarly, using measured reverse dFRF, $H^e_{p1\hat{g}1}(\omega)$, and analytical dFRFs, the updating can be formulated as

$$S_2\Delta\phi = \varepsilon_2, \tag{26}$$

where

$$S_2 = -\{H^a_{c_1}(\omega_k)\}^{\mathrm{T}}\frac{\partial[Z_c(\omega_k)]}{\partial\phi}\{H^a_{c_3}(\omega_k)\} \text{ and } \varepsilon_2 = H^e_{p1\hat{g}1}(\omega_k) - H^a_{p1\hat{g}1}(\omega_k). \tag{27}$$

Here, similarly, the analytical dFRFs, $\left\{H^a_{c_1}(\omega_k)\right\}^{\mathrm{T}}$ and $\left\{H^a_{c_3}(\omega_k)\right\}$, are the first row and third column vectors of $[H_c(\omega_k)]$ in Eq. (22), respectively. Combining the two equations in Eqs. (23) and (26), the updating of a set of parameters, $\{\phi\}$, can be formulated as

$$\begin{bmatrix} S_1 \\ \kappa S_2 \end{bmatrix}\{\Delta\phi\} = \begin{Bmatrix} \{\varepsilon_1\} \\ \kappa\{\varepsilon_2\} \end{Bmatrix} \quad \text{or} \quad [S]\{\Delta\phi\} = \{\varepsilon\}, \tag{28}$$

where κ is the weighting factor. Here, weighting factor, κ, is considered in order to balance the two different dFRFs. Note that the magnitude of normal dFRFs is much larger than that of reverse dFRFs. One the other hand, reverse dFRFs are sensitive to slight variation of bearing parameters from isotropic condition [21,22].

4.2.3. Simulation results

To demonstrate the effectiveness of the proposed method, two identical rigid rotor–bearing models were used to simulate the experimental and analytical models. Here, the following parameters for analytical model in Eqs. (16)–(18) are assumed to be known exactly:

$$m = 5\,\text{kg},\ l_1 = 0.4,\ l_2 = 0.6,\ L = 0.8\,\text{m},\ J_t = 12.8\,\text{kg-m}^2,\ J_p = 30.72\,\text{kg/m}^2,$$
$$k_{yz1} = 0,\ k_{yz2} = 0,\ k_{zy1} = 0,\ k_{zy2} = 0,\ c_{yy1} = 10\,\text{N s/m},\ c_{zz1} = 10\,\text{N s/m},\ c_{yy2} = 9\,\text{Ns/m},$$
$$c_{zz2} = 12\,\text{N s/m},\ c_{yz1} = 0,\ c_{yz2} = 0,\ c_{zy1} = 0,\ c_{zy2} = 0,\ \Omega = 15\,\text{rad/s}.$$

The other analytical parameters, which is going to be updated, have initial values as

$$k_{yy1} = 150\,\text{kN/m},\quad k_{zz1} = 145\,\text{kN/m},\quad k_{yy2} = 149\,\text{kN/m},\quad k_{zz2} = 150\,\text{kN/m}.$$

However, the exact values of the parameters used in the experimental model are assumed to be the following

$$k_{yy1} = 157\,\text{kN/m},\quad k_{zz1} = 140\,\text{kN/m},\quad k_{yy2} = 154\,\text{kN/m},\quad k_{zz2} = 146\,\text{kN/m}.$$

Then, the updating problem is to update the 4 parameters of the analytical model in order to minimize the difference between analytical and experimental FRFs. It is assumed that only

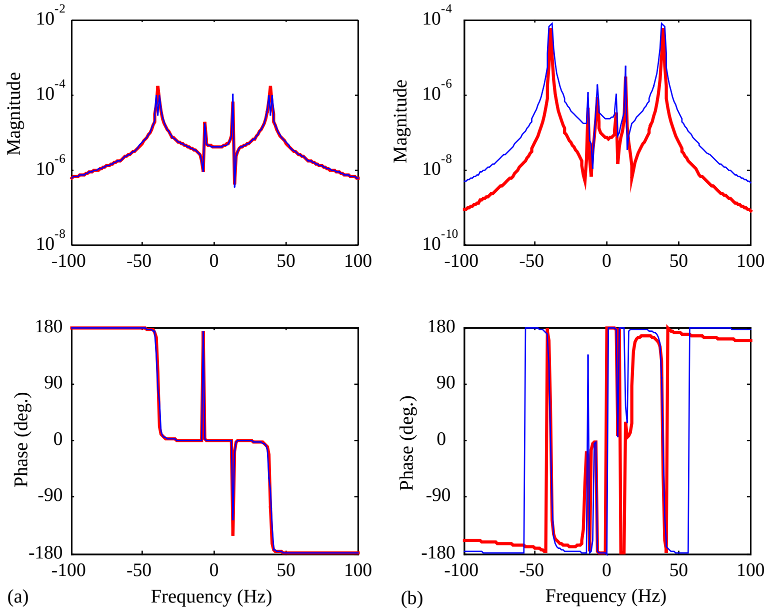

Fig. 5. Direction frequency response functions of a rigid rotor–bearing system: normal dFRF, (a) $H_{p1g1}(\omega)$; (b) reverse dFRF, $H_{p1\hat{g}1}(\omega)$; —— analytical; ▬▬ experimental.

bearing co-ordinate #1 is excited and measured for experimental FRFs. Therefore, the only available dFRFs from experiment are $H^e_{p_1 g_1}(\omega)$ and $H^e_{p_1 \hat{g}_1}(\omega)$. Fig. 5 shows the initial analytical dFRFs and simulated experimental dFRFs. As seen in Fig. 5, the magnitude of normal dFRF is larger than that of reverse dFRF. On the other hand, the reverse dFRF is more sensitive to small parameter deviation from the exact values. For simulation of measurement errors, 1% random noise is added in each experimental FRF.

In this study, 400 frequencies, which are equally spaced along both normal and reverse dFRFs, are considered as candidate reference frequencies for updating the 4 parameters concerned. In order to select optimum frequency points from total candidate frequencies, the linear dependency among row vectors in sensitivity matrix is investigated using MDI values defined by Eq. (13). As discussed before, frequencies close to resonances might be sensitive to damping and subject to various errors. Therefore, in this example, the frequencies near resonances are deliberately excluded from the total candidate frequencies such that the frequencies at least 2 Hz away from the resonances can be selected. Then, the optimum frequencies for updating are selected such that MDI values associated with the row vectors in the sensitivity matrix become less than 0.8. Here, the threshold value of 0.8 is used by considering independency among row vectors and the number of selected frequencies. Note that the independency among row vectors and the number of selected frequencies have trade-off relationship. Thus, 6 frequency points are selected such that 3 frequencies are selected from normal dFRF and 3 frequencies are selected from reverse dFRF as seen in case 4 of Table 2. The selected 6 frequencies may not lead to sufficiently over-determined set of equations for updating 4 parameters. The limited number of the selected frequencies comes from the fact that the degrees of the freedom of the model are not large. In addition, there may be some loss of information due to the elimination of the frequencies near resonances.

Table 2
Comparison of various frequency sets for updating of rigid rotor-bearing system

	Case	Frequencies used for updating (Hz)		Condition number of $[S_r]^T[S_r]$
1.	Arbitrary selected 6 frequencies	$H_{p_1 g_1}$	−50, −25, 50	2.4E+4 (276.72)[a]
		$H_{p_1 \hat{g}_1}$	−25, 0, 25	
2.	Evenly spaced 20 frequencies (resonance frequencies are avoid)	$H_{p_1 g_1}$	−99, −80, −60, −50, −20, 0, 20, 50, 60, 80, 100	795.2 (13.44)[a]
		$H_{p_1 \hat{g}_1}$	−99, −80, −60, −50, −20, 0, 20, 50, 60, 80, 100	
3.	400 total candidate frequencies	$H_{p_1 g_1}$	−99, −98, −97…0…98, 99, 100	1.8E+5 (10.50)[a]
		$H_{p_1 \hat{g}_1}$	−99, −98, −97…0…98, 99, 100	
4.	6 frequencies selected using the proposed method	$H_{p_1 g_1}$	−5, 15, 41	3.2E+4 (1.48)[a]
		$H_{p_1 \hat{g}_1}$	−41, −5, 15	

[a] Condition numbers in ( ) where weightings on all information are used.

For comparison, different sets of frequencies shown in Table 2 are used for updating. In case 1 of Table 2, 6 frequencies are arbitrarily selected avoiding resonances. For comparison with a large number of frequencies, 20 frequencies and total 400 candidate frequencies are used for updating as seen in cases 2 and 3 of Table 2, respectively.

In order to balance the sensitivity matrix using two different dFRFs, the weighting factor of 10 on reverse dFRF, that is $\kappa = 10$ in Eq. (28), is used considering the magnitude of normal and reverse dFRFs seen in Fig. 5. On the other hand, for comparison with a different weighting scheme, weightings on all information are used and compared with the case of the weighting ($\kappa = 10$) on reverse dFRF. As discussed before, it may be advantageous in terms of numerical stability to consider weightings on all equations such that all equations can have equal contribution to least-square solutions. Table 2 shows that conditioning of sensitivity matrix can be improved significantly by using such weightings. Here, the condition number of $[S_r]^T[S_r]$, where $[S_r]$ is the sensitivity matrix with real components, is used for comparison of numerical stability.

Table 3 shows the MDI values of the two different sets of 6 frequencies for updating. In case of using 6 frequencies of case 4 in Table 2, all row vectors in sensitivity matrix tend to be linearly independent one another with the MDI values below 0.8. On the other hand, in case of the arbitrary selected frequencies of case 1 in Table 2, there exists linear dependency among some of row vectors. The linear dependency among row vectors may result in insufficient information for updating 4 parameters concerned.

Table 4 shows the PDI values of each case, which represent the linear dependency among column vectors in sensitivity matrix. Note that the selection of frequency points can affect PDI values associated with column vectors in sensitivity matrix. It can be also noted that PDI values can be influenced by weightings whereas MDI values are not affected. As seen in PDI values in

Table 3
Measurement dependency index (MDI) of rotor-bearing example

Case 1							Case 4						
Frequency (Hz)	−50	−25	50	**(−25)**	**(0)**	**(25)**		−5	15	41	**(−41)**	**(−5)**	**(15)**
−50	**1.0000**	**0.9911**	**1.0000**	0.0075	0.0097	0.0012	−5	**1.0000**	0.6236	0.5808	0.2258	0.0203	0.0448
−25	**0.9911**	**1.0000**	**0.9909**	0.0412	0.0307	0.0411	15	0.6236	**1.0000**	0.2614	0.0730	0.0006	0.0161
50	**1.0000**	**0.9909**	**1.0000**	0.0009	0.0098	0.0075	41	0.5808	0.2614	**1.0000**	0.0441	0.0761	0.0100
(–25)	0.0075	0.0412	0.0009	**1.0000**	0.5985	**0.9988**	**(−41)**	0.2258	0.0730	0.0441	**1.0000**	0.4823	0.5071
(0)	0.0097	0.0307	0.0098	0.5985	**1.0000**	0.5590	**(−5)**	0.0203	0.0006	0.0761	0.4823	**1.0000**	0.4971
(25)	0.0012	0.0411	0.0075	**0.9988**	0.5590	**1.0000**	**(−15)**	0.0448	0.0161	0.0100	0.5071	0.4971	**1.0000**

Frequencies in ( ) are selected from reverse dFRF, $H_{p1\hat{g}1}(\omega)$, whereas other frequencies are selected from normal dFRF, $H_{p1g1}(\omega)$.

Table 4
Parameter dependency index (PDI) of rotor bearing example

		k_{yy1}	k_{zz1}	k_{yy2}	k_{zz2}
Case 1	k_{yy1}	1.0000 (1.0000)[a]	0.9805 (0.0921)[a]	0.8158 (0.8192)[a]	0.7969 (0.1809)[a]
	k_{zz1}	0.9805 (0.0921)[a]	1.0000 (1.0000)[a]	0.7877 (0.1801)[a]	0.8062 (0.8000)[a]
	k_{yy2}	0.8158 (0.8192)[a]	0.7877 (0.1801)[a]	1.0000 (1.0000)[a]	0.9817 (0.0962)[a]
	k_{zz2}	0.7969 (0.1809)[a]	0.8062 (0.8000)[a]	0.9817 (0.0962)[a]	1.0000 (1.0000)[a]
Case 2	k_{yy1}	1.0000 (1.0000)[a]	0.9789 (0.0020)[a]	0.7945 (0.8614)[a]	0.7680 (0.0004)[a]
	k_{zz1}	0.9789 (0.0020)[a]	1.0000 (1.0000)[a]	0.7621 (0.0005)[a]	0.7675 (0.8431)[a]
	k_{yy2}	0.7945 (0.8614)[a]	0.7621 (0.0005)[a]	1.0000 (1.0000)[a]	0.9794 (0.0020)[a]
	k_{zz2}	0.7680 (0.0004)[a]	0.7675 (0.8431)[a]	0.9794 (0.0020)[a]	1.0000 (1.0000)[a]
Case 3	k_{yy1}	1.0000 (1.0000)[a]	0.4095 (0.0014)[a]	0.9993 (0.8043)[a]	0.4100 (0.0351)[a]
	k_{zz1}	0.4095 (0.0014)[a]	1.0000 (1.0000)[a]	0.4074 (0.0285)[a]	0.9975 (0.7822)[a]
	k_{yy2}	0.9993 (0.8043)[a]	0.4074 (0.0285)[a]	1.0000 (1.0000)[a]	0.4105 (0.0009)[a]
	k_{zz2}	0.4100 (0.0351)[a]	0.9975 (0.7822)[a]	0.4105 (0.0009)[a]	1.0000 (1.0000)[a]
Case 4	k_{yy1}	1.0000 (1.0000)[a]	0.9799 (0.0031)[a]	0.9962 (0.1534)[a]	0.9751 (0.0788)[a]
	k_{zz1}	0.9799 (0.0031)[a]	1.0000 (1.0000)[a]	0.9748 (0.0787)[a]	0.9934 (0.0004)[a]
	k_{yy2}	0.9962 (0.1534)[a]	0.9748 (0.0787)[a]	1.0000 (1.0000)[a]	0.9798 (0.0039)[a]
	k_{zz2}	0.9751 (0.0788)[a]	0.9934 (0.0004)[a]	0.9798 (0.0039)[a]	1.0000 (1.0000)[a]

[a] PDI values in ( ) where weightings on all information are used.

Table 4, the numerical stability of updating associated with PDI values is improved significantly by using weightings on all information.

Figs. 6 and 7 show the convergence plots of each case for comparison of two different weighting schemes as well as different frequency sets.

Figs. 6(a) and 7(a) show the convergence plots of updating 4 parameters, k_{yy1}, k_{zz1}, k_{yy2}, and k_{zz2} using the arbitrarily selected 6 frequencies of case 1 in Table 2. Since some of row vectors in

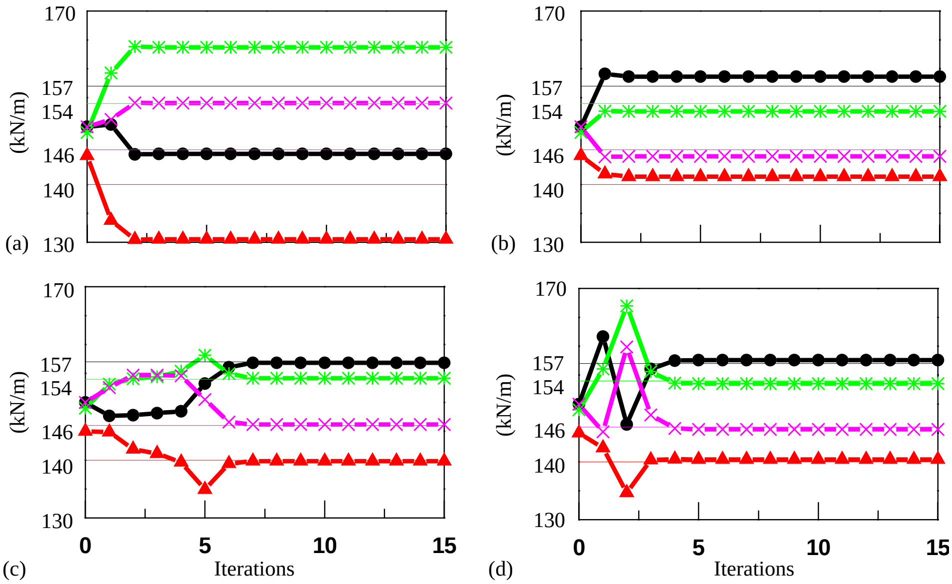

Fig. 6. Convergence plot of updated parameters using weighting, $\kappa = 10$: case 1 (arbitrary selected); (b) case 2 (20 frequencies); (c) case 3 (total frequencies); (d) case 4 (proposed method); $kyy1$; $kzz1$; $kyy2$; $kzz2$.

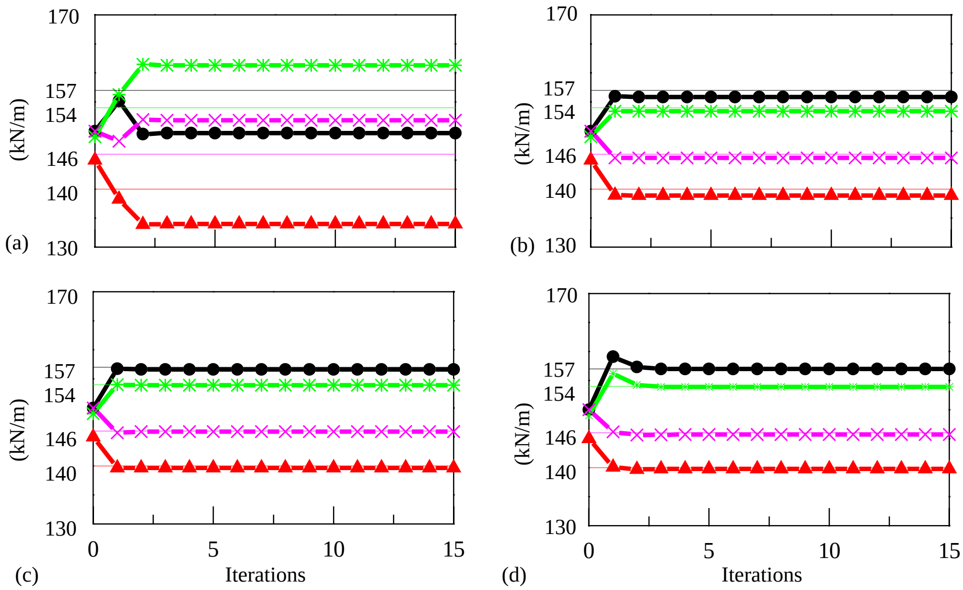

Fig. 7. Convergence plot of updated parameters with weightings on all information: (b) case 1 (arbitrary selected); (b) case 2 (20 frequencies); (c) case 3 (total frequencies); (d) case 4 (proposed method); $kyy1$; $kzz1$; $kyy2$; $kzz2$.

sensitivity matrix associated with the 6 frequencies are nearly linearly dependent as seen in Table 3, the set of equations may not have enough information for updating 4 parameters concerned. As a result, the updated parameters are deviated from the exact parameters as seen in Figs. 6(a) and 7(a). Note that numerical stability of case 1 is significantly improved from condition number of 2.4E+4 to 276.72 as seen in Table 2 by weightings on all information. However, the actual information remains the same nonetheless.

Figs. 6(b) and 7(b) show the convergence plots of updated parameters using 20 frequencies of case 2 in Table 2. Note that the updated results can be improved by using more frequency data. Nevertheless, updated parameters are slightly deviated from the exact values because the selected frequencies may not have sufficient information for updating the 4 parameters in spite of using 20 frequencies.

Figs. 6(c) and 7(c) show convergence plots where all information available is used for updating. In this case, updated parameters are likely to be least deviated from the exact parameter values in presence of measurement noise since the noises in experimental dFRFs can be averaged out using a lot of equations. However, a lot of information does not necessarily mean better-conditioning and fast convergence as seen in Fig. 6(c). In addition, the use of all frequencies available is not efficient in terms of computational point of view either. Note that the two parameters, k_{yy1} and k_{yy2}, which have almost the same effect with high PDI value, are updated in similar manner at first few iterations as seen in Fig. 6(c). Here, least-square solutions are likely to be dominated by a few frequency points near resonances, where non-linear effects are large. However, by using proper weightings on total equations, these effects can be reduced and all frequencies can have equal contributions in least-square solutions. Therefore, the convergence can be improved significantly in case of using weightings on all information as seen in Fig. 7(c).

Figs. 6(d) and 7(d) shows the updated results using 6 frequencies selected by the proposed method. The updated parameters are more close to the exact parameter values than that of using

Table 5
Updated results of rigid rotor-bearing system using different sets of frequencies

		k_{yy1}	k_{zz1}	k_{yy2}	k_{zz2}
Case 1	Converged parameters	145.30 (149.65)[a]	130.50 (133.99)[a]	163.74(161.30)[a]	154.08 (151.85)[a]
	Initial errors	−7	5	−5	4
	Errors after updating	−11.7 (−7.53)[a]	−9.5 (−6.01)[a]	9.74 (7.30)[a]	8.08 (5.85)[a]
Case 2	Converged parameters	158.65 (155.85)[a]	141.34 (138.87)[a]	152.63 (153.40)[a]	144.87(145.36)[a]
	Initial errors	−7	5	−5	4
	Errors after updating	1.65 (−1.15)[a]	1.34 (−1.13)[a]	−1.37 (-0.6)[a]	−1.13 (−0.64)[a]
Case 3	Converged parameters	156.85 (156.64)*	139.82(139.67)*	154.16(153.93)*	146.17 (145.90)*
	Initial errors	−7	5	−5	4
	Errors after updating	−0.15 (−0.36)[a]	−0.18 (−0.33)[a]	0.16 (−0.07)[a]	0.17 (−0.10)[a]
Case 4	Converged parameters	157.58 (157.01)[a]	140.43 (139.77)[a]	153.51(153.93)[a]	145.63 (145.68)[a]
	Initial errors	−7	5	−5	4
	Errors after updating	0.58 (0.01)[a]	0.43 (−0.23)[a]	−0.49 (−0.07)[a]	−0.37 (−0.32)[a]

[a] Results obtained in ( ) using weightings on all information.

either arbitrary selected 6 frequencies or 20 frequencies because most informative frequencies are used for updating. By using the proposed method, 400 candidate frequencies for updating can be significantly reduced to 6 frequencies minimizing the loss of information.

The updated results for each case are summarized in Table 5.

5. Concluding remarks

A new frequency selection method for FRF based model updating is proposed. The proposed method has following advantages:

1. Automatic selection of frequency points for FRF based model updating is possible with minimum engineering judgment.
2. Selected frequency points are informative because the row vectors in sensitivity matrix are apt to be linearly independent.
3. Computational efforts can be reduced effectively.
4. The proposed concept of MDI and PDI can be effectively used for the understanding of ill conditioning in updating problem.

However, the selection of frequencies can affect the linear dependency among column vectors of sensitivity matrix, which are associated with updating parameters. Therefore, it is recommended to check the condition number or PDI values in order to ensure stable convergence of updating parameters. Proper weightings on equations associated with selected frequencies can improve the numerical condition for updating by lowering PDI values. However, MDI values, which are used for selection of optimum frequency points, are not affected by such weightings.

Acknowledgements

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Appendix A. Nomenclature

$[\ ]$, $\{\ \}$	matrix; column vector
$[H]$	frequency response matrix
$[Z]$	dynamic stiffness matrix
$[M]$	mass matrix
$[C]$	damping matrix
$[K]$	stiffness matrix
$[G]$	gyroscopic matrix
ϕ, $\{\phi\}$	an updating parameter; updating parameter vector
S_k, $[S]$	sensitivity; sensitivity matrix
$[S_r]$	sensitivity matrix with real component
ε_k,$\{\varepsilon\}$	output residue; output residue vector
$\{X_i\}$	ith column vectors in sensitivity matrix

$\{Y_i\}^{\mathrm{T}}$ ith row vectors in sensitivity matrix
$\{q\}, \{f\}$ real co-ordinate vector; real force vector
$\{p_c\}, \{g_c\}$ complex co-ordinate vector; complex force vector
$[T]$ transformation vector
$[H^a], [H^e]$ analytical/experimental FRF

Subscripts
c complex formulation
r real formulation

Superscripts
$*$ complex conjugate transpose of matrix/vector
-1 inverse of a square matrix
$+1$ pseudo-inverse of a matrix

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