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Nyquist—overcoming the limitations

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Abstract

In digital signal analysis it is not possible by normal methods to retrieve components of frequency in excess of half the sampling frequency, generally referred to as the Nyquist frequency. Any frequency components that exist in excess of this value will give errors in their frequency determination because they will appear as false or 'aliased' signals. A new technique has been developed to unambiguously retrieve signals many thousands times greater than this hitherto stated limitation. A feature of the system is its ability to sample at low frequency and, without further sampling, be capable of unambiguously determining frequency components far in excess of the initial low-frequency sampling rates. Although the approach has its origins in the area of machinery condition monitoring it has applications in numerous other fields, such as lowering the bandwidth of frequency spectra within communication systems. The technique can also be used in reverse to synthesis high-frequency signals from transmitted lower frequencies.

1. Introduction

It has been accepted as a 'fact' in signal analysis that it is not possible, when considering periodic broadband signals, to retrieve frequency components greater in frequency than half the sampling frequency. This premise is clearly stated in all textbooks on this subject as the 'Nyquist criterion' or 'Shannon's sampling theorem' [1–4]. This maximum retrievable frequency is often called the Nyquist frequency. For example, with a sampling frequency of 2 KHz the Nyquist frequency is 1 KHz and so it was not thought possible to retrieve signals greater in frequency than this 1 KHz limit. Any frequencies greater than this 1 KHz signal will appear as false or aliased signals, which will confuse the analysis [4–7]. Consequently, considerable research effort and

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commercial activity has emerged to eliminate these aliased signals by filtering out all components which cause aliasing within the incoming signal [5]. *Quite clearly, the filtering could inadvertently remove and discard signals of practical importance as they may be the result of a defect (in condition monitoring for example). Valuable information would then be lost from any subsequent diagnostic analysis.*

It has now been proved through the work reported here that the limitation imposed by the Nyquist criterion on periodic broadband signals can be overcome and the risk incurred by low-pass filtering effectively removed in that the value of the low-pass filtering can be substantially increased beyond the highest frequency which the system is currently capable of identifying. A theoretical study combined with a simple capturing device has introduced the possibility of retrieving periodic signals in excess of 2500 times the Nyquist frequency. Initial experimental work has determined frequencies up to 1.4 MHz when using a maximum sampling frequency of 2 KHz. Additionally the technique is capable of reanalyzing systems at very much higher frequencies than the low-frequency sampling without further data sampling and is able to unambiguously detect multiple aliased overlapping frequencies and their respective amplitudes over the complete range of frequencies.

The system is in principle capable of the inverse action of generating high-frequency signals from lower-frequencies.

2. Analytical procedure

The method proposed here of retrieving frequency components in excess of the Nyquist frequency depends on leaving the incoming signal unfiltered and then interpreting the change in indicated frequency, which results from different sampling rates. The technique can best be described by using a specific example in which an incoming sinusoidal frequency of 1100 Hz is sampled at 2000 Hz as shown in Figs. 1 and 2. Although a sine wave consists of two components with a positive and negative frequency, respectively, in order to simplify the explanations, the following graphical representations have been drawn showing the absolute values of the frequencies throughout, since absolute values are all that are generally displayed by methods such as by an FFT.

As shown in Fig. 1, sampling at 2000 Hz gives a Nyquist or ‘fold over’ frequency at 1000 Hz. A signal at 1100 Hz, which is 100 Hz greater than the Nyquist value, will be reflected back from the Nyquist frequency by 100 Hz so as to appear as a false or ‘aliased’ signal at 900 Hz within the

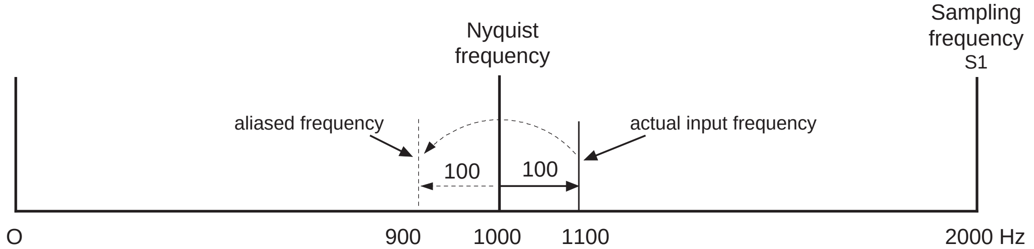

Fig. 1. Single fold-over diagram of aliased signal sampled at 2000 Hz.

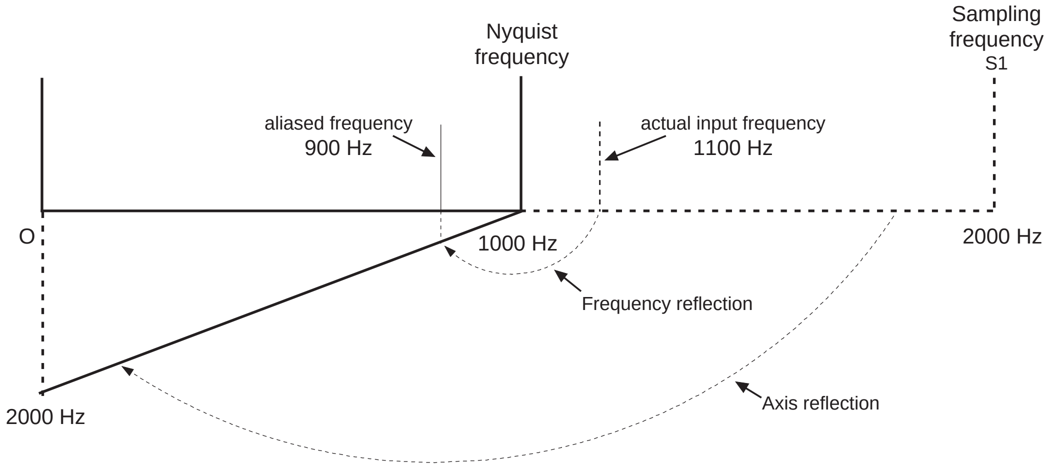

Fig. 2. Single fold-over sawtooth diagram of aliased signal sampled at 2000 Hz.

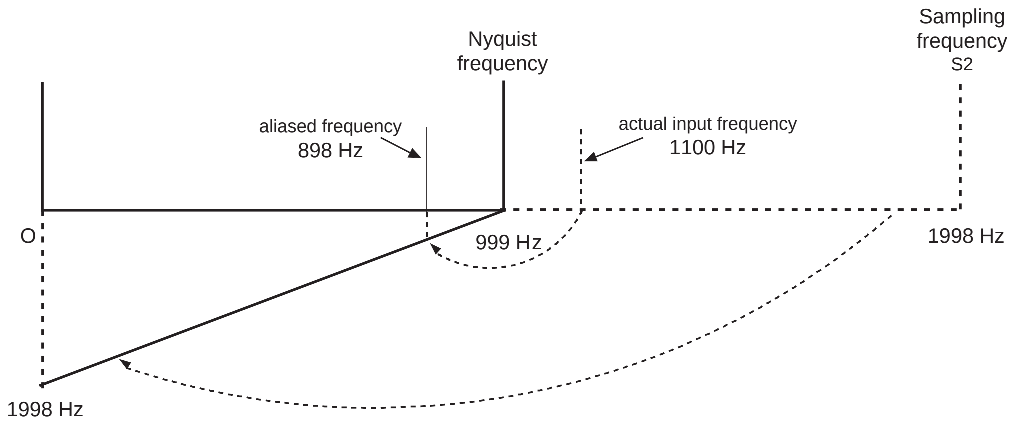

Fig. 3. Single fold-over diagram of aliased signal sampled at 1998 Hz.

perceived spectrum. Fig. 2 represents an alternative view of the fold over principle with the range beyond 1000 Hz being pivoted about the Nyquist frequency location to give a more meaningful illustration showing the reflection of the 1100 Hz incoming signal as 900 Hz.

As shown in Fig. 3, sampling the same incoming 1100 Hz signal at a frequency of 1998 Hz will give a Nyquist frequency of 999 Hz and thus an aliased frequency of 898 Hz.

The previous diagrams can be expanded for higher orders of incoming frequencies as illustrated in Fig. 4, where a sampling frequency of 2000 Hz is shown by way of example.

It can be seen from the multiple fold-over example in Fig. 4 that a frequency appearing at 200 Hz can signify an aliased frequency of 1800, 2200, 3800, 4200 Hz, etc. A frequency appearing as 800 Hz can signify an aliased frequency of 1200, 2800, 3200, 4800 Hz, etc. Hence the aliased frequency can be obtained from the equation

$$F_{alias} = NS - F_{act},$$

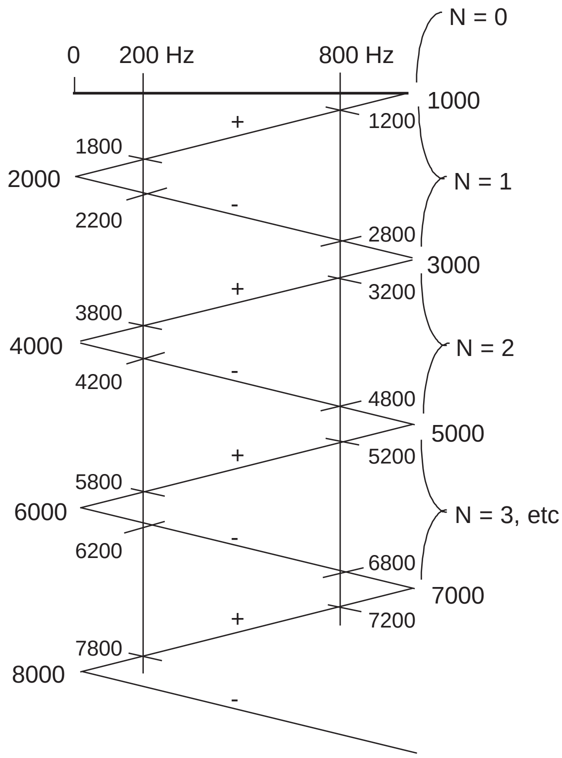

Fig. 4. Sawtooth diagram for multiple fold-overs sampled at 2000 Hz.

thus

$$F_{act} = NS - F_{alias}, \tag{1}$$

where F_{act} is the actual incoming frequency, F_{alias} the projection of F_{act} on basic spectrum (aliased frequency), S the sampling frequency and N the Nyquist band.

2.1. Recovery when actual frequency is on a +Nyquist slope

For illustrative purposes, the following example as shown in Fig. 5 displays two sampling frequencies, S_1 and S_2 (from channel 1 and 2), giving two aliased frequencies, F_1 and F_2 from an actual frequency, F_{act}, lying in the +slope of the Nyquist band where $N = 1$.

From Eq. (1):

$$\begin{aligned} &\textit{for channel } 1\text{: } F_{act} = NS_1 - F_1, \\ &\textit{for channel } 2\text{: } F_{act} = NS_2 - F_2 \end{aligned} \tag{2}$$

therefore

$$\begin{gathered} NS_1 - F_1 = F_{act} = NS_2 - F_2, \\ N = (F_1 - F_2)/(S_1 - S_2). \end{gathered}$$

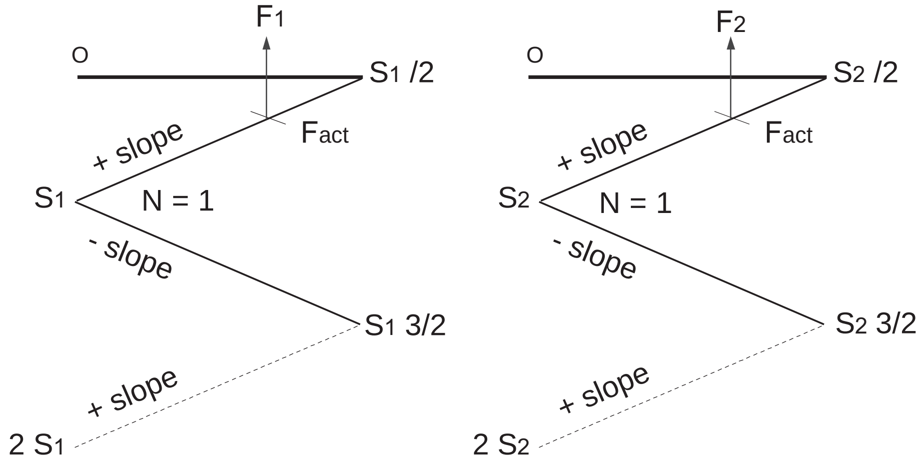

Fig. 5. Frequency recovery from +Nyquist slope.

Substituting N into Eq. (2)

$$F_{act} = (F_1 - F_2)S_1/(S_1 - S_2) - F_1,$$
$$F_{act} = (F_1 - F_2)Q - F_1,$$

where

$$Q = S_1/(S_1 - S_2). \tag{3}$$

2.2. Recovery when actual frequency is on a −Nyquist slope

Similarly an actual frequency, F_{act}, when lying in the negative slope of the same Nyquist band will produce

$$-F_{act} = (F_1 - F_2)Q - F_1. \tag{4}$$

Since the polarity of F_{act} is not relevant, Eqs. (3) and (4) may be expressed as

$$F_{act} = |(F_1 - F_2)Q - F_1|, \tag{5}$$

where

$$Q = S_1/(S_1 - S_2), \tag{6}$$

F_1 and F_2 are the observed (aliased) frequencies and S_1 and S_2 are the sampling frequencies.

For example, if a 3800 Hz signal were sampled at 2000 and 2002 Hz, it would produce two aliased frequencies of 200 and 204 Hz, respectively.

$$Q = 2000/(2000 - 2002) = -1000,$$
$$F_{act} = |(200 - 204) - 1000) - 200| = 3800 \text{ Hz}.$$

This gives the true and unambiguous F_{act} as 3800 Hz by using only the 2000 and 2002 Hz sampling frequencies. Due to the requirement that F_{act} has to lie at no greater a distance than one slope on any + or − slope of the same Nyquist band of the sawtooth diagrams, any fold-over frequencies from two sampling frequencies that do not fulfil this requirement will not produce an unambiguous F_{act}. This condition shall occur, when F_{act} moves from one slope (in sampling channel one), to the adjoining slope (in sampling channel two) at a distance greater than half of the sampling frequency S_1. This restriction can be overcome by an appropriate change in the sampling frequencies but this would eliminate the automatic frequency detection process. A more effective method can be utilized by introducing a third sampling channel S_3 to ensure that two sampling channels (either S_1 and S_2 or S_2 and S_3) will keep F_{act} on the same slope of the same Nyquist band. It therefore follows that the largest value of F_{act} (calculated from S_1 and S_2 and S_2 and S_3) will reliably provide an accurate unambiguous assessment of the incoming frequency. This comparison technique can be dispensed with and the frequency conditions, which satisfy both criteria, can be used. This latter method of using a third sampling channel S_3 has an advantage in that it is capable of detecting a number of frequency components, which coexist at the same frequency within a particular frequency spectra and can thereby be missed by larger overlapping amplitude signals.

2.3. Maximum recoverable frequency when using three sampling channels

For any configuration, the maximum value that can be retrieved depends upon at least two of the fold-over frequencies lying at no greater a distance than the Nyquist band as shown previously, thus when using three channels of sampled data the maximum length of a Nyquist band is from 0 to the Nyquist frequency (of the lower of the sampling frequencies). At the maximum frequency, F_1 and F_2 will lie on the same Nyquist band and will be at the extremities, i.e., F_1 will be at 0 and F_2 will be at the Nyquist frequency ($S_1/2$, where S_1 is the sampling frequency.) The maximum unambiguously retrievable frequency $F_{\mathrm{max},}$ which is valid over the complete sampling range is derived from Eqs. (5) and (6) for F_{act}:

$$F_{act} = |(F_1 - F_2)Q - F_1|.$$

Since at the maximum frequency

$$F_{act} = F_{max} \quad \text{and} \quad F_1 = 0 \quad \text{and} \quad F_2 = S_1/2$$

substituting in Eq. (5) gives

$$F_{max} = |-S_1^2/(2(S_1 - S_2))|,$$

which can be rewritten as

$$F_{max} = S^2/(2R), \tag{7}$$

where

$$R = |S_1 - S_2|,$$

where S is the maximum sampling frequency of the slowest clocking A/D device compatible with obtaining three distinct sampling frequencies at R Hz apart, where $R = |S_1 - S_2|$. The smaller the value of R the greater the maximum frequency that can be determined.

Any frequencies greater in value than F_{max} will give a fold-over. Such occurrences may be prevented by the usual method of low-pass filtering at the F_{max} frequency. However, with current analogue-to-digital converters this would only be necessary for incoming signals within the THz region.

Using the above sampling frequencies of 1998, 2000 and 2002 Hz it is possible to unambiguously retrieve frequency signals around 1 MHz, which is 1000 times the universally accepted limitation imposed by the Nyquist criterion. Reducing the difference between the sampling frequencies can retrieve even higher frequencies. Current analogue-to-digital converters can easily give capture frequencies of 1 MHz. Hence by using a sampling frequency difference of 0.25 Hz, retrievable frequencies within the THz range are possible.

3. Development of hardware and software

Early implementation of the techniques described in the previous section did show that the proposals, within this research programme, gave results confirming the validity of the method. This required an incoming signal to be analyzed at three different sampling rates, sequentially and therefore not simultaneously. The A/D system had to be reprogrammed to capture the same incoming signal twice more at slightly different sampling frequencies. During this process, if the incoming signal varied in any significant way, the accuracy of the results could not be relied upon. Subsequent transfer of the data to analogue tape and subsequently replaying the same section of this tape for the three sampling frequencies provided better results but was never considered a realistic approach to capturing and analysing real time data. In addition, any recording unit such as a tape unit will impose its own maximum frequency limitation.

3.1. Available equipment

There are many methods of sampling data and returning the results to a PC in order to analyze the data, with each having its advantages and disadvantages. The main requirement was to obtain data from three A/D channels operating at slightly different sampling frequencies.

Currently available A/D sampling units for the PC could not be used because the sampling clock for each A/D interface card could not hold, to adequately close limits, the required sampling rates. Hence, the differences between the sampling rates, on which the method depends, would not be held to the very fine limits needed. Also, if the A/D system is arranged as a multiplexed input unit, there will be only one internal A/D converter using only one sampling clock, which is incapable of generating the required three slightly different sampling frequencies.

3.2. Development of asynchronous data logger (ADL) equipment

Due to the above limitations it was necessary to design a hardware unit capable of maintaining the difference between the sampling rates to a very high order of accuracy. The solution adopted was to build a multi-channel data logging unit independent of PC technology, consisting of separate A/D circuits for each channel, with each channel having its own digital signal processor (DSP) microcontroller dedicated to maintaining the high accuracy independent clocking of its A/D facility; each channel also has an additional microcontroller dedicated to storing data into its own data buffer. A separate master microcontroller circuit is used to communicate with the

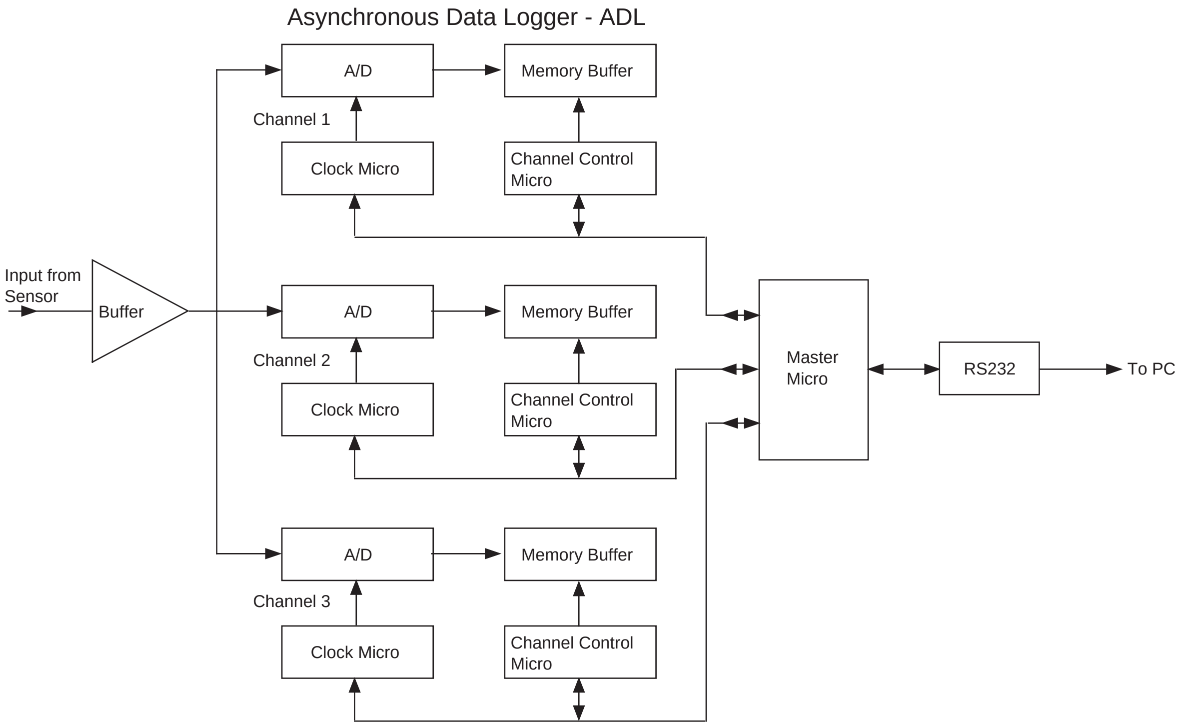

Fig. 6. Asynchronous data logger (ADL)—block diagram.

separate A/D circuits for sending data to a PC and for receiving commands from the PC. This was called an asynchronous data logger (ADL).

Fig. 6 shows the arrangement of the ADL, developed to sample the incoming signal simultaneously at the three slightly different sampling frequencies. The incoming signal from a sensor or transducer passes, without filtering, to the three sampling channels, which are running at slightly different sampling rates.

3.3. *Analytical software—ADL FFT analyzer (ADLFFT)*

The analytical procedure as outlined in Fig. 7 was devised to make use of the information for all three captured channels and to automatically select the correct channel pair needed to predict the actual value of the incoming frequency. This was performed within a specially developed program called ‘asynchronous data logger FFT’ (ADLFFT). A procedure within this program, given the name ‘alias hunt’, calculates the possible aliased signals by comparing channel pair 1 and 2 with channel pair 2 and 3. The true incoming frequency is then obtained as described in Section 2.

3.4. *Accuracy of calculating the aliasing frequency*

The accuracy of the calculated aliased and true frequency components depends on several factors.

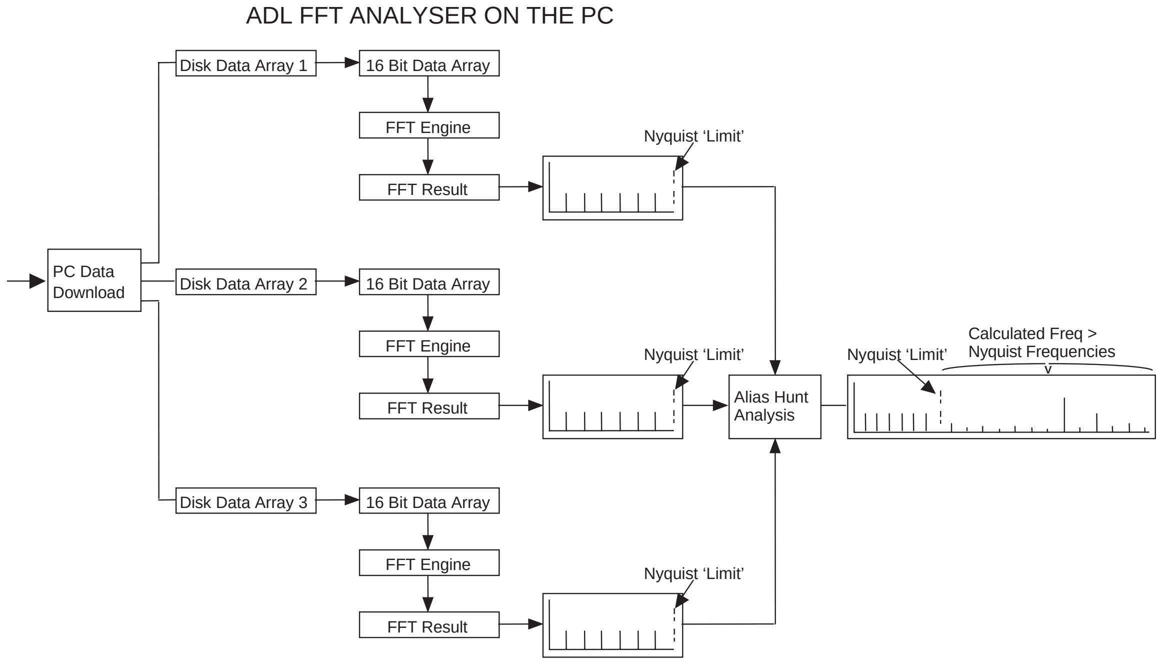

Fig. 7. Asynchronous data logger FFT (ADLFFT) and alias hunt—block diagram.

3.4.1. *The absolute frequency accuracy of each channel's A/D sampling clocks within the ADL*

A single master crystal oscillator was used as a common oscillator to feed all three clocking DSP microprocessors in the A/D channels so that any master frequency drift was common to all channels. An accurate internally temperature stabilized crystal oscillator was used to obtain a high level of stability in preference to such as an RC oscillator, which is prone to frequency drift.

3.4.2. *The accuracy of the difference between each channel's A/D sampling clocks within the ADL*

A single crystal oscillator, as explained above, was used in place of three separate crystal oscillators. Any drift in the master oscillator would be common to all three channels. To maintain relative accuracy, the pulses from this single master oscillator are digitally counted by each channel's clocking DSP microprocessor. When a digital count value reaches a set number, the channel's microprocessor outputs the sampling pulse to the channel's sampling A/D unit. The alternative would be to use an oscillator for each channel, but the possibility of differing drift rates rules this out. In contrast, the use of one master oscillator, using digital counting methods reduces frequency drift in the difference between the sampling rates, R, to insignificant levels. Any difference between the channel sampling rates would be due to differences in the switching speeds of the channel's DSP microprocessor, e.g., if one DSP microprocessor took 5 nS longer than another to physically change state on a digital line, this 5 nS would occur each time it changed state and so would cancel out. This DSP microprocessor would, in effect, be simply 5 nS out of phase with the others, which would not introduce significant error to the analytical procedure.

3.4.3. The size of the FFT

In order to discern an aliased signal occurring in different channels, the frequency resolution of the FFT result must be sufficient to enable the observed aliased frequencies to be seen as being different in frequency. For example, when sampling at 2000 Hz a 1K FFT would give a frequency resolution of 1.953125 Hz per FFT data element and would as a result, show frequencies having a difference smaller than this value as being at the same frequency. The frequency difference between the aliased frequencies F_1 and F_2 as used in this project is adequately discerned since a 512K FFT is used in conjunction with a sampling rate of 2000 Hz, gives a frequency resolution of 0.003815 Hz per FFT data element. Hence, F_1 and F_2, being 1 Hz apart, occupy separate data elements.

The ADLFFT program was designed to enable up to 512K sized FFTs to be chosen by the operator as deemed appropriate for the circumstances. The software, as currently developed, requires trial and error 'alias hunts' to be performed with different sized FFTs, until aliased signals are either seen and eliminated by analysis, or none are found. This operation can be repeated on the stored data without the need to re-sample.

3.4.4. The mathematical accuracy of the FFT in determining the amplitudes of the signals

Once analyzed by the ADLFFT program, aliased signals are paired by matching the amplitudes of the signals between the channels. The use of 10 byte extended variables within the FFT program was used to maximize amplitude accuracy.

3.4.5. The accuracy of the amplitude of the sampled data obtained from the A/D units

As previously stated, within the alias hunt section of the ADLFFT program, aliased signals are paired by matching the relative amplitudes of the signals between the channels. The A/D devices used were 16-bit precision A/D units, which give one part in 65 535 resolution, as opposed to 8-bit A/D devices, which only give one part in 255 resolution. Each of these A/D devices has an internal zero and offset voltage calibration feature and all are fed by a common voltage reference, so that any voltage drift is common to all channels, so as to achieve maximum relative amplitude accuracy within the A/D converters.

3.4.6. Matching the amplitudes between FFT channels within the Alias Hunt procedure

As previously stated, within the alias hunt section of the ADLFFT program, aliased signals are paired by matching the amplitudes of the signals between the channels. The largest amplitude component to result from the FFT operation in all channels is scaled to be the 100% amplitude level and subsequent results are scaled to their respective channel's 100% level, rather than using absolute levels. This is done to equalize the analogue and digital gain within each channel's A/D unit, to minimize amplitude comparison errors.

4. Experimental results

To illustrate the operational features and the scope of application of the ADL system and the ADLFFT program experimental results were produced. When a frequency is displayed, the following factors need to be considered.

The resolution of each element of data in an FFT analysis equals the Nyquist frequency divided by the number of data elements used to store up to Nyquist. The number of data elements used to store up to Nyquist frequency equals half the FFT size (because the other half of the FFT data array contains a 'mirror image' of the results, which are not used), therefore

$$\begin{aligned} \textit{FFT resolution per data element} &= (\textit{sampling frequency}/2)/(\textit{FFT size}/2)\ \text{Hz} \\ &= \textit{sampling frequency}/\textit{FFT size}\ \text{Hz}. \end{aligned}$$

Therefore, when an FFT size of 8K is used, a frequency resolution of 0.244335938, 0.244140625 and 0.243945313 Hz per FFT data element (point) is obtained, when sampling at 2001.6, 2000 and 1998.4 Hz, respectively. For the case in which the true incoming frequency is 737.714 Hz.

Sampling frequency (Hz)	Resolution (Hz)/data element	FFT display (Hz)
2001.6	0.244335938	737.6501968
2000.0	0.244140625	737.7929688
1998.4	0.243945313	737.6906265

Therefore, this true 737.714 Hz signal would be slightly misrepresented. This effect is due to the use of the nearest applicable data element. It is sometimes called the 'picket gate' effect and can be minimized by increasing the size of the FFT so as to increase the frequency resolution.

When the frequency and amplitude results of an FFT were displayed in the following examples, the results were left displaying eight decimal places on the graphical output. The reader should not infer the accuracy of the FFT and frequency calculations from this since the overall accuracy is dependent on the factors previously explained in Section 3.4.

4.1. 74 512 *Hz sine wave input*

A sine wave of 74 512 Hz was applied to the experimental apparatus with sampling frequencies of 2001.6, 2000 and 1998.4 Hz. The three channels simultaneously captured the signals and the relevant values of the frequency determined from the FFT diagrams are displayed as shown in the following table.

Sampling frequency (Hz)	Aliased frequency component (Hz)
2001.6	453.4875
2000.0	512.6935
1998.4	571.8078

Fig. 8 shows the comparison of the frequency determined from the 2001.6/2000 Hz capture frequency combination to give a predicted input frequency of 74 512.6875 Hz.

Fig. 9 gives a corresponding value of 74 512.6953 Hz for the capture combination of 2000/1998.4 Hz.

Fig. 10 shows the result of the selection and calculation to determine the correct value of the actual frequency obtained from Figs. 8 and 9 to be the higher of the two values, as detailed in

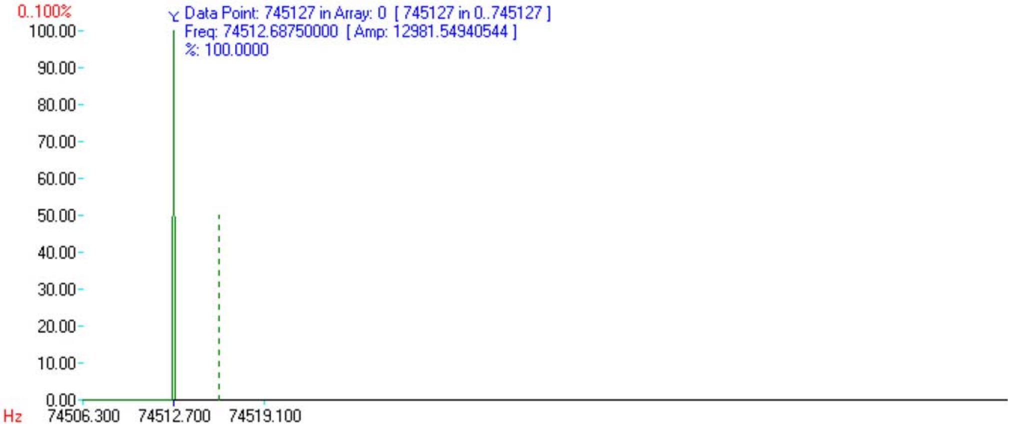

Fig. 8. Combined FFT analysis for 2001.6 and 2000 Hz.

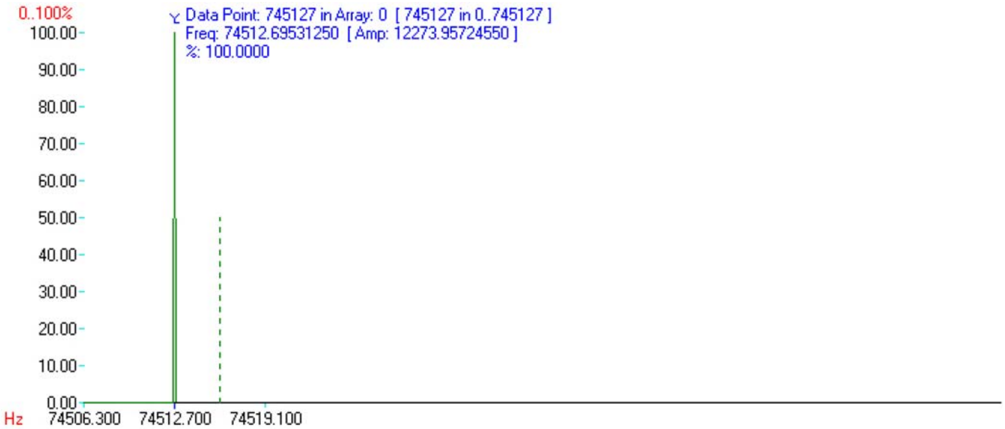

Fig. 9. Combined FFT analysis for 2000 and 1998.4 Hz.

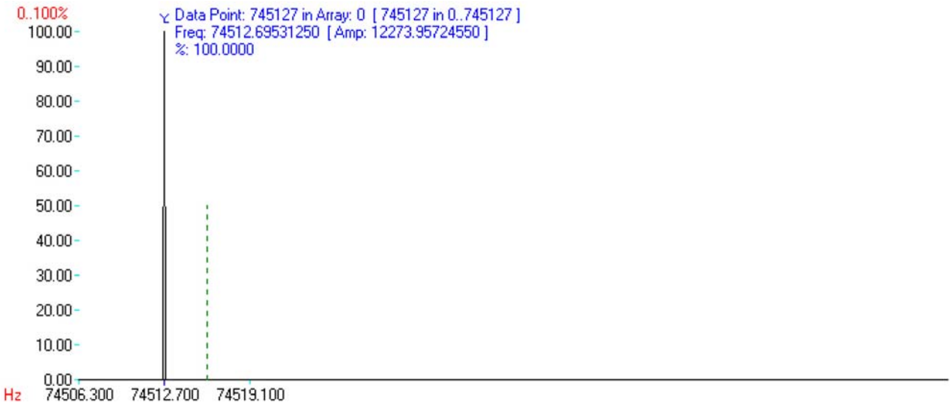

Fig. 10. Overall combination of 2001.6, 2000 and 1998.4 Hz.

Section 2. In this case, the differences are negligible and both lie within the resolution of the analysis since all of the three sampling frequencies give aliased frequencies, which lie within the same frequency band.

When any signal is analyzed by techniques such as an FFT, the calculated value will be determined by the size of the FFT analysis. For example, when sampling at 2000 Hz and using a 8192 byte sized FFT analysis, each FFT data segment would represent a frequency band 0.24414 Hz wide. Calculated and displayed frequencies will have to fall into the nearest 0.24414 Hz data segment. Therefore, the values shown for the aliased frequencies will determine by the nearest FFT data segment. The frequency band, N, is calculated from Eqs. (5) and (6) as

$$(F_1 - F_2)/(S_1 - S_2). \tag{8}$$

In order to compensate for the FFT data segment size; because the frequency band, N, must be an integer value, Eq. (8) can be modified to give the correct frequency band from

$$\mathrm{int}[(F_1 - F_2)/(S_1 - S_2) + 0.5]. \tag{9}$$

Therefore, Eq. (5) can be modified as follows:

$$F_{act} = |(\mathrm{int}[(F_1 - F_2)/(S_1 - S_2) + 0.5])\, S_1 - F_1|. \tag{10}$$

This factor is included within the ADLFFT analysis software. Using Eq. (10) to assess the aliased frequencies gives values for F_{act} of 74 512.6875 and 74 512.6953 Hz for the 2001.6/2000 and 2000/1998.4 Hz sampling frequency combinations, respectively.

4.2. *Square wave input of* 1922 *Hz*

To consider the effect of multiple input frequencies a square wave of 1922 Hz was applied to the system, again using sampling frequencies of 2001.6, 2000 and 1998.4 Hz. Space limitations within this paper preclude the insertion of all of the analysis and diagrams, but Figs. 11–13 show the resulting frequency diagrams, which did correctly predict the first three components of the input frequency at 1921.946, 5766.379 and 9610.709 Hz.

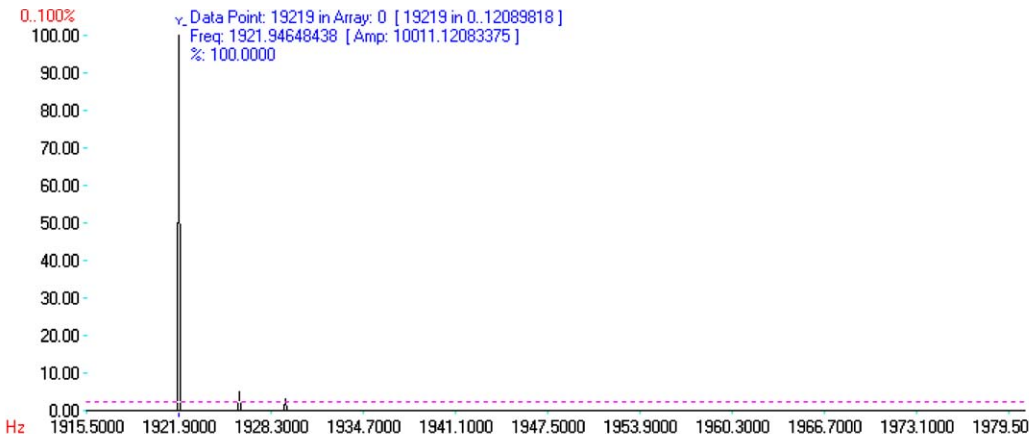

Fig. 11. Fundamental component of 1922 Hz square wave.

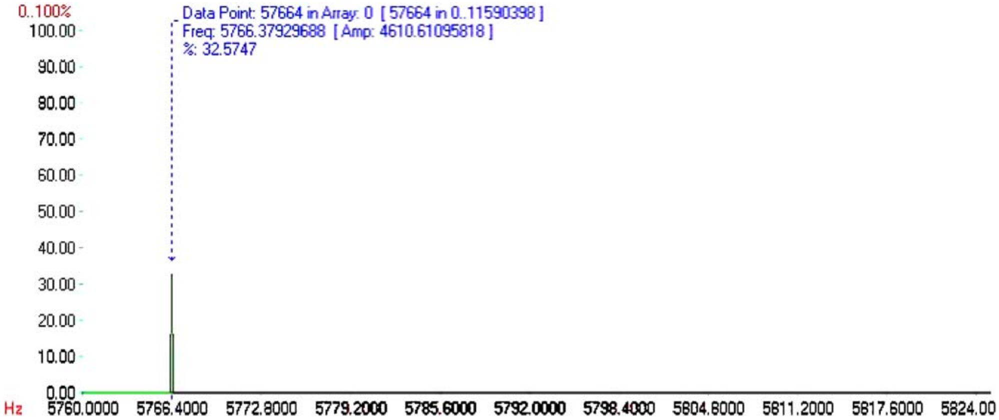

Fig. 12. Third harmonic of 1922 Hz square wave.

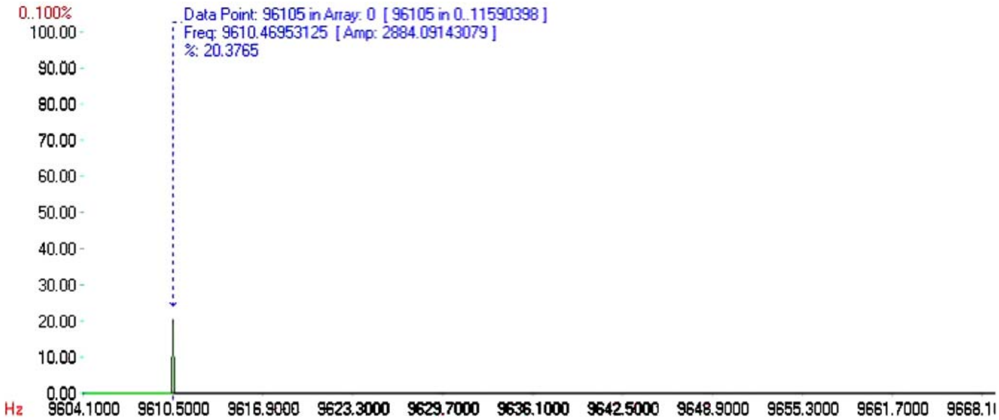

Fig. 13. Fifth harmonic of 1922 Hz square wave.

4.3. *Multiple coincident frequency components*

The previous section showed that data captured at any specific sampling frequency can contain multiple frequencies coinciding at the same location on a frequency spectrum. Whilst differences in amplitude can exist for each of these coincident frequency components, only the largest of these will be shown with the lesser amplitudes being obscured by the more dominant amplitudes. Utilizing Eq. (5) and comparing all combinations of the frequencies evident on each spectra such overlapping frequencies can be retrieved in both frequency and amplitude. Since it is unusual to get practical examples of such multiple overlapping of frequencies exactly coincident on a spectrum, a theoretically generated signal was composed to give a multitude of sine wave components all coinciding on 1000 Hz spectra. Frequencies of 200 and 800 Hz were chosen, together with the first seven components of each of these chosen spectra, as shown in Fig. 4, are utilized with Table 1 showing the corresponding amplitudes for the three chosen sampling frequencies of 1999, 2000 and 2002 Hz. Also shown in this table are the frequency components

Table 1

Selected		Aliased frequencies			Detected	
Freq (Hz)	Amplitude	1999	2000	2001	Freq (Hz)	Amplitude
800.00	60.00	800.00	800.00	800.00	800.00	60.00
1200.00	100.00	799.00	800.00	801.00	1200.00	100.00
2800.00	20.00	801.00	800.00	799.00	2800.00	20.00
3200.00	30.00	798.00	800.00	801.00	3200.00	30.00
4800.00	80.00	802.00	800.00	798.00	4800.00	80.00
5200.00	40.00	797.00	800.00	803.00	5200.00	40.00
6800.00	50.00	806.00	800.00	797.00	6800.00	50.00
7200.00	30.00	796.00	800.00	804.00	7200.00	30.00
200.00	38.00	200.00	200.00	200.00	200.00	38.00
1800.00	46.00	199.00	200.00	201.00	1800.00	46.00
2200.00	73.00	201.00	200.00	199.00	2200.00	73.00
3800.00	41.00	198.00	200.00	202.00	3800.00	41.00
4200.00	63.00	202.00	200.00	198.00	4200.00	63.00
5800.00	35.00	197.00	200.00	203.00	5800.00	35.00
6200.00	38.00	203.00	200.00	197.00	6200.00	38.00
7800.00	33.00	196.00	200.00	204.00	800.00	33.00

Table 2

Selected		Aliased frequencies			Detected	
Freq (Hz)	Amplitude	1998	2000	2002	Freq (Hz)	Amplitude
1234.00	22.00	764.00	766.00	769.00	1234.00	66.00
5678.00	55.00	316.00	322.00	328.00	5678.00	55.00
91 011.00	77.00	897.00	989.00	921.00	91 011.00	77.00
76 5432.00	25.00	198.00	568.00	668.00	765 432.00	25.00
		198.00	766.00	668.00	567 234.00	25.00

found by the anti-aliasing technique and shows complete agreement in both frequency and amplitude resolution.

A selected set of frequencies can be chosen to show lack of uniqueness in the detected data. In Table 2, frequencies of 1234, 5678, 91 011 and 765 432 Hz are shown with the aliased and detected frequencies.

It can be seen from Table 2 that an additional frequency of 567234 Hz appears. Eq. (1) was used to calculate the aliased frequencies associated with this frequency and, as shown in the table, identical component aliased frequencies exist at 1234, and 765 432 Hz. If all of the above selected frequencies in Table 2 are added to Table 1 only one additional frequency of 438 766 Hz appears due to an identical aliased frequency of 794 Hz being repeated at 5200 Hz. These additional frequencies disappear when the selected sample frequencies are changed to 1999/2000/2001 Hz. The additional frequencies appear as a consequence of cross matching of aliased components and can be overcome by increasing the sampling rate. Such coincident components will be a rare occurrence since all of the aliased frequencies need to be cross matched to within the frequency resolution of the FFT which is likely to be less than a fraction of 1 Hz. However, such occurrences may be dealt with by two methods. The first is to use a higher sampling rate but this would require resampling of the data which is usually not possible in practical situations. An alternative elimination of these additional frequencies can be achieved by the inclusion of a fourth sampling channel, in this case at 2004 Hz. It is not necessary to evolve additional techniques to utilize the four channels as this can be achieved by using the results obtained from using channels 1–3 together with the results obtained from using channels 2–4 and discarding the frequencies which are not common to both sets of results. This will depict all of the real unambiguous frequencies and discard all of the erroneous frequencies. This method is preferable to increasing the sampling rate since it removes the need for human intervention and requires only the addition of another sampling channel, the cost of which is low.

4.4. Industrial application

A simple, but practical application of the proposed technique was used to diagnose a problem, which arose within a system comprising a variable speed motor and a rotating drum coupled through a single reduction gear train.

A typical spectrum, taken with on-site monitoring equipment, from the drive train on 3 January 2001, is shown in Fig. 14. The sampling frequency was 2000 Hz, which gave an effective bandwidth up to the Nyquist frequency of 1000 Hz. The most significant component of vibration was seen to have an amplitude of 15.91 m/s^2 at 668.4 Hz.

To ensure that no higher components of vibration existed within the system, a second spectrum was obtained as shown in Fig. 15 at a sampling frequency of 4200 Hz, so as to give a bandwidth to

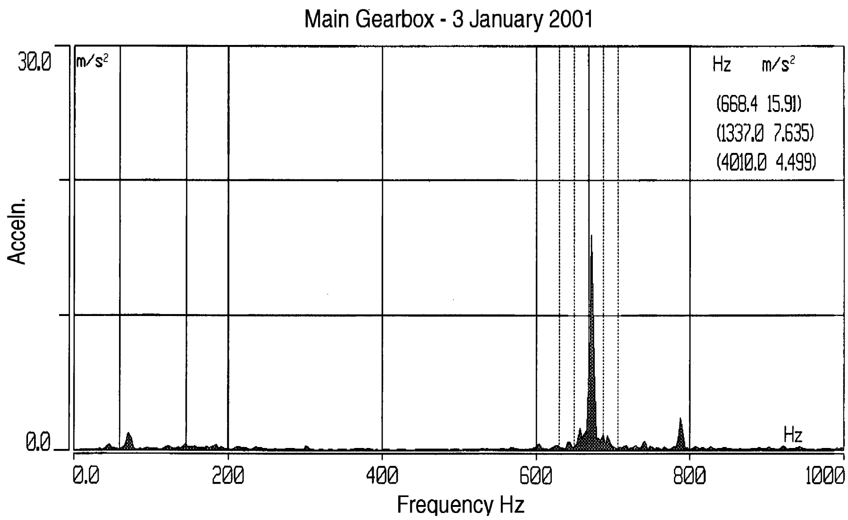

Fig. 14. Drive train spectrum sampled at 2000 Hz.

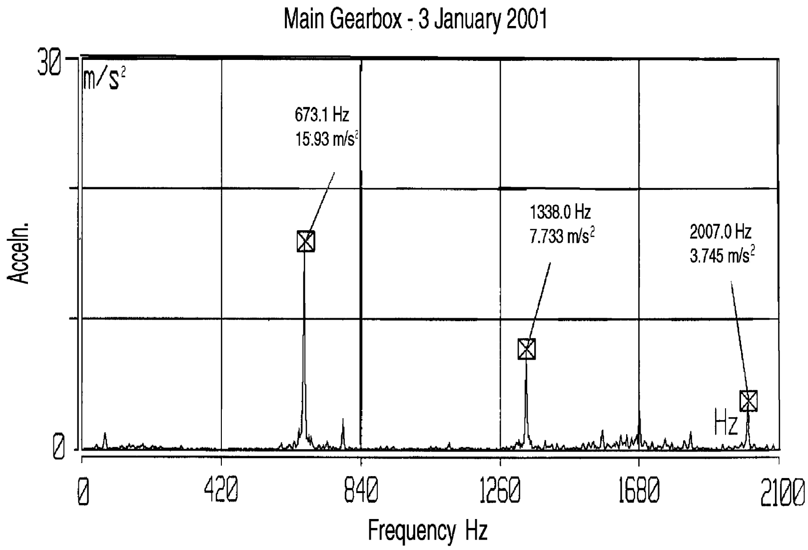

Fig. 15. Spectrum sampled at 4200 Hz.

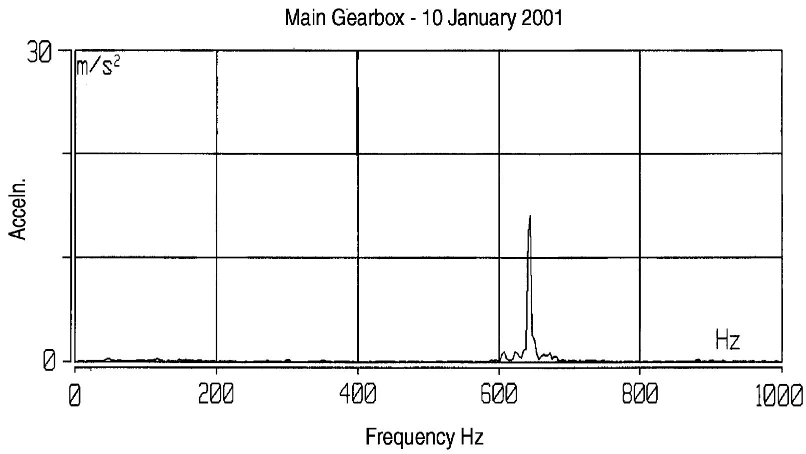

Fig. 16. Spectrum sampled at 2000 Hz.

2100 Hz. This showed the first three harmonics of the gearmesh frequency, with the most dominant being the fundamental at 673.1 Hz.

Subsequent monitoring was therefore undertaken with the bandwidth set to 1000 Hz, i.e., a sampling frequency of 2000 Hz was used.

On 10 January 2001, the machine operators reported a deterioration in product quality, which instigated further vibration sampling. This is shown in Fig. 16, which indicates the gearmesh frequency to be at a lower frequency of 642.9 Hz and with a reduction in vibration amplitude to 13.9 m/s^2.

It was therefore considered that gearmesh vibration was not the cause of the problem. Further investigation, using the ADL system as developed within this research project, was performed to measure the true actual frequency from the aliased frequencies, which it had detected. This showed that an actual signal was present at 1920.8 Hz, as depicted in Fig. 17. This result was obtained using the same initial sampling rate of 2000 Hz together with 1999.2 and 2000.8 Hz, within the ADL unit. This signal also showed a considerable increase in vibration level.

As a check on this finding, further sampling was undertaken at a sampling frequency of 4000 Hz using the existing conventional on-site monitoring equipment. The results were as shown in Fig. 18, which confirmed the above results from the ADL unit.

In Fig. 18, the frequency shown by the graph marker, as manually placed by the operator, is seen to be 1925 Hz, however on the console section the frequency was actually listed as 1920.0 Hz.

Further investigations showed that the unpredicted increase in the amplitude of vibration at the third harmonic of gearmesh frequency was due to the presence of a resonant frequency close to

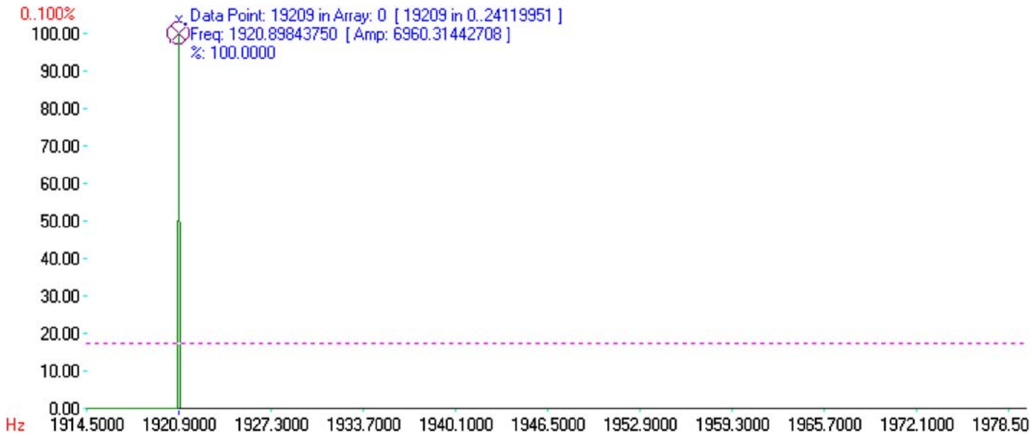

Fig. 17. ADL analysis showing the actual signal to be at 1920.8 Hz.

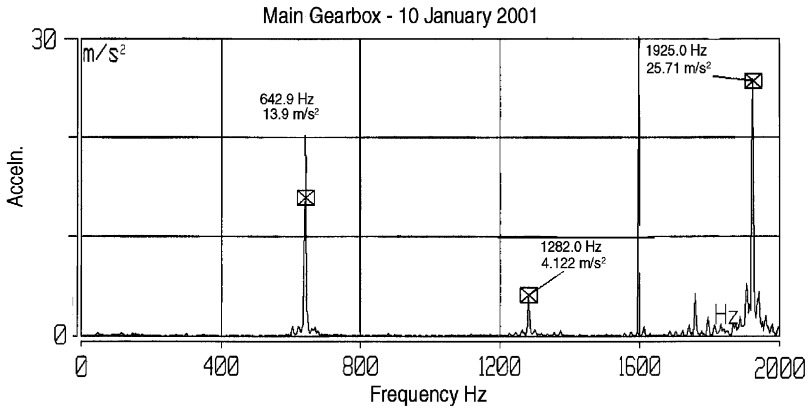

Fig. 18. Check spectrum sampled at 4000 Hz.

the 1920 Hz frequency range. This resonant frequency was speed dependent and did not show at the higher speed condition as may be seen in Fig. 15.

5. Conclusions

The system developed and described within this paper illustrates that the established theory, which states that the highest frequency that can be retrieved from an incoming periodic broadband signal is half of the sampling frequency and that the input must be filtered to eliminate higher frequencies from appearing as false aliased frequencies, can be overcome. Experimental verification has shown that frequencies at least 2000 times this limitation can be unambiguously identified.

The system as tested was inexpensively constructed to prove the technique. The system can also be used to invert the technique to enable high frequencies to be generated from low-frequency signal generating equipment.

The technique has been used in vibration condition monitoring and it removes the need for the analyst to predict the maximum frequency of interest in a signal. Of particular relevance is when an unexpectedly high frequency exists within an incoming signal. The proposed method enables the event to be immediately re-examined for the presence of much higher frequency components without the conventional need to resample using higher cut-off filters and higher sampling frequencies—this is especially important in situations when the problematic operating conditions may be intermittent. The system is also capable of extending the bandwidth of existing or future sampling rates with its overlapping principle and provides a high degree of frequency resolution up to its maximum attainable frequency. It is also possible, due to its high-frequency potential, for the method to be used within the field of communication, data encryption and associated disciplines.

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