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Transient responses of an axially accelerating viscoelastic string constituted by a fractional differentiation law

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Abstract

This paper deals with the transverse vibration of an initially stressed moving viscoelastic string obeying a fractional differentiation constitutive law. The governing equation is derived from Newtonian second law of motion, and reduced to a set of non-linear differential–integral equations based on Galerkin's truncation. A numerical approach is proposed to solve numerically the differential–integral equation through developing an approximate expression of the fractional derivatives involved. Some numerical examples are presented to highlight the effects of viscoelastic parameters and frequencies of parametric excitations on the transient responses of the axially moving string.

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1. Introduction

Axially moving strings can represent many engineering devices such as power transmission belts, plastic films, magnetic tapes, paper sheets, and textile fibers. Much research has been done to study transverse vibrations of such systems, which has been reviewed by Wickert and Mote [1] and Chen and Zu [2]. One major problem is the occurrence of large transverse vibrations due to tension or axial speed variation termed as parametric vibrations.

As viscoelastic damping is becoming widely applied for vibration and noise suppression in various industries, there are several papers dealing with transverse parametric vibrations of axially moving viscoelastic strings. Fung et al. [3] studied the transient motion of an axially moving viscoelastic string constituted by the Boltzmann superposition principle. They applied the Galerkin truncation based on

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the basis of stationary string eigenfunctions to obtain a set of ordinary differential–integral equations, and then used the finite difference integration to calculate approximately the related integrals to transform the ordinary differential–integral equations into the ordinary differential equations. Fung et al. [4] used the Galerkin method to study numerically the case that the standard linear solid model constitutes the accelerating string. Zhang and Zu [5,6] presented closed-form solutions for the amplitude and the existence conditions of non-trivial solutions of the summation resonance of the axially moving viscoelastic string described by the Kelvin model, and determined the stability of the trivial solution and the non-trivial solutions. Investigating the same problem in Ref. [3], Zhang and Zu [7] used the 1-term Galerkin method to discretize the governing equation based on the translating eigenfunctions instead of stationary eigenfunctions. Zhao and Chen [8] developed a numerical algorithm to simulate the non-linear parametric vibration of an axially moving viscoelastic string constituted by the standard linear solid model or the Maxwell model. They used the finite difference to discrete spatial variables, and deduced a model defined by a large set of differential–algebraic equations. Chen and Zu [9] and Chen et al. [10] studied analytically parametric vibration at the principal resonance and the summation resonance, respectively, of the axially moving viscoelastic string constituted by the Boltzmann superposition principle. Chen et al. [11,12] investigated bifurcation and chaos in transverse motion of an axially moving viscoelastic string based on 2- and 4-term Galerkin truncations, respectively.

All available studies [3–12] are concentrated on axially moving strings constituted by the differential relationships such as the Kelvin model [5,6,11,12], the Maxwell model [8] and the standard linear solid model [4,8] and the integral relationship, the Boltzmann superposition principle [3,7,9,10]. In fact, in Refs. [3,7,9,10] all used the exponential function as the relaxed function in the integral constitution law. For certain viscoelastic materials such as synthetic rubber and synthetic fiber, the relaxed function of the Abelian type with a weak singularity describes the features of viscoelasticity more appropriately [13]. Therefore, the fractional differentiation law, a integral relationship with a weakly singular kernel, is used to constitute those materials. Although many vibration and wave problems were investigated for the continua constituted by the fractional differentiation [14], the literature that is specially related to axially moving strings is very limited. To address the lack of research in this aspect, this paper studies the transient responses of an axially accelerating viscoelastic string constituted by a fractional differentiation law.

This paper adopts the fractional differentiation relationship to constitute the axially moving string. The governing equation of transverse vibration is derived from Newton's second law. Lagrangian strain is employed to account non-linearity due to finite stretching. Galerkin's method is applied to truncate the governing equation, a non-linear partial-differential–integral equation, into a set of differential–integral equations. Based on approximate calculations of the fractional derivatives concerned, a numerical approach is developed to solve those differential–integral equations. The numerical approach is used to analyze the effects of viscoelastic parameters on the transient responses of axially moving strings.

2. Problem formulations

Consider a uniform, flexible, axially moving viscoelastic string of density ρ , area of cross-section A , initial tension P , and uniform transport speed c that travels between two fixed ends

separated by distance L . The speed c is not constant, but a prescribed function of time T . Several simplifying assumptions are made as follows: (1) only transverse motion in the y direction is taken into consideration; (2) Lagrangian strain for string is employed as a finite measure of the strain; (3) the viscoelastic string is in a state of uniform initial stress, and the initial tension is rather large; (4) only geometric non-linearity due to finite stretching is considered through Lagrangian strain.

Based on the above assumptions, the equation of motion in the transverse direction can be derived from Newton’s second law

$$\begin{aligned} \rho A \left(\frac{\partial^2 U(X, T)}{\partial T^2} + 2c \frac{\partial^2 U(X, T)}{\partial X \partial T} + \frac{dc}{dT} \frac{\partial U(X, T)}{\partial X} + c^2 \frac{\partial^2 U(X, T)}{\partial X^2} \right) \\ = P \frac{\partial^2 U(X, T)}{\partial X^2} + \frac{\partial}{\partial X} \left(A \sigma(X, T) \frac{\partial U(X, T)}{\partial X} \right), \end{aligned} \tag{1}$$

where $U(X, T)$ is the displacement in the transverse direction, X is the spatial Cartesian coordinate in the axial direction and $\sigma(X, T)$ is the perturbed stress. The transverse acceleration is compounded by the relative acceleration, the Coriolis acceleration, and the convected acceleration.

A fractional differentiation constitutive law is chosen to describe the viscoelastic property of the string material. Experiments demonstrated that it is a good model for many materials such as synthetic rubbers and synthetic fibers [13]. For such materials, the stress–strain relation is

$$\sigma(X, T) = E_0 \varepsilon_L(X, T) + \eta_0 D_T^\alpha (\varepsilon_L(X, T)) \quad (0 < \alpha < 1), \tag{2}$$

where $\sigma(X, T)$ is the perturbed strain in the axial direction, $\varepsilon_L(X, T)$ is the perturbed Lagrangian strain component, E_0 is the stiffness constant of the string, η_0 is the dynamic viscosity and the Riemann–Liouville α order fractional differentiation operator with respect to T is defined by [15]

$$D_T^\alpha (f) = \frac{1}{\Gamma(1 - \alpha)} \frac{d}{dT} \int_0^T \frac{f(\tau) d\tau}{(T - \tau)^\alpha} \tag{3}$$

in which Γ is the Gamma function. For strings with finite amplitude, the perturbed Lagrangian strain component in the axial direction related to the transverse displacement is given by

$$\varepsilon_L(X, T) = \frac{1}{2} \left(\frac{\partial U(X, T)}{\partial X} \right)^2. \tag{4}$$

Substituting Eqs. (2)–(4) into Eq. (1) leads to the governing equation of transverse vibration of the string:

$$\begin{aligned} \rho \frac{\partial^2 U(X, T)}{\partial T^2} + 2\rho c \frac{\partial^2 U(X, T)}{\partial T \partial X} + \left(\rho c^2 - \frac{P}{A} \right) \frac{\partial^2 U(X, T)}{\partial X^2} + \rho \frac{dc}{dT} \frac{\partial U(X, T)}{\partial X} \\ = \left(\frac{3E_0}{2} \left(\frac{\partial U(X, T)}{\partial X} \right)^2 + \frac{\eta_0}{2} D_T^\alpha \left(\left(\frac{\partial U(X, T)}{\partial X} \right)^2 \right) \right) \frac{\partial^2 U(X, T)}{\partial X^2} \\ + \eta_0 \frac{\partial U(X, T)}{\partial X} D_T^\alpha \left(\frac{\partial U}{\partial X} \frac{\partial^2 U(X, T)}{\partial X^2} \right). \end{aligned} \tag{5}$$

The boundary conditions are assumed to be homogeneous

$$U(0, T) = U(L, T) = 0. \quad (6)$$

To model the parametric vibration experimentally observed, some researchers introduced the initial tension as a parametric excitation [3–12]. Following their practices, this paper assumes that the initial tension $P(t)$ is characterized as a small periodic perturbation $P_1 \cos(\Omega t)$ superimposed on the steady state tension P_0 , i.e. $P = P_0 + P_1 \cos(\Omega t)$, which is the same as that in previous researches.

For the convenience of analysis, introduce the following non-dimensional variables and parameters:

$$\begin{aligned} u &= \frac{U}{L}, & x &= \frac{X}{L}, & t &= \frac{T}{L} \sqrt{\frac{P_0}{\rho A}}, & \gamma &= c \frac{\rho A}{P_0}, & \omega &= \Omega L \sqrt{\frac{\rho A}{P_0}}, \\ v &= \frac{P_1}{P_0}, & \eta &= \frac{\eta_0}{L^\alpha} \sqrt{\left(\frac{P_0}{\rho A}\right)^\alpha}, & e_0 &= \frac{A}{P_0} E_0. \end{aligned} \quad (7)$$

Then one obtains the non-dimensional governing equations of transverse motion

$$\mathbf{L}u = 0, \quad (8)$$

where

$$\begin{aligned} \mathbf{L}u &= \frac{\partial^2 u(x, t)}{\partial t^2} + 2\gamma \frac{\partial^2 u(x, t)}{\partial t \partial x} + (\gamma^2 - 1 - v \cos(\omega t)) \frac{\partial^2 u(x, t)}{\partial x^2} + \frac{d\gamma}{dt} \frac{\partial u(x, t)}{\partial x} \\ &\quad - \frac{3e_0}{2} \left(\frac{\partial u(x, t)}{\partial x} \right)^2 \frac{\partial^2 u(x, t)}{\partial x^2} - \frac{\eta}{2} D_t^\alpha \left(\left(\frac{\partial u(x, t)}{\partial x} \right)^2 \right) \frac{\partial^2 u(x, t)}{\partial x^2} \\ &\quad - \eta \frac{\partial u(x, t)}{\partial x} D_t^\alpha \left(\frac{\partial u(x, t)}{\partial x} \frac{\partial^2 u(x, t)}{\partial x^2} \right). \end{aligned} \quad (9)$$

The boundary conditions in non-dimensional form are

$$u(0, t) = u(1, t) = 0. \quad (10)$$

3. Method of solution

Eq. (8) is a non-linear partial differential–integral equation. It is impossible to get its exact analytical solution. To obtain its numerical solution, the Galerkin method is applied to discretize the spatial variable. Under the homogenous boundary condition (6), the solution of Eq. (8) can be expanded into the following trial function:

$$u(x, t) = \sum_{n=1}^{\infty} \varphi_n(t) \sin(n\pi x), \quad (11)$$

where the $\varphi_n(t)$ are generalized displacements, and $\sin(n\pi x)$ is the n th eigenfunction of the simply supported stationary string. Define the inner product

$$\langle f, g \rangle = \int_0^1 f(x)g(x) dx. \tag{12}$$

If one chooses also the stationary string eigenfunctions as the weighting functions $\psi_m(\xi)$

$$\psi_m(x) = \sin(m\pi x). \tag{13}$$

The Galerkin method implies that the generalized displacements $\varphi_n(t)$ can be obtained by solving the following ordinary differential–integral equation system:

$$\langle Lu, \psi_m \rangle = 0 \quad (m = 1, 2, \dots). \tag{14}$$

Inserting Eqs. (11) and (13) into Eq. (14) and calculating the resulting inner products defined by Eq. (12), one gets the explicit form of Eq. (14):

$$\begin{aligned} &\ddot{\varphi}_m + 4\gamma \sum_{j \neq m} b_{mj} \dot{\varphi}_j - m^2 \pi^2 (\gamma^2 - 1 - v \cos(\omega t)) \varphi_m + 2\gamma \sum_{j \neq m} b_{mj} \varphi_j \\ &= 3e_0 \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{l=1}^{\infty} d_{mijl} \varphi_i \varphi_j \varphi_l \\ &\quad + \eta \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{l=1}^{\infty} (d_{mijl} + 2d_{milj}) \varphi_l D_t^\alpha(\varphi_i \varphi_j), \quad (m = 1, 2, \dots), \end{aligned} \tag{15}$$

where

$$b_{ml} = \begin{cases} \frac{ml(1 - \cos(m\pi) \cos(l\pi))}{m^2 - n^2}, & m \neq l, \\ 0, & m = l, \end{cases} \quad d_{mijl} = \begin{cases} \frac{1}{8} ij l^2 \pi^4, & m = l \pm i \pm j, \\ -\frac{1}{8} ij l^2 \pi^4, & m = i \pm j - l \text{ or } j - i - l, \\ 0 & \text{all other cases.} \end{cases} \tag{16}$$

To transform the differential–integral equations (15) into a set of ordinary differential equation, one has to deal with the terms $D_t^\alpha(\varphi_i \varphi_j)$ included in Eq. (15). Integrating by parts, one can rewrite the fractional derivative terms as

$$D_t^\alpha(\varphi_i \varphi_j) = \frac{1}{\Gamma(1 - \alpha)} \int_0^t \frac{(\varphi_i \varphi_j)'(t - \tau)}{\tau^\alpha} d\tau + \frac{\varphi_i(0)\varphi_j(0)}{\Gamma(1 - \alpha)t^\alpha}. \tag{17}$$

Consider a subinterval $[t_k, t_{k+1}] \subset [0, 1]$. For $t_k < t \leq t_{k+1}$, the central difference approximation leads to

$$\begin{aligned} \int_0^t \frac{(\varphi_i \varphi_j)'(t - \tau)}{\tau^\alpha} d\tau &= \sum_{s=0}^{k-1} \int_{t_s}^{t_{s+1}} \frac{(\varphi_i \varphi_j)'(t - \tau)}{\tau^\alpha} d\tau + \int_{t_k}^t \frac{(\varphi_i \varphi_j)'(t - \tau)}{\tau^\alpha} d\tau \\ &= \sum_{s=0}^{k-1} (\varphi_i \varphi_j)[t_{k-s-1}, t_{k-s}] A_s + \int_{t_k}^t \frac{(\varphi_i \varphi_j)'(t - \tau)}{\tau^\alpha} d\tau + O(kh^3 + h^{3-\alpha}), \end{aligned} \tag{18}$$

where

$$\begin{aligned}
 (\varphi_i \varphi_j)[t_{k-s-1}, t_{k-s}] &= \frac{\varphi_i(t_{k-s-1})\varphi_j(t_{k-s-1}) - \varphi_i(t_{k-s})\varphi_j(t_{k-s})}{t_{k-s-1} - t_{k-s}}, \\
 A_s &= \int_{t_s}^{t_{s+1}} \tau^{-\alpha} d\tau = \frac{t_{s+1}^{1-\alpha} - t_s^{1-\alpha}}{1 - \alpha}, \quad h = \max_k(t_{k+1} - t_k).
 \end{aligned}
 \tag{19}$$

Denote

$$h_k(t) = \int_{t_k}^t \frac{(\varphi_i \varphi_j)'(t - \tau)}{\tau^\alpha} d\tau.
 \tag{20}$$

Then $h_k(t_k) = 0$. The Taylor series expansion at $t = t_k$ yields

$$h_k(t) = \frac{(\varphi_i \varphi_j)'(0)}{t_k^\alpha} (t - t_k) + O(h^2).
 \tag{21}$$

Substituting of Eqs. (18) and (21) into Eq. (17) and dropping the higher order terms, and then inserting the resulting equation into Eq. (15) give

$$\begin{aligned}
 &\ddot{\varphi}_m + 4\gamma \sum_{j \neq m} b_{mj} \dot{\varphi}_j - m^2 \pi^2 (\gamma^2 - 1 - v \cos(\omega t)) \varphi_m + 2\dot{\gamma} \sum_{j \neq m} b_{mj} \varphi_j \\
 &= 3e_0 \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{l=1}^{\infty} d_{mijl} \varphi_i \varphi_j \varphi_l + \frac{\eta}{\Gamma(1 - \alpha)} \sum_{i=1}^{\infty} \sum_{j=1}^{\infty} \sum_{l=1}^{\infty} (d_{mijl} + 2d_{milj}) \varphi_l \left\{ \frac{\varphi_i \varphi_j(0)}{t^\alpha} \right. \\
 &\quad \left. + (\varphi_i \varphi_j)'(0) \frac{t - t_k}{t_k^\alpha} - \sum_{s=0}^{k-1} A_s (\varphi_i \varphi_j)[t_{k-s-1}, t_{k-s}] \right\} \\
 &(m = 1, 2, \dots, \quad t_k < t \leq t_{k+1}, \quad k = 0, 1, 2, \dots).
 \end{aligned}
 \tag{22}$$

Although the difference quotient terms $(\varphi_i \varphi_j)[t_{k-s-1}, t_{k-s}]$ make Eq. (22) not a usual set of ordinary differential equations, the convenient numerical solution methods, such as the fourth order Runge–Kutta routine, can be applied for given initial conditions providing that $t_{k-s-1} - t_{k-s}$ is treated as an integration step.

If the initial values of Eq. (8) are given as

$$u(x, 0) = a(x), \quad \frac{\partial u(x, 0)}{\partial t} = b(x),
 \tag{23}$$

then the initial values of Eq. (22) are

$$\varphi_m(0) = \sqrt{2} \int_0^1 f(x) \sin(m\pi x) dx, \quad \dot{\varphi}_m(0) = \sqrt{2} \int_0^1 f_1(x) \sin(m\pi x) dx \quad (m = 1, 2, \dots).
 \tag{24}$$

4. Numerical results and discussions

In this section, a few numerical examples are presented to demonstrate the effects of related parameters on transient responses of transverse vibration of the axially moving viscoelastic string.

In all cases, the initial conditions of Eq. (8) are prescribed as

$$u(x, 0) = 0.1x(1.0 - x), \quad \frac{\partial u(x, 0)}{\partial t} = 0. \tag{25}$$

Some researches [3,4,7] assumed that the transport speed is characterized as a small simple harmonic variation about the constant mean speed, i.e.

$$\gamma(t) = \gamma_0 + \gamma_1 \cos(\omega_0 t). \tag{26}$$

The assumption has its physical meaning. For example, if the axially moving string models a belt on a pair of rotating pulleys, the rotation vibration of the pulleys will result in a small fluctuation in the axial speed of the belt. Thus the authors adopt the assumption and consider the axial speed given by Eq. (26).

First of all, the authors try to find how many terms in Eq. (11) are needed to be obtained plausible results. To do this, we compare the transient responses numerically calculated from the 1-, 2-, 3- and 4-term Galerkin truncation. Substitution of Eq. (25) into Eq. (24) yields the initial conditions of the truncated systems as

$$\varphi_1(0) = \frac{\sqrt{2}}{5\pi^3}, \quad \varphi_2(0) = 0, \quad \varphi_3(0) = \frac{\sqrt{2}}{35\pi^3}, \quad \varphi_4(0) = 0, \quad \dot{\varphi}_i(0) = 0 \quad (i = 1, 2, 3, 4). \tag{27}$$

The time histories of the center displacement of the string with a constant speed and tension or a varying speed and tension are, respectively, illustrated in Figs. 1 and 2, in which the parameters are, respectively, chosen as

$$e_0 = 0.1, \quad \alpha = 0.5, \quad \eta = 0.1, \quad \gamma_0 = 0.5, \quad \gamma_1 = 0, \quad v = 0, \tag{28}$$

$$e_0 = 10, \quad \alpha = 0.5, \quad \eta = 2.0, \quad \gamma_0 = 0.8, \quad \gamma_1 = 0.1, \quad \omega_0 = 0.2, \\ v = 0.2, \quad \omega = 1.0. \tag{29}$$

In both Figs. 1 and 2, the time histories obtained from 1-, 2-, 3- and 4-term Galerkin truncated systems are, respectively, represented by dotted, dash-dot, dashed and solid lines. The results

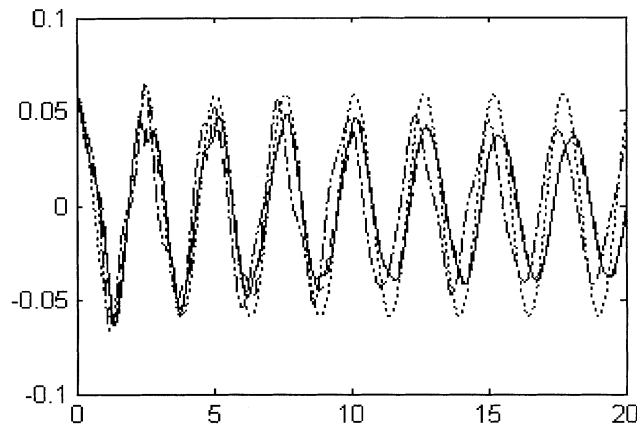


Fig. 1. The time histories based on 1-, 2-, 3- and 4-term truncated systems with parameters (28).

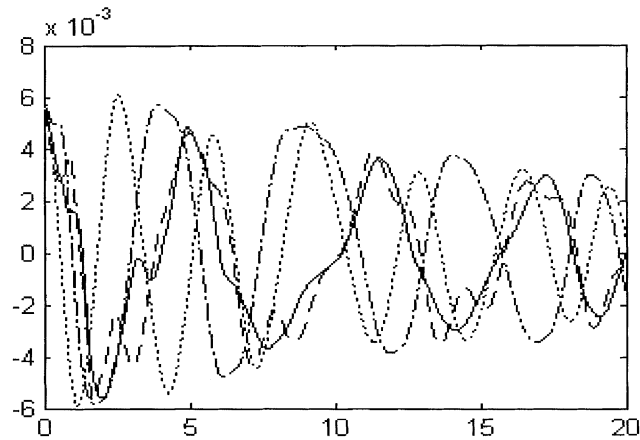


Fig. 2. The time histories based on 1-, 2-, 3- and 4-term truncated systems with parameters (29).

indicate that the difference between the dashed lines and the solid lines are rather small. Therefore, it is inferred that the 4-term Galerkin truncation yields good enough approximation. In the following calculations, the 4-term Galerkin truncation will be used.

Now we analyze numerically the effects of viscoelastic power parameter α and viscoelastic coefficient η , in their non-dimensional forms, on the transient responses of the center of the axially moving strings. Fig. 3 shows the time histories of the center displacements for parameter α at the different values 0.1 (solid line), 0.5 (dashed-dot line) and 0.9 (dashed line), while other parameters are fixed as

$$e_0 = 10, \quad \eta = 50, \quad \gamma_0 = 0.5, \quad \gamma_1 = 0.1, \quad \omega_0 = 0.4, \quad v = 0.2, \quad \omega = 1.0. \quad (30)$$

Fig. 4 shows the time histories of the center displacements for different values of parameter η at the different values 5 (solid line), 25 (dash-dot line) and 50 (dashed line), while other parameters are fixed as

$$e_0 = 10, \quad \eta = 0.1, \quad \gamma_0 = 0.5, \quad \gamma_1 = 0.1, \quad \omega_0 = 0.4, \quad v = 0.2, \quad \omega = 1.0. \quad (31)$$

In both cases, the larger viscoelastic parameters result in the smaller amplitudes of the transient responses, which is physically sound since the larger viscoelasticity causes more energy dissipation. Besides, Fig. 4 indicates that the periods of the transient responses decrease with the increase of viscoelastic coefficient η , but, in Fig. 3, viscoelastic power parameter α has no significant effect on the periods.

Finally, we study numerically the effects of frequencies of parametric excitations on the transient responses of the center of the axially moving strings. In the present investigation, the parametric excitations are modeled as harmonic variations of the tension and the axial speed. Time histories of the center displacements are depicted in Fig. 5 for varying frequencies of axial speed fluctuation $\omega_0 = 0.4, 1.6, 2.8$, represented, respectively, by the solid, dash-dot, and dashed lines, in which all other parameters are chosen as

$$e_0 = 10, \quad \eta = 50, \quad \alpha = 0.1, \quad \gamma_0 = 0.5, \quad \gamma_1 = 0.1, \quad v = 0.2, \quad \omega = 1.0. \quad (32)$$

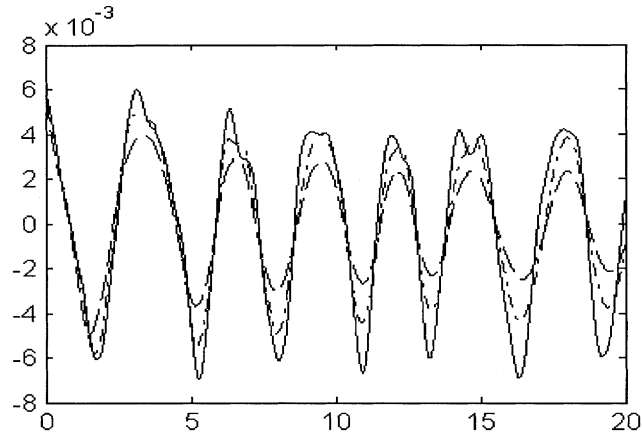


Fig. 3. Effects of the viscoelastic power parameter on the transient responses.

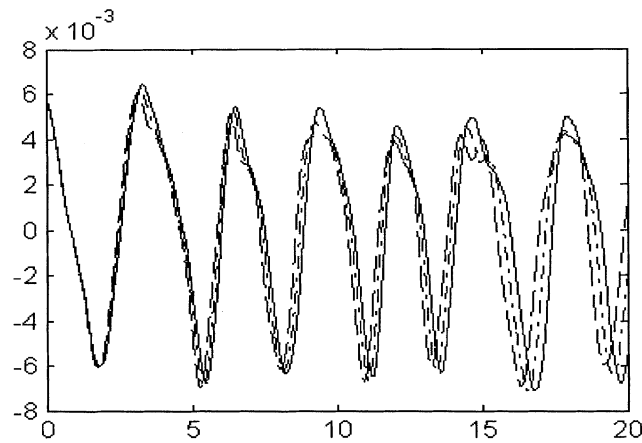


Fig. 4. Effects of the viscoelastic coefficient on the transient responses.

Time histories of the center displacements are depicted in Fig. 6 for varying frequencies of tension fluctuation $\omega = 0.4, 1.6, 2.8$, represented, respectively, by the solid, dash-dot, and dashed lines, in which all other parameters are chosen as

$$e_0 = 10, \quad \eta = 50, \quad \alpha = 0.1, \quad \gamma_0 = 0.5, \quad \gamma_1 = 0.1, \quad \omega_0 = 1.6, \quad \nu = 0.2. \quad (33)$$

Numerical results indicate that the frequencies of parametric excitations influence both the amplitudes and the periods of the transient responses. Fig. 6 also indicates that instability may occur for a certain set of parameters, which is the dramatic difference between the free vibration and the parametric vibration. In the case that both the axial speed and the initial tension are constant, due to the damping of the viscoelasticity, the free vibration of the string will die out as shown in Fig. 1. Contrastively, the amplitude of the parametric vibration of the string, resulted from the time-varying axial speed or tension, may increase with the time. Fung et al. [3] first found the unstable phenomenon in the parametric vibration of a viscoelastic moving string constituted

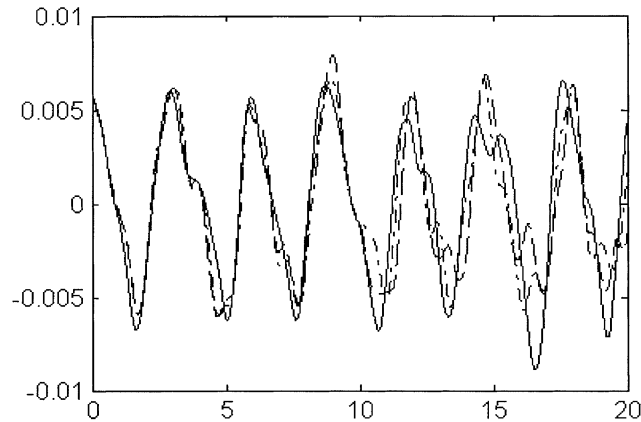


Fig. 5. Effects of the frequencies of the axial speed variations on the transient responses.

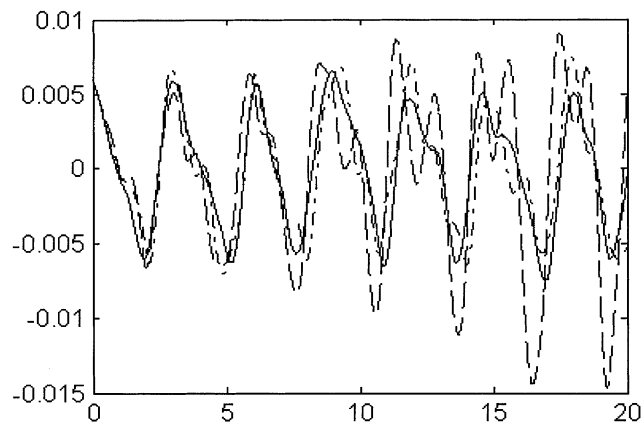


Fig. 6. Effects of the frequencies of the tension variations on the transient responses.

by the Boltzmann superposition principle. Present investigation also found such instability in the parametrically excited viscoelastic string constituted by the fractional differentiation constitutive law.

5. Conclusions

This paper treats the transient transverse response of a moving viscoelastic string constituted by a fractional differentiation relationship. Lagrangian strain is used to account geometric non-linearity due to the finite deflection of the string. The governing equation is derived from Newtonian second law of motion, and reduced to a set of non-linear differential–integral equations based on Galerkin’s truncation. A numerical approach is proposed to solve numerically

the differential–integral equation through developing an approximate expression of the fractional derivatives involved. Some numerical examples are presented to demonstrate the effects of viscoelastic parameters and frequencies of parametric excitations on the transient responses of the axially moving string. Numerical results indicate that the amplitudes of transient responses decrease with the increase of both the viscoelastic power parameter and the viscoelastic coefficient, while the periods of transient response decrease only with the increase of the viscoelastic coefficient. In addition, numerical results indicate that transverse vibrations may become unstable when the frequencies of parametric excitations change.

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