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Short Communication

Quenching mode-coupling friction-induced instability using high-frequency dither

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1. Introduction

For self-excited friction-induced oscillations, essentially four different excitation mechanisms have been described in literature: First, a friction coefficient decreasing with relative sliding velocity may lead to negative damping and consequently to an oscillatory instability of the steady sliding state. Second, mode-coupling (sometimes also referred to as binary flutter or displacement-dependent friction force instability) may destabilize the steady sliding state also for constant friction coefficients. Third, sprag slip, and fourth the follower force nature of the friction force have been identified as fundamental mechanisms for friction self-excited vibrations. All of these mechanisms are amply described in literature [1–8], a recent survey on the excitation mechanisms may be found in Ref. [9], a further discussion is therefore not given here.

In the course of searching for measures to suppress friction self-excited vibrations arising from these mechanisms, the idea of active control by superimposing some sort of force- or amplitude-excitation has always been a very attractive one; indeed it has been known for a long time that superimposing ultrasonic vibrations markedly affects the friction characteristics of many technical systems. Rather recently the approach of this so-called dither, i.e. the application of superimposed high-frequency excitation to affect the low-frequency behaviour of the

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system, has been applied successfully—at least in theoretical modelling—to suppress or quench the friction self-excited vibrations arising from a falling friction characteristic (confer especially the pioneering work of Thomsen [10]) and to stabilize a friction affected externally driven nonlinear oscillator [11]. Also experimental validation for the effect of dither has recently been obtained [12]. Therefore, in the present paper the effect of dither on the mode-coupling friction induced instability is investigated.

2. The model problem

Mode-coupling instability is essentially a two-degree-of-freedom phenomenon, it is therefore sufficient to formulate a simple two-degree-of-freedom model and investigate its dynamics and response to dither. A graphical interpretation of the model of the present paper is given in Fig. 1. The model may be thought of as a single-point mass sliding over a conveyor belt, mainly held in position by two linear springs k_1 and k_2 parallel and normal to the belt surface, k_2 may be regarded as the physical contact stiffness between the objects in relative sliding motion. Moreover, there is another linear spring k (oriented at an oblique angle of 45° relative to the normal direction) leading to off-diagonal entries in the model's stiffness matrix, which has already earlier turned out to be necessary for the appearance of mode-coupling instability [6]. For the friction a Coulomb model is assumed, where the frictional force F_t is proportional to the normal force F_n exerted at the friction interface, $F_t = \mu F_n$, where μ is the kinetic coefficient of friction taken to be constant. Since the normal force at the friction interface is linearly related to the displacement x_2 of the mass normal to the contact surface, the following

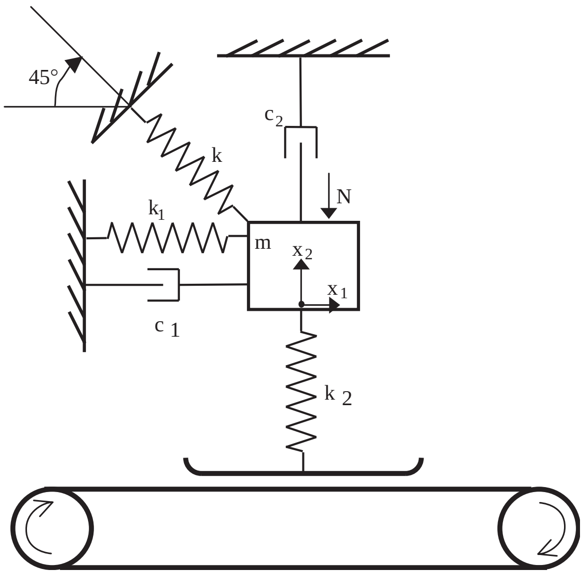

Fig. 1. Two-degree-of-freedom model.

equations result:

$$\begin{bmatrix} m & 0 \\ 0 & m \end{bmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{bmatrix} c_1 & 0 \\ 0 & c_2 \end{bmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{bmatrix} k_1 + \frac{1}{2}k & -\frac{1}{2}k \\ -\frac{1}{2}k & k_2 + \frac{1}{2}k \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\mu k_2 x_2 \operatorname{sgn}(v_B - \dot{x}_1) \\ N \end{pmatrix} + \begin{pmatrix} am\Omega^2 \sin(\Omega t) \\ 0 \end{pmatrix}, \quad (1)$$

where N denotes a constant normal load, v_B stands for the belt speed, linear viscous damping has been assumed and an external forcing parallel to the moving belt with frequency Ω and amplitude a is applied to the mass. To bring the equations into a more generic form it is convenient to divide by m and to use the relative (Lehr's) damping coefficients $D_i = c_i/(2\omega_i m)$ with $\omega_i^2 = (k_i + k/2)/m$, $i = 1, 2$:

$$\begin{bmatrix} 1 & 0 \\ 0 & 1 \end{bmatrix} \begin{pmatrix} \ddot{x}_1 \\ \ddot{x}_2 \end{pmatrix} + \begin{bmatrix} 2D_1\omega_1 & 0 \\ 0 & 2D_2\omega_2 \end{bmatrix} \begin{pmatrix} \dot{x}_1 \\ \dot{x}_2 \end{pmatrix} + \begin{bmatrix} \omega_1^2 & -\frac{k}{2m} \\ -\frac{k}{2m} & \omega_2^2 \end{bmatrix} \begin{pmatrix} x_1 \\ x_2 \end{pmatrix} = \begin{pmatrix} -\mu(\omega_2^2 - k/2m) x_2 \operatorname{sgn}(v_B - \dot{x}_1) \\ N/m \end{pmatrix} + \begin{pmatrix} a\Omega^2 \sin(\Omega t) \\ 0 \end{pmatrix}. \quad (2)$$

To investigate the effect of the external forcing on the system, Eq. (2) are brought into the standard (vector-)form

$$\ddot{\mathbf{X}} + \mathbf{S}(\mathbf{X}, \dot{\mathbf{X}}) = \begin{pmatrix} a\Omega^2 \sin(\Omega t) \\ 0 \end{pmatrix}, \quad (3)$$

where $\mathbf{X} = (x_1, x_2)^{\mathrm{T}}$ and $\mathbf{S}$ collects all terms not explicitly depending on time.

3. Derivation of effective slow-timescale equations

To derive effective slow-timescale equations the approach of Thomsen from [10] is applied to the two-degree-of-freedom problem at hand. Due to the conceptual clarity of the approach of Ref. [10], formal analogy is kept as far as possible. This also allows the reader quick comparison of the results of the present work in a two-degree-of-freedom mode-coupling instability context with the results of Thomsen's analysis in the context of a single-degree-of-freedom problem with a friction characteristic depending explicitly on the differential sliding velocity. In the following it is assumed that $\Omega \gg \omega_{1,2}$ and the goal is to derive effective equations of motion for dynamical processes on the slow timescale corresponding to ω_1 or ω_2. For this purpose solutions of Eq. (3) of the form

$$\mathbf{X}(t) = \mathbf{Z}(t) + \Omega^{-1}\boldsymbol{\phi}(t, \Omega t) \quad (4)$$

are considered, where $\Omega^{-1}\boldsymbol{\phi}(t, \Omega t)$ represents the fast timescale component of the response $\mathbf{X}$. The reason for the pre-factor Ω^{-1} is, that $\dot{\mathbf{Z}}$ is assumed to be of the same order of magnitude as ϕ, since only then can the dither vibrations 'activate' the inherent friction nonlinearity, as will become clear from the results of the analysis. Also it is enforced that the high-frequency component does have a zero mean over the high-frequency period

$$\bar{\boldsymbol{\phi}} \equiv \langle \boldsymbol{\phi} \rangle = \frac{1}{2\pi} \int_0^{2\pi} \boldsymbol{\phi}(t, \Omega t) \, \mathrm{d}(\Omega t) = 0, \tag{5}$$

since otherwise the mean component could be attributed to the slow timescale component $\mathbf{Z}$. As usual in multiple-timescale analyses, the two timescales t and Ωt are in the following treated as independent variables, such that $\mathrm{d}\mathbf{X}/\mathrm{d}t = \dot{\mathbf{Z}} + \Omega^{-1}\dot{\boldsymbol{\phi}} + \boldsymbol{\phi}'$, where $\dot{\boldsymbol{\phi}} = \partial\boldsymbol{\phi}/\partial t$ and $\boldsymbol{\phi}' = \partial\boldsymbol{\phi}/\partial(\Omega t)$. To simplify the analysis further, it is assumed that the high-speed component does—in the first order of approximation—not show slow timescale behaviour, i.e. $\boldsymbol{\phi}(t, \Omega t) \approx \boldsymbol{\phi}(\Omega t)$. Using the decomposition, Eq. (3) reduces to

$$\boldsymbol{\phi}'' = \begin{pmatrix} a\Omega \sin(\Omega t) \\ 0 \end{pmatrix} - \Omega^{-1}\left[\ddot{\mathbf{Z}} + \mathbf{S}(\mathbf{Z} + \Omega^{-1}\boldsymbol{\phi}, \dot{\mathbf{Z}} + \boldsymbol{\phi}')\right]. \tag{6}$$

To first order the solution for $\boldsymbol{\phi}$ results in

$$\boldsymbol{\phi}(\Omega t) = \begin{pmatrix} -a\Omega \sin(\Omega t) \\ 0 \end{pmatrix} + O(\Omega^{-1}), \tag{7}$$

and by that, making use of Eq. (4)

$$\mathbf{X}(t) = \mathbf{Z}(t) + \begin{pmatrix} -a \sin(\Omega t) \\ 0 \end{pmatrix} + O(\Omega^{-2}). \tag{8}$$

To obtain equations of motion for the slow timescale variables Eq. (6) is averaged over one fast vibration cycle

$$\ddot{\mathbf{Z}} + \langle \mathbf{S}(\mathbf{Z} + \Omega^{-1}\boldsymbol{\phi}, \dot{\mathbf{Z}} + \boldsymbol{\phi}') \rangle = 0, \tag{9}$$

and Eq. (7) can be used to evaluate the average of $\mathbf{S}$ over one period of the fast vibration.

It is obvious, that all linear terms in $\mathbf{S}$ are unaltered in form by the averaging procedure, since they either lead to terms constant with respect to Ωt, or to terms proportional to $\sin(\Omega t)$ which have vanishing high-frequency average. The only term that has to be calculated explicitly is the nonlinear term comprising the sgn-function arising from the friction force

$$\begin{aligned} &\langle -\mu(\omega_2^2 - k/2m) z_2 \, \mathrm{sgn}(v_B - \dot{z}_1 + a\Omega \cos(\Omega t)) \rangle \\ &\quad = (\omega_2^2 - k/2m) z_2 \mu \langle \mathrm{sgn}(v_B - \dot{z}_1 + a\Omega \cos(\Omega t)) \rangle \\ &\quad \equiv (\omega_2^2 - k/2m) z_2 \bar{\mu}, \end{aligned} \tag{10}$$

where for abbreviation $\bar{\mu}$ has been introduced, which can be obtained from simple analytical integration as

$$\bar{\mu} = \mu \, \mathrm{sgn}(v_B - \dot{z}_1) \quad \text{for } |v_B - \dot{z}_1| > a\Omega \tag{11}$$

and

$$\bar{\mu} = \mu\left(-1 + \frac{2}{\pi}\arccos\left(-\frac{v_B - \dot{z}_1}{a\Omega}\right)\right) \quad \text{for } |v_B - \dot{z}_1| < a\Omega, \tag{12}$$

which effectively describes a modified friction characteristic, depicted in Fig. 2. Obviously high-frequency dither leads to two effects for $|v_B - \dot{z}_1| < a\Omega$: (1) the absolute level of the friction

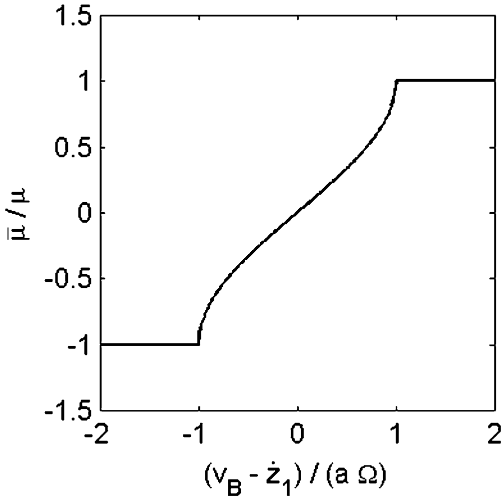

Fig. 2. Effective friction characteristic for the low-speed dynamics resulting from high-frequency dither.

coefficient is reduced and (2) an effective viscous damping results from the positive slope of the friction characteristic. It should be noted here, that these results have already been obtained in Thomsen's single-degree-of-freedom problem [10], which is in fact not really surprising, since the basic effect of dither lies in directly changing the slow-timescale friction characteristic, independent of the dimensionality of the dynamical problem at hand.

The focus of the present paper however lies in investigating how this change in the slow-timescale friction characteristic influences the stability characteristics of a system faced with mode-coupling instability, which is rather intricately dependent on the specific friction and damping situation (compare e.g. Ref. [7]). Using $\bar{\mu}$ to write down the final equations for the slow timescale it turns out that they are—apart from a modified friction characteristic and the missing high-frequency forcing—formally identical to the original Eq. (2)

$$\begin{pmatrix} \ddot{z}_1 \\ \ddot{z}_2 \end{pmatrix} + \begin{bmatrix} 2D_1\omega_1 & 0 \\ 0 & 2D_2\omega_2 \end{bmatrix} \begin{pmatrix} \dot{z}_1 \\ \dot{z}_2 \end{pmatrix} + \begin{bmatrix} \omega_1^2 & -\frac{k}{2m} \\ -\frac{k}{2m} & \omega_2^2 \end{bmatrix} \begin{pmatrix} z_1 \\ z_2 \end{pmatrix} = \begin{pmatrix} -\bar{\mu}(\omega_2^2 - k/2m)z_2 \\ N/m \end{pmatrix}. \tag{13}$$

For all subsequent exemplary calculations the following parameters will be used, which allow to show all relevant phenomena:

$$\omega_1 = 4\,\mathrm{s}^{-1}, \quad \omega_2 = 5\,\mathrm{s}^{-1}, \quad \frac{k}{2m} = 5\,\mathrm{s}^{-2}, \quad N/m = -10\,\mathrm{N\,kg}^{-1},$$
$$\Omega = 50\,\mathrm{s}^{-1}, \quad v_B = 10\,\mathrm{m\,s}^{-1}, \quad D_1 = 0.05, \quad D_2 = 0.04, \tag{14}$$

where the damping constants have been chosen such as to obtain mass-proportional damping.

To roughly estimate the modelling quality of the approach chosen, first the results from the full set of Eq. (2) are compared with the results from the effective slow timescale Eq. (13), see Fig. 3. It turns out that the averaged equations capture the effective average slow-timescale dynamics surprisingly well, as can be seen from Fig. 3.

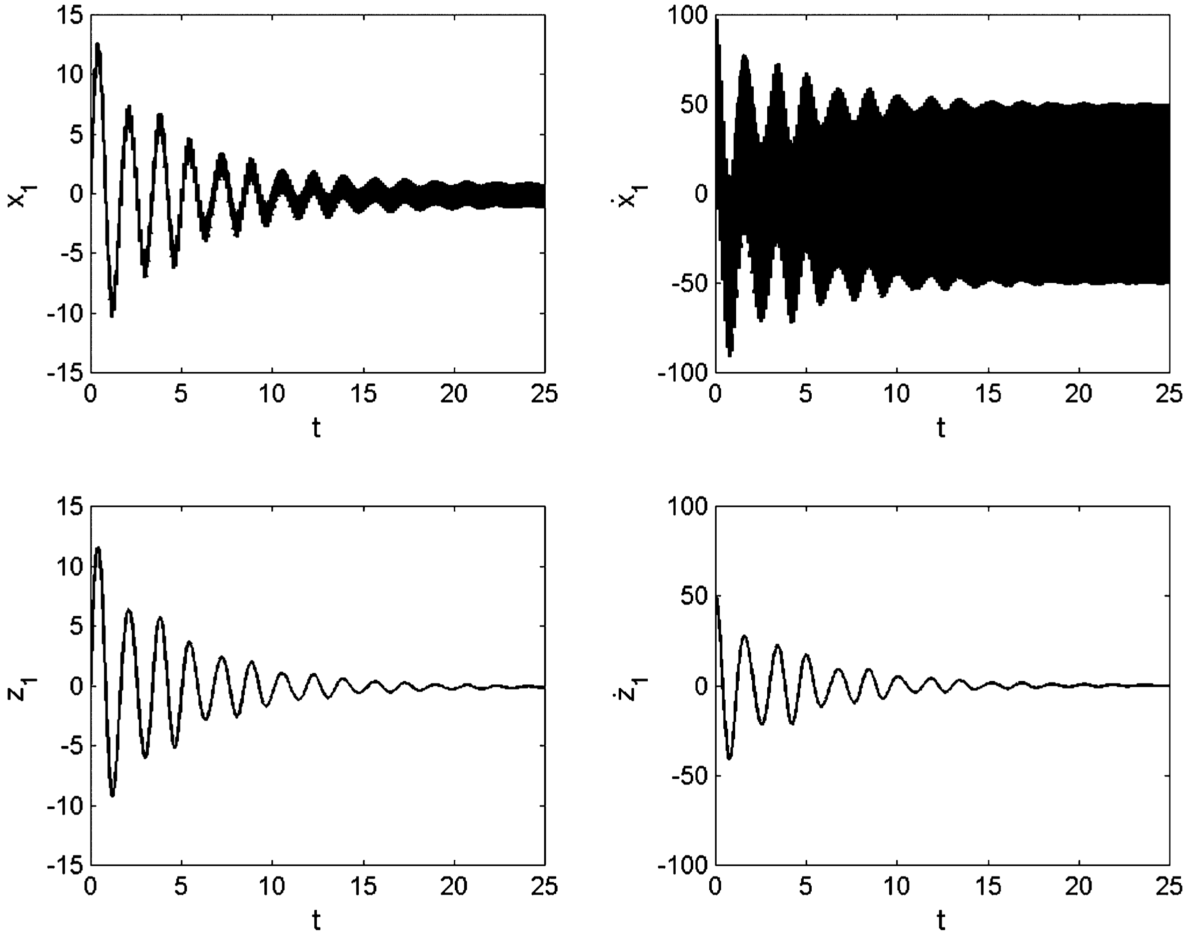

Fig. 3. Comparison of the results of time-marching the full Eq. (2) in the upper row and the effective averaged Eq. (11) in the lower row. Parameters: $\mu = 0.1$, $a = 1\,\text{m}$.

4. Effect of dither on the steady sliding state

Now consider the effects that dither has on the static solutions, i.e. the slow timescale steady sliding state determined from Eq. (13) by setting all temporal derivatives to zero and solving the remaining linear inhomogeneous equations

$$\begin{bmatrix} \omega_1^2 & -\frac{k}{2m} + \bar{\mu}^0(\omega_2^2 - k/2m) \\ -\frac{k}{2m} & \omega_2^2 \end{bmatrix} \begin{pmatrix} z_1^0 \\ z_2^0 \end{pmatrix} = \begin{pmatrix} 0 \\ N/m \end{pmatrix}, \tag{15}$$

where $\bar{\mu}^0$ denotes the effective friction coefficient evaluated for $\dot{z}_1 = 0$. For relative velocities (which equal the belt velocities for static solutions) larger than $a\Omega$ the static solution is obviously unchanged by the superimposed dither. For relative velocities smaller than $a\Omega$ the static

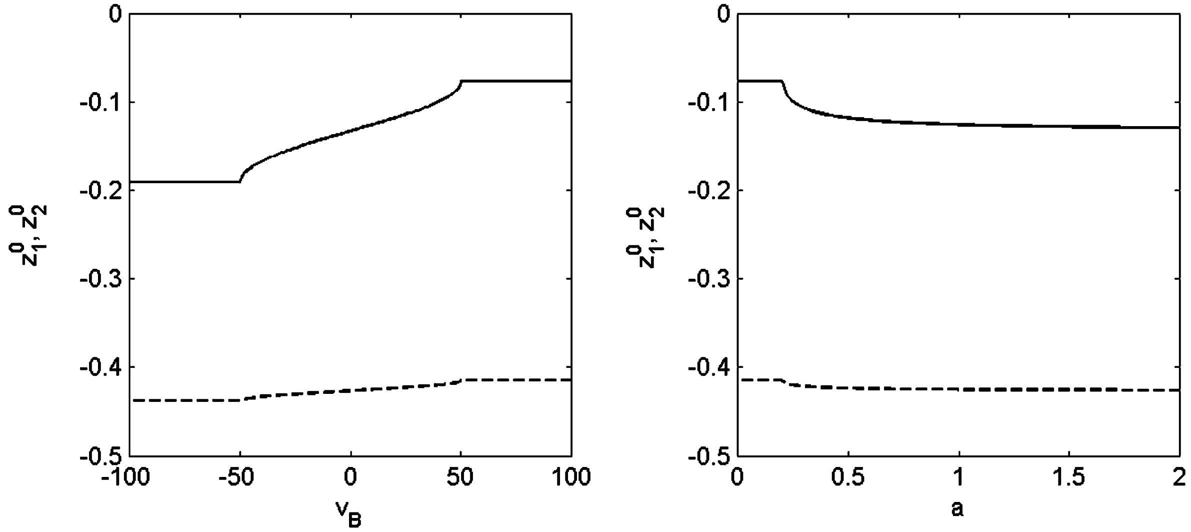

Fig. 4. The static solutions of Eq. (11) (z_1^0 solid, z_2^0 dashed) vs. the belt velocity v_B (left) and vs. dither amplitude a (right).

displacement of the sliding mass goes continuously to the value corresponding to $\mu = 0$ for decreasing relative velocities, as can be seen from the left graph of Fig. 4. This is a direct consequence of the effective reduction of the effective friction coefficient in the situation of superimposed dither. Fig. 4 also shows the effect of increasing the dither amplitude a: dither has no effect at all, until $a\Omega$ reaches v_B, then a further increase of a leads asymptotically to the values corresponding to the frictionless system.

5. Effect of dither on the stability of the steady sliding state

Now the slow timescale Eq. (13) are linearized around the static solution corresponding to the steady sliding state and the steady sliding state's linear stability will be considered. For that purpose equations for the 'perturbations' $\mathbf{Y}$ around the static solution (steady sliding state) $\mathbf{Z}^0$ are obtained by inserting $\mathbf{Z} = \mathbf{Z}^0 + \mathbf{Y}$ into Eq. (13) and subtracting Eq. (15) from the result to eliminate inhomogeneous terms. The resulting equation is then linearized, where it has to be taken into account that $\bar{\mu}$ depends on the relative sliding velocity $v_B - \dot{z}_1$ and thus has to be taken into account correctly in the linearization process. Writing

$$\bar{\mu} = \bar{\mu}^0 + \left.\frac{\partial \bar{\mu}}{\partial \dot{z}_1}\right|_{\dot{z}_1=0} \dot{z}_1 + O(\dot{z}_1^2) = \bar{\mu}^0 + \bar{\mu}' \dot{z}_1 + O(\dot{z}_1^2), \tag{16}$$

where

$$\bar{\mu}^0 = \bar{\mu}(\mathbf{Z}^0), \quad \mu' = \mu \frac{-2}{\pi a \Omega} \frac{1}{\sqrt{1 - (v_B/a\Omega)^2}}, \tag{17}$$

the linearized homogeneous equations representing the dynamics of **Y** read

$$\begin{pmatrix} \ddot{y}_1 \\ \ddot{y}_2 \end{pmatrix} + \begin{bmatrix} 2D_1\omega_1 + \mu'(\omega_2^2 - k/2m)z_2^0 & 0 \\ 0 & 2D_2\omega_2 \end{bmatrix} \begin{pmatrix} \dot{y}_1 \\ \dot{y}_2 \end{pmatrix} + \begin{bmatrix} \omega_1^2 & -\frac{k}{2m} + \bar{\mu}^0(\omega_2^2 - k/2m) \\ -\frac{k}{2m} & \omega_2^2 \end{bmatrix} \begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = 0. \quad (18)$$

It should be noted that the effect of dither consists in (1) a modified stiffness matrix, which has basically been expected from the type of system under consideration, and (2) a modified damping matrix, in which additional effective viscous damping appears.

Following the usual procedure the homogeneous Eq. (18) may be rewritten as a system first order in $\mathrm{d}/\mathrm{d}t$ and solved by an eigenanalysis with an ansatz proportional to $\exp(\sigma + \mathrm{i}\omega t)$, where ω represents the resulting oscillation frequency and σ the corresponding growth rate. Fig. 5 shows the results: For sufficiently small dither amplitudes a (such that $a\Omega < v_B$) the stability behaviour is identical to the system without dither (solid line graphs in Fig. 5), cf. e.g. Ref. [7]: the frequencies of two modes of the system approach each other when the control parameter (i.e. the friction coefficient μ in the present context) is increased, until the modes coalesce, rendering a defective system. A further increase of the control parameter leads to a spectral unfolding again, such that a more strongly damped mode and an unstable mode result (compare the middle graph of Fig. 5 where the growth rates are depicted vs. μ).

When the dither amplitude exceeds a critical value ($a = 0.2$ in the present example), corresponding to the dither velocity first reaching the band velocity, the mode-coupling diagram (left and middle graph of Fig. 5) is distorted. For the parameters used, the whole system is stabilized, which can be seen from the right graph of Fig. 5 that depicts the critical friction coefficient μ_c forming the borderline between a stable and unstable steady sliding state. Note for completeness that $\mu_c(a)$ is indeed not continuous at $a = 0.2$, which goes back to the fact that the present approach has yielded $\bar{\mu}$ not continuously differentiable there. Nevertheless it is clear that an increasing dither amplitude stabilizes the present system substantially.

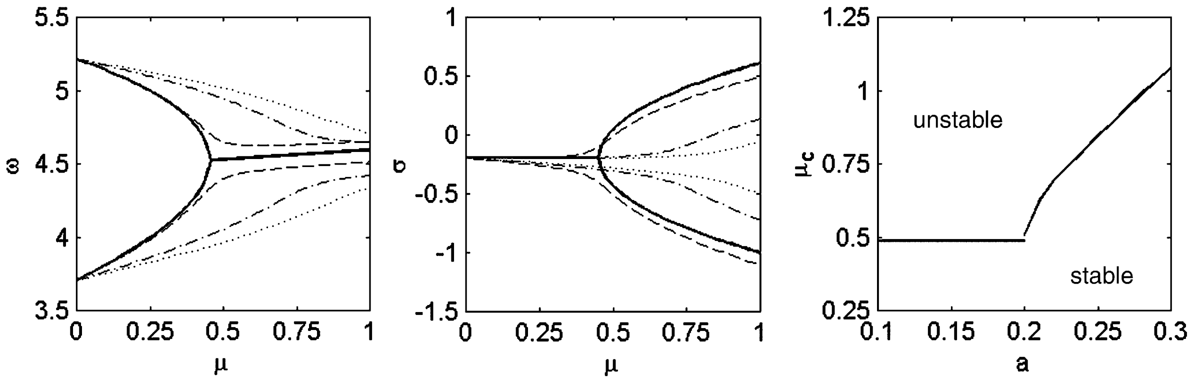

Fig. 5. Effect of dither on mode-coupling. Natural frequencies ω vs. μ (left) and growth rates σ (middle) for $a < 0.2$ m (solid), $a = 0.2001$ m (dashed), $a = 0.25$ m (dash-dotted) and $a = 0.30$ m (dotted). The right graph shows the critical friction coefficient μ_c vs. the dither amplitude a.

However it should also be noted that systems in slightly subcritical situations might also be destabilized by dither, as can be seen by looking at the growth rates σ for $a = 0.2001$ m: there is a small interval of friction coefficients around $\mu = 0.4$ in which the more unstable mode has larger growth rates than both stable modes of the original system without the superposition of dither. Conceptually this effect goes back to the additional viscous damping added to the system by dither: for mode-coupling unstable systems it is known (cf. Ref. [7]) that an increase in viscous damping can, under certain circumstances, destabilize the system. Since dither also leads to a change in the system's damping matrix, this leads to the conclusion that dither may—under very specific conditions (cf. Ref. [7])—lead to a destabilization of systems endangered by mode-coupling instability.

An important question that also remains to be answered is, how much energy would be needed to quench mode-coupling instability by dither. The above considerations have shown that dither affects the slow timescale vibrations only, if the velocity corresponding to the superimposed dither reaches the belt velocity: $a\Omega = v_B$, i.e. the dither has to bring the system into a state of intermittent change of the direction of the friction force. This sets the requirement on the dither amplitude to actually affect the stability properties of the steady sliding state.

6. Conclusions

It has been shown that in close analogy to single-degree-of-freedom cases (cf. e.g. to the original work by Thomsen [10]) dither in general has a stabilizing effect on the mode-coupling instability in friction-induced oscillations, if the dither amplitude is sufficiently large. For the slow-timescale dynamics dither results in a continuous friction characteristic for low relative velocities with effectively lowered absolute friction coefficients and an additional viscous damping effect. It should also be noted that the stability equations for the slow-timescale dynamics do explicitly depend on the steady sliding state, which is usually not the case when typical mode-coupling stability calculations are performed.

Future work should focus on three aspects: First, experimental evidence for the phenomenon described only theoretically up until now should be gathered. Second, limit cycle behaviour of mode-coupling induced vibrations under the influence of dither should be studied. Third, the influence of further system nonlinearities, e.g. due to nonlinear contact stiffness, etc., should be taken into account into extended models, to better approximate real-life engineering applications plagued by mode-coupling-induced vibration and noise.

References

[1] R.A. Ibrahim, Friction-induced vibration, chatter, squeal and chaos—part II: dynamics and modeling, *Applied Mechanics Reviews* 47 (1994) 227–253.

[2] J. Wallaschek, K.-H. Hach, U. Stolz, P. Mody, A survey of the present state of friction modelling in the analytical and numerical investigation of brake noise generation, *Proceedings of the ASME Vibration Conference*, Las Vegas, 1999, pp. 12–15.

[3] K. Popp, P. Stelter, Stick-slip and chaos, *Philosophical Transactions of the Royal Society, London A* 332 (1990) 89–105.

[4] L. Gaul, R. Nitsche, Role of friction in mechanical joints, *Applied Mechanics Reviews* 54 (2001) 93–105.
[5] A. Gasparetto, Eigenvalue analysis of mode-coupling chatter for machine-tool stabilization, *Journal of Vibration and Control* 7 (2001) 181–197.
[6] N. Hoffmann, M. Fischer, R. Allgaier, L. Gaul, A minimal model for studying properties of the mode-coupling type instability in friction induced oscillations, *Mechanics Research Communications* 29 (2002) 197–205.
[7] N. Hoffmann, L. Gaul, Effects of damping on mode-coupling instability in friction induced oscillations: imperfect merging of modes and viscous instability, *Zeitschrift für Angewandte Mathematik und Mechanik* 83 (2003) 524–534.
[8] R.T. Spurr, A theory of brake squeal, *Proceedings of the Automotive Division, Institute of Mechanical Engineers,* (AD) 1 (1961) 33–40.
[9] K. Popp, M. Rudolph, M. Kröger, M. Lindner, Mechanisms to generate and to avoid friction induced vibration, VDI report 1736, Düsseldorf, 2002, pp. 1–15.
[10] J.J. Thomsen, Using fast vibrations to quench friction-induced oscillations, *Journal of Sound and Vibration* 228 (1999) 1079–1102.
[11] B.F. Feeny, F.C. Moon, Quenching stick-slip chaos with dither, *Journal of Sound and Vibration* 237 (2000) 173–180.
[12] W. Littmann, H. Storck, J. Wallaschek, Sliding friction in the presence of ultrasonic oscillations: superposition of longitudinal oscillations, *Archives of Applied Mechanics* 71 (2001) 549–554.