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Short Communication

Determining the extension of a hydraulic cylinder using spectral estimation

B. Yagci[a], A.E. Frazho[a], H. Sumali[b,*]

[a]School of Aeronautical and Astronautical Engineering, Purdue University, West Lafayette, IN 47907, USA
[b]Department of Agricultural and Biological Engineering, Purdue University, West Lafayette, IN 47907, USA

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1. Introduction

In this study the extension of a telescopic structure is determined using an estimate of the spectrum of the structure. As an example a hydraulic cylinder is chosen. The hydraulic cylinder is a nonlinear structure with damping. This makes it difficult to formulate an analytical model for this structure. In this note it is assumed that the damping of the cylinder is low enough to use a sinusoid estimation algorithm to obtain an estimate of its spectrum. The aim of this study is to determine the extension of the cylinder from the natural frequencies of the structure. In other words a relation between the natural frequencies and the extension of the cylinder real time is investigated. Such a relation could be used for control purposes.

Two different methods to estimate the natural frequencies of the cylinder are used. The first method is based on Capon–Geronimus algorithm presented in Refs. [1,2,8–10]. The second method uses stochastic techniques along with the Kalman–Ho algorithm to estimate the eigenvalue of a certain matrix. These eigenvalues correspond to the natural frequencies of the cylinder. Both of these methods yield very close results. In certain situations, the methods used in this note yield a more accurate estimate of the natural frequencies than a widely used fast Fourier transform(FFT) analyzer.

*Corresponding author. Current address: Sandia National Laboratories, P.O. Box 5800, Albuquerque, NM 87185-1310, USA

E-mail address: hsumali@sandia.gov (H. Sumali).

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2. The experiment

A hydraulic cylinder is used for the experimental analysis. The aim of the study is to detect changes in the shape of a structure. In this note two different statistical methods are used to determine the length of a hydraulic cylinder. A hydraulic cylinder is chosen because one can easily change its length. Moreover, it is nonlinear and thus there are no easy methods to model the cylinder. Finally, a hydraulic cylinder is a real-life structure on which it is relatively easy to collect data. The length of the cylinder is determined from the point spectrum of the system.

Free boundary conditions are used on both ends of the cylinder. In other words the cylinder is suspended using soft elastic cords, i.e. bungee cords. The cylinder is connected to a hydraulic pump so that the extension of the cylinder can be changed during the experiment. The hydraulic hoses are kept connected at all times. In this case the effect of vibrations induced by the hydraulic pump is also taken into account. It is noted that the power spectrum of the hydraulic pump is very small for the frequency range of interest. The signals of interest we will discuss later are two orders of magnitude higher than the noise shown in Fig. 1. In other words the natural frequency of the pump do not interfere with the experimental analysis.

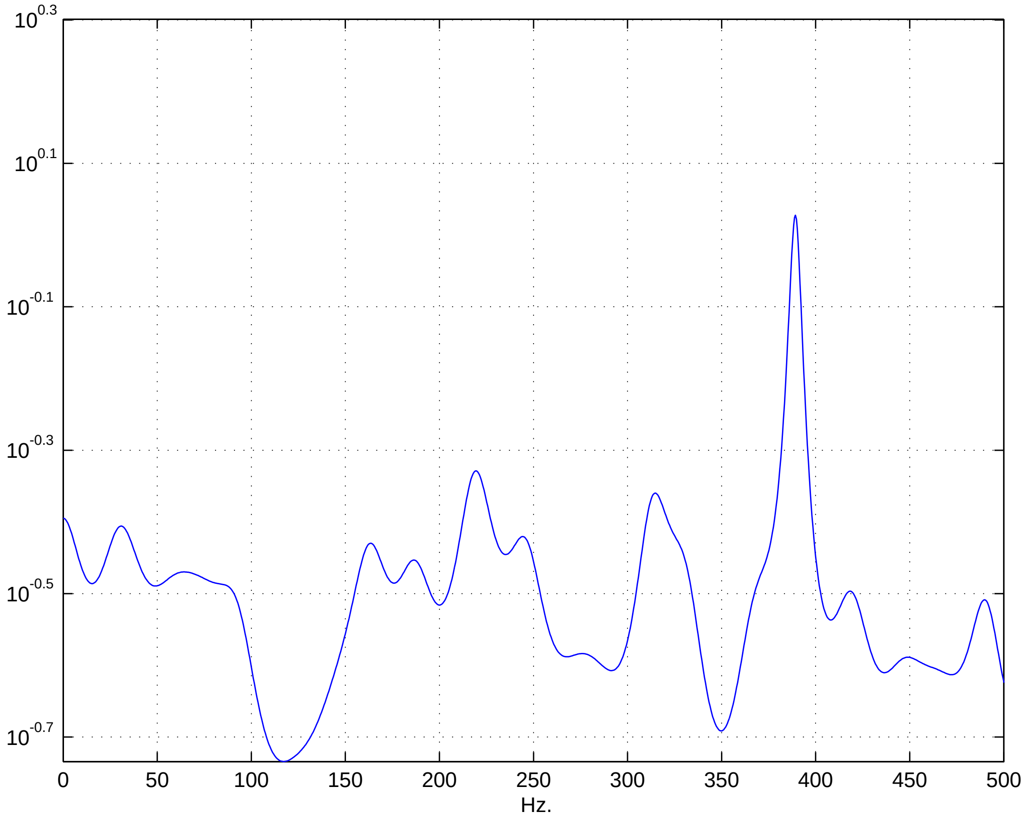

Fig. 1. Spectrum of the hydraulic bench, arbitrary unit.

To excite the cylinder a piezoelectric stack is used. A piezoelectric stack is light weight and can be easily integrated into the structure. Since a piezoelectric stack delivers higher force under compression, it is squeezed between two small metal blocks welded to the rod; see Fig. 2. This configuration does not add too much weight to the system. Using a piezoelectric stack has also the advantage of being a stand alone system. For example a shaker needs to be fixed to the ground or supported appropriately to obtain a reliable excitation, whereas the piezoelectric stack is attached directly to the system. Furthermore, it is easier to isolate the direction of excitation when using a piezoelectric stack. Because the stack is a small device it is portable and suitable for real-life applications.

A piezoelectric stack is used to excite the cylinder from the end of the rod. Previous tests showed that the first mode is best measured at the end of the rod. The cylinder is excited using white noise. Here white noise means that the spectrum of the input is constant over the frequency range of interest.

Previous testing showed that the first natural frequency of the cylinder is well separated from the second and the third natural frequencies; see Fig. 3. The second and the third natural frequencies are changing between 650 and 1200 Hz as the rod moves through its full range of extension. At around 40% extension, the second and third natural frequencies tend to overlap. This makes it difficult to track a certain mode. On the other hand, the first natural frequency changes between 150 and 400 Hz so it is always well separated from the second and the third

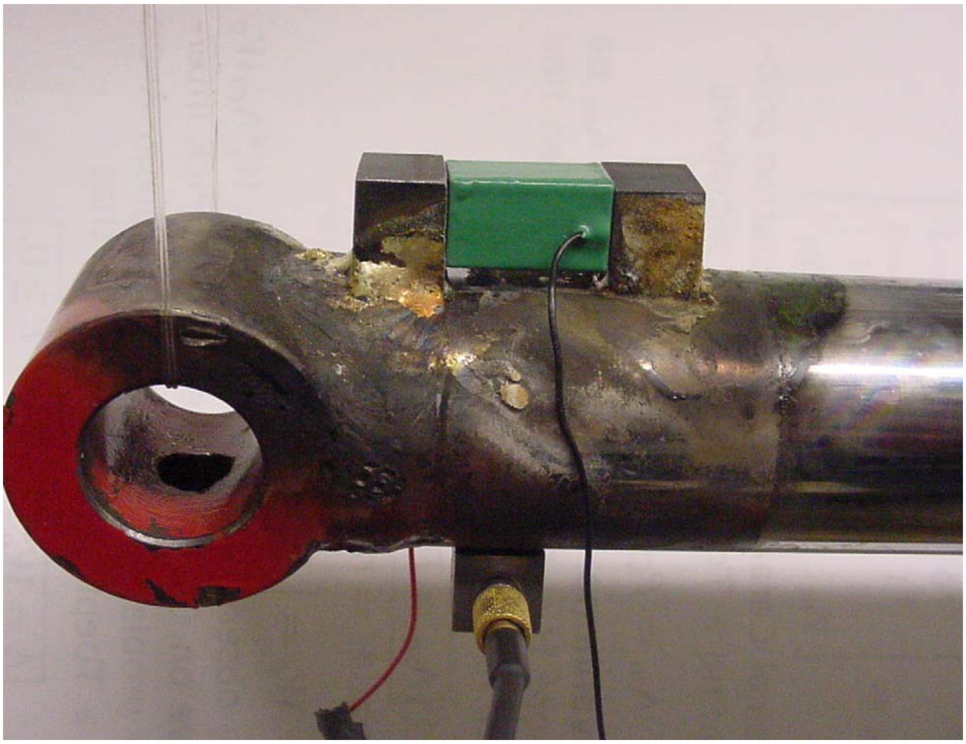

Fig. 2. The piezoelectric actuator and the cylinder.

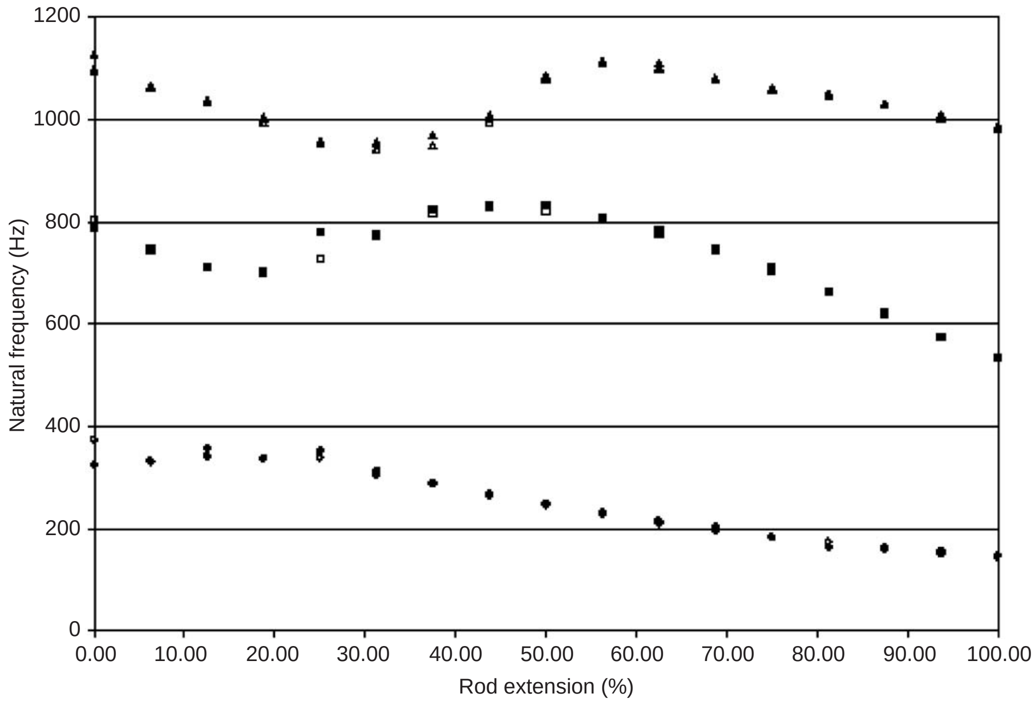

Fig. 3. Natural frequencies of the cylinder at different rod extensions. The circle, rectangle, and triangle correspond to the first, the second and the third modes. Hollow markings indicate retraction and full markings indicate extension of the cylinder (data from F.M. Monroig).

modes. Hence, the first natural frequency of the cylinder is monitored to determine the extension of the cylinder.

The piezoelectric stack is excited with white noise, then the acceleration data is collected at the end of the rod. To analyze this data two different statistical techniques are used. First, the Capon–Geronimus method is used. In this method one obtains an estimate of the spectrum directly from the acceleration data. This estimate is used to determine the natural frequencies of the system; see Section 2.1.

The second method used is based on combining some statistical methods with the Kalman–Ho algorithm to obtain a reduced order model of the system; see Section 2.2. Then this model is employed to calculate the natural frequencies of the system.

2.1. Capon–Geronimus method

In this section a sketch of Capon–Geronimus algorithm is presented which is used to compute the natural frequencies of the cylinder. For more detailed discussion of this topic; see Refs. [1–3].

To apply this method the acceleration data is modeled as a wide sense stationary signal of the form

$$y(n) = \sum_{k=1}^{v} a_k e^{i(\omega_k n + \phi_k)} + \zeta(n), \tag{1}$$

where v is some positive integer and $\{\omega_k\}_1^v$ are the sinusoid frequencies of the cylinder with corresponding weights $\{a_k\}_1^v$. Moreover, $\{\phi_k\}_1^v$ are mutually independent uniformly distributed random variables over $[0, 2\pi]$. Finally, $\zeta(n)$ is additive stationary measurement noise. The aim is to estimate the frequencies ω_k to determine the natural frequencies of the system.

To apply the Capon–Geronimus method the response of the cylinder is modelled as a sinusoidal wide sense stationary process of the form

$$b(n) = \sum_{k=1}^{v} a_k \mathrm{e}^{\mathrm{i}(\omega_k n + \phi_k)}.$$

In this model ω_k/γ is the natural frequency of the cylinder where γ is the sampling rate. Moreover, $\{a_k\}_1^v$ are the amplitude at the corresponding natural frequency $\{\omega_k/\gamma\}_1^v$. Furthermore, the ϕ_k are independent uniformly distributed random variables over $[0, 2\pi]$. In this case $b(n)$ is a wide sense stationary process. In this model $b(n)$ is a real signal because the sinusoid frequencies come in complex conjugate pairs.

To implement the Capon–Geronimus method the data is collected with an accelerometer. Moreover, it is assumed that this data is corrupted by additive noise $\zeta(n)$. To be precise $y(n) = b(n) + \zeta(n)$, where $\zeta(n)$ is purely nondeterministic wide sense stationary process, independent of $b(n)$. For example $\zeta(n)$ could be white noise. In Capon–Geronimus method one does not need to know any statistical information about $\zeta(n)$. This method will obtain the natural frequencies and amplitudes without any prior knowledge of $\zeta(n)$. The Capon–Geronimus method provides an algorithm to extract the spectrum of $b(n)$ from $y(n)$. That is, the algorithm yields the point spectrum of $y(n)$ which has magnitude a_k at the sinusoid frequencies and zero everywhere else. So when applying the Capon–Geronimus method for estimating the natural frequencies the peaks in the graphs indicate the natural frequencies.

Now the implementation of the Capon–Geronimus algorithm will be explained. Recall that $y(n) = b(n) + \zeta(n)$ is the data collected from the experiment. One uses standard correlation techniques to compute the autocorrelation matrix associated with the data. Let T_m be the $m \times m$ Toeplitz matrix corresponding to the data. To be precise the entries of T_m are given by

$$(T_m)_{i,j} = E(y(i)\overline{y(j)}),$$

where E denotes the expectation of a random variable. In this method one uses the Levinson polynomials to estimate the spectrum of the process. Notice that T_k is the upper left $k \times k$ corner of T_m for $k \leqslant m$. Then the Levinson system is given by

$$[\alpha_{k,1}\alpha_{k,2} \cdots \alpha_{k,k-1}\alpha_{k,k}]T_k = [0\ 0 \cdots 0\ \varepsilon_k], \tag{2}$$

where the scalars $\{\alpha_{k,j}\}_{j=1}^k$ are the Levinson coefficients and ε_k is the estimation error. Set $\alpha_{k,k} = 1$ for all k. These coefficients can be obtained using the *levinson* command in Matlab. Define the polynomials $p_k(\omega) = \sum_{j=1}^k \alpha_{kj} \mathrm{e}^{-k\omega(j-1)}/\sqrt{\varepsilon_k}$. In this case the estimate of the spectrum $\mu_m(\omega)$ of $\{y(n)\}_1^\infty$ is given by

$$\mu_m(\omega) = \frac{1}{\sum_{k=1}^m |p_k(\omega)|^2}.$$

One can combine the Levinson algorithm with standard FFT techniques to evaluate $|p_k(\omega)|^2$ around the unit circle. The estimate of the point spectrum $\mu_m(\omega)$ is calculated for different values of m and plotted in a graph. In such a plot the curves converge to zero at all frequencies except the sinusoid frequencies. At sinusoid frequencies $\mu_m(\omega)$ converges to the corresponding amplitudes of the sinusoids as m tends to infinity. In other words,

$$\lim_{m\to\infty} \mu_m(\omega) = \begin{cases} |a_k|^2 & \text{for } \omega = \omega_k,\ 1 \geqslant k \geqslant v, \\ 0 & \text{otherwise.} \end{cases} \tag{3}$$

The natural frequencies of the system are determined from the peaks in the plot of $\mu_m(\omega)$ vs. ω.

Recall that during the experimental work the acceleration data is collected by exciting the cylinder by a piezoelectric stack with white noise. To have real-time measurements a dSPACE digital signal processor is used. A simple Simulink program generates random noise. To concentrate the energy of this noise to lower frequencies a sixth-order low-pass Butterworth filter is used with corner frequency at 400 Hz in the Simulink program.

The hydraulic cylinder is retracted slowly starting from the totally extended position to the fully retracted position in a continuous manner. It takes approximately 85 s for the cylinder to be fully retracted. A Matlab code excites the cylinder, collects data, and then waits 2 s to excite the system again. This way we have estimates of the spectrum of the cylinder at different extensions. The length of the rod is measured using a ruler drawn on the rod.

The acceleration data collected is passed through an anti-aliasing filter with a corner frequency at 1 kHz. The output of the filter is amplified before feeding into the digital processor. At this point now one has $y(n) = b(n) + \zeta(n)$. The spectrum estimate of the signal is then calculated using the Capon–Geronimus algorithm on $y(n)$. To have reliable results the experiments are repeated ten times, that is the rod is retracted from full extension to retracted position ten times. The estimates of the spectrum at different extension lengths for these ten experiments are plotted in Fig. 4. In this figure the graph closest to the frequency axis corresponds to the fully extended position. The peaks obtained in different experiments are consistent; see also Fig. 5. As the piston is retracted the first natural frequency of the structure shifts right as expected.

In the Capon–Geronimus method the sinusoid frequencies are computed where the estimate of the spectrum converges to a finite value. In this example the spectrum is estimated only once and the highest peak is used as an indication of natural frequency. In the frequency range 0–400 Hz there is only one natural frequency. Hence the highest peak is used to determine the first natural frequency of the system.

2.2. Kalman–Ho algorithm

In this section some state-space statistical methods are used with the Kalman–Ho algorithm to estimate the natural frequencies of the cylinder; see Refs. [4,5]. The Kalman–Ho algorithm uses the impulse response of the system to generate a state-space model. However, in this example the system is excited with white noise. In this case one can use the autocorrelation sequence of the acceleration data as an input to the Kalman–Ho algorithm. From this the natural frequencies of the system are obtained. To this end assume that the cylinder can be modeled as a discrete time linear system of the form

$$x(n+1) = Ax(n) + Bu(n) \quad \text{and} \quad y(n) = Cx(n), \tag{4}$$

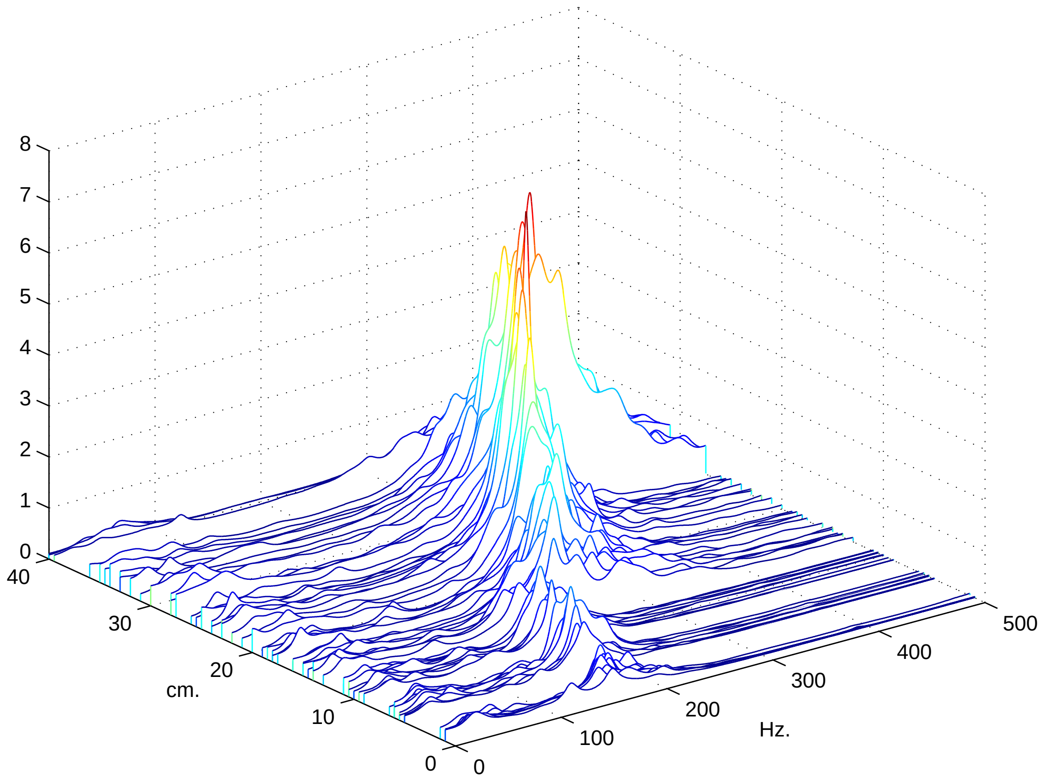

Fig. 4. Estimate of the spectrum at different locations when the piston is extending. The unit is the same as in Fig. 1.

where A is an operator on some finite-dimensional space $\mathbb{C}^m$, the operator B takes $\mathbb{C}$ into $\mathbb{C}^m$, and C maps $\mathbb{C}^m$ into $\mathbb{C}$. In this model the eigenvalues of A correspond to the natural frequencies of the cylinder, that is the natural frequencies of the cylinder are given by

$$\omega_n = \frac{1}{2\pi\gamma}\log(\lambda_d). \tag{5}$$

Here λ_d are the eigenvalues of A and ω_n are the natural frequencies of the system. The constant γ is the sampling rate of the digital processor in seconds.

In the experiments the input $u(n)$ is white noise and the output $y(n)$ is the acceleration data. It is well known that the model (4) driven by white noise produces a steady-state wide sense stationary random process. The autocorrelation function for this random process is given by

$$R_y(n) = \begin{cases} CA^nPC^*, & n \geqslant 0, \\ CPA^{*|n|}C^*, & n \leqslant 0. \end{cases} \tag{6}$$

Here P is the controllability Grammian, that is

$$P = APA^* + BB^*.$$

For more details; see Refs. [6,7].

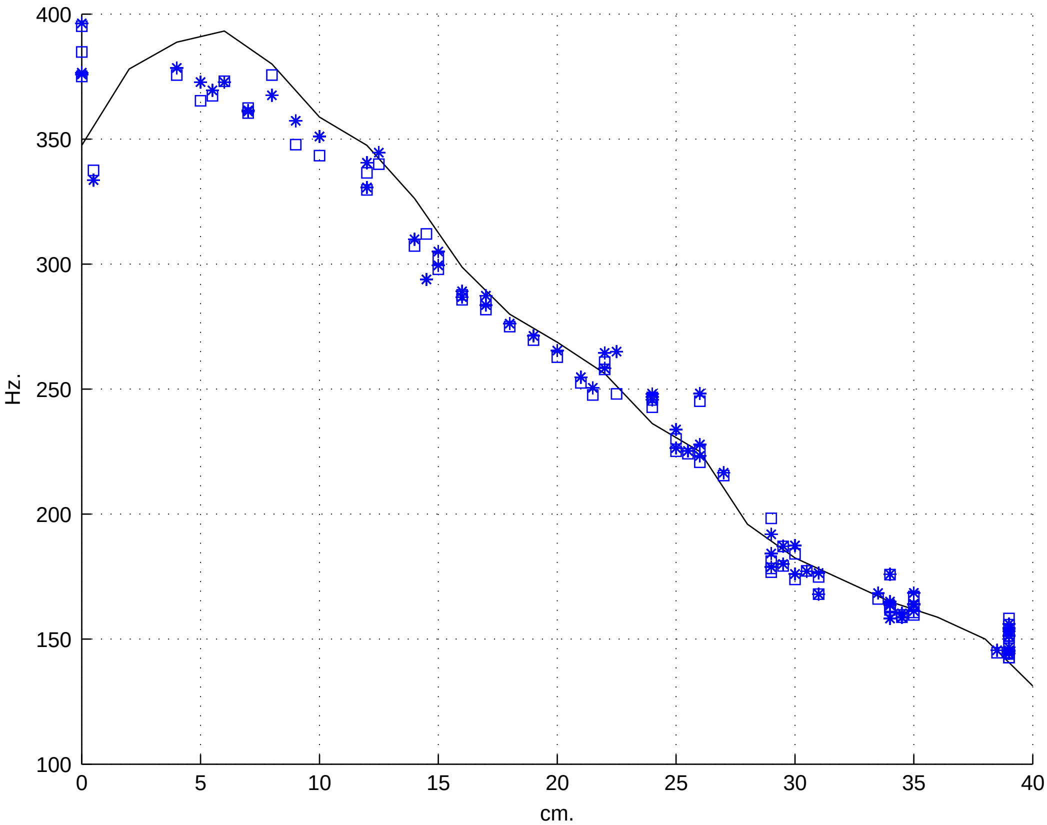

Fig. 5. Change of first natural frequency vs. time while the cylinder is retracting. The solid line, square markings and * correspond to the result of experiments with the FFT analyzer, Capon's method and the Kalman–Ho algorithm.

Now the problem of obtaining the natural frequencies from the data $y(n)$ is equivalent to finding the eigenvalues of A given the sequence $\{CA^nPC^*\}_{n=0}^{\infty}$. It is well known that one can apply the Kalman–Ho algorithm to find the matrix A up to a similarity transformation from any sequence of the form $\{CA^nQ\}_{n=0}^{\infty}$. In particular with $Q = PC^*$ one can use the Kalman–Ho algorithm to find A up to the similarity transformation from the correlation data $R_y(n) = CA^nPC^*$. Then using Eq. (5) the eigenvalues of this A yield the natural frequencies for the cylinder.

To find the natural frequencies of the system the following experimental procedure is used. First the acceleration data is collected. Then the autocorrelation sequence is formed from the data. At this point the Kalman–Ho algorithm is applied on the autocorrelation sequence to find the eigenvalues of A. Finally, the natural frequencies of the system are computed using Eq. (5).

Since the system is assumed to have low damping, the imaginary parts of the eigenvalues of the continuous system are the natural frequencies of the system. In Fig. 5 the results from Kalman–Ho and the locations of the peaks from the Capon–Geronimus method are plotted. The percent difference between the results of the two methods is around 6% for all locations. The experimental setup is illustrated in Fig. 6.

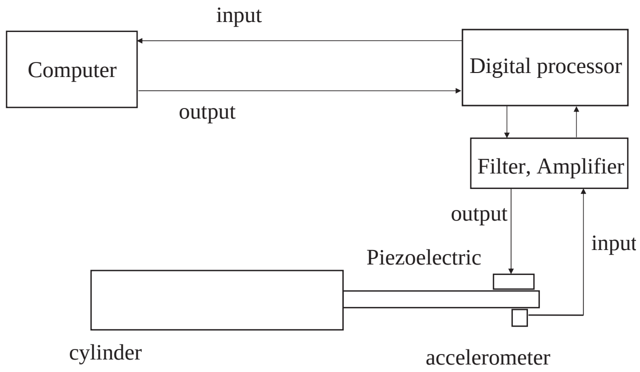

Fig. 6. Experimental setup.

Because the cylinder is a complicated structure which involves seals, hydraulic oil, a barrel and a rod the experimental results found in this note are compared with the results of a widely used experimental method instead of an analytical model. The results are checked using a commercially available FFT analyzer. The frequency response function calculated by the FFT analyzer is used to find the natural frequencies of the system. The commercially available analyzer uses the ratio of the spectrum of the input and the output to find the frequency response function. Using an impact hammer the cylinder is excited from the tip of the rod. The accelerometer is also placed at the tip of the rod. The rod is retracted from the fully extended position to the fully retracted position by steps of 2 cm. A frequency response function is calculated at each point. The natural frequencies of the cylinder at an extension is obtained by determining the frequency where the peaks occur. To compare the results obtained from Kalman–Ho algorithm and the results from the FFT analyzer the natural frequency versus extension graph is plotted in Fig. 5. The results obtained using Kalman–Ho algorithm are close to the one obtained using the commercial FFT analyzer. The percent error between the two results is less than 10%. Notice that the results obtained using Capon–Geronimus and Kalman–Ho algorithms are more consistent with each other than the results picked up by the FFT analyzer. Recall that the percent error between the Kalman–Ho and Capon–Geronimus methods was less than 6% for any location. Therefore, one can use the natural frequencies found in real-time using the digital processor together with the Kalman–Ho or Capon–Geronimus method to determine the extension of the cylinder with the help of Fig. 5.

3. Conclusion

The results of the experiments showed that using both the Capon–Geronimus method and the statistical method of Section 2.2 one could detect the length of the cylinder under test using Fig. 5 accurately. The estimates using Capon–Geronimus and Kalman–Ho algorithms are more consistent with each other when compared to the results from the FFT analyzer. Finally, in many situations our methods agree with the FFT analysis. Because the statistical methods we use are better suited to handle random data, we believe our results are more accurate than the FFT analyzer in certain cases.

Acknowledgements

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