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An experimental investigation of Curle's theory of aerodynamic noise generation by a stationary body in a turbulent air stream

David Alan Bies*

4 Parslow Road, Marion, South Australia 5043, Australia

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Abstract

Bies et al. (J. Sound Vib. 204(4) (1997) 631) investigated Curle's theory (Proc. R. Soc. Ser. A 231 (1955) 505) published in 1955 over a wide range of flow speeds from about 50–200 m/s and found only partial agreement with the experimental data. Here the experimental investigation has been repeated allowing the data to be recorded in a format amenable to analysis, which was not previously possible.

Reintroduction of a term neglected by Curle has been found necessary as Curle's compact source condition ensures so low a radiation impedance that the effect cannot be detected in the jet background noise. The reintroduction of the term, which has been neglected, allows his analysis to include radiation from sources not compact but less than half a wavelength in characteristic dimension. It is shown that the power ratio defined as the measured sound power divided by Curle's amended prediction converges to about 3 whereas Curle predicts that the power ratio should converge to 1 as the wave number converges to zero. The introduction of the empirically determined constant 3 into Curle's prediction brings the measurements into very good agreement with prediction over the entire range of the non-dimensional wave numbers from about 0.4 to 3.2.

1. Introduction

In a paper by Bies et al. [1] an apparatus for experimental investigation of aerodynamic noise generation by a stationary body was described. Using this apparatus it was possible for the first time to investigate Curle's [2] well-known theory of aerodynamic noise generation over an

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*Tel.: +61-882-960-720.

E-mail address: dabies@optusnet.com.au (D.A. Bies).

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extended flow speed range in the frequency domain. The theory due to Curle describes noise generation by a compact stationary body in a turbulent air stream.

The apparatus described by Bies et al. [1] overcame difficulties encountered with the previous experimental investigation of Curle's theory which apparently thwarted earlier attempts at verification. Consequently, only two other experimental investigations have been found in the published literature. Clark and Ribner [3] reported a single measurement, which was 2.7 dB below Curle's prediction. Heller and Widnall [4] attempted to find confirmation in experiments, but found none, and reported that they had no explanation for their lack of success.

Curle's theory has been widely accepted as mathematically correct [5–8]. However, extensive search of the published literature for reports of experimental investigation of Curle's theory has yielded only the three mentioned [1,3,4] of which none provides convincing verification. The work reported here was designed to investigate the reasons for the seeming partial agreement with and the reasons for its departure from Curle's theory reported in the earlier work of Bies et al. [1].

Surprisingly, as will be shown, the experimental confirmation of Curle's analysis was not obtained and it has been shown elsewhere that Curle's analysis is incorrect [9] as concluded here. However, as shown here, Curle's analysis may readily be corrected empirically to bring the reported experimental data into agreement with the prediction for the case considered.

The partial agreement previously reported [1] apparently was the result of two errors. The source used in the previously reported work, though small compared to the radiated sound wavelength, was not compact as required by Curle and his prediction is incorrect. These two errors explain the apparent partial agreement between the theory and the experiment reported earlier [1]. This conclusion readily follows from a plot of the power ratio, defined in Eq. (2) below, as a function of the non-dimensional wave number published in Fig. 10 elsewhere [10].

2. The test

The test apparatus used for the experimental investigation reported here has previously been described and a review of literature, adequate for the present purpose, has been provided [1]. The test apparatus consists of an upstream blade with an elliptic leading edge and a blunt trailing edge which generates a vortex street. A downstream block of the same thickness as the upstream blade is placed co-linear with the upstream blade in its wake on taut wires.

As shown in the reference the response of the downstream block is essentially mass controlled. The impingement of successive vortices on the leading edge of the downstream block produces an intense tone which radiates from the downstream block. The tone is generated in the same way as the whistling noise of an idling circular saw, which observation suggested the test apparatus [11].

As shown in the reference the tone emitted by the downstream block is directly proportional to the speed of the air stream suggesting that the scale of each successive vortex remains fixed being a function of the characteristic dimension of the upstream blade. The shedding of each successive vortex is a random event within limits, thus the tone produced by the wake impingement is characterised by a finite band width which, in turn accounts for the scatter in the level of the noise generated at the downstream block. As will be shown a scatter of level of about 6 dB was observed about the mean level, which is characteristic of the noise-generating mechanism used here to investigate Curle's theory.

In this report the apparatus used in the reference is unchanged but the calibrations have all been repeated with some improvements and with great care. The data reported here are new. The data of the earlier work, though in agreement with that of the present work, could not be included as they were not recorded in a format suitable for the present analysis.

The basis for the suggestion that Curle's theory is wrong will now be explained. Following procedures basically the same as those of the previous investigation, the force, which a small square steel block $30 \times 30\,\mathrm{mm}^2$ and 8 mm thick exerted on a passing air stream, was determined and used to predict the radiated sound power of the block according to Curle.

The force was determined by measuring the acceleration of the block with a calibrated accelerometer imbedded in the block. Integration of the block acceleration showed that the motion of the block was effectively mass controlled and was quite negligible over the entire range of the investigation so that noise radiation from the block due to its induced motion was negligible.

The test apparatus was mounted in a reverberation chamber allowing experimental determination of the radiated sound power. Division of the measured sound power by the predicted sound power provided the power ratio [4]. The power ratio forms the data of this investigation. Curle's theory predicts that the power ratio should asymptotically approach 1 in the limit as the non-dimensional wave number tends to zero. As will be shown, a plot of the power ratio as a function of the wave number tends to about 3.

3. Analysis

The test apparatus allowed investigation of a tone emitted by the test block which was 15–20 dB above the background level at all reported flow speeds at non-dimensional wave numbers greater than 0.4. The background level included a tone at the same frequency associated with a vortex street generator used to drive the test block. Consequently, the frequency of the tone was readily tracked and measured. The measured room temperature and frequency, f, allowed calculation of the wavelength, λ, of the emitted tone and the associated wave number $k = 2\pi/\lambda$. The wave number multiplied by the stream-wise length (characteristic dimension, l) of the small block used as a source provided the variable non-dimensional wave number, kl, used to describe the variation in the observed sound power, W_X.

For the purpose of this investigation into radiation from a compact source, Curle's predicted radiated power, W_C, may be written in terms of the force, F, exerted on the air stream by the block, the density of the air, ρ, the speed of sound, c, and alternatively the sound pressure, p, exerted by the block on the air stream as follows:

$$W_C = \frac{\pi f^2 |F|^2}{6\rho c^3} = \frac{l^2 |p|^2}{24\rho c}(kl)^2. \tag{1a, b}$$

Following a practice proposed by Heller and Widnall [4] the power ratio, P, will be defined as the experimentally determined radiated sound power, W_X, divided by Curle's prediction, W_C, for a compact source as follows:

$$P = \frac{W_X}{W_C}, \tag{2}$$

A careful review of Curle's theory has shown that the condition for compactness is very restrictive. However, the latter restriction may be relaxed by the inclusion of a term, which Curle neglects. To account for the fact that the source used in this investigation is not compact at the higher flow speeds (frequencies) tested, Curle's expression will be amended. The amended expression, W_{CA}, will allow its application over the whole range of flow speeds which were tested. Introducing the non-dimensional wave number, kl, Curle's amended prediction is as follows:

$$W_{CA} = W_C[1 + (kl)^2]^{-1}. \tag{3}$$

Forming W_X/W_{CA} using Eqs. (2) and (3) gives, what will be called here, the normalised power ratio, P_N, expressed, for convenience, in logarithmic form as follows:

$$P_N = 10\log_{10} P + 10\log_{10}[1 + (kl)^2]. \tag{4}$$

When P_N is plotted as a function of non-dimensional wave number, kL, Curle predicts that in the limit of small wave numbers, as kL approaches zero and the source then becomes compact as required by Curle, the power ratio, P, should asymptotically approach 1 and the normalised power ratio P_N should asymptotically approach 0 dB.

4. Results

The experiment has been carefully carried out with the results shown in Fig. 1. In the figure, 368 data points have been recorded over a range of non-dimensional wave numbers from about 0.4 to 3.2. The lower wave number bound was determined by the weakness of the source noise, which could not be detected in the background noise of the jet. In this case as the characteristic dimension of the block became very much less than the radiated wavelength the radiation efficiency of the block steadily diminished until the radiated sound could not be detected in the background noise of the jet.

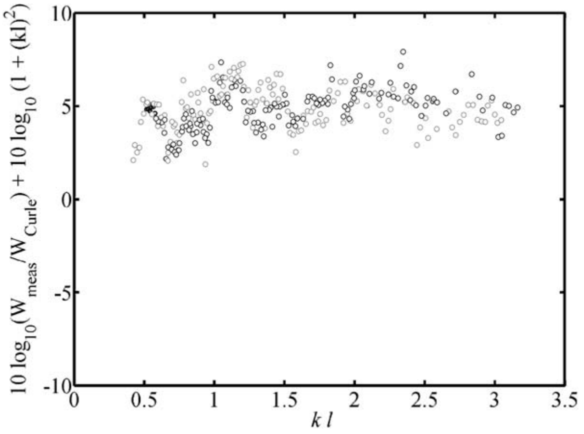

Fig. 1. The normalised power ratio according to Eq. (4) is plotted as a function of the non-dimensional wave number, kl, $l = 30$ mm. Curle predicts that the normalised power ratio should tend to 0 dB as kl tends to zero.

The upper wave number bound was determined by the arbitrary requirement that the wavelength of the radiated sound should be greater than twice the characteristic dimension of the radiating block. In passing it should be observed that satisfaction of Curle's prediction, without the amendment proposed here, would require measurements restricted to wave numbers less than 0.4 where measurements were not possible. At low wave numbers the radiation impedance of any object will tend to that of a point source. Consequently, what has been observed here may be considered true in any case.

The value of the normalised power ratio is most certainly not 0 dB in the limit as the wave number correction proposed here tends to zero and as predicted by Curle. However, as shown here, over the entire range of the data the normalised power ratio may be approximated as 5 dB.

Careful counting of the data points shown in the figure leads to the conclusion that the greatest number of data points (139) are within plus or minus 0.5 dB of 5 dB. The second highest number (102) are within plus or minus 0.5 dB of 4 dB. The third highest and the fourth highest numbers (74 and 53) are within plus or minus 0.5 dB of 6 dB and 3 dB, respectively. A numerical value of about 3 for P in Eq. (2) seems to be indicated to put all the data in agreement with Curle's prediction over the entire non-dimensional wave number range which could be investigated from about 0.4 to 3.2.

5. Discussion

The boundary condition that the surface of the source is rigid and immovable requires that there can be no net fluid flow across the surface. This important point, concerned with Curle's model, seems to have been overlooked in Curle's analysis. In his analysis Curle replaces the rigid surface with a dipole sheet implying radiation through the solid body.

Here it is suggested that the turbulent boundary layer over the surface of the block may be thought of as a distribution of very small nodules of vorticity effectively forming a distribution of very small spheres capable of deformation under the force applied to them by the external field. For example, the distribution of very small spheres would each be capable of forced modal response as monopoles, dipoles and quadruples [9].

Reference to Morse [12] shows the following. The ratio of the sound power radiated as a dipole, W_D, by a sphere of radius, a, vibrating transversely at velocity amplitude U_D along one axis to the sound power radiated as a monopole, W_M, by the same sphere vibrating radially at velocity amplitude U_M, has the following form:

$$\frac{W_D}{W_M} = \frac{1}{12}\frac{U_D^2}{U_M^2}(ka)^2. \tag{5}$$

In the case that the volume displacement of the dipole is just compensated by the in-phase volume displacement of the monopole there will be no net volume flow in the direction of the in-phase compensation. When the direction of compensation is normal to a rigid surface the requirement that there be no net flow across the rigid surface is satisfied.

The monopole radiates hemispherically while the dipole radiates through a circular cross-section. This leads to the conclusion that in order that there be no net volume flow across the rigid surface of a body (source) the amplitude of the monopole must be half of the amplitude of the dipole. In turn this leads to the conclusion that the power ratio takes the following form:

$$\frac{W_D}{W_M} = \tfrac{1}{3}(ka)^2. \tag{6}$$

In Eq. (6) a is the radius of a hypothetical sphere which will be very small compared to the characteristic dimension, l, of the radiating block. Consequently, over the range of wave numbers investigated, including $kl = 0.4$, it may be assumed that $(ka)^2 \ll 0.16$. Eq. (6) shows that the sound power radiated by the dipole sources will be negligible compared to the sound power radiated by the monopole sources radiating in opposite phase from the opposite sides of the radiating rigid body as a dipole [9].

In the case under consideration the source is driven by a vortex street, which impinges alternately upon the upper and the lower edges of a small steel block. This consideration suggests that the monopoles on opposite faces of the block will radiate in opposite phases in what will be called here as baffled dipoles. The empirically determined constant 3 when inserted into Eq. (1a) shows that the sound power of the baffled dipoles will radiate sound power as follows:

$$W_D = \frac{\pi f^2 |F|^2}{2\rho c^3} = 3.0\,W_C \tag{7a, b}$$

6. Conclusion

Direct measurement of the force exerted by a small steel block on an impinging turbulent air stream has allowed calculation of the predicted sound power according to Curle. Direct measurement of the radiated sound power in a reverberation chamber has provided the means for investigation of Curle's prediction over an extended frequency range. Over a non-dimensional wave number ranging from about 0.4 to 3.2, a good agreement is found when Curle's prediction is multiplied by 3 and amended to include radiation from a small source compared to the radiated sound wavelength but not compact as defined by Curle. It is suggested that the apparent error in Curle's analysis lies with the implied model with which he begins.

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