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Updating 2D acoustic models with the constitutive relation error method

V. Decouvreur[a,1], Ph. Bouillard[a,*], A. Deraemaeker[b], P. Ladevèze[b]

[a] *Structural and Material Computational Mechanics Department, Université Libre de Bruxelles, av. F.D. Roosevelt 50, CP 194/05, Brussels 1050, Belgium*

[b] *Laboratoire de Mécanique et Technologie, ENS-Cachan Université Paris VI/CNRS, 61 av. Président Wilson, Cachan 94235, France*

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Abstract

Nowadays, the increasing importance of acoustic noise in industry makes it essential to establish reliable simulation tools. Furthermore, many industries need to know the acoustic performances of the products that they achieve or use. Indeed, these components are often parts of larger set-ups (like cars, airplanes, concert halls, theatres, etc.) for which numerical acoustic simulations are run from the earliest design stage. In that framework, this paper proposes a new updating technique for acoustic simulations, which is based on the constitutive relation error (CRE) proposed by Ladevèze in structural dynamics.

The technique consists of improving the quality of acoustic models by reducing the constitutive relation error below a prescribed level.

The CRE updating method aims at minimizing a cost function with respect to physical parameters of the model. Both modelling error (i.e., the error related to the approximation of physical phenomena) and measurement error are taken into account. Particular attention is paid to the admittance coefficient, which is probably the most important and the least known acoustic parameter, and the application to two-dimensional finite element numerical simulations is presented showing how promising the technique is.

The ultimate goal of the approach should be to improve the numerical simulations of the acoustic pressure level of real-life complex set-ups like cars, aircrafts, satellite launchers, etc.

*Corresponding author. Tel.: +32-2-650-2720; fax: +32-2-650-2789.

E-mail addresses: vdecouvr@smc.ulb.ac.be, pbouilla@smc.ulb.ac.be (Ph. Bouillard), ladeveze@lmt.ens-cachan.fr (P. Ladevèze).

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1. Introduction

Nowadays, many manufacturing companies have to control acoustic noise either to improve user comfort or to decrease environmental pollution. Most of the acoustic simulations are performed using finite element or boundary element software.

While computers become faster and faster, allowing decreasing computational time together with smaller calculation error, the acoustic models remain unchanged making the simulations unreliable in many cases due to the complexity of the physical phenomena. For instance, the approximative evaluation of the admittance coefficients that are used to run numerical simulations is an example of the reason why simulations deviate from experimental data. Indeed, in most of the cases, admittance coefficients are evaluated by achieving experimental measurements on a few samples of a material for which such coefficient is needed. The most classical way to evaluate that coefficient uses a laboratory set-up of a Kundt duct type. The admittance coefficient A_n is generally assumed real, which is obviously wrong, since phase shift occurs for waves propagating in porous media. Presently, manufacturers' data are limited to the absorption coefficient, which is equivalent to giving the modulus of the impedance or admittance coefficient.

In this paper, the idea is to evaluate in situ frequency-dependent complex admittance coefficients on the base of a validation stage for a complex incident field. Sound pressure measurements are achieved at a few points of the acoustic domain and admittance coefficients are tuned to verify admittance relations as closely as possible with respect to the physical phenomena.

The values of the admittance coefficients obtained after updating can be used in future numerical simulations. For example, these parameters should be useful in a prototyping phase, when changing the configuration (e.g., the shape of the acoustic domain). The updated values are introduced in the new numerical model of the acoustic domain enabling a good prediction of the acoustic pressure level without having to build a new prototype of the studied set-up. Three different kinds of models are to be considered: the continuous model, the numerical model, and the experimental one. While the continuous and the numerical models are usually the reference and the approximate models, respectively, a significant difference appears in the following approach. Indeed, one updates here the continuous model (that is approximated by a numerical model) with respect to experimental data, which constitute the reference.

Concerning the acoustics, the literature treating updating techniques seems to be poor so that one has to refer to structural dynamics to get an overview of the existing possibly applicable updating methods. Indeed, the governing equations in dynamics are very similar to those of acoustics; the acoustic pressure and velocities are analogous to the displacement and stresses, respectively in structural dynamics. That domain offers more bibliographic sources, leading one to distinguish between direct and parametric updating techniques. Direct techniques mainly consist of modifying the mass and stiffness matrices so that numerically simulated and experimentally measured frequency response functions agree as well as possible in terms of natural frequencies. Such modifications lack physical meaning, making the validity domain thin if the configuration changes. In the case of parametric updating techniques, one has to minimize a cost function by tuning physical parameters of the model. The sensitivity of the cost function with respect to the different parameters allows one to choose which of these have to be tuned. That choice can also vary with the geometrical localization in the studied domain. A more extended state of the art on validation methods is presented in Refs. [1,2].

The present paper proposes the application of a particular parametric updating technique based on the constitutive relation error to acoustics, and to make use of it to obtain accurate evaluation of complex admittance coefficients. The fundamentals of the CRE were first developed by Ladevèze in structural dynamics (see Refs. [3–5]), and Ref. [6] introduced the idea of applying the CRE to acoustics. The main idea in the CRE technique consists of splitting the data and equations of the model into reliable and less reliable information. Whether one trusts given data or equation has to be related to the assumptions made to establish it.

The paper is organized as follows. Firstly, the CRE is applied to acoustics, and reliable and less reliable data are set. Admissible pressure and velocity fields verifying the reliable equations are built and used to define the CRE. Secondly, measurements related errors are discussed, which leads on to considering the modified CRE. Afterwards, the paper deals with a particular numerical approximation of the continuous model, the finite element discretization. Finally, simulations are run on a 2D car cabin to validate the method.

2. The CRE applied to acoustics

2.1. Principles

One deals with an acoustic problem that is defined on a domain Ω with boundary $\partial\Omega$. In linear acoustics, one assumes small harmonic perturbations of the particle velocity $\vec{v}$, the pressure p and the density ρ of the isotropic medium so that these oscillations around steady values are, respectively, written as follows:

$$\begin{aligned} \vec{v} &= \vec{v}' \mathrm{e}^{\mathrm{j}\omega t}, \\ p &= p' \mathrm{e}^{\mathrm{j}\omega t}, \\ \rho &= \rho' \mathrm{e}^{\mathrm{j}\omega t}, \end{aligned} \tag{1}$$

where $\mathrm{j} = \sqrt{-1}$, ω the angular frequency and t the time.

Consider that the reliable equations are the wave equation, called the Helmholtz equation in the frequency domain, and the Dirichlet boundary condition defined on $\partial_1\Omega$ (see Fig. 1 for an

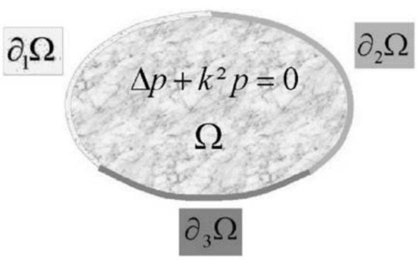

Fig. 1. Studied domain and its boundaries.

illustration of the boundaries):

$$\begin{aligned}&\text{Helmholtz}:\boldsymbol{\Delta} p + k^2 p = 0,\\&\text{Dirichlet B.C.}:p_{|_{\partial_1\Omega}} = \bar{p},\end{aligned} \tag{2}$$

where c is the sound velocity, and $k = \omega/c$ is the wave number.

The less reliable equations were originally the constitutive relations [3]. Here, it will be assumed that the mixed Robin boundary condition defined on $\partial_3\Omega$, which links the pressure to the normal velocity by an impedance coefficient Z_n, and the Neumann B.C. defined on $\partial_2\Omega$ are the less reliable data. That latter boundary condition is rewritten in what follows using the Euler equation, which links the pressure gradient to the velocity vector. The resulting two equations are:

$$\begin{aligned}&\text{Robin B.C.}:v_{n_{|_{\partial_3\Omega}}} = A_n p,\\&\text{Neumann B.C.}:v_{n_{|_{\partial_2\Omega}}} = \frac{\mathrm{j}}{\omega\rho}\frac{\partial p}{\partial n}|_{\partial_2\Omega} = \bar{v}_n,\end{aligned} \tag{3}$$

where $A_n = Z_n^{-1}$ is the complex admittance coefficient, and $\bar{v}_n$ is the prescribed velocity on $\partial_2\Omega$ which is known either by measurement or by structural dynamic computation. Discussions are still open concerning the most appropriate form of admittance relation (3). More details can be found in Ref. [7].

One has chosen here to express the impedance relation under the form:

$$v_n = c_1 p + c_2 \frac{\partial p}{\partial t}, \tag{4}$$

where c_1 and c_2 are constants and not functions of time. Eq. (4) is equivalent in the frequency domain to

$$v_n = (c_1 + \mathrm{j}\omega c_2)p = A_n p. \tag{5}$$

2.2. Admissibility

Now define two Hilbert spaces V_1 and V_2 of square-integrable functions together with their first derivatives in $\bar{\Omega} = \Omega \cup \partial\Omega$:

$$\begin{aligned}V_1 &= H_D^1(\bar{\Omega}) = \{p \in H^1(\bar{\Omega}) | p = \bar{p} \text{ on } \partial_1\Omega\},\\V_2 &= H_0^1(\bar{\Omega}) = \{w \in H^1(\bar{\Omega}) | w = 0 \text{ on } \partial_1\Omega\}.\end{aligned}$$

The variational formulation corresponding to the Helmholtz equation with associated boundary conditions as given in Eqs. (2) and (3) is expressed by

$$\text{Find } p \in V_1 | \int_\Omega (\boldsymbol{\nabla} p \boldsymbol{\nabla} w^* - k^2 p w^*)\mathrm{d}\Omega + \mathrm{j}\omega\rho \int_{\partial_3\Omega} v_n w^* \mathrm{d}\Gamma + \mathrm{j}\omega\rho \int_{\partial_2\Omega} \bar{v}_n w^* \mathrm{d}\Gamma = 0 \quad \forall w \in V_2, \tag{6}$$

where $*$ denotes the complex conjugate. The solution $s(p, v_n, \bar{v}_n)$ (where $p, v_n, \bar{v}_n$ are independent fields) $\in \mathbf{S}_{ad}$ (is admissible) if $p \in V_1$ and Eq. (6) is verified.

2.3. Definition of the CRE

The CRE is an error which measures the verification of the less reliable equations defined by Eq. (3). Its value is always positive or equal to zero. It is equal to zero if the Neumann and the Robin equations are satisfied. The following expression for the CRE will be used:

- error from the Robin BC: $\omega^2\rho^2\int_{\partial_3\Omega}(v_n - A_n p)^*(v_n - A_n p)\mathrm{d}\Gamma$,
- error from the Neumann BC

$$\omega^2\rho^2\int_{\partial_2\Omega}\left(\bar{v}_n - \frac{\mathrm{j}}{\omega\rho}\frac{\partial p}{\partial n}\right)^*\left(\bar{v}_n - \frac{\mathrm{j}}{\omega\rho}\frac{\partial p}{\partial n}\right)\mathrm{d}\Gamma.$$

The CRE ξ_ω^2 measuring the modelling error at angular frequency ω is the sum of the errors related to the poorly reliable relations:

$$\begin{aligned}\xi_\omega^2(p, v_n, \bar{v}_n) = {} & \frac{L_2}{(L_2+L_3)}\gamma\omega^2\rho^2\int_{\partial_2\Omega}\left(\bar{v}_n - \frac{\mathrm{j}}{\omega\rho}\frac{\partial p}{\partial n}\right)^*\left(\bar{v}_n - \frac{\mathrm{j}}{\omega\rho}\frac{\partial p}{\partial n}\right)\mathrm{d}\Gamma \\ & + \frac{L_3}{(L_2+L_3)}(1-\gamma)\omega^2\rho^2\int_{\partial_3\Omega}(v_n - A_n p)^*(v_n - A_n p)\mathrm{d}\Gamma \end{aligned} \tag{7}$$

$$\text{where}\begin{cases} L_2 = \int_{\partial_2\Omega}\mathrm{d}\Gamma, \\ L_3 = \int_{\partial_3\Omega}\mathrm{d}\Gamma. \end{cases}$$

The factor $\gamma(0 \leqslant \gamma \leqslant 1)$ allows one to weight differently the error related to the impedance relation (Robin B.C.) and the one related to the system excitation (Neumann B.C.). The factor γ is to be adjusted by taking into account the *a priori* knowledge of the studied set-up. For example, if the set-up excitation is very complex and it is known to be not reliable, the parameter γ should be tuned in the function (i.e., γ should tend to zero) so that the updating does not focus on the error on that B.C. only since it is dominant in that case. If no information is available at this subject, γ is set to 0.5.

The interest of using such coefficient is explained by the following example. Suppose that the updating process is stopped when reaching a 9% residual CRE level at the end of the validation step (one assumes that 9% is the accuracy level needed for the problem studied). Analyzing the contribution of the error on the Neumann B.C.; (ξ_ω^N) and the one on the Robin B.C. (ξ_ω^R) at the end of the optimization shows:

$$\xi_\omega = 0.5{*}\xi_\omega^N + 0.5{*}\xi_\omega^R = 0.5{*}12\% + 0.5{*}6\% = 9\%. \tag{8}$$

The corresponding updated parameters verify the Neumann B.C. with an error of 12% and the Robin B.C. with an error of 6%.

Though, one would prefer to get updated parameters that correspond to an equally distributed error on both B.C. (i.e., something like $\xi_\omega^N \approx \xi_\omega^R \approx 9\%$). Assuming an *a priori* knowledge of the set-up of the type $\xi_\omega^N \approx 2{*}\xi_\omega^R$, the coefficient γ is set to 0.33 so that the previously updated parameters that satisfied the CRE threshold now yield:

$$\xi_\omega = \gamma{*}\xi_\omega^N + (1-\gamma){*}\xi_\omega^R = 0.33{*}12\% + 0.67{*}6\% = 4\% + 4\% \tag{9}$$

One can see that using the γ weight forces the updating process to admit only parameters that equally distribute the CRE on both boundary conditions. If no information is available enabling one to determine the value of γ before starting the optimization step, the ratio of the errors on the Neumann and Robin B.C. is evaluated at the end of the minimization procedure. It allows one to assess the value of the factor γ which is used to run a new optimization process.

3. The modified CRE

Since one would like to update a continuous model with reference to experimental measurements, an additional measurement error adds to the error caused by the model formulation itself. Just as for the model, it is necessary to define the reliable and less reliable equations for the measurements and to build an error measure on the less reliable experimental quantities. Measurement errors are among others due to the positioning of the sensors and microphones, their accuracy, calibration, measurement orientation, reproductivity and repeatability of the measurements [8].

For instance, measurement errors occur for two types of data:

- pressure measurement by using microphones,
- velocity measurement by using accelerometers or velocity transducers.

3.1. The measurement error

In what follows, one assumes that reliable experimental information is:

- the measurement of the angular frequency,
- the positioning of the sensors and microphones,
- the calibration of the sensors and microphones,
- the directions of the measurements and excitations.

These define the admissibility $\mathbf{S}_{ad}$ for the measurements. Considering the two types of measurement error described in Section 3, the measurement errors at a given frequency are described as follows:

- pressure measurement (amplitude and phase): $|\Pi_1 p - \Pi_1 \tilde{p}|^2$,
- velocity measurement (amplitude and phase): $||\Pi_2 \bar{v}_n - \Pi_2 \tilde{v}_n||^2$.

where $|| \; ||^2$ and $| \; |^2$ denote energy norms, Π_1 and Π_2 are projection operators that give the value of the pressure and normal velocity, respectively, at the corresponding sensors, and $\tilde{p}$ and $\tilde{v}_n$ are the measured pressure and normal velocity.

A projection operator Π is a matrix defined by

$$\begin{aligned} &\Pi_{ii} = 1 \text{ if the dof } i \text{ is measured,} \\ &\Pi_{ii} = 0 \text{ if the dof } i \text{ is not measured,} \\ &\Pi_{ij} = 0 \text{ if } i \neq j. \end{aligned} \tag{10}$$

3.2. Quality of a model with respect to measurements: the modified CRE

By summing the constitutive relation error and the measurement error at angular frequency ω, the modified CRE e^2_ω is obtained:

$$e^2_\omega = \xi^2_\omega + \frac{r}{1-r}\{\zeta|\Pi_1 p - \Pi_1\tilde{p}|^2 + (1-\zeta)||\Pi_2\bar{v}_n - \Pi_2\tilde{v}_n||^2\}, \tag{11}$$

where $0 \leqslant \zeta \leqslant 1$ and $0 \leqslant r < 1$. The weighting factor $r/(1-r)$ translates the trueness in the measurements with respect to the model accuracy. If the error on the measurements is known to be smaller than the modelling error, the parameter r should be consequently adjusted to a value that is greater than 0.5. Indeed, $r = 0.5$ weights equally the modelling error (ξ_ω) and the measurement error.

Similarly, ζ allows to weight the relative importance of the pressure and velocity measurement errors. Indeed, it is assumed that pressure as well as velocity measurements are performed, each of those being polluted. The factor ζ is tuned according to the relative trust that one places in the pressure and velocity measurements. For example, if the pressure measurements are known to be much more polluted than the velocity ones, ζ should tend to 0. Otherwise, ζ is set to 0.5.

The use of these two coefficients r and ζ can be explained in the same way as that done for the weight γ.

The modified CRE is an indicator of the verification of the less reliable quantities and equations of the problem. Now the problem becomes

$$\text{Find } s_\omega(p, v_n, \bar{v}_n) | \begin{cases} s_\omega \in \mathbf{S}_{ad}, \\ e^2(s_\omega) \text{ is minimum.} \end{cases} \tag{12}$$

The solution s_ω will thus verify the reliable equations and quantities exactly by satisfying the admissibility. It will satisfy the less reliable quantities and equations as well as possible by minimizing e^2_ω.

The study of an acoustic system being usually led in a finite frequency range $[\omega_{\min}, \omega_{\max}]$, a weighting function $z(\omega)$ is defined so that

$$\int_{\omega_{\min}}^{\omega_{\max}} z(\omega)\mathrm{d}\omega = 1 \quad z(\omega) \geqslant 0, \tag{13}$$

and the mean modified CRE in the interval $[\omega_{\min}, \omega_{\max}]$ is then given by

$$e^2 = \int_{\omega_{\min}}^{\omega_{\max}} e^2_\omega z(\omega)\mathrm{d}\omega. \tag{14}$$

If the same weight is attributed to each updating frequency, the function $z(\omega)$ is given by

$$z(\omega) = \frac{1}{(\omega_{\max} - \omega_{\min})}. \tag{15}$$

More complex functions can be used to focus on a given zone of interest of the frequency range.

4. Finite element discretization

The method proposed in this paper is very general and can be applied to all kinds of numerical approximations such as the finite element method, boundary element method, meshless method [9], etc. The CRE method is illustrated here for the case of a finite element discretization. It is assumed in what follows that the interpolation and the pollution errors are kept under control by adapting the mesh size to the frequency [10]. Indeed, the sum of these errors has to be sufficiently small compared to the modelling error described before, otherwise the updating presented here does not make sense. It is first necessary to introduce a pressure formulation by introducing pressure variables (P, Q, R) as follows:

$$p = P, \tag{16}$$

$$v_n = A_n Q, \tag{17}$$

$$\bar{v}_n = \frac{\mathrm{j}}{\omega\rho}\frac{\partial R}{\partial n}. \tag{18}$$

The CRE becomes

$$\begin{aligned}\xi^2_\omega(P, Q, R) = {} & \gamma\omega^2\rho^2 \int_{\partial_2\Omega}\left(\frac{\mathrm{j}}{\omega\rho}\frac{\partial P}{\partial n} - \frac{\mathrm{j}}{\omega\rho}\frac{\partial R}{\partial n}\right)^* \left(\frac{\mathrm{j}}{\omega\rho}\frac{\partial P}{\partial n} - \frac{\mathrm{j}}{\omega\rho}\frac{\partial R}{\partial n}\right)\mathrm{d}\Gamma \\ & + (1-\gamma)\omega^2\rho^2 \int_{\partial_3\Omega}(A_n P - A_n Q)^*(A_n P - A_n Q)\mathrm{d}\Gamma.\end{aligned} \tag{19}$$

Nodal unknowns are associated to the pressure fields as follows:

Pressure field	Nodal unknown
P	$\mathbf{P}$
Q	$\mathbf{Q}$
R	$\mathbf{R}$

Note that fields Q and R are only defined on $\partial_3\Omega$ and $\partial_2\Omega$, respectively. From the variational formulation (6), one writes the corresponding discrete matrix equation:

$$[\mathbf{K}]\mathbf{P} + \mathrm{j}\omega\rho[\mathbf{C}]\mathbf{Q} - \omega^2[\mathbf{M}]\mathbf{P} = [\mathbf{E}]\mathbf{R}, \tag{20}$$

where

- $p^h = \mathbf{N}^t\mathbf{P}$ is the finite element approximation of the pressure,
- $[\mathbf{M}] = 1/c^2 \int_\Omega \mathbf{N}^t\mathbf{N}\mathrm{d}\Omega$ is the mass matrix,
- $[\mathbf{K}] = \int_\Omega \boldsymbol{\nabla}^t\mathbf{N}\nabla\mathbf{N}\mathrm{d}\Omega$ is the stiffness matrix,
- $[\mathbf{C}] = \int_{\partial_3\Omega} A_n\mathbf{N}^t\mathbf{N}\mathrm{d}\Gamma$ is the impedance matrix,
- $[\mathbf{E}] = \int_{\partial_2\Omega} \boldsymbol{\nabla}^t_n\mathbf{N}\mathbf{N}\mathrm{d}\Gamma$ is the system excitation matrix due to normal velocities imposed on boundary $\partial_2\Omega$.

CRE (7) is written for the FE discretization:

$$\xi_\omega^2(\mathbf{P},\mathbf{Q},\mathbf{R}) = \gamma(\mathbf{R}-\mathbf{P})^*[\mathbf{K_n}](\mathbf{R}-\mathbf{P}) + (1-\gamma)\rho^2\omega^2(\mathbf{Q}-\mathbf{P})^*[\mathbf{D}](\mathbf{Q}-\mathbf{P}), \tag{21}$$

where

- $[\mathbf{K_n}] = \int_{\partial_2\Omega} \nabla_n^t \mathbf{N} \nabla_n \mathbf{N} \mathrm{d}\Gamma$,
- $[\mathbf{D}] = \int_{\partial_3\Omega} A_n^* A_n \mathbf{N}^t \mathbf{N} \mathrm{d}\Gamma$.

Problem (12) to be solved is rewritten:

$$\text{Find } s'_\omega = (\mathbf{P},\mathbf{Q},\mathbf{R}) \left| \begin{cases} [\mathbf{K}]\mathbf{P} + \mathrm{j}\omega\rho[\mathbf{C}]\mathbf{Q} - \omega^2[\mathbf{M}]\mathbf{P} = [\mathbf{E}]\mathbf{R}, \\ \mathrm{e}_\omega^2(s') \text{ is minimum.} \end{cases} \right. \tag{22}$$

5. Two-dimensional numerical applications

At this stage, only two-dimensional numerical simulations are run. Such problems are of course non-realistic (because reality is three-dimensional) so that experimental data acquisition is not possible. Consequently, error evaluations are made by comparison with numerical results that are known as being accurate and reliable instead of experimental data.

The updating is performed at a few points located inside the acoustic domain. For the two following cases studied one limits oneself to the modelling error related to the impedance relation (3) and to the pressure measurement error. Pressure measurement is computed at a few points distributed inside the acoustic domain. In practice, it is important to have enough measurement points to be able to filter the noise on these measurements, and to avoid being in a situation where all measurement points would coincide with pressure nodes, at a given frequency.

There is only one value of $\bar{v}_n$ given on $\partial_2\Omega$, so that the pressure value can be normalized for a unit value of $\bar{v}_n$. In that case, the measurement error is on the pressure only, and the error described in Eq. (11) reduces to

$$\mathrm{e}_\omega^2 = \omega^2\rho^2 \int_{\partial_3\Omega} (v_n - A_n p)^*(v_n - A_n p)\mathrm{d}\Gamma + \frac{r}{1-r}|\Pi_1 p - \Pi_1 \tilde{p}|^2. \tag{23}$$

The corresponding relative error for each frequency ω is obtained by dividing e^2 by σ^2, that is for instance

$$\sigma^2 = \frac{\omega^2\rho^2}{2} \int_{\partial_3\Omega} ((A_n p)^* A_n p + v_n^* v_n)\mathrm{d}\Gamma. \tag{24}$$

The relative modified CRE is then written $\mathrm{e}_{\mathrm{rel}} = \mathrm{e}/\sigma$.

5.1. First application: pressure field inside a car cabin with 2 real A_n

Fig. 2 presents a mesh of the car cabin that has been studied. The mesh comprises 298 nodes and is made up of linear elements with 4 nodes. The excitation of the car structure is caused by the vibration of the firewall. The corresponding boundary condition is represented by a dotted bold

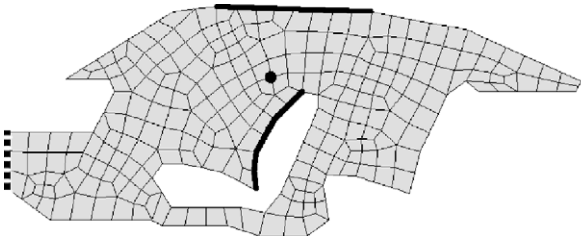

Fig. 2. 2D mesh of a car cabin.

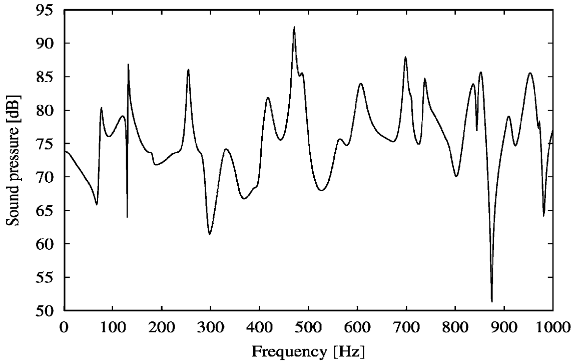

Fig. 3. Sound pressure FRF at the ear of the driver for mesh (Fig. 2).

line on the mesh. It is assumed in this simulation that only two parts of the cabin are covered by absorbing materials and cause the attenuation of the ambient noise inside the car. The first absorbing material overlays a part of the top of the car (see the heavy line in Fig. 2) with an impedance value $Z_{n1} = 600$ N s^{-1} m^{-3}. The second absorbing material corresponds to the front side of the back of the driver seat ($Z_{n2} = 800$ N s^{-1} m^{-3}). At this time, impedance values are supposed real for that first application.

The frequency response function of such a set-up calculated at the ear of the driver (see the bullet on the car mesh (2) for the location) is shown in Fig. 3: the frequency range goes from 0 up to 1000 Hz, and the ordinate corresponds to the sound pressure FRF in dB when the firewall is excited with a normal velocity equal to 1 mm s^{-1}. The FRF is computed using the *ACTRAN*© software developed by Free Field Technologies [11]. The updating algorithm is run for five different frequencies in the range 0–1000 Hz : $\{20, 100, 300, 600, 1000\}$Hz.

The updating parameters are the impedances Z_{n1} and Z_{n2} of the absorbing materials described before. Initial values for these unknowns are set to 1000 N s^{-1} m^{-3}. Fig. 4 illustrates the modified CRE to be minimized with respect to Z_{n1} and Z_{n2} at each updating frequency. The error shown is frequency averaged and clearly indicates the values of the impedances minimizing the function.

If there are many different admittance coefficients, it is no more possible to examine the shape of the function to be minimized. That is the reason why the addressed numerical example presents only two admittance coefficients. In the framework of updating models, the unknowns are assumed to be sufficiently close to the initial values that are used at the first iteration of the

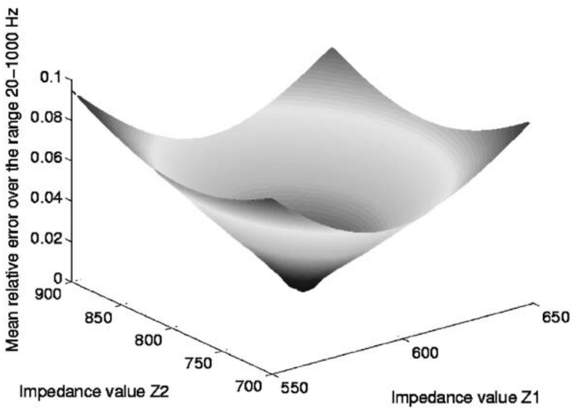

Fig. 4. Frequency averaged modified CRE of the car versus (Z_1, Z_2).

optimization procedure, so that a local minimization algorithm is used to find the minimum of the error function.

Besides, if one cannot guarantee that the global minimum was found, a CRE level after updating that is lower than the one before running the optimization process certifies that the model was improved by the updating procedure.

The optimization algorithm that has been used in the numerical examples is a multidimensional unconstrained non-linear minimization algorithm of Nelder–Mead [12] type.

Running the modified CRE technique implemented in a MATLAB© environment with stopping criterion $e_{rel} \leqslant 10^{-4}$ yields the two following values for the updated impedances:

$$Z_{n1} = 600.007\ \mathrm{N\ s^{-1}\ m^{-3}},$$
$$Z_{n2} = 799.995\ \mathrm{N\ s^{-1}\ m^{-3}}.$$

The relative modified CRE for the initial values of the two impedance coefficients Z_{n1} and Z_{n2} was about 38%. After updating the acoustic model, that error diminished below the prescribed value of 0.01%, which shows that the updating technique effectively validates the acoustic model.

For sure, such error level of 0.01% can only be reached for ideal study cases, i.e., without measurement noise and when referring to simulated acoustic fields. Real-life cases should exhibit error values that rarely decay below the 5% barrier.

5.2. Second application: updating a 2D car cabin with 5 frequency dependent complex An

The studied set-up is the same as before, but the pressure field is now attenuated by the contribution of 5 absorbing materials covering the seats, the roof, the floor and the dashboard of the car. These materials are characterized by complex frequency dependent admittance coefficients of the form: $A_n = C_1 + \mathrm{j}\omega C_2$, where C_1 and C_2 are constant values and ω is the angular frequency.

The mesh of the set-up is identical to the previous one, but more absorbing materials are now covering the boundaries, as can be seen in Fig. 5 where the bold lines correspond to the regions covered by one of the absorbing materials.

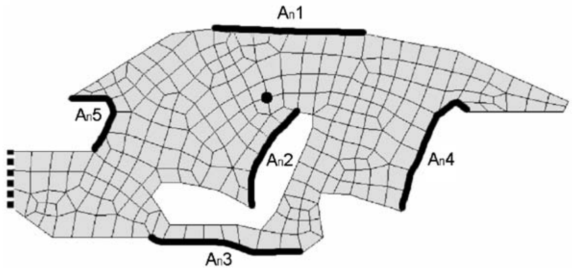

Fig. 5. 2D mesh of the car cabin with 5 absorbing materials.

Table 1
Frequency average CRE and error on updated admittance coefficients

Unity	10^{-3} N^{-1} s^{-1} m^3	(%)
CRE		0.05
A_n1	$2 + 0.002\omega j$	0.20
A_n2	$1 - 0.0015\omega j$	0.41
A_n3	$3 + 0.001\omega j$	0.37
A_n4	$5 + 0.009\omega j$	3.47
A_n5	$4 - 0.008\omega j$	0.77

5.2.1. *Updating without measurement noise*

In this application, reference measurements are computed from a FE simulation with the exact values of the 5 updated parameters. Table 1 shows the reference values of the admittance coefficients and the error on each of these after updating the model from 0 up to 500 Hz. These errors are frequency average values, i.e., each average error is the sum of the errors at each updating frequency divided by the total number of updating frequencies, which is 100 since the set-up was updated at each multiple of 5 Hz.

The initial values of the parameters to be updated were set to twice the exact values. The results of Table 1 are quite satisfying.

Note that the number of frequencies at which the set-up is to be updated depends on factors like the type of material that is characterized by the updated parameter. Indeed, some materials exhibit high frequency dependence (and thus need lots of updating frequencies) while others present quasi-frequency independent behaviour.

5.2.2. *Updating with measurement noise*

In this section, simulated noise is added to the computed measurements. The noise is obtained by multiplying the real and imaginary parts of each measurement by $(1 + w{*}N)$, where N is a random number chosen from a normal distribution with mean zero and variance one, and w is the weight applied to the normal distribution, and so the average noise level. The noise affects both the amplitude and the phase of the reference pressure field.

Table 2
Frequency average CRE and error on updated admittance coefficients with measurement noise

Unity	10^{-3} N^{-1} s^{-1} m^{3}	(%)
CRE		4.82
A_n1	$2 + 0.002\omega j$	3.39
A_n2	$1 - 0.0015\omega j$	4.55
A_n3	$3 + 0.001\omega j$	2.42
A_n4	$5 + 0.009\omega j$	4.05
A_n5	$4 - 0.008\omega j$	5.08

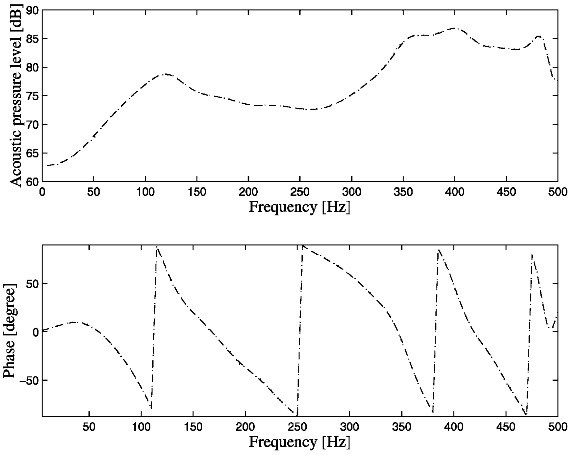

Fig. 6. Sound pressure FRF at the ear of the driver for 5 A_n with polluted measurements. Dashed line: reference FRF, dotted line: updated FRF.

Updating the set-up presented above with an average noise level of 5% (i.e., $w = 0.05$) generated results of Table 2. One observes that the error levels after updating are of the order of growth of the average noise level on the measurements.

Fig. 6 plots the amplitude and the phase of the FRF from 0 up to 500 Hz of the 2D car with the 5 previously defined admittance coefficients. The exact FRF together with that coming from the updating process with polluted data are plotted. As one can see, the 5% modified CRE level allows for quite a good match with the reference curve.

6. Conclusions

A new updating technique inspired from structural dynamics has been adapted to acoustics. The goal here is to update a continuous model with reference experimental data.

Based on the constitutive relation error that basically separates data into reliable and less reliable ones, the paper discusses this splitting in what concerns acoustic relations, boundary conditions, and experimental information. Attention is paid to the error coming from experimental measurements that is integrated to the technique which becomes the modified CRE.

The exposed technique applying to every kind of numerical approaches, yet the paper deals with one of these: the finite element formulation. The implemented technique is applied to simulate the sound pressure inside a two-dimensional car cabin with absorbing materials on the top and on the driver seat, which constitutes a first validation of the modified CRE technique in acoustics: the impedance values of the absorbing materials are accurately updated. Then, the modified CRE updating technique is successfully applied to simulate the sound pressure inside the same car cabin but with 5 absorbing materials defined by frequency-dependent complex admittance coefficients covering the boundaries.

The same validation is performed when adding simulated noise to the measurements, allowing the technique to still successfully fit the reference FRF with the updated one.

Since the modified CRE updating technique and its application for determining frequency dependent complex admittance coefficients is promising, three-dimensional real-life test cases are planned to be achieved, using more realistic models for the admittance coefficient. But model size reduction should precede the application of the technique presented in this paper to 3D model updating, due to the highly increasing computational time with the number of degrees of freedom, as shown in Ref. [13].

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