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Letter to the Editor

Coupled vibration control with tuned mass damper for long-span bridges

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1. Introduction

Long-span bridges undergoing wind excitation exhibit complex dynamic behaviors. Buffeting vibrations induced by wind turbulence happen throughout the full range of wind speed. As the wind speed increases, aerodynamic instabilities such as flutter may occur at high wind speed [1]. Much research effort has been made in mitigating excessive buffeting vibrations and improving aerodynamic stabilities for long-span bridges during construction [2,3] and at service [4–6]. Among all of the control procedures, dynamic energy absorbers such as tuned mass dampers (TMDs) have been studied in suppressing the excessive dynamic buffeting [7] or enhancing the flutter stability of bridges [4,8]. As traditional control devices, the dynamic energy absorbers dissipate external energy through providing supplemental damping to the modes of concern [9–11].

In a conventional TMD control design, the TMD frequency is designed or tuned to the modal frequency of the fundamental mode [12] in order to reduce the so-called resonant vibration and this method is thus called resonant-suppression approach here. When the modal coupling among the modes is weak, the bridge can be regarded as a simple combination of many single degree-of-freedom (d.o.f.) systems and single mode analysis is usually applicable [13].

It is well-known that wind-induced aeroelastic effects result in additional aerodynamic damping and stiffness for long-span bridges [14]. The TMD control efficiency decreases with the increase of modal damping ratio. This implies that, for coupled mode vibrations of long-span bridges, the control efficiency of buffeting response of bending mode decreases with the increase of wind velocity since the aerodynamic damping of bending modes usually increases with the wind speed. Since bending modes usually contribute significantly to the overall buffeting response among all of the modes, the decreased control efficiency in bending modes may deteriorate the overall control efficiency of the bridge vibration.

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The adoption of a slender deck and the increase of bridge span lengths tend to make the frequencies of modes closer, which increases modal coupling effects through aeroelastic effects in high wind velocity [15–19]. Modal coupling effects due to strong wind may result in a significant additional component to the buffeting response of each individual mode, compared with the cases of weak modal coupling. Accordingly, a more efficient control approach than the traditional resonant suppression method may exist for the coupled buffeting control of bridges in strong winds.

The present study aims at introducing an alternative TMD design approach, which is based on suppression of modal coupling effect among modes under strong wind. With the proposed control approach, a well-designed TMD system can efficiently suppress wind-induced vibrations for the strongly coupled modes even at high wind speed. Poorer control performance may otherwise be anticipated for TMDs designed based on the conventional resonant suppression approach.

2. Analytical formulation of bridge–TMD system

To better understand the coupled vibrations and the interaction of the bridge–TMD system, closed-form solutions are derived below. This derivation will give insights and facilitate the discussion in developing a new TMD control approach for coupled vibrations in strong winds.

For a bridge under wind action with a displacement of $r(x,t)$, the buffeting and aeroelastic forces are expressed as functions of the displacement $r(x,t)$ and location ordinate x as $f_b(x,t)$ and $f_s(x,r,\dot{r})$, respectively. Assume that a total number of n_2 TMDs are attached to the bridge at the location of x_p ($p=1$ to n_2), then the equation of motion is derived as

$$\mathbf{L}[r(x,t)] + \mathbf{D}[\dot{r}(x,t)] + \rho(x)\ddot{r}(x,t) = f_b(x,t) + f_s(x,r,\dot{r}) + \sum_{p=1}^{n_2} \delta(x-x_p) f_{TMD_p}(t), \tag{1}$$

where $\mathbf{L}[\cdot]$ and $\mathbf{D}[\cdot]$ are the elastic and viscous damping operators, $\rho(x)$ the mass density, $\delta(\cdot)$ the Dirac delta function, and f_{TMD_p} the reaction force from the pth TMD on the bridge.

By representing the deflection components of the bridge in terms of the mode shapes and generalized co-ordinate, Eq. (1) can be expressed in the generalized co-ordinate system as

$$\mathbf{I}\xi'' + \mathbf{A}\xi' + \mathbf{B}\xi = \mathbf{Q}_b + \mathbf{F}_{TMD}, \tag{2}$$

where ξ is the generalized co-ordinate vector, the superscript prime "′" represents a derivative with respect to dimensionless time $s = Ut/b$, $\mathbf{I}$ the identity matrix, $\mathbf{Q}_b$ the excitation force vector normalized to the generalized mass inertia, $\mathbf{F}_{TMD}$ the reaction force vector of TMD on the bridge normalized to the generalized mass inertia, and $\mathbf{A}$ and $\mathbf{B}$ the total damping ratio matrix and total stiffness matrix, respectively. The general terms of matrices $\mathbf{A}$ and $\mathbf{B}$ are

$$A_{ij}(K) = 2\zeta_i K_i \delta_{ij} - J_i K Z_{ij}, \quad B_{ij}(K) = K_i^2 \delta_{ij} - J_i K^2 T_{ij}, \tag{3,4}$$

where ζ_i is the damping ratio for the ith mode, δ_{ij} the Kronecker delta function that is equal to 1 if $i=j$ and equal to 0 if $i \neq j$, $K = b\omega/U$ the reduced frequency, $K_i = b\omega_i/U$ the ith reduced frequency, b the bridge width, ω_i the ith circular natural frequency, U the mean velocity of the oncoming wind; and

$$J_i = \frac{\rho b^4 l}{2I_i}, \tag{5}$$

$$Z_{ij} = H_1^* G_{h_i h_j} + H_2^* G_{h_i \alpha_j} + H_5^* G_{h_i p_j} + P_1^* G_{p_i p_j} + P_2^* G_{p_i \alpha_j} + P_5^* G_{p_i h_j} + A_1^* G_{\alpha_i h_j} + A_2^* G_{\alpha_i \alpha_j} + A_5^* G_{\alpha_i p_j}, \tag{6}$$

$$T_{ij} = H_4^* G_{h_i h_j} + H_3^* G_{h_i \alpha_j} + H_6^* G_{h_i p_j} + P_3^* G_{p_i \alpha_j} + P_4^* G_{p_i p_j} + P_6^* G_{p_i h_j} + A_3^* G_{\alpha_i \alpha_j} + A_4^* G_{\alpha_i h_j} + A_6^* G_{\alpha_i p_j}, \tag{7}$$

where I_i is the generalized mass inertia for the ith mode, ρ the air density, l the bridge length, H_i^*, P_i^*, A_i^* ($i = 1$–6) the experimentally determined flutter derivatives for the bridge deck; and the modal integrals ($G_{r_i s_j}$) are computed as

$$G_{r_i s_j} = \int_0^l r_i(x) s_j(x) \frac{\mathrm{d}x}{l}, \tag{8}$$

where $r_i = h_i,\ p_i$ or α_i; and $s_j = h_j, p_j$ or α_j.

To solve the equation of motion, Eq. (2) is Fourier transformed in the reduced frequency K domain and the ith equation is written as

$$(K_i^2 - K^2)\bar{\xi}_i(K) + \sum_{j=1}^{n_1} iKK_j D_{ij}(K)\bar{\xi}_j(K) = \bar{Q}_{b_i}, \tag{9}$$

where

$$D_{ij}(K) = \left(\frac{A_{ij}}{K_j} + \frac{\sum_{p=1}^{n_2} \mu_{ij}^p R_p^2(K)}{K_j K} \right) + \left(\frac{J_i K T_{ij}}{K_j} - \frac{\sum_{p=1}^{n_2} \mu_{ij}^p R_p^1(K)}{K_j K} \right) i. \tag{10}$$

In Eq. (10), the term associated with R represents the contribution of the TMDs to the bridge vibrations; the expression is not given here for the sake of brevity. If the cross-modal buffeting spectrum is omitted [1], the power spectral density (PSD) for the generalized displacements of the ith mode, ξ_i, is derived, following a similar procedure as in Ref. [20], as

$$S_{\xi_i \xi_i}(K) \approx S_{\xi_i \xi_i}^{un}(K) + \sum_{j \neq i}^{n_1} \gamma_{ij}(K) S_{\xi_j \xi_j}^{un}(K), \tag{11}$$

where

$$S_{\xi_i \xi_i}^{un} = \frac{S_{Q_{b_i} Q_{b_i}}}{[(B_{ii} - K^2 + \sum_{p=1}^{n_2} \mu_{ii}^p R_p^1(K))^2 + (KA_{ii} + \sum_{p=1}^{n_2} \mu_{ii}^p R_p^2(K))^2]}, \tag{12}$$

$$\gamma_{ij}(K) = \frac{[(-J_i K^2 T_{ij} + \sum_{p=1}^{n_2} \mu_{ij}^p R_p^1(K))^2 + (-J_i K^2 Z_{ij} + \sum_{p=1}^{n_2} \mu_{ij}^p R_p^2(K))^2]}{[(K_i^2 - K^2 - J_i K^2 T_{ii} + \sum_{p=1}^{n_2} \mu_{ii}^p R_p^1(K))^2 + (2\zeta_i K K_i - J_i K^2 Z_{ii} + \sum_{p=1}^{n_2} \mu_{ii}^p R_p^2(K))^2]}. \tag{13}$$

Eq. (11) indicates that the coupled response of each mode mainly consists of two parts. The first part (the first term) is the uncoupled response of the current ith mode, namely the *resonant component* of the ith mode buffeting. The second part (the second term) is due to the modal coupling between the *i*th mode and other modes and is called here the *coupled component* of the *i*th mode buffeting. It can be seen from these equations that including TMDs may affect not only the first term of Eq. (11)—the resonant component, but also the coupling component of the second term. The traditional control approach of resonant suppression that targets the first term may not control the coupling component directly. A new control approach may be naturally inspired to optimize the control efficiency by reducing the total response, not just the resonant vibration. This new control approach will be discussed below with numerical examples.

3. Coupled vibration control with a typical 2D.O.F. model

As discussed above, conventional TMD control strategy is to suppress resonant vibration that is essentially represented by the first term of Eq. (11). If the modal coupling among the current *i*th mode and the other modes is very weak, the second term of Eq. (11) will be trivial. In that case, conventional single-mode-based control analysis without considering the effect from the second term of Eq. (11) could lead to acceptable results. However, for the modes with strong modal coupling, the contribution of the second term to the total response can be significant. It becomes necessary to consider both the resonant vibration and that from coupling effects to achieve the optimal performance.

To examine this concept and verify the closed-form derivation conducted above, a simple 2d.o.f. system attached with two identical TMDs was considered as shown in Figure 1, where the parameters associated with masses M_1, M_2 and M_p represent the 1st d.o.f., the 2nd d.o.f., and the TMD d.o.f., respectively. The springs between the mass blocks are used to simulate the coupling effects between the different modes of the bridge. The parameters are defined in Table 1.

For TMD vibration control, multiple TMDs can be connected to any mass block [21]. However, the simple 2d.o.f. model with only one TMD attached on each mass block in the present

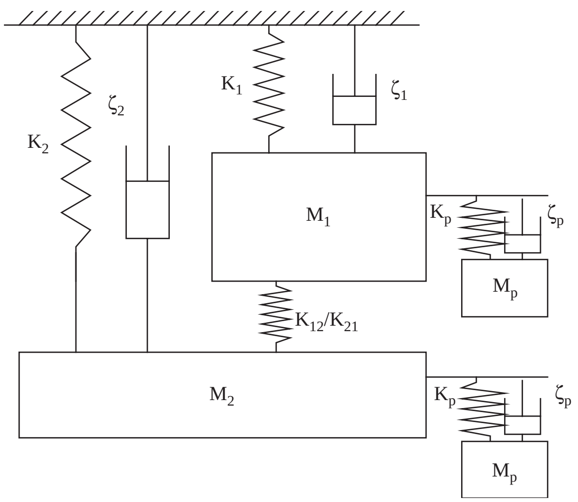

Fig. 1. 2-d.o.f. model.

Table 1
Parameters for the 2-d.o.f. system attached with two identical TMDs

2 d.o.f. system		TMD system	
$M_1,\ M_2$ (kg)	1	M_p (kg)	0.01
K_1 (N/m)	$(2\pi \times 1)^2$	K_p (N/m)	$(2\pi \times 0.1)^2$ $(2\pi \times 0.15)$
K_2 (N/m)	$(2\pi \times 1.5)^2$	$\omega_p = 2\pi f_p$ (rad)	$2\pi \times 1$ $2\pi \times 1.5$
$K_{12},\ K_{21}$ (N/m)	$(2\pi \times 0.1)^2$	Generalized inertia ratio $\mu_{11},\ \mu_{22}$	0.01
$\omega_1 = 2\pi f_1$ (rad)	$2\pi \times 1$	ζ_p	0.04
$\omega_2 = 2\pi f_2$ (rad)	$2\pi \times 1.5$		
$\zeta_1,\ \zeta_2$	0.005		
S_0	1		

study is chosen mainly due to its simplicity. It is also necessary to mention that the single TMD is the simplest but most representative case compared with the cases of multiple TMDs. While the former has accurate closed-form solutions in most cases, the latter does not have. Coupled vibration with two modes is the most typical example whose closed-form results can be more conveniently derived. Using two identical TMDs makes it easy to distinguish clearly the control effect on any part of the vibrations. For simplicity but without losing generality, it is assumed that the external excitation is white noise with a power spectral density of S_0.

According to Eqs. (11)–(13), the solution of the 2d.o.f. model ($n_1 = 2$) may reduce to

$$S_{\xi_i\xi_i}(\omega) \approx S^{un}_{\xi_i\xi_i}(\omega) + \gamma_{ij}(\omega) S^{un}_{\xi_j\xi_j}(\omega), \tag{14}$$

where

$$S^{un}_{\xi_1\xi_1}(\omega) = \frac{S_0}{[(\omega_1^2 - \omega^2 + \mu_{11} R_p^1(\omega))^2 + (2\omega\omega_1\zeta_1 + \mu_{11} R_p^2(\omega))^2]}, \tag{15}$$

$$S^{un}_{\xi_2\xi_2}(\omega) = \frac{S_0}{[(\omega_2^2 - \omega^2 + \mu_{22} R_p^1(\omega))^2 + (2\omega\omega_2\zeta_2 + \mu_{22} R_p^2(\omega))^2]}, \tag{16}$$

$$\gamma_{12}(\omega) = \frac{[(\omega_{12}^2 + \mu_{12} R_p^1(\omega))^2 + (\mu_{12} R_p^2(\omega))^2]}{[(\omega_1^2 - \omega^2 + \mu_{11} R_p^1(\omega))^2 + (2\zeta_1\omega\omega_1 + \mu_{11} R_p^2(\omega))^2]}. \tag{17}$$

Eqs. (15) and (16) are typical response spectra of the main oscillator attached with one TMD, which can be easily derived. The R terms in Eqs. (15)–(17) will disappear in the cases without TMDs and these equations will thus reduce to those common response spectra for 1d.o.f. system. Since Eqs. (15) and (16) are accurate results without approximation, same formulations can be found from literature like Refs. [7,22]. By assuming that the structural damping ratios of both the 1st d.o.f. (M_1) and 2nd d.o.f. (M_2) are as low as 0.5% (a typical value for aerodynamic analysis of

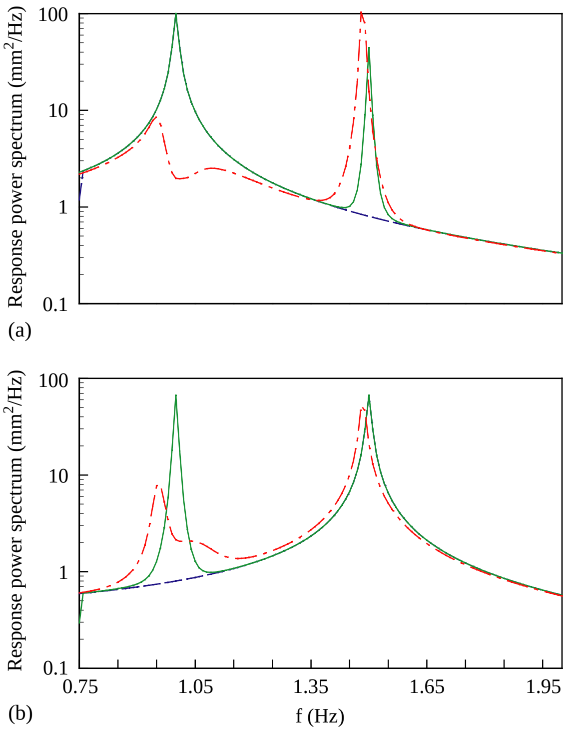

Fig. 2. Response spectra of 2-d.o.f. model with $\zeta_1 = 0.005$ (TMDs optimally designed for M_1): (a) first d.o.f. M_1; (b) second d.o.f. M_2. - - -, Uncoupled response w/o control; ——, coupled response w/o control; – - –, coupled response with control.

long-span bridges), the response power spectra were calculated with the above formulas and shown in Figs. 2 and 3. In these figures, the top half is the spectra for the 1st d.o.f. and the bottom half is for the 2nd d.o.f.

Two identical TMDs are still considered here. In Fig. 2, the two identical TMDs are conventionally designed to suppress the resonant vibration of the 1st d.o.f. For comparison, both coupled and uncoupled analyses were conducted. It can be seen that when uncoupled vibration analysis is conducted, the vibration power spectrum for each d.o.f. has only one peak due to resonant vibration. However, there exist two peaks when coupled analysis is conducted. One peak is induced by resonant vibration corresponding to its modal frequency, while the other is due to the modal coupling effect between the 1st d.o.f. and the 2nd d.o.f. The modal coupling effects are significant to the dynamic response.

It is shown in Fig. 2 that the TMDs designed for the 1st d.o.f. have good control efficiency for the resonant vibration of the 1st d.o.f. (the first peak of Fig. 2(a)), and also has some effect on the first peak of Fig. 2(b) that is the contribution of the 1st d.o.f. to the 2nd d.o.f. due to modal coupling. However, this design of TMDs does not help reduce the vibrations due to the modal coupling from the 2nd d.o.f. (the second peak of Fig. 2(a)) and the resonant vibration of the 2nd d.o.f. (the second peak of Fig. 2(b)).

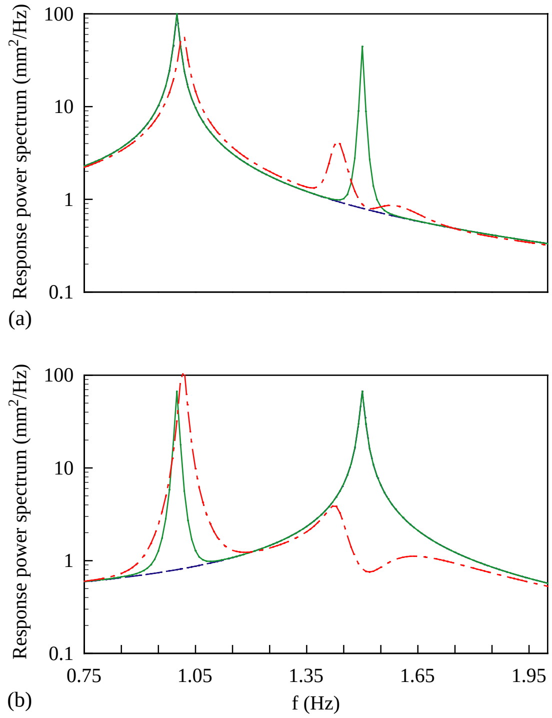

Fig. 3. Response spectra of 2-d.o.f. model with $\zeta_1 = 0.005$ (TMDs optimally designed for M_2): (a) first d.o.f. M_1; (b) second d.o.f. M_2. - - -, Uncoupled response w/o control; ——, coupled response w/o control; – - –, coupled response with control.

Fig. 3 shows the vibration power spectra when the TMDs are designed for the 2nd d.o.f. Similarly, the TMD helps reduce only the second peak values that are caused by the 2nd d.o.f., but not the peak values that are caused by the 1st d.o.f. (the first peak of both Figs. 3(a) and (b)). Figs. 2 and 3 suggest that the TMDs should be optimally designed to suppress either the resonant vibration (first part in Eq. (11)), or the vibration due to modal coupling (second part in Eq. (11)) through weakening the modal coupling. When the overall response of the structure other than any single mode is considered, multiple TMDs can be designed to achieve the best control performance under any particular condition.

As stated before, wind-induced vibration results in aeroelastic damping so that the total vibrational damping of some modes may be large in strong wind. To simulate such a case that is common for modern long-span bridges, it is arbitrarily assumed that the damping ratio of the 1st d.o.f. is 3%, while that of the 2nd d.o.f. remains to be 0.5%. The corresponding vibration spectral density results are shown in Figs. 4 and 5.

It can be found that when the 1st d.o.f. vibrates with high damping ratio, the TMDs designed for the 1st d.o.f. (Fig. 4) have less control efficiency for its resonant component (the first peak of Fig. 4(a)) than that of its counterpart when the TMDs are designed for the 2nd d.o.f. (the second peak of Fig. 5(b)). The component of the 2nd d.o.f. due to coupling even increases slightly as observed from the first peak of Fig. 4(b). In comparison, it can be seen from Fig. 5 that when

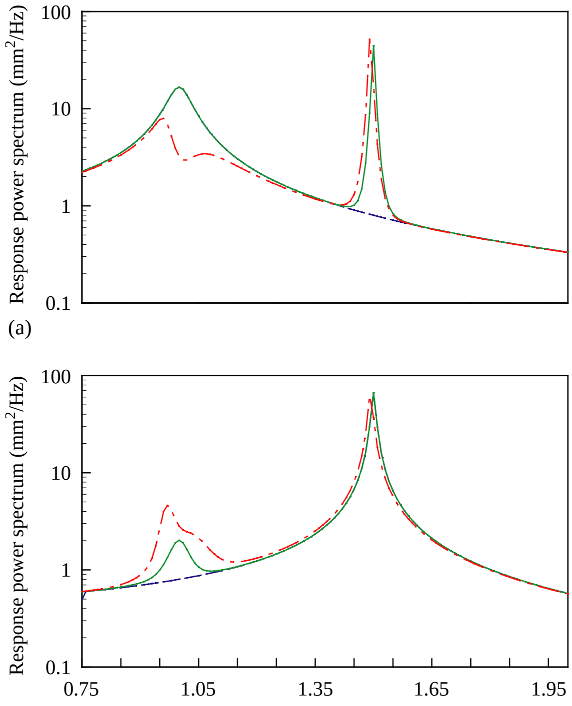

Fig. 4. Response spectra of 2-d.o.f. model with $\zeta_1 = 0.03$ (TMDs optimally designed for M_1): (a) first d.o.f. M_1; (b) second d.o.f. M_2. - - -, Uncoupled response w/o control; ——, coupled response w/o control; – - –, coupled response with control.

TMDs are designed for the 2nd d.o.f. with low damping ratio, the control efficiencies of its resonant component (the second peak of Fig. 5(b)) and the component of the 1st d.o.f. due to coupling (the second peak of Fig. 5(a)) are still high, even though the 1st d.o.f. has a very high damping ratio.

For coupled vibrations, these observations have confirmed that the total modal vibration consists of mainly one portion from resonant vibration and another portion caused by coupling effects with other modes. The frequency of conventionally designed TMDs is tuned to that of the targeted mode to control the resonant vibrations and they may not achieve an efficient control especially when the coupling effect is significant. An optimal control strategy should aim at not only the resonant vibration, but also the vibration from modal coupling. Especially for some strongly coupled modes vibrating in high wind velocity with high damping ratios, there exists a possibility that the vibration can be optimally suppressed even when the TMD is not designed around the natural modal frequency of the targeted mode. For example, to control the vibration of the 1st d.o.f. in strongly coupled vibration, the TMD frequency needs to be tuned to the natural frequency of the 2nd d.o.f. rather than that of the 1st d.o.f. In other words, weakening the coupling effects may sometimes be more efficient than reducing the resonant vibrations when strong modal coupling exists (for maximum efficiency, both resonant and coupling components should be suppressed, but certainly that will be also more costly).

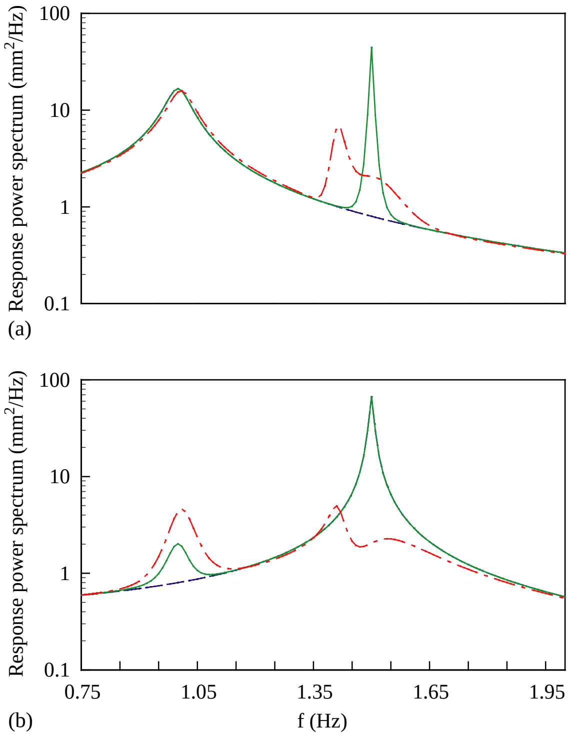

Fig. 5. Response spectra of 2-d.o.f. model when $\zeta_1 = 0.03$ (TMDs optimally designed for M_2): (a) first d.o.f. M_1; (b) second d.o.f. M_2. - - -, Uncoupled response w/o control; —, coupled response w/o control; – - –, coupled response with control.

4. Concluding remarks

The conventional TMD control approach usually focuses on suppressing the resonant vibration by supplying additional damping to the concerned modes. This approach could be inefficient for coupled vibration of long-span bridges in strong wind due to two reasons.

The first is the strong modal coupling effects in strong wind. For slender long-span bridges, the aeroelastic forces from the wind action often cause several vibration modes to couple together. Such coupling effect increases with the increase of wind speed. The coupled buffeting response of each mode usually consists of two major parts: one is a resonant component associated with its modal frequency; the other part of response is due to the modal coupling with other modes. For bridges with weak modal coupling effects, the second part is trivial. However, for long-span bridges at high wind speed, modal coupling effects may become quite strong. The latter part of the response is no longer negligible and a control approach focusing on the first part may be inefficient.

The second reason is the increased total modal damping, caused by aeroelastic effects in strong wind, of the concerned modes. Even though damping helps reduce bridge vibration, satisfactory control performance may be extremely difficult to achieve by supplying additional damping using the conventionally designed TMDs, since damping of those concerned modes is already high compared with the additional damping provided by the TMDs.

The present study proposes a new control approach that is to attenuate the modal coupling effects, in addition to suppressing the resonant vibration with TMDs. The vibration contributions to the total response from modal coupling (second part of Eq. (11)) can be significant at high wind velocity. Weakening the coupling effects with TMDs can significantly reduce the overall responses. The newly introduced control approach also enables a well-designed TMD system to be efficient in controlling buffeting vibration of coupled modes even with high modal damping under high wind velocity. For optimal control efficiency in applications, the TMD frequency needs to be adaptable in order to switch from resonant suppression to coupling suppression, or multiple frequency TMDs are needed in order to control both resonant and coupling effects.

The effects of TMDs on reducing both resonant and coupled vibrations have been demonstrated through the analytically derived closed-form solutions. Numerical analyses on a 2 d.o.f. model have validated that the new control approach may lead to more efficient control performance than the conventional resonant suppression strategy when the coupling effects are significant and when the damping ratios of those modes of concern are high.

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