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Letter to the Editor

Re-analysis of free vibration of annular plates by the new version of differential quadrature method ☆

Xinwei Wang*, Yongliang Wang

Aeronautical Science Key Laboratory for Smart Materials and Structures, College of Aerospace Engineering, Nanjing University of Aeronautics & Astronautics, Nanjing 210016, PR China

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Laura et al. [1] pointed out that the question of determining the fundamental frequency of transverse vibration of a circular annular plate simply supported at the outer boundary and free at inner contour is not a trivial numerical problem when b/a is small (say, $b/a \leqslant 0.1$) at least judging from the different values in the open literature [1–6]. Although the DQ results in Ref. [6] have been improved over the earlier DQ results [5], but they are still quite different from the accurate data in Refs. [1–4].

As is well known, Rayleigh–Ritz method is one of the most powerful numerical tools. The rate of convergent, however, depends highly on the choice of admissible functions in the deflection series [7,8]. The DQ method is even simpler than the Rayleigh–Ritz method in numerical implementation. Han and Liew [9] have demonstrated the versatility and simplicity of their DQ method for analyzing thick annular plates and pointed out that their DQ results are very accurate and could serve as a benchmark for future reference. It is, however, that the accuracy of DQ method may be affected by the way to apply the multi-boundary conditions for thin annular plates [5,6]. A new version of differential quadrature method has been recently developed and shown great potential for applying various boundary conditions [10,11]. The purpose of this letter is to demonstrate the improvement of the accuracy of the new version of DQ method for analyzing thin annular plates.

Thus, the method is used to re-analyze the title problems for two boundary conditions, namely, free at the inner contour (F) and clamped (C) or simply supported (SS) at the outer edge. The two cases are denoted by F–C and F–SS, respectively. In the analyses, the grid points are computed by

$$r_i = b + \frac{(a-b)}{2}\left\{1 - \cos\frac{(i-1)\pi}{N-1}\right\}, \quad i = 1, 2, \ldots, N, \tag{1}$$

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*Corresponding author.

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where b, a, N are radius of the inner and outer boundary, and the number of grid points, respectively.

Denote the fundamental frequency $\Omega_1 = \omega_1 a^2\sqrt{\rho h/D}$. Figs. 1 and 2 show the convergence study of the Ω_1 with the number of grid points N for the cases of annular plates with F–C and F–SS boundary conditions and $b/a = 0.1$. The Poisson ratio is set to $\frac{1}{3}$. It can be seen that accurate results can be obtained with $N \geqslant 20$, similar to the cases of thick annular plates [9]. Figs. 3 and 4 show the variation of Ω_1 with the ratio of b/a for the cases of annular plates with F–C and F–SS boundary conditions, where solid lines represent the present DQ data, symbols are results cited from Refs. [2,4,9]. The Poisson ratio is set to 0.3 for all cases except for the data from Ref. [2] where $\frac{1}{3}$ is used. It can be seen that the Poisson ratio has only small effect on the results, observed by Vogel and Skinner [12]. The data for circular plates ($b/a = 0$) are cited from Ref. [4]. It can be seen that the present DQ results are compared well with existing data for $0.1 \leqslant b/a \leqslant 0.9$. It can also be seen that the DQ data of annular plates and the data of the circular plate form smooth curves. In other words, the present DQ results and the data in Ref. [2] for small ratios of b/a are reliable and accurate. It should be pointed out that the DQ results are sensitive to the grid spacing for small number of grid points, but are all reached the same accuracy for the various non-uniform grid points. However, the convergence rate for uniform grid spacing is slightly slower than that for non-uniform grid spacing. No numerical instability arose during the analyses. Table 1 summarized various results for comparisons. It should be mentioned that most data are copied from Ref. [1]. It can be seen that the present results agree very well with the data in Refs. [2,13], DQM results for thick plates [9] (thickness to outer radius ratio: $h/a = 0.001$, shear correction factor: $\kappa = \pi^2/12$, and number of grid points: $N = 22$) and the finite element results in reference [3]. Polynomial functions and optimized Rayleigh–Ritz method (ORRM) was used and all boundary conditions were satisfied in Ref. [2]. Very fine mesh (661 elements to model a half of the plate) was used in the finite element analysis [3]. The number of equations involved in Ref. [9] is almost twice of the number of present DQ method for the cases of thin plates. Based on the numerical results reported herein, one may conclude that great improvement of the accuracy of the results has been achieved by using the new version of differential quadrature method proposed recently [10,11].

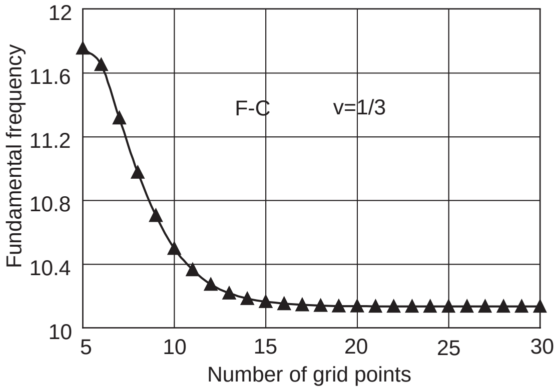

Fig. 1. Convergence study of annular plate with F–C boundary.

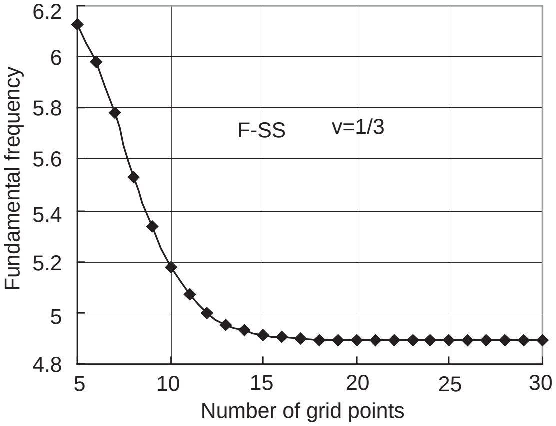

Fig. 2. Convergence study of annular plate with F–SS boundary.

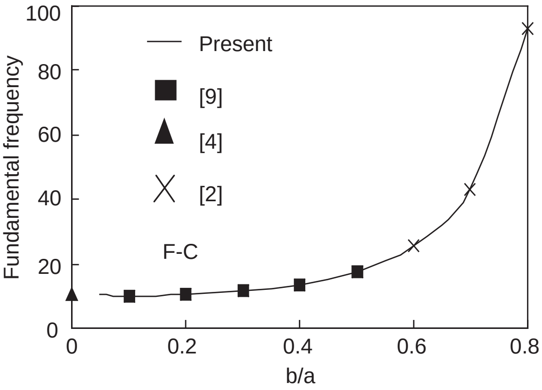

Fig. 3. Variation of Ω_1 with ratio of b/a (F–C plates).

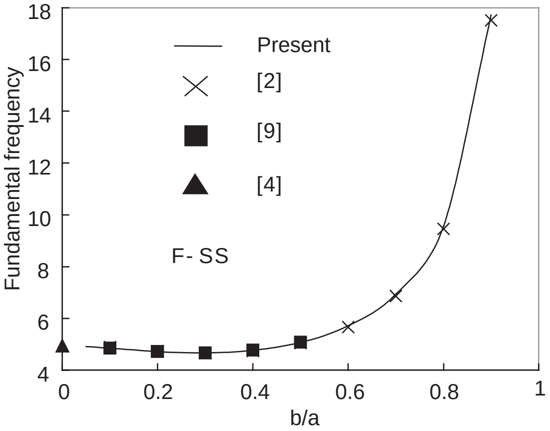

Fig. 4. Variation of Ω_1 with ratio of b/a (F–SS plates).

Table 1
Comparisons of fundamental frequency Ω_1 obtained by various methods

b/a	The Poisson ratio (ν)	Methods	F–C	F–SS
0.1	$\frac{1}{3}$	Exact [4]	10.18	4.983: 4.86
	$\frac{1}{3}$	ORRM [2]	10.13	4.890
	$\frac{1}{3}$	ORRM [3]	9.996	4.9316;4.8569
	$\frac{1}{3}$	DQM [5]	13.41	7.138
	$\frac{1}{3}$	DQM [6]	10.13	5.198
	$\frac{1}{3}$	Present	10.1348	4.89033
	0.3	FEM [3]	10.1376	4.8534
	0.3	DQM [9]	10.159	4.8533
	0.3	Present	10.1592	4.85328
	0.3	Vogel et al. [12]	10.16	4.858
	0.3	Gorman [13]	10.16	4.854
0.3	$\frac{1}{3}$	Exact [4]	11.37	4.654
	$\frac{1}{3}$	ORRM [2]	11.34	4.659
	$\frac{1}{3}$	ORRM [3]	11.331	4.6194
	$\frac{1}{3}$	DQM [5]	11.31	4.633
	$\frac{1}{3}$	DQM [6]	11.34	4.661
	$\frac{1}{3}$	Present	11.3380	4.65935
	0.3	FEM [3]	11.396	4.6644
	0.3	DQM [9]	11.424	4.6641
	0.3	Present	11.4238	4.66408
	0.3	Vogel et al. [12]	11.417	4.659
	0.3	Gorman [13]	11.424	4.663

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