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Estimation of the absorption performance of multiple layer perforated panel systems by transfer matrix method

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Abstract

The absorption performance of single layer perforated panel system has been usually estimated by equivalent electro-acoustic circuit analysis based upon the analogy between electric circuit and acoustic system. In the case of multiple layer perforated panel system, however, the transfer matrix method is more convenient than the equivalent circuit analysis.

Hence, in this paper the transfer matrix method widely used for the one-dimensional acoustic analysis of engine exhaust muffler is presented. The absorption coefficient is estimated from the overall transfer matrix obtained by multiplying unit transfer matrices for perforated panels or airspaces. The proposed transfer matrix method is confirmed by comparing the estimated absorption coefficient with the measured value. In addition, the effect of dimension and arrangement of the perforated panels on the acoustical performance is discussed.

1. Introduction

Conventional sound absorbing materials such as glass fiber, polyester, and polyurethane foam have some disadvantages like hygiene, secondary pollution and fire problems. In order to overcome these problems, the perforated panel with airspace has been used.

The absorption performance of the perforated panel system depends upon the dimensions such as thickness, hole diameter and porosity, the depth of airspace and the number of the perforated panels [1–5]. Until recently, the absorption performances of perforated panel systems have been estimated by analytical approach [6–8] or equivalent electro-acoustic circuit approach [1,3,4,9]

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based upon the electro-acoustic analogy. By means of the circuit analysis, the resultant surface acoustic impedance is derived, and the absorption coefficient is estimated. In the case of multiple layer perforated panel system consisting of more than two perforated panels, however, the circuit analysis becomes very complicated.

Hence, the objective of this study is to employ the transfer matrix method [10] that has been widely used for the one-dimensional acoustic analysis of engine exhaust muffler. Using the method, the resultant surface acoustic impedance is estimated from the overall transfer matrix obtained by multiplying unit transfer matrices for perforated panels or airspaces. For the transfer matrix of perforated panel, the impedance has been obtained by empirically correcting the linear model by Rao and Munjal [11] developed for a grazing flow. The transfer matrix method is validated by comparing the calculated absorption coefficients with the values measured by the two-microphone impedance tube method for various design parameters. Both the analysis and the experiment are performed in the range of sound pressure level where the linear impedance model is valid. In addition, effect of the dimensions of perforated panel as well as the depth of airspace and the number of perforated panel on the acoustical performance is discussed.

2. Transfer matrix method

The schematic diagram for a transfer matrix representation of one-dimensional acoustical system element is shown in Fig. 1. With sound pressure p and particle velocity u at the upstream r and downstream $r+1$, the unit transfer matrix can be written as follows [10]:

$$\begin{bmatrix} p_r \\ u_r \end{bmatrix} = \begin{bmatrix} E_{11} & E_{12} \\ E_{21} & E_{22} \end{bmatrix} \begin{bmatrix} p_{r+1} \\ u_{r+1} \end{bmatrix}, \tag{1}$$

where E_{11}, E_{12}, E_{21} and E_{22} are the four-pole parameters (or transfer matrix elements).

For an airspace of depth l, as can be seen from Fig. 2, the transfer matrix is given by

$$\begin{bmatrix} S_{11} & S_{12} \\ S_{21} & S_{22} \end{bmatrix} = \begin{bmatrix} \cos kl & (\mathrm{j}\rho_0 c_0)\sin kl \\ (\mathrm{j}/\rho_0 c_0)\sin kl & \cos kl \end{bmatrix}, \tag{2}$$

where ρ_0, c_0 and k are the density of air, the speed of sound in air and the wave number.

Fig. 1. Four-pole parameters of an acoustic element.

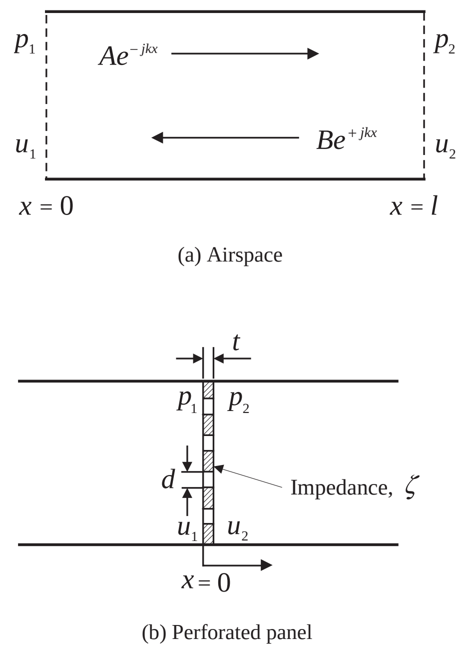

Fig. 2. Airspace and perforated panel.

In the case of a perforated panel, the transfer matrix can be expressed by

$$\begin{bmatrix} P_{11} & P_{12} \\ P_{21} & P_{22} \end{bmatrix} = \begin{bmatrix} 1 & \rho_0 c_0 \zeta \\ 0 & 1 \end{bmatrix}, \tag{3}$$

where ζ is the normalized acoustic impedance of the panel defined by $\rho_0 c_0 \zeta = \Delta p/u$.

The above expression is based upon the assumption that the panel thickness is so thin, compared with the acoustic wavelength, that phase difference of the particle velocity between both sides of the panel can be neglected.

For the case of a perforated panel with grazing flow, there is a impedance model well established by Rao and Munjal [11] as follows:

$$\begin{aligned} \zeta = {} & [7.337 \times 10^{-3}(1 + 72.23M) \\ & + \mathrm{j}2.2245 \times 10^{-5}(1 + 51t)(1 + 204d)f]/\sigma, \end{aligned} \tag{4}$$

where σ is porosity, t panel thickness in mm, d hole diameter in mm, f frequency in Hz, and M Mach number. The above model is valid in the linear range, where the particle velocity is not so high. The present situation is different from the case for the above model, in that the particle motion is normal to the plate with no mean flow.

Fig. 3 shows the experimental results for the absorption coefficients compared with estimations based on various corrections to the reaction part of the above impedance model by multiplication factor δ_X. From the figure, it is shown that estimations and experimental results show the best agreement for the correction factor of $\delta_X = 1.3$. Hence, impedance of the perforated panel has

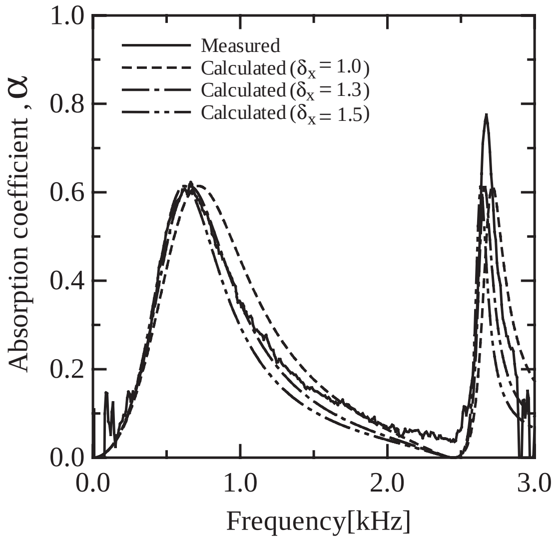

Fig. 3. Comparison of the estimated absorption coefficients using the multiplication correction factors with experimental results.

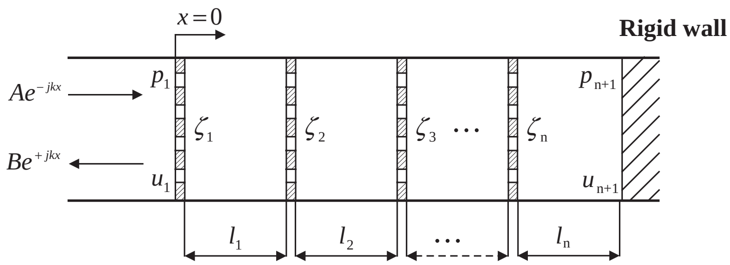

Fig. 4. Configuration of the multiple layer perforated panel system.

been fixed as follows for the present analysis:

$$\zeta = [7.337 \times 10^{-3} + j\delta_X \times 2.2245 \times 10^{-5}(1 + 51t)(1 + 204d)f]/\sigma. \tag{5}$$

The overall transfer matrix $[T]$ for a multiple layer perforated panel system shown in Fig. 4 can be obtained by multiplying all the unit transfer matrices for panel $[P]_n$ or airspace $[S]_n$ as follows:

$$[T] = [P]_1\,[S]_1 \cdots [P]_n\,[S]_n. \tag{6}$$

Then the state variables on the surface of the left end panel, number 1, can be expressed in terms of the overall transfer matrix and the variables at the right end rigid wall, number $n+1$, as follows:

$$\begin{bmatrix} p_1 \\ u_1 \end{bmatrix} = \begin{bmatrix} T_{11} & T_{12} \\ T_{21} & T_{22} \end{bmatrix} \begin{bmatrix} p_{n+1} \\ u_{n+1} \end{bmatrix}. \tag{7}$$

When the pressure amplitudes for the incident and reflected sound waves on the surface are A and B, respectively, the complex amplitudes of the pressure and particle velocity, that is, the state variables, on the surface of the acoustic system can be expressed in terms of matrix elements and the p_{n+1} and u_{n+1} for the right end plate, as follows:

$$A + B = p_1 = T_{11}p_{n+1} + T_{12}u_{n+1}, \tag{8a}$$

$$(A - B)/\rho_0 c_0 = u_1 = T_{21}p_{n+1} + T_{22}u_{n+1}. \tag{8b}$$

Since the particle velocity $u_{n+1} = 0$ on a rigid wall, the pressure reflection coefficient $\gamma = B/A$ can be expressed by the transfer matrix elements as

$$\gamma = \frac{T_{11} - \rho_0 c_0 T_{21}}{T_{11} + \rho_0 c_0 T_{21}}. \tag{9}$$

The reflection coefficient and the normalized acoustic impedance for normal incident wave, $z = (p_1/u_1)\rho_0 c_0$, has the following relationship:

$$\frac{z}{\rho_0 c_0} = \frac{1+\gamma}{1-\gamma}. \tag{10}$$

Since the sound absorption coefficient is defined by $\alpha = 1 - |\gamma|^2$, it can be expressed in terms of the normalized surface impedance as follows:

$$\alpha = \frac{4\,\mathrm{Re}(z/\rho_0 c_0)}{[1 + \mathrm{Re}(z/\rho_0 c_0)]^2 + [\mathrm{Im}(z/\rho_0 c_0)]^2}, \tag{11}$$

where Re and Im represent the real and imaginary parts, respectively.

Since the reactance is a function of frequency, the absorption coefficient becomes maximum at the frequency to make the reactance, $\mathrm{Im}(z/\rho_0 c_0)$, equal to zero, that is, the resonance frequency of the system.

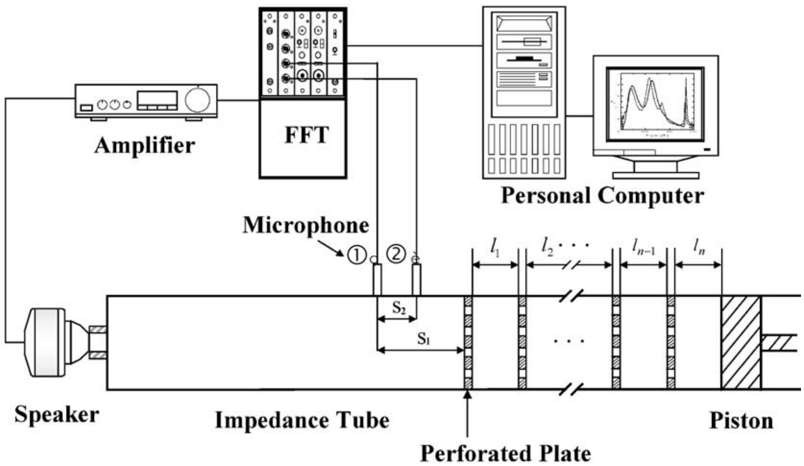

Fig. 5. Experimental set-up for normal sound absorption coefficient by impedance tube method.

3. Experimental set-up

Fig. 5 shows the experimental set-up for measuring the normal incident absorption coefficients of sound absorbing systems by the impedance tube method. The impedance tube is a rectangular acrylic pipe of cross-section 60×60 mm, length of 1000 mm, and thickness of 10 mm. The first cut-off frequency of the tube is 2970 Hz. Two microphones of 1/4 in pressure type (B&K Type 4938) are mounted flush with the inner surface of the tube. A loudspeaker is located at one end of the tube, and the perforated panel system is attached at the other end. A random sound generator is used to provide sound signal through an amplifier to the loudspeaker. The maximum frequency of the analyzer (B&K Type 2825) is 3200 Hz with 8 Hz resolution.

The transfer function of sound pressure is measured using two microphones mounted at two locations of the tube. The resultant surface acoustic impedance z of the multiple layer perforated

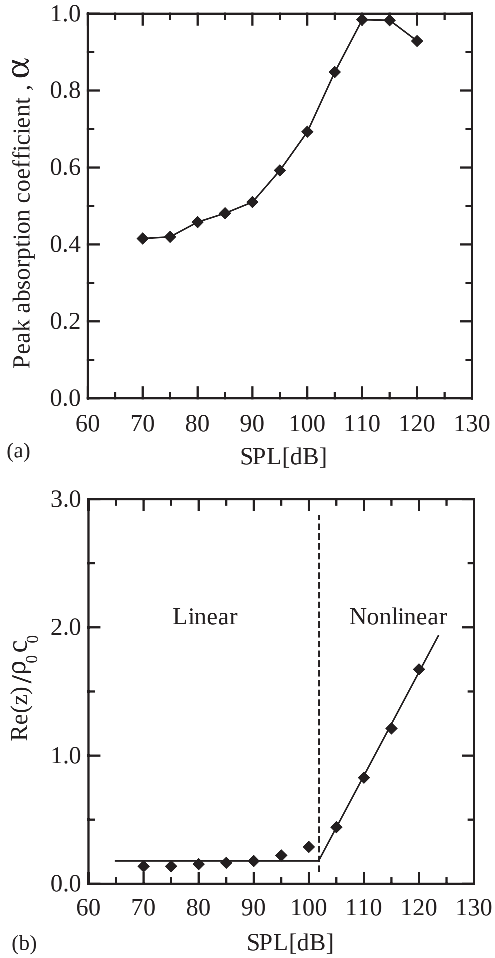

Fig. 6. Variation of the measured peak absorption coefficients and acoustic resistances as functions of incident sound pressure levels for a single layer perforated panel system.

panel system is obtained by substituting the measured transfer function $H(f)$ in the following expression [12]:

$$\frac{z}{\rho_0 c_0} = \mathrm{j}\frac{\sin[k(s_2+s_1)] - H(f)\sin(ks_1)}{H(f)\cos(ks_1) - \cos[k(s_2+s_1)]}, \tag{12}$$

where s_1 is the distance between the first microphone and the first perforated panel and s_2 is the distance between two microphones. In the present experiment, $s_1 = 100$ and $s_2 = 40$ mm are used.

By substituting Eq. (12) into Eq. (11), the sound absorption coefficients of multiple layer perforated panel systems are obtained. The perforated panels are made of steel plate and the holes are arranged in square arrays.

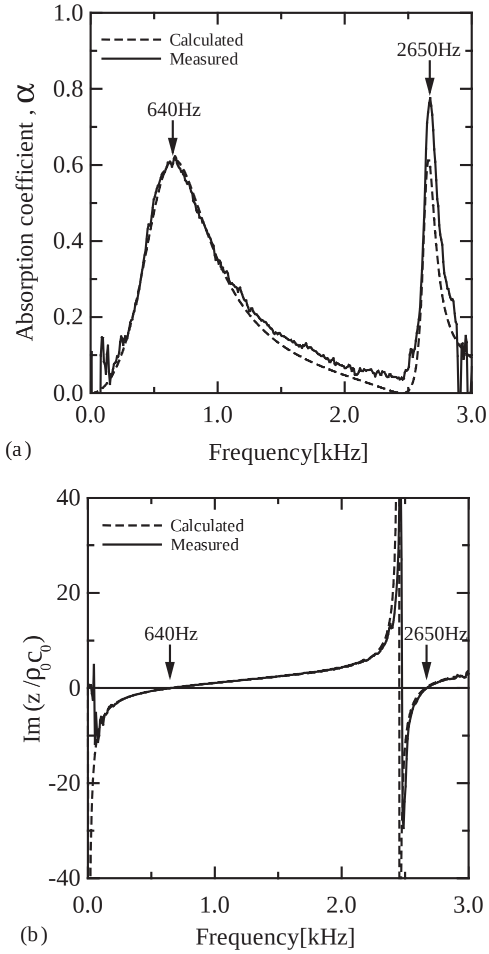

Fig. 7. Comparison of the calculated and measured values for absorption coefficient and acoustic reactance of a single layer perforated panel system.

4. Results and discussions

Fig. 6 shows the measured peak absorption coefficients and acoustic resistances as a function of incident sound pressure levels for a single layer perforated panel system, where thickness, hole diameter and porosity of the perforated panel are $t = 1\,\text{mm}$, $d = 2\,\text{mm}$ and $\sigma = 3.14\%$, respectively, and depth of the airspace is $l = 70\,\text{mm}$. In Fig. 6(a), the peak absorption coefficients increase with increasing the incident sound pressure levels up to 115 dB. The increase of absorption coefficients with sound pressure is due to the non-linear phenomenon at holes of the perforated panel. The relationship between the incident sound pressure levels and the normalized acoustic resistances is shown in Fig. 6(b). It can be shown that the acoustic resistances remain almost constant below the sound pressure levels of about 102 dB, but at higher levels they proportionately increase with increasing the incident sound pressure levels. It has been found in

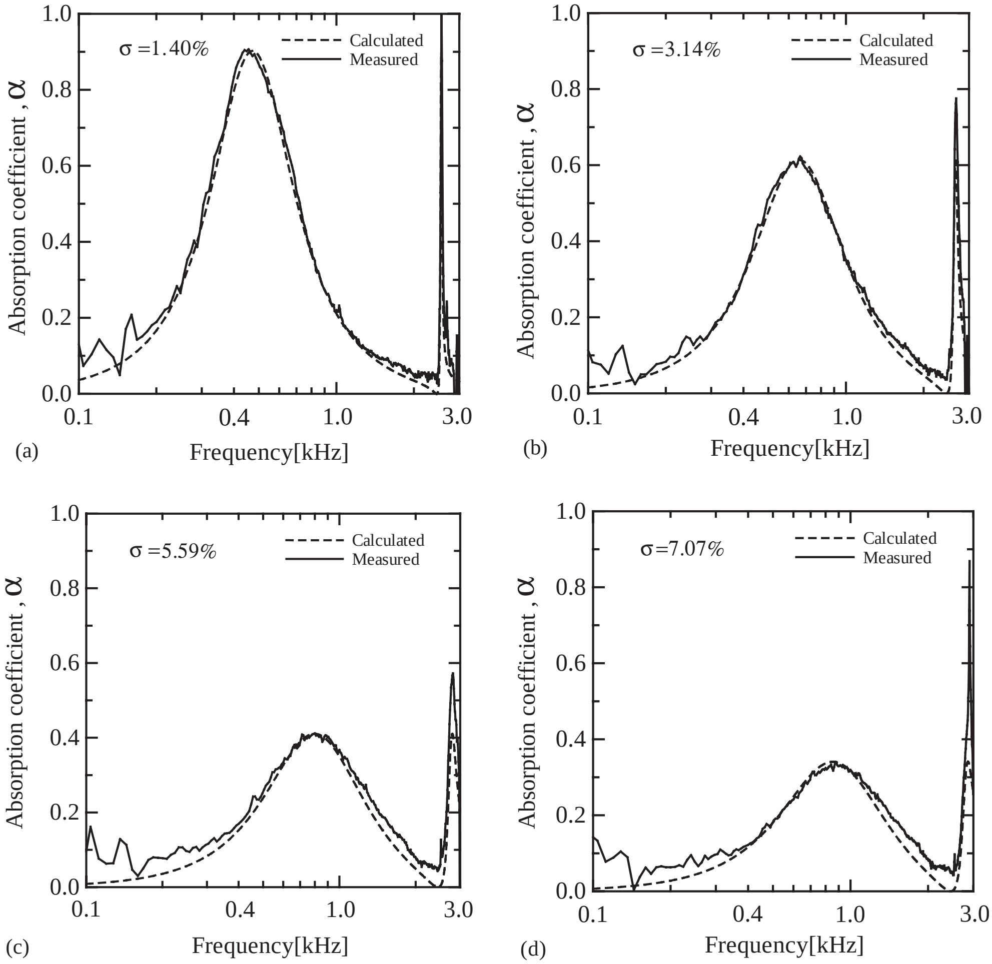

Fig. 8. Comparison between the measured and calculated absorption coefficients of a single layer perforated panel system for various porosities.

the previous research [13] that the boundary between linear and non-linear acoustic resistances depends on dimensions of the perforated panel such as hole diameter, porosity and thickness. Since the acoustic analysis of the present study is based upon a linear model, the incident sound pressure level in the impedance tube has been kept below about 102 dB.

In order to check the relationship between the resonance frequency and the absorption coefficient, the calculated and measured absorption coefficients as well as acoustic reactance are shown in Fig. 7, for the same system as that for Fig. 6. In Fig. 7(a), we can see that the peak absorption coefficients appear at 640 and 2650 Hz, which are exactly coincident with the resonance frequencies that make the acoustic reactance equal to zero in Fig. 7(b).

Fig. 8 shows the measured and calculated absorption coefficients of a single layer perforated panel system with the airspace of $l = 70$ mm for various porosities. It is shown that reducing porosity of the perforated panel system yields higher absorption coefficient and lower acoustic resonance frequency. As can be seen from Fig. 8, the calculated absorption coefficients show a

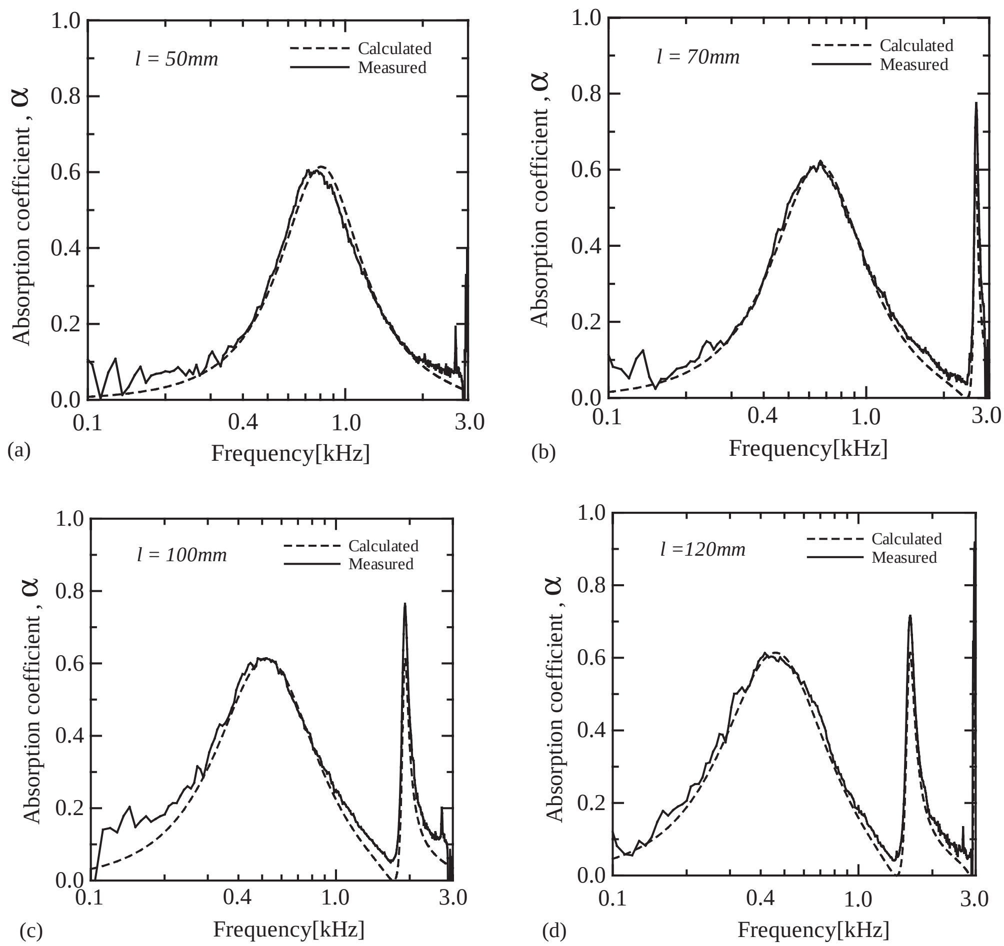

Fig. 9. Comparison between the measured and calculated absorption coefficients of a single layer perforated panel system for various depths of airspace.

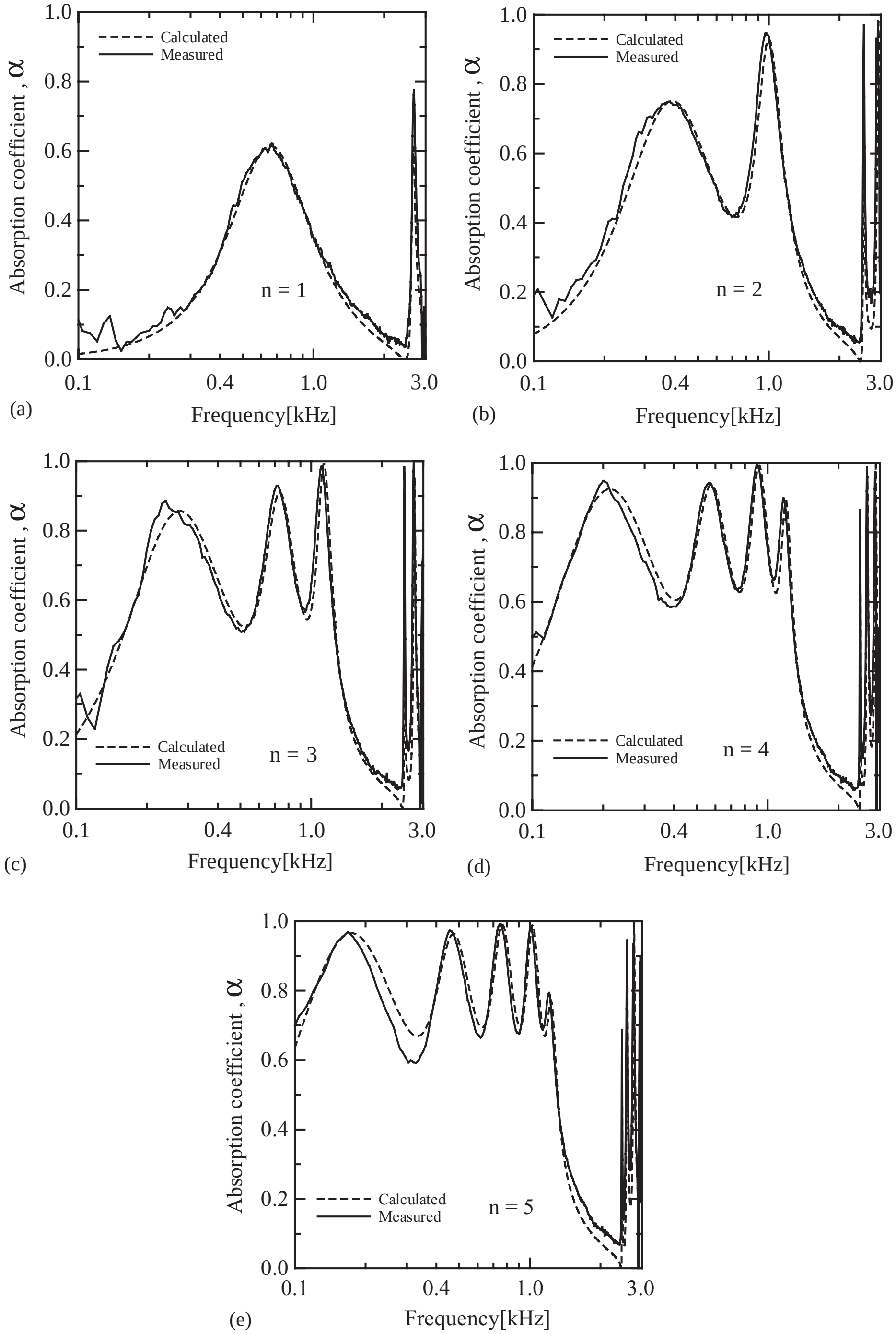

Fig. 10. Comparison between the measured and calculated absorption coefficients by varying the number of perforated panels.

good agreement with the measured values except for the results below about 200 Hz, due to phase error associated with spacing between the two microphones.

Fig. 9 shows the measured and calculated absorption coefficients for a single layer perforated panel system with porosity of $\sigma = 3.14\%$ for various depths of airspace. From the figure, we can see that the calculated results also agree well with the measured values. Only the fundamental resonance frequency decreases as the depth increases, while the peak absorption coefficient remains almost constant.

For the multiple layer perforated panel system, the calculated and measured absorption coefficients are compared in Fig. 10, for various number of perforated panels with porosity of 3.14% and airspace of $l = 70$ mm. As shown in Fig. 10, the measured absorption coefficients agree well with the calculated ones with a good accuracy. Increasing the number of perforated panels results in better absorption performance over broadband frequency ranges. It can be seen that the number of acoustic resonance frequencies at which acoustic absorptions become maximum is same as the number of panels.

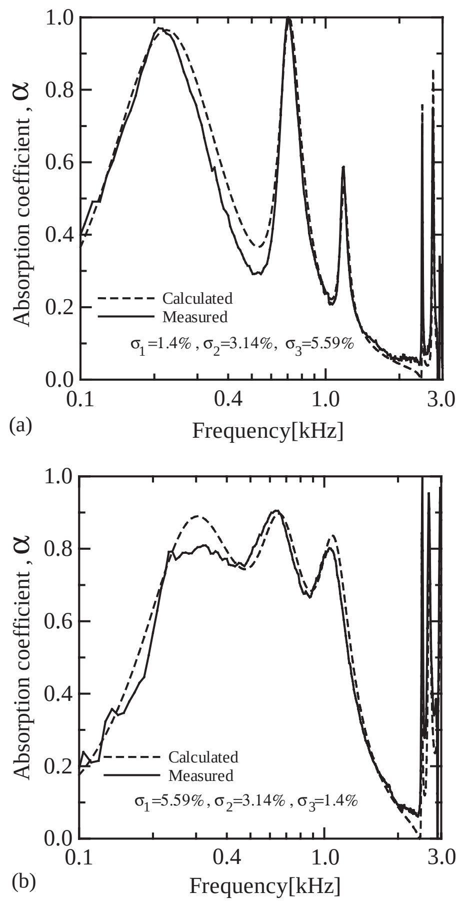

Fig. 11. Effect of the array of perforated panels on the absorption coefficient of three-layer perforated panel system.

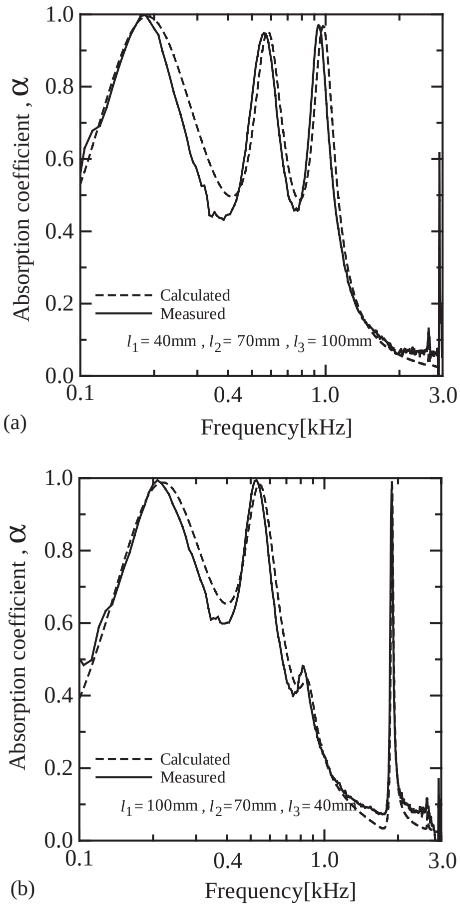

Fig. 12. Effect of the array of depths of airspace on the absorption coefficient of three-layer perforated panel system.

Fig. 11 shows the effect of the arrangement of panels with different porosity on the absorption coefficient of three-layer perforated panel system with the same airspaces of $l = 70\,\text{mm}$. The case for Fig. 11(b) shows better performance than that for Fig. 11(a). This phenomenon can be explained by the impedance-matching effect. Since the acoustic impedance of the panel increases with inverse proportion to the porosity, decrease in porosity results in increase of impedance of the system, so that sound reflections occur step-by-step through the panels and the absorption is maximized in the system. From the results discussed in Fig. 11, it is known that the appropriate arrangement of perforated panels with different porosity plays an important role for the performance of multiple layer perforated panel system.

Fig. 12 shows the effect of arrangement of the airspace on the absorption coefficient of the three-layer perforated panel system with same porosity of 1.40%. The performances are significantly influenced, especially in frequency characteristics, by the airspace arrangement, too.

5. Conclusions

The transfer matrix method has been employed to estimate the sound absorption coefficient for the perforated panel system. Formulations are derived for the absorption coefficient based upon impedance of the panel obtained by empirically correcting the existing model valid for grazing incidence. The method has been validated by comparing the calculated absorption coefficients with measured values by the two-microphone impedance tube method for various panel systems in the linear range with low- amplitude sound pressure. For all conditions examined, frequency spectrum of the calculated absorption coefficient has shown a close agreement with experimental results. Arrangement of the panels and the airspaces has been shown to have significant effects on the absorption performance of perforated panel system.

Appendix A. Nomenclature

c_0 speed of sound
d hole diameter
f frequency
$H(f)$ transfer function
j complex number ($=\sqrt{-1}$)
k wave number
l length of an acoustic element
p sound pressure
$[P]$ transfer matrix for a perforated panel
$[S]$ transfer matrix for an airspace
s_1 distance from the first microphone to the first perforated panel
s_2 distance between two microphones
t panel thickness
$[T]$ overall transfer matrix for a multiple layer perforated panel system
u acoustic particle velocity
z resultant surface acoustic impedance of a perforated panel system
α normal sound absorption coefficient
γ pressure reflection coefficient
δ_X multiplication correction factor of the acoustic reactance for a perforated panel
ζ specific acoustic impedance of a perforated panel
ρ_0 density in air
σ porosity

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