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A comparison of semi-active damping control strategies for vibration isolation of harmonic disturbances

Y. Liu*, T.P. Waters, M.J. Brennan

Institute of Sound and Vibration Research, University of Southampton, Southampton SO17 1BJ, UK

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Abstract

Active vibration isolation systems are less commonly used than passive systems due to their associated cost and power requirements. In principle, semi-active isolation systems can deliver the versatility, adaptability and higher performance of fully active systems for a fraction of the power consumption. Various semi-active control algorithms have been suggested in the past, many of which are of the "on–off" variety. This paper studies the vibration isolation characteristics of four established semi-active damping control strategies, which are based on skyhook control and balance control. A semi-active damper is incorporated into a single-degree-of-freedom (s.d.o.f.) system model subject to base excitation. Its performance is evaluated in terms of the root-mean-square (r.m.s.) acceleration transmissibility, and is compared with those of a passive damper and an ideal skyhook damper. The results show that the semi-active system always provides better isolation at higher frequencies than a conventional passively damped system.

1. Introduction

A passive vibration isolation system is the simplest way to protect a dynamical system from vibration inputs. There is a trade-off with this system, however, between the control of vibration at resonance, when a highly damped isolator is desirable, and the higher frequency isolation performance, when low damping is required [1]. Active isolation systems can be used to overcome this limitation [2]. They generally fall into three categories: adaptive-passive, semi-active and fully active [3]. Fully active isolation systems apply dynamic forces at the same frequency as the primary excitation and can provide superior performance, but the system becomes more complex and there are a number of issues that need to be addressed. These include the selection of

*Corresponding author. Tel.: +44-23-8059-2276; fax: +44-23-8059-3190.
E-mail address: lyy@isvr.soton.ac.uk (Y. Liu).

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actuators and sensors, weight constraints, power requirements, stability, robustness, closed-loop performance and potential failure [3]. Adaptive-passive and semi-active vibration isolation involve changing the system properties, such as damping and stiffness as a function of time. In an adaptive-passive system, the properties are changed relatively slowly, but in a semi-active system, the properties are changed within a cycle of vibration. Adaptive-passive control has been used successfully in isolating harmonic rather than random disturbances [3], and semi-active control has been proposed as an alternative to fully active control for some applications [4,5].

The advantages of semi-active dampers over traditional passive dampers have been addressed in many studies [4,6–11]. For example, Karnopp et al. [4] studied the performance of a skyhook controlled semi-active damper, and compared the performance with that of a conventional passive damper [4,12]. The name "skyhook" is derived from the fact that it is a passive damper hooked to an imaginary inertial reference point. Semi-active control strategies can maintain the reliability of passive devices using a very small amount of energy, yet provide the versatility, adaptability and higher performance of fully active systems. The particular benefits of semi-active methods are that (as with adaptive-passive methods) the parameters of the system can be changed with time to retain optimal performance and (unlike adaptive-passive methods) higher levels of optimisation can be achieved due to the rapid time-variation achievable.

There exists a large number of control strategies for semi-active damping control in the literature [4,8,12,13]. Karnopp studied the performance of skyhook damping control [4,12]. Alanoly and Sankar studied the performance of balance control for vibration and shock isolation [8,13]. Carter studied the effectiveness of semi-active damping for vibration control of the car suspension [14]. Many of these control strategies are of the "on–off" variety. Although complicated control strategies may offer some advantages, significant performance gains can still be realized with more basic control strategies. The aim of this paper is to compare some of these basic control strategies in the vibration isolation of harmonic disturbances. Continuous skyhook control [4,15], on–off skyhook control [16], on–off balance control [6] and continuous balance control [8] are investigated and are compared with an adaptive-passive damping strategy. Numerical simulations are carried out on a single-degree-of-freedom (s.d.o.f.) system and results are presented to evaluate the suitability of these basic algorithms for vibration isolation of harmonic disturbances. The performance is evaluated in terms of the root-mean-square (r.m.s.) acceleration of the suspended mass, and compared with that of conventional and skyhook passive dampers.

2. Description of the control stategies

A s.d.o.f. system with a semi-active damper installed in place of the conventional passive damper subject to base excitation is shown schematically in Fig. 1. The vibration of the base and the mass are measured and fed into a controller, which tunes the damper coefficient such that the damping force, which is proportional to the relative velocity, $\dot{x} - \dot{x}_0$, can be varied as a function of time. The controller unit in Fig. 1 can represent any control strategy.

Semi-active dampers may be of the on–off type or of the continuously variable type. A damper of the first type is switched between "on" and "off" damping states in accordance with a suitable control algorithm. In its "on" state, the damping coefficient is relatively high, and in its "off" state, it is relatively low. Ideally the off-state damping should be zero, but in practical situations this is not possible. A continuously variable semi-active damper is also switched between "on" and "off" states. However, in its "on" state, the damping coefficient and corresponding damping force are varied. The concept of semi-active damping is illustrated in Fig. 2, which shows the force–velocity characteristics for an on–off and a continuously variable damper. The shaded part of the graph in Fig. 2(b) represents the range of achievable damping coefficients for a continuously variable damper.

This section describes four semi-active damping control strategies which are on–off and continuously variable implementations of skyhook and balance control. In Section 4 these methods are benchmarked against passive control and a simple adaptive-passive algorithm which is described next.

2.1. Adaptive-passive damping control

The first control algorithm considered is an adaptive-passive method. A passive system can only provide isolation in the frequency range $\omega/\omega_n > \sqrt{2}$, where ω is the excitation frequency and ω_n is the natural frequency. Increasing the damping coefficient in the frequency range $\omega/\omega_n \leqslant \sqrt{2}$ will reduce the resonance peak, while the isolation performance in the frequency range $\omega/\omega_n > \sqrt{2}$ will be degraded. Thus for harmonic vibration isolation the damping coefficient should be large

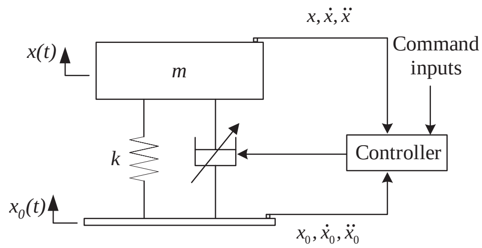

Fig. 1. Schematic of a s.d.o.f. system with a semi-active damper.

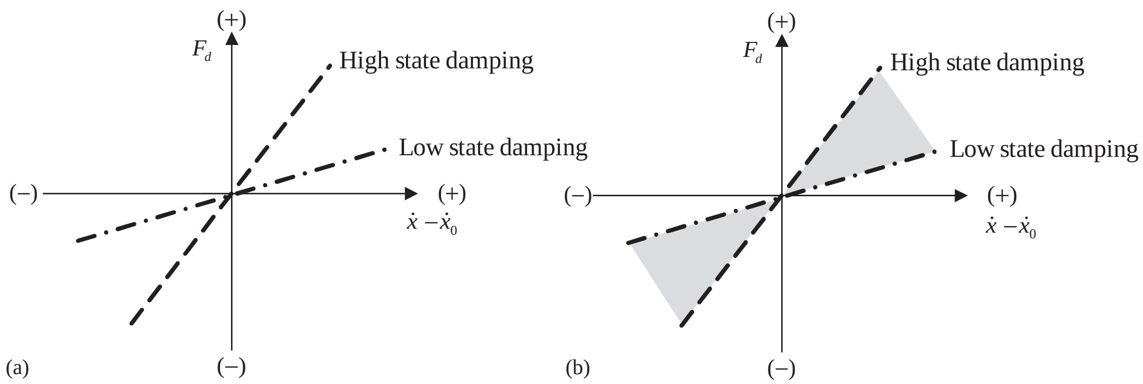

Fig. 2. Semi-active damper concepts (a) on–off damper; (b) continuously variable damper (the shaded part in (b) represents the range of the achievable damping coefficients).

when $\omega/\omega_n \leqslant \sqrt{2}$, and as small as possible when $\omega/\omega_n > \sqrt{2}$. The following control algorithm will ensure these conditions:

$$c = \begin{cases} c_{\max}, & r.m.s.(\ddot{x}) \geqslant r.m.s.(\ddot{x}_0), \\ c_{\min}, & r.m.s.(\ddot{x}) < r.m.s.(\ddot{x}_0). \end{cases} \tag{1}$$

The control algorithm uses the r.m.s. values, calculated over a time period much longer than the period of excitation, as the condition function to adjust the damping. When the r.m.s. value of the acceleration response $\ddot{x}$ is greater than the r.m.s. value of the base acceleration $\ddot{x}_0$, amplification occurs and the damper is switched to its maximum value. Otherwise, the damper is switched off so that minimal damping is present in the system. For this adaptive-passive control algorithm, the damper works in a bi-state (on–off) manner and does not require the damper to switch rapidly. It is potentially well suited to vibration isolation of rotating machines. The disadvantage of this control algorithm is that it is only applicable to harmonic vibration isolation.

2.2. Semi-active damping control

This section considers four semi-active control algorithms, namely continuous and on–off skyhook control, and continuous and on–off balance control.

2.2.1. Continuous skyhook control

Considering a s.d.o.f. system with a skyhook damper, the damping force can be written as

$$F_{sky} = c_{sky}\dot{x}, \tag{2}$$

where F_{sky} is the skyhook damping force, $\dot{x}$ is the velocity response of the mass, and c_{sky} is the damping coefficient of the skyhook damper. The intention is to replicate such a skyhook damping force with a semi-active damper mounted conventionally between the base and the mass. However, since a passive damper can only absorb vibration energy, the product of the semi-active damping force, F_{sa}, and the relative velocity, $\dot{x} - \dot{x}_0$, across the damper must satisfy the inequality

$$F_{sa}(\dot{x} - \dot{x}_0) \geqslant 0. \tag{3}$$

The desired force is $c_{sky}\dot{x}$, but the semi-active damper can only generate this force when $\dot{x}$ and $\dot{x} - \dot{x}_0$ have the same sign. When $\dot{x}$ and $\dot{x} - \dot{x}_0$ are of opposite sign, the semi-active damper can only provide a force opposite to the desired force. In this situation, it is better to supply no force at all. The continuous semi-active skyhook control algorithm is thus given by

$$F_{sa} = \begin{cases} c_{sky}\dot{x}, & \dot{x}(\dot{x} - \dot{x}_0) \geqslant 0, \\ 0, & \dot{x}(\dot{x} - \dot{x}_0) < 0. \end{cases} \tag{4}$$

The switching of the device is controlled by the term $\dot{x}(\dot{x} - \dot{x}_0)$, which is called the condition function. When the damper is on, the damping force can be written as

$$F_{sa} = c_{sa}(\dot{x} - \dot{x}_0), \tag{5}$$

where c_{sa} is the semi-active damping coefficient. The value that c_{sa} must take to emulate a skyhook damper may be found by setting Eq. (5) equal to Eq. (4), giving

$$c_{sa} = \begin{cases} \dfrac{c_{sky}\dot{x}}{(\dot{x} - \dot{x}_0)}, & \dot{x}(\dot{x} - \dot{x}_0) \geqslant 0, \\ 0, & \dot{x}(\dot{x} - \dot{x}_0) < 0. \end{cases} \tag{6}$$

One can see from Eq. (6) that when the relative velocity is very small, the required damping coefficient increases abruptly and tends to infinity. However, in practice the semi-active damper coefficient is limited by the physical parameters of the conventional damper, which means that there is both an upper bound, c_{max}, and a lower bound, c_{min}. The damping coefficient in Eq. (6) can thus be rewritten as

$$c_{sa} = \begin{cases} \max\left[c_{min}, \min\left[\dfrac{c_{sky}\dot{x}}{(\dot{x} - \dot{x}_0)}, c_{max}\right]\right], & \dot{x}(\dot{x} - \dot{x}_0) \geqslant 0, \\ c_{min}, & \dot{x}(\dot{x} - \dot{x}_0) < 0. \end{cases} \tag{7}$$

2.2.2. *On–off skyhook control*

The control algorithm given in Eq. (7) requires the damper coefficient to be continuously variable. To simplify the operation an on–off scheme has been proposed [16]. The on–off damper acts as a conventional passive damper during the vibration attenuation portion of the vibration cycle, but a zero damping coefficient is assumed when the damping force generated by the semi-active damper is in the opposite direction to the ideal skyhook damping force. For on–off control, the damping force is governed by the control algorithm

$$F_{sa} = \begin{cases} c_{on}(\dot{x} - \dot{x}_0), & \dot{x}(\dot{x} - \dot{x}_0) \geqslant 0, \\ 0, & \dot{x}(\dot{x} - \dot{x}_0) < 0, \end{cases} \tag{8}$$

where c_{on} is the on-state damping constant of the on–off damper. In practice, a zero damping coefficient is impossible when the damper is switched off. Therefore, the damping coefficient is switched between a high value and a low value, and the control algorithm in Eq. (8) can be rewritten as

$$c_{sa} = \begin{cases} c_{max}, & \dot{x}(\dot{x} - \dot{x}_0) \geqslant 0, \\ c_{min}, & \dot{x}(\dot{x} - \dot{x}_0) < 0, \end{cases} \tag{9}$$

where c_{max} and c_{min} are the maximum and minimum coefficients of the on–off damper, respectively. The on-state damping c_{max} should be much greater than the off-state damping c_{min}, which should be as small as possible.

Both the continuous skyhook control and on–off skyhook control algorithms attempt to produce the effect of skyhook damping from a conventionally mounted damper. However, there are differences between them, which can be found by interpreting the damping force in terms of the phase and amplitude. The continuous skyhook control attempts to provide the same amplitude and phase in its on state as those of a skyhook damper. However, due to the practical limitations of physical systems it can only provide the same amplitude during part of the on state period. In addition, there are non-zero off-state damping effects. Conversely, the on–off skyhook control can only ensure that the semi-active damping force has the same sign as the desired skyhook damping force, and does not attempt to replicate the magnitude of the skyhook damping force. It has been shown in Ref. [17] that on–off and continuously variable skyhook control can give comparable performance.

2.2.3. On–off balance control

Balance control is based on an alternative physical criterion to skyhook control, and is so-called because it attempts to cancel the spring force in part by the damping force. It is sometimes termed "relative control" since the variables in the condition function are the relative displacement and the relative velocity between the mass and the base [18]. Both on–off and continuous versions of balance control are discussed here.

Considering a passive s.d.o.f. system subject to base excitation, the acceleration response of the suspended mass due to base excitation can be expressed as

$$\ddot{x} = -\frac{1}{m}(F_k + F_d), \tag{10}$$

where F_k and F_d are the spring and damping forces, respectively, which are given by

$$F_k = k(x - x_0) \tag{11}$$

and

$$F_d = c(\dot{x} - \dot{x}_0), \tag{12}$$

where k and c are the constant spring rate and damping coefficients, respectively. The amplitude of the acceleration of the mass due to harmonic base excitation can be expressed in terms of the spring and damping forces [6]

$$|\ddot{x}| = \frac{|F_k| + |F_d|}{m}\begin{cases} t_0 < t < t_0 + \dfrac{\tau}{4}, \\ t_0 + \dfrac{\tau}{2} < t < t_0 + \dfrac{3\tau}{4}, \end{cases} \tag{13}$$

$$|\ddot{x}| = \frac{|F_k| - |F_d|}{m}\begin{cases} t_0\dfrac{\tau}{4} < t < t_0 + \dfrac{\tau}{2}, \\ t_0 + \dfrac{3\tau}{4} < t < t_0 + \tau, \end{cases} \tag{14}$$

where t_0 is the time point at which $\ddot{x} = 0$ and is increasing, and τ is the period of vibration. It is evident from Eq. (13) that the damping force tends to increase the acceleration of the mass during two quarters of a cycle. In the remaining part of the cycle the damping force tends to decelerate the mass, which can be seen by Eq. (14). Poor vibration isolation performance of heavily damped passive systems is attributed to this phenomenon, where the magnitude of the damping force is dominant at high frequencies.

The damping force will increase the acceleration of the mass whenever forces due to the spring and the damper have the same sign, or equivalently when the relative velocity and relative displacement have the same sign. A control algorithm to ensure that this does not occur is [6]

$$F_{sa} = \begin{cases} c_{on}(\dot{x} - \dot{x}_0), & (x - x_0)(\dot{x} - \dot{x}_0) \leqslant 0, \\ 0, & (x - x_0)(\dot{x} - \dot{x}_0) > 0, \end{cases} \tag{15}$$

where c_{on} is the on-state damping constant of the on–off damper. The corresponding semi-active damping coefficient considering non-zero off-state damping is given by

$$c_{sa} = \begin{cases} c_{\max}, & (x - x_0)(\dot{x} - \dot{x}_0) \leqslant 0, \\ c_{\min}, & (x - x_0)(\dot{x} - \dot{x}_0) > 0, \end{cases} \tag{16}$$

where $c_{\max}$ and $c_{\min}$ are the maximum and minimum damping coefficients of the on–off semi-active damper.

2.2.4. Continuous balance control

The control algorithm given in Eqs. (15) and (16) has the potential for improvement. During the on state, the instantaneous damping force is seldom exactly equal in magnitude to the instantaneous spring force. Consequently, the surplus force will still accelerate the mass. In Ref. [8], a continuously variable control algorithm has been proposed, which can be considered as a further development of the control algorithm given in Eq. (15). The damping coefficient can be varied continuously, depending on the relative displacement and the relative velocity, such that the spring and damper forces balance exactly during the "on" part of the cycle. The required force is

$$F_{sa} = \begin{cases} -k(x - x_0), & (x - x_0)(\dot{x} - \dot{x}_0) \leqslant 0, \\ 0, & (x - x_0)(\dot{x} - \dot{x}_0) > 0. \end{cases} \tag{17}$$

In this formulation, the damper is attempting to behave like a spring with a negative stiffness coefficient during the on part, where the damping force is adjusted to equal the magnitude of the spring force in order to produce zero acceleration. The semi-active damping coefficient required for this control algorithm can be written as

$$c_{sa} = \begin{cases} \dfrac{-k(x - x_0)}{(\dot{x} - \dot{x}_0)}, & (x - x_0)(\dot{x} - \dot{x}_0) \leqslant 0, \\ 0, & (x - x_0)(\dot{x} - \dot{x}_0) > 0. \end{cases} \tag{18}$$

It can be seen from Eq. (18) that the damping coefficient tends to infinity as $\dot{x} - \dot{x}_0 \to 0$, which cannot be implemented in practice. Similarly to Eq. (7), the damper constant c_{sa} saturates at the upper and lower bounds imposed by the physical parameters of the damper. Considering practical constraints, the damping coefficient can be rewritten as

$$c_{sa} = \begin{cases} \max\left[c_{\min}, \min\left[\dfrac{-k(x - x_0)}{(\dot{x} - \dot{x}_0)}, c_{\max}\right]\right], & (x - x_0)(\dot{x} - \dot{x}_0) \leqslant 0, \\ c_{\min}, & (x - x_0)(\dot{x} - \dot{x}_0) > 0. \end{cases} \tag{19}$$

Both the on–off and continuously variable balance control algorithms set the damping force to cancel the spring force to some extent, whenever the damping force and the spring force have the opposite sign. Since the on–off balance control can only produce a damping force proportional to the relative velocity across the damper in its on state, it cannot ensure the damping force is exactly equal to the spring force. As can be seen by Eqs. (16) and (19), for the on–off balance control the spring force can be partly cancelled or even over-cancelled depending on values of $c_{\min}$, $c_{\max}$ and the frequency. For the continuous balance control, the spring force can be partly or totally cancelled.

Table 1 summarizes the semi-active damping coefficients required to achieve the desired damping force for each of the four control algorithms discussed in this section. Also shown are the practically achievable damping values given that the damping coefficient of the semi-active damper must lie in the range $[c_{\min}, c_{\max}]$. For the continuous skyhook control and continuous balance control algorithms, the denominator in the expressions of the damping equation introduces a high degree of non-linearity into the system. This raises two issues that need to be addressed when considering implementation of the semi-active dampers: chatter and jerk. These are studied in the next section.

3. Controller development

This section describes the numerical problems met when simulating the response of a system with a semi-active damper: chatter and jerk. Chatter refers to the phenomenon whereby the system switches rapidly between different states, and jerk is defined as a sharp change in the acceleration response of the system. There is potential for chatter and jerk to occur in semi-active systems since the damping coefficient switches between on and off states. The chatter problems that occur with on–off and continuous skyhook dampers are addressed and an "anti-jerk" control algorithm is proposed to avoid the sharp change in the acceleration response.

3.1. Chatter of the semi-active skyhook system

Semi-active skyhook systems can chatter between the on and off states, and Fig. 3 shows typical time histories of the damping and spring forces when chatter occurs. Since a semi-active damper system is energetically passive, the chatter depends totally on the instantaneous states of the system. Donald and Mehrnaz studied the occurrence of chatter for on–off skyhook control and suggested a modified logic to cure it [19]. For the semi-active skyhook control, switches due to changes in the sign of the mass velocity, $\dot{x}$, are defined as "$\dot{x}$ switches", while those due to changes in the sign of $\dot{x} - \dot{x}_0$ are called "$\dot{x} - \dot{x}_0$ switches". Recalling the form of damping forces for the on–off and continuous skyhook dampers in Eqs. (4) and (8), it is noted that only $\dot{x}$ switches are important with respect to the potential of chatter for the on–off skyhook control and $\dot{x} - \dot{x}_0$ switches are important for the continuous skyhook control. This is because for the on–off skyhook control, $\dot{x}$ switches can be associated with large relative velocity, $\dot{x} - \dot{x}_0$, and thus large damping forces, while $\dot{x} - \dot{x}_0$ switches are always associated with small damping forces. Conversely, for continuous skyhook control, $\dot{x} - \dot{x}_0$ switches can be associated with large damping forces.

Also, chatter can only occur if the damper and spring force are in opposition, and if the on-state damping force is of larger magnitude than the instantaneous spring force. If the damping force is not greater than the spring force, then the damper does not change the direction of the acceleration and does not initiate chatter [19].

The three conditions for chatter to occur are summarized in Table 2. If these conditions are met, chatter will be initiated and continue until an $\dot{x} - \dot{x}_0$ (on–off skyhook control) or an $\dot{x}$

Table 1
Damping characteristics of some semi-active dampers

Damper type	Original control algorithm	Semi-active damping required	Semi-active damping in practice
Continuous skyhook	$F_{sa} = \begin{cases} c_{sky}\dot{x}, & \dot{x}(\dot{x}-\dot{x}_0) \geqslant 0 \\ 0, & \dot{x}(\dot{x}-\dot{x}_0) < 0 \end{cases}$	$c_{sa} = \begin{cases} \dfrac{c_{sky}\dot{x}}{\dot{x}-\dot{x}_0}, & \dot{x}(\dot{x}-\dot{x}_0) \geqslant 0 \\ 0, & \dot{x}(\dot{x}-\dot{x}_0) < 0 \end{cases}$	$c_{sa} = \begin{cases} \max\left[c_{\min}, \min\left[\dfrac{c_{sky}\dot{x}}{\dot{x}-\dot{x}_0}, c_{\max}\right]\right], & \dot{x}(\dot{x}-\dot{x}_0) \geqslant 0 \\ c_{\min}, & \dot{x}(\dot{x}-\dot{x}_0) < 0 \end{cases}$
On–off skyhook	$F_{sa} = \begin{cases} c_{on}(\dot{x}-\dot{x}_0), & \dot{x}(\dot{x}-\dot{x}_0) \geqslant 0 \\ 0, & \dot{x}(\dot{x}-\dot{x}_0) < 0 \end{cases}$	$c_{sa} = \begin{cases} c_{on}, & \dot{x}(\dot{x}-\dot{x}_0) \geqslant 0 \\ 0, & \dot{x}(\dot{x}-\dot{x}_0) < 0 \end{cases}$	$c_{sa} = \begin{cases} c_{\max}, & \dot{x}(\dot{x}-\dot{x}_0) \geqslant 0 \\ c_{\min}, & \dot{x}(\dot{x}-\dot{x}_0) < 0 \end{cases}$
On–off balance	$F_{sa} = \begin{cases} c_{on}(\dot{x}-\dot{x}_0), & (x-x_0)(\dot{x}-\dot{x}_0) \leqslant 0 \\ 0, & (x-x_0)(\dot{x}-\dot{x}_0) > 0 \end{cases}$	$c_{sa} = \begin{cases} c_{on}, & (x-x_0)(\dot{x}-\dot{x}_0) \leqslant 0 \\ 0, & (x-x_0)(\dot{x}-\dot{x}_0) > 0 \end{cases}$	$c_{sa} = \begin{cases} c_{\max}, & (x-x_0)(\dot{x}-\dot{x}_0) \leqslant 0 \\ c_{\min}, & (x-x_0)(\dot{x}-\dot{x}_0) > 0 \end{cases}$
Continuous balance	$F_{sa} = \begin{cases} -k(x-x_0), & (x-x_0)(\dot{x}-\dot{x}_0) \leqslant 0 \\ 0, & (x-x_0)(\dot{x}-\dot{x}_0) > 0 \end{cases}$	$c_{sa} = \begin{cases} \dfrac{-k(x-x_0)}{\dot{x}-\dot{x}_0}, & (x-x_0)(\dot{x}-\dot{x}_0) \leqslant 0 \\ 0, & (x-x_0)(\dot{x}-\dot{x}_0) > 0 \end{cases}$	$c_{sa} = \begin{cases} \max\left[c_{\min}, \min\left[\dfrac{-k(x-x_0)}{\dot{x}-\dot{x}_0}, c_{\max}\right]\right], & (x-x_0)(\dot{x}-\dot{x}_0) \leqslant 0 \\ c_{\min}, & (x-x_0)(\dot{x}-\dot{x}_0) > 0 \end{cases}$
Adaptive-passive damping	$F_{sa} = \begin{cases} c_{on}(\dot{x}-\dot{x}_0), & r.m.s.(\ddot{x}) \geqslant r.m.s.(\ddot{x}_0) \\ 0, & r.m.s.(\ddot{x}) < r.m.s.(\ddot{x}_0) \end{cases}$	$c_{sa} = \begin{cases} c_{on}, & r.m.s.(\ddot{x}) \geqslant r.m.s.(\ddot{x}_0) \\ 0, & r.m.s.(\ddot{x}) < r.m.s.(\ddot{x}_0) \end{cases}$	$c_{sa} = \begin{cases} c_{\max}, & r.m.s.(\ddot{x}) \geqslant r.m.s.(\ddot{x}_0) \\ c_{\min}, & r.m.s.(\ddot{x}) < r.m.s.(\ddot{x}_0) \end{cases}$

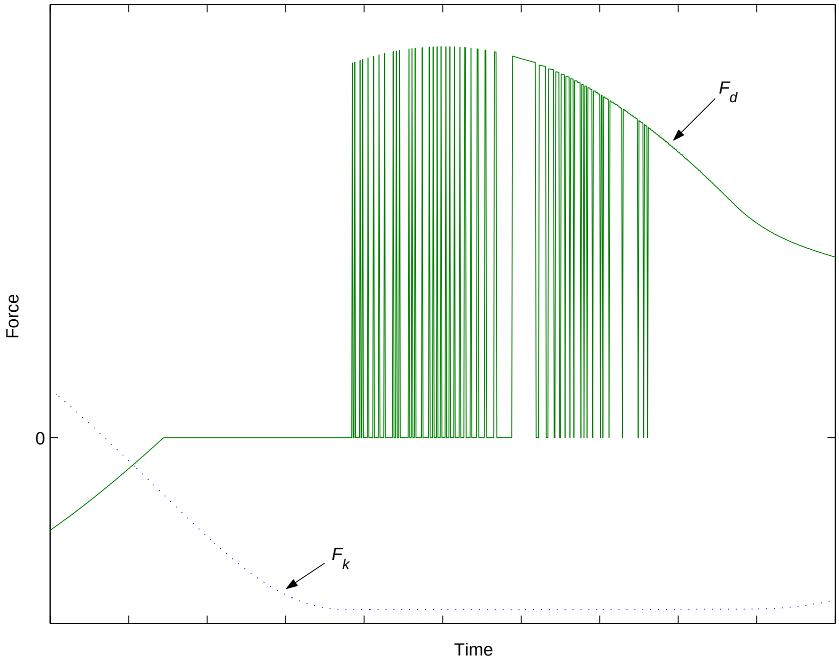

Fig. 3. Typical time history of spring and damper forces during chatter.

Table 2
Conditions for chatter of semi-active skyhook damper to occur

(1)	An $\dot{x}$ switch (on–off skyhook control) or an $\dot{x}-\dot{x}_0$ (continuous skyhook control) switch has taken place
(2)	The damper force, F_d, if on, is of opposite sign to the spring force
(3)	The damper force is of larger amplitude than the instantaneous spring force

(continuous skyhook control) switch takes place; or either condition (2) or (3) in Table 2 is not met. Several attempts have been proposed to eliminate the chatter problem. For example, Karnopp et al. use the ‘‘lock up’’ force [4,15], and Tudoruse uses a fuzzy logic controller to reduce chatter [18]. However, practical semi-active devices will have some time delays and the damping properties cannot be changed instantaneously. The chatter and jerk problems may be suppressed to some degree by the time delays. A first order time delay can be included in the control force to study the effects of time delay on suppressing chatter and jerk [20]. A modified logic is used here to eliminate the chatter, which is given in the flowchart in Fig. 4. This modified logic is used in the numerical simulations in Section 4.

3.2. Anti-jerk control

The damping force exhibits discontinuities at the time of switching. Thus, a significant change in acceleration may be experienced by the suspended mass, which is undesirable. Figs. 5(a)–(d)

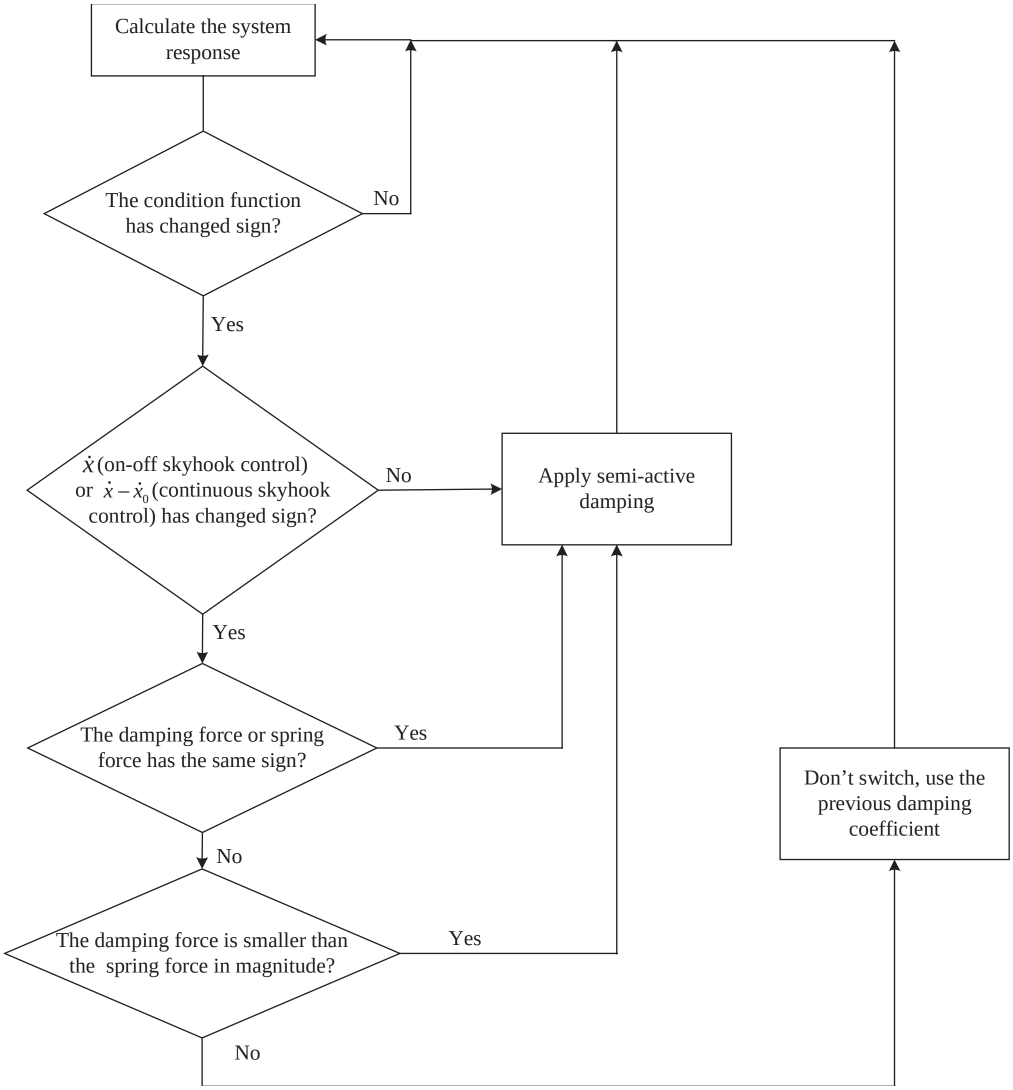

Fig. 4. Flow chart of the modified logic for cure of chatter in semi-active skyhook damping control.

show three-dimensional control surface plots of the damping force F_{sa} as a function of the variables in the condition function defined by Eqs. (4), (8), (15) and (17). Surface discontinuities are present in the control surfaces at $\dot{x}-\dot{x}_0=0$ in Fig. 5(a), $\dot{x}_0=0$ in Fig. 5(b), $x-x_0=0$ in Fig. 5(c), and $\dot{x}-\dot{x}_0=0$ in Fig. 5(d). All these surface discontinuities may lead to jerk, and some may cause chatter as described in Section 3.1.

To reduce this, a shaping function can be introduced to smooth the introduction of a damping force [21]. The shaping function can be written as a function of the variables defining the condition function, i.e., $F(x-x_0,\dot{x},\dot{x}-\dot{x}_0)$, and is chosen to modify the overall shape of the three-dimensional control surface to remove discontinuities. When choosing the shaping function,

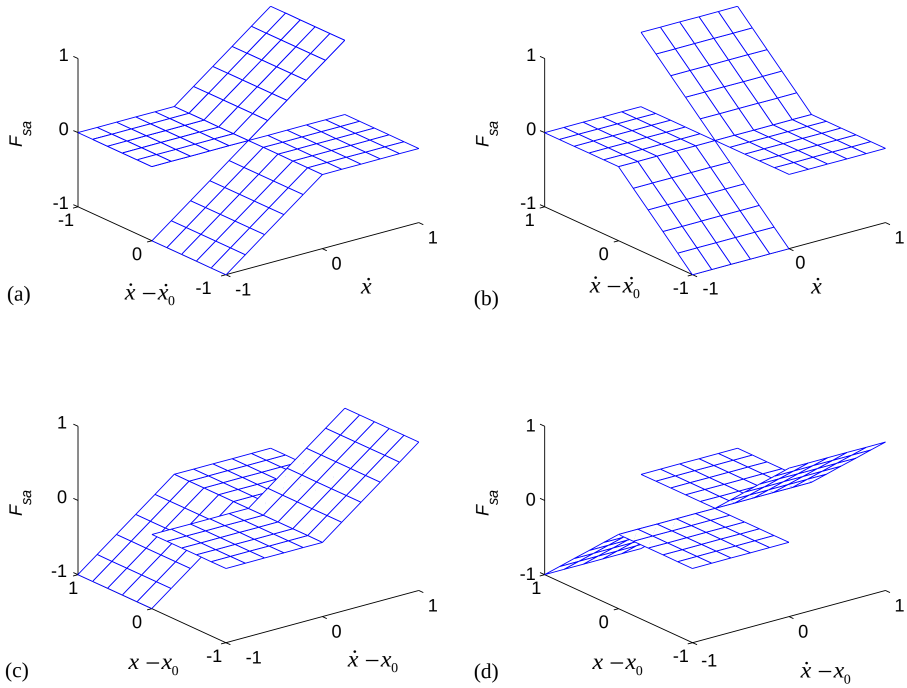

Fig. 5. Three-dimensional control surface plot of the semi-active damper force as a function of the variables defining the condition function: (a) continuously variable skyhook control; (b) on–off skyhook control; (c) on–off balance control; (d) continuous balance control.

the following guidelines must be observed: (1) $F(x-x_0, \dot{x}, \dot{x}-\dot{x}_0)$ is a continuous function, and both $F(x-x_0, \dot{x}, \dot{x}-\dot{x}_0)$ and the "shaped" control surface both include continuous first derivatives for all values of $x-x_0$, $\dot{x}$ and $\dot{x}-\dot{x}_0$; (2) $F(x-x_0, \dot{x}, \dot{x}-\dot{x}_0)$ is equal to 0 whenever a variable in the condition function will result in the occurrence of surface discontinuities. One can see that the shaping function for a particular semi-active control algorithm is not unique.

For the continuous skyhook control algorithm, the shaping function can be simply chosen as

$$F(\dot{x}, \dot{x}-\dot{x}_0) = |\dot{x}-\dot{x}_0|. \tag{20}$$

With this shaping function, the control strategy becomes

$$F_{sa} = \begin{cases} G \cdot |\dot{x}-\dot{x}_0| \cdot \dot{x}, & \dot{x}(\dot{x}-\dot{x}_0) \geqslant 0, \\ 0, & \dot{x}(\dot{x}-\dot{x}_0) < 0, \end{cases} \tag{21}$$

where G is a gain factor. After some manipulation and taking into consideration the constraints of practical implementation, the following algorithm can be used to implement the anti-jerk continuous skyhook control

$$c_{sa} = \begin{cases} \max[c_{\min}, \min[G|\dot{x}|, c_{\max}]], & \dot{x}(\dot{x}-\dot{x}_0) \geqslant 0, \\ c_{\min}, & \dot{x}(\dot{x}-\dot{x}_0) < 0. \end{cases} \tag{22}$$

The shaping function for continuous balance control can be chosen as

$$F(x - x_0, \dot{x} - \dot{x}_0) = |\dot{x} - \dot{x}_0|. \tag{23}$$

Correspondingly, the damping force is

$$F_{sa} = \begin{cases} -G \cdot |\dot{x} - \dot{x}_0| \cdot (x - x_0), & (x - x_0)(\dot{x} - \dot{x}_0) \leqslant 0, \\ c_{\min}(\dot{x} - \dot{x}_0), & (x - x_0)(\dot{x} - \dot{x}_0) > 0. \end{cases} \tag{24}$$

Following the same procedure as for continuous skyhook control, the damping coefficient can be written as

$$c_{sa} = \begin{cases} \max[c_{\min}, \min[G|x - x_0|, c_{\max}]], & (x - x_0)(\dot{x} - \dot{x}_0) \leqslant 0, \\ c_{\min}, & (x - x_0)(\dot{x} - \dot{x}_0) > 0. \end{cases} \tag{25}$$

Fig. 6 shows the control surface plot for the control algorithms defined by Eqs. (21) and (24) with anti-jerk modification. It can be seen that surface discontinuities are avoided for both of the two continuous control strategies, thus jerk can be reduced.

On–off skyhook control and on–off balance control are not amenable to anti-jerk control since only two states of damping are possible. Consequently, jerk might occur, but since they are relatively simple, they are implemented numerically and studied here. Table 3 summarizes the four control algorithms used to study the vibration isolation performance of semi-active dampers, which are labelled for convenience as SA-1 to SA-4. Also listed is the adaptive-passive damping algorithm described in Section 2.1, which works in a bi-state manner. It works as a common passive damper, which can be switched from one value to the other. There is no jerk associated with this control algorithm since the switch time can be chosen to occur when the damping force equals zero. This algorithm is subsequently labelled as AP.

4. Numerical simulations

The equation of motion describing a base excited s.d.o.f. system can be written as

$$m\ddot{x}(t) + c(\dot{x}(t) - \dot{x}_0(t)) + k(x(t) - x_0(t)) = 0, \tag{26}$$

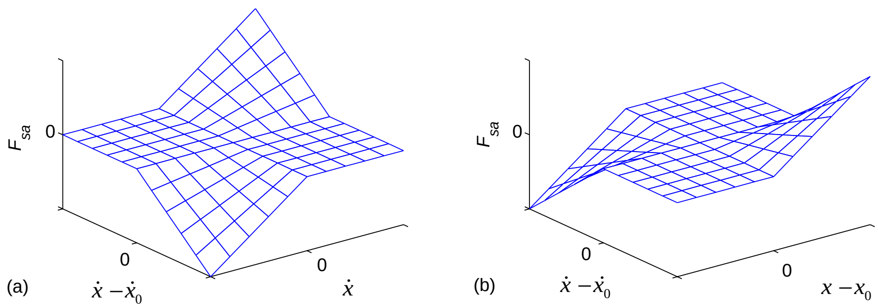

Fig. 6. Three-dimensional control surface plot of the semi-active damper force as a function of the variables defining the condition function with anti-jerk modification: (a) continuously skyhook control; (b) continuous balance control.

Table 3
Semi-active control algorithms implemented numerically

Semi-active damper type		Damping coefficient		Condition function
SA-1	Continuous skyhook control with anti-jerk modification	On state	$\max[c_{\min}, \min[G\|\dot{x}\|, c_{\max}]]$	$\dot{x}(\dot{x}-\dot{x}_0)\geqslant 0$
		Off state	$c_{\min}$	$\dot{x}(\dot{x}-\dot{x}_0)<0$
SA-2	On–off skyhook control	On state	$c_{\max}$	$\dot{x}(\dot{x}-\dot{x}_0)\geqslant 0$
		Off state	$c_{\min}$	$\dot{x}(\dot{x}-\dot{x}_0)<0$
SA-3	Continuous balance control with anti-jerk modification	On state	$\max[c_{\min}, \min[G\|x-x_0\|, c_{\max}]]$	$(x-x_0)(\dot{x}-\dot{x}_0)\leqslant 0$
		Off state	$c_{\min}$	$(x-x_0)(\dot{x}-\dot{x}_0)>0$
SA-4	On–off balance control	On state	$c_{\max}$	$(x-x_0)(\dot{x}-\dot{x}_0)\leqslant 0$
		Off state	$c_{\min}$	$(x-x_0)(\dot{x}-\dot{x}_0)>0$
AP	Adaptive-passive control	On state	$c_{\max}$	$r.m.s.(\ddot{x})\geqslant r.m.s.(\ddot{x}_0)$
		Off state	$c_{\min}$	$r.m.s.(\ddot{x})<r.m.s.(\ddot{x}_0)$

where c is the damping parameter which for the semi-active control algorithms considered in this paper, is given in Table 3. Eq. (26) is solved for harmonic excitation to establish the vibration isolation performance. The response of the system can be obtained by directly integrating Eq. (26). Simulations were carried out in MATLAB and SIMULINK using a fourth order Runge–Kutta integration scheme. The parameters of the system are chosen as $m=1$ kg, $k=4\pi^2$ N/m and $c_{\min}=0$.

The acceleration response will have discontinuities since the semi-active system is non-linear with step changes in damping force, so previous researchers have used displacement transmissibility to characterize the isolator performance [6,8]. The characterization in terms of acceleration would be more appropriate since the human body or a suspended mass is sensitive to inertial forces. In this study, the ratio of the r.m.s. of the response acceleration to the r.m.s. of the input acceleration is used as a performance index to evaluate vibration isolation performance, which is defined by

$$TR=\frac{r.m.s.(\ddot{x})}{r.m.s.(\ddot{x}_0)}. \tag{27}$$

When simulating continuously variable control in Eqs. (22) and (25), a gain factor G is chosen such that the maximum damping ratio is equal to that of the on–off strategies.

4.1. Skyhook control

This section presents the results of the simulations using the continuously variable and on–off skyhook control (SA-1 and SA-2).

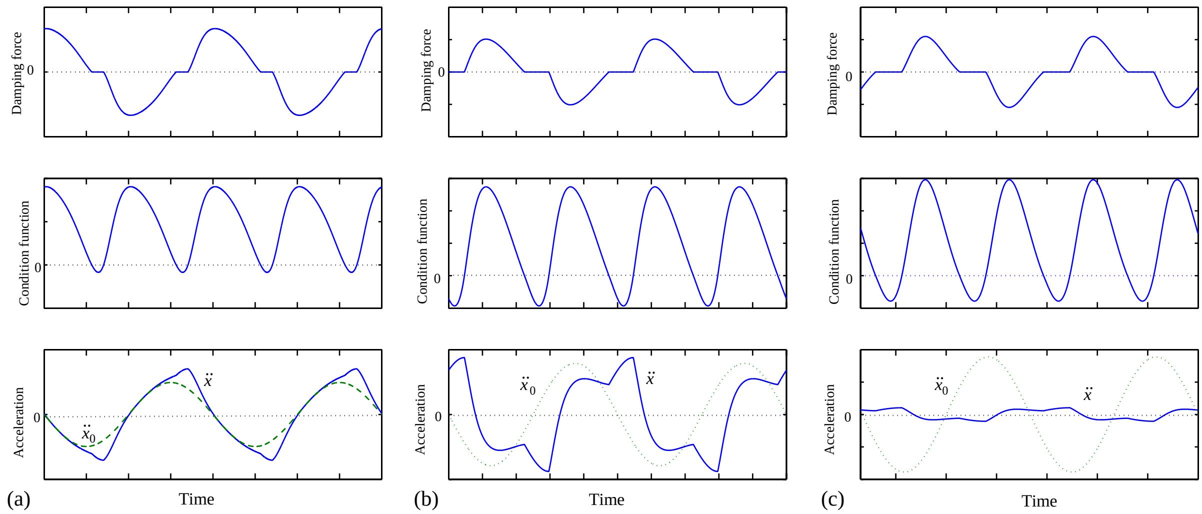

Fig. 7. Steady state response of continuous skyhook semi-active s.d.o.f. system (a) $\omega/\omega_n = 0.5$; (b) $\omega/\omega_n = 1$; (c) $\omega/\omega_n = 3$.

Figs. 7(a)–(c) show the time histories of the damping force, the condition function and the acceleration response with $G = 185$, which gives the maximum damping ratio to be unity. The results correspond to frequency ratios of $\omega/\omega_n = 0.5$, $\omega/\omega_n = 1.0$, and $\omega/\omega_n = 3.0$. The system was allowed to run until steady state was reached, but only the last few cycles are plotted in the figures. Note that the damping forces in the figure are not to the same scale. The time histories of the damping force show clearly the damping being switched on and off. The acceleration plots show the somewhat non-harmonic mass acceleration due to the non-harmonic force generated by the semi-active damper.

Figs. 8(a)–(c) show the time histories of the damping force, the condition function and the acceleration responses for the SA-2 control algorithm with the on-state damping ratio $\zeta_{\max} = 1$. Again, frequency ratios of 0.5, 1 and 3 have been chosen. The acceleration response of the on–off damper consistently reveals four jerks during each vibration cycle irrespective of the excitation frequency. The four jerks occur at the instances at which the damper is switched on and off. With the increase of excitation frequency, the duration of the off cycle of SA-2 system increases.

4.2. Balance control

This section presents the results of the simulations using the continuous variable and on–off balance control (SA-3 and SA-4) semi-active control algorithms.

Figs. 9(a)–(c) show the steady state response of an SA-3 system at three different frequencies for a gain of $G = 140$, which gives the maximum damping ratio to be unity. The results correspond to frequency ratios of $\omega/\omega_n = 0.5$, $\omega/\omega_n = 1.0$, and $\omega/\omega_n = 3.0$. One can see that whenever the relative displacement and the relative velocity bear different signs, the semi-active damper is switched on to partially cancel the spring force.

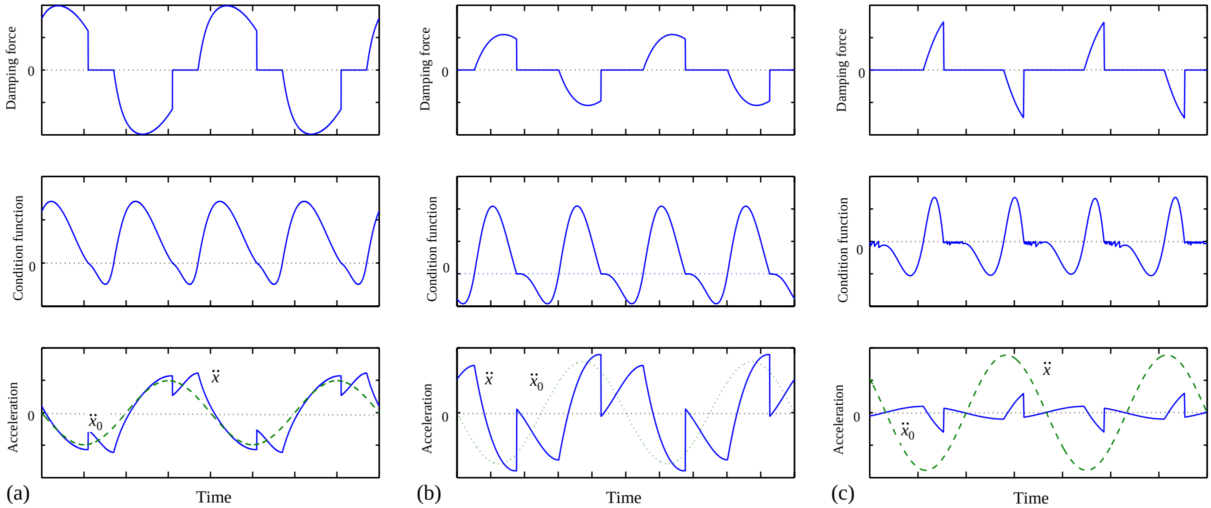

Fig. 8. Steady state response of on–off skyhook semi-active s.d.o.f. system at (a) $\omega/\omega_n = 0.5$; (b) $\omega/\omega_n = 1$; (c) $\omega/\omega_n = 3$.

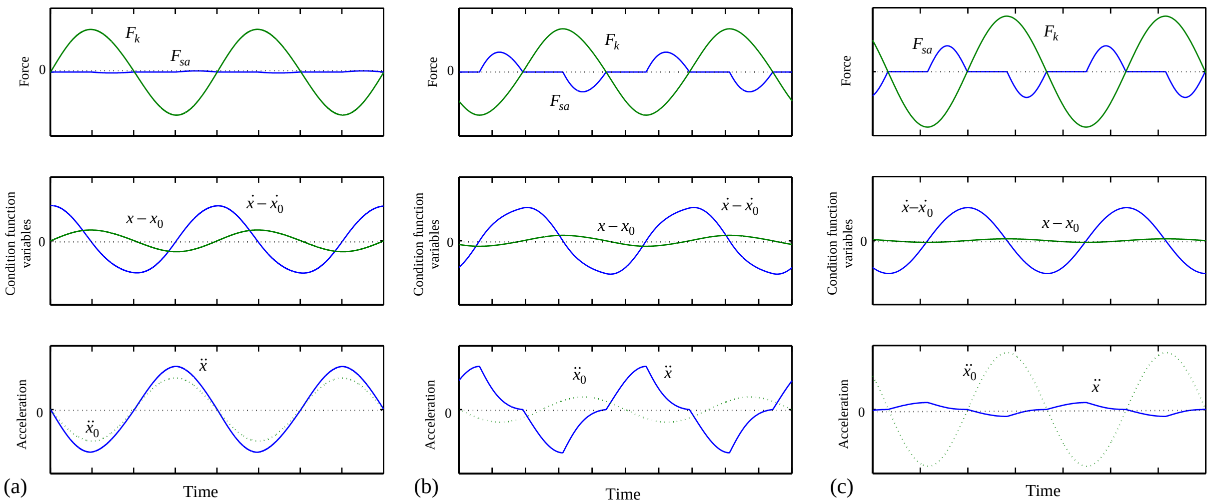

Fig. 9. Steady state response of continuous balance semi-active s.d.o.f. system at (a) $\omega/\omega_n = 0.5$; (b) $\omega/\omega_n = 1$; (c) $\omega/\omega_n = 3$.

The steady state response of the SA-4 system with $\zeta_{max} = 1.0$ is shown in Figs. 10(a)–(c) for three excitation frequencies. It can be seen that the damper assumes a zero damping force whenever the spring and the damping forces bear the same sign. The acceleration response features four jerks associated with the switching of the damper. It can be seen from Fig. 10(c) that at higher frequencies, the SA-4 system changes from its equilibrium position. In the extreme case shown, the relative displacement does not change sign, such that the switch of the semi-active damper is determined solely by the sign of relative velocity.

4.3. Comparison of semi-active and passive dampers

To evaluate the suitability of these semi-active control algorithms for vibration isolation of harmonic disturbances, the r.m.s. acceleration transmissibility of the s.d.o.f. system with a semi-active damper is compared with that of a passive damper and an ideal skyhook damper in the frequency domain. The frequency response is obtained by carrying out the simulations at discrete excitation frequencies.

Figs. 11(a) and (b) are a comparison of the r.m.s. acceleration transmissibility defined by Eq. (27), of the passive, skyhook and semi-active dampers subject to harmonic inputs. The

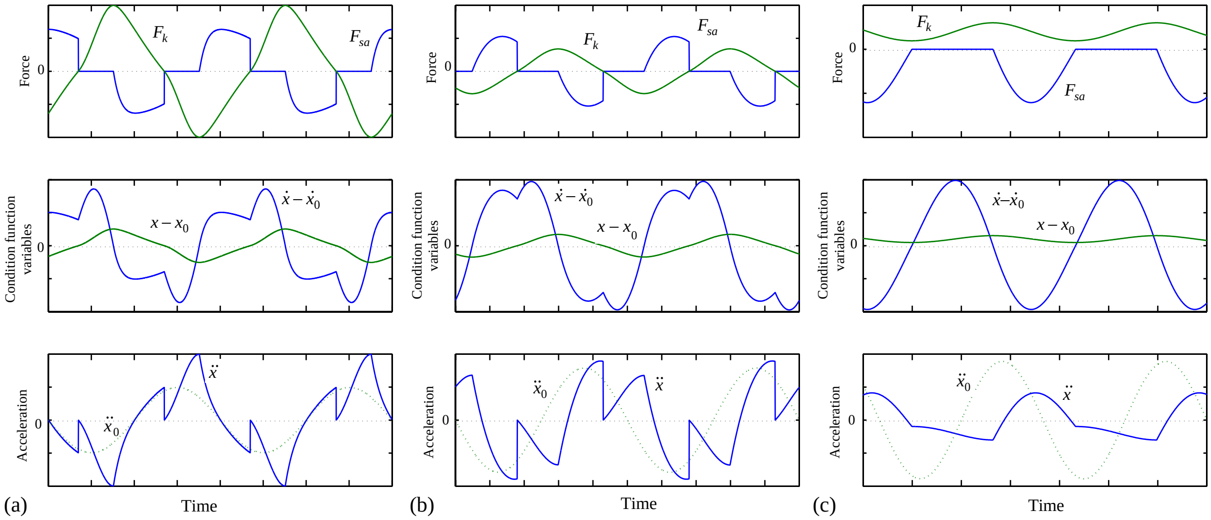

Fig. 10. Steady state response of on–off semi-active s.d.o.f. system at (a) $\omega/\omega_n = 0.5$; (b) $\omega/\omega_n = 1$; (c) $\omega/\omega_n = 3$.

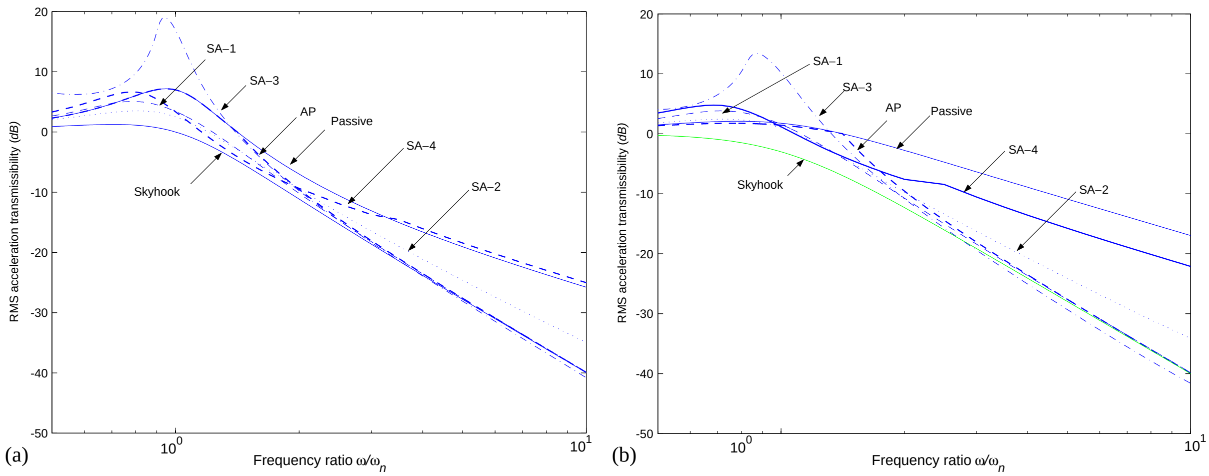

Fig. 11. Comparison of the transmissibility of a s.d.o.f. system with semi-active dampers: (a) small damping ($\zeta_{max} = 0.25$); (b) moderate damping ($\zeta_{max} = 0.50$).

damping ratios of the passive damper and the skyhook damper are chosen to be 0.25 and 0.5 in the simulations. The on-state damping ratios of the semi-active damper ζ_{max} are chosen to be equal to 0.25 and 0.5, respectively, and the off-state damping is $\zeta_{min} = 0$.

It can be established from the results that

1. The semi-active system always provides better isolation at higher frequencies than a conventional passive damped system. Fig. 11(b) shows that the difference between the two systems becomes more obvious as the damping ratio increases.
2. The compromise between resonance control and isolation that is inherent in a conventional passive system does not exist for the semi-active systems. The reduction in the resonance peak does not necessarily occur at the cost of reduced isolation at high frequencies. In fact, with a sufficiently large damping ratio, one can eliminate the resonance peak and actually achieve better isolation across the whole frequency spectrum. This is particularly useful for sensitive machinery that cannot tolerate any overshoot in power-up or power-down, and yet must have good isolation during normal operation. With the increase of damping of a semi-active damper, both the high-frequency isolation and resonance response are improved. However, this also leads to deterioration at very low frequencies due to the abrupt discontinuities in the damping force.
3. The skyhook damper system always provides the best performance but it is only an ideal case. Adaptive-passive damping control is possibly the simplest way to implement a control algorithm for harmonic vibration isolation, but is not applicable to random excitations. SA-1 and SA-2 provide similar performance but SA-2 is much simpler to implement. SA-3 and SA-4 systems can provide superior isolation performance at higher frequencies at the cost of a large resonance peak.

5. Conclusions

A model of a s.d.o.f. system subject to harmonic base excitation has been used to study the vibration isolation performance of four semi-active dampers. Five control algorithms have been studied, which are based on skyhook control, balance control, and adaptive damping control. The chatter and jerk problems that arise when simulating semi-active dampers numerically have been investigated and anti-jerk semi-active control algorithms have been suggested. With the suggested modifications, discontinuities of the damping force for the two continuous variable semi-active dampers can be avoided, thus jerk can be significantly reduced. Using a s.d.o.f. model, the vibration isolation performance of the five control algorithms has been analyzed and compared with passive and skyhook systems. It can be concluded from the results that the semi-active systems considered can always provide better isolation at higher frequencies than a conventional passive damped system. As the damping ratio increases, the difference between the two systems becomes more obvious. The skyhook damper system always provides the best performance but it is only an ideal case. The skyhook control systems provide similar performance but the on–off skyhook system is much simpler than the continuously variable system. The SA-3 and SA-4 systems are good at reducing acceleration response at higher frequencies. The adaptive-passive damper is the simplest control algorithm to achieve a better performance over conventional passive systems. However, there is a narrow frequency range just above the resonance frequency where semi-active dampers provide better performance than adaptive-passive systems.

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