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Short Communication

# Displacement amplitudes and flexural moments for a rectangular plate with a rectangular cutout under a uniformly distributed static load

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## 1. Introduction

Several publications dealing with free transverse vibrations of doubly connected rectangular and circular plates with free-edge holes have recently appeared in the open literature [1–10] and some additional complexities have been taken into account: variable thickness, anisotropy, presence of concentrated masses, etc.

In the case of rectangular plates the studies have been mainly concerned with the case where the outer boundary is simply supported. When applying the classical Rayleigh–Ritz method it is then possible to express the displacement amplitude in terms of a double Fourier series deducting then, from the total energy of the primitive simple supported system, the energy of the non-existing portion.

In order to characterize in a more complete fashion the response of a simply supported rectangular plate with free-edge holes, the authors have tackled the present analysis which can be considered as an extension of the famous Navier solution for the simply connected rectangular plate. Obviously if the uniformly distributed excitation is  $p_0 \cos(\omega t)$  one can determine approximate values of the dynamic parameters multiplying the static value by the amplification factor  $1/[1 - (\omega/\omega_1)^2]$  where  $\omega_1$  is the fundamental frequency for the system under analysis. On

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the other hand, the present mathematical model does not take into account additional structural complications such as stress concentration phenomena which are rather appreciable at the corners of the holes.

### 2. Approximate analytical solution

For the rectangular plate under study, depicted in Fig. 1, the Rayleigh–Ritz variational approach requires minimization of the functional

$$J[W'] = U[W'] - L[W'], \tag{1}$$

where  $U[W']$  is the maximum strain energy and  $L[W']$  is the maximum work done by the distributed load for the (true) displacement amplitude  $W'$  of the plate.

As has been shown elsewhere, see, for example, Ref. [11], in the case of a plate of general anisotropy, that the first term in Eq. (1) can be written as

$$U[W'] = \frac{1}{2} \int \int \left\{ D_{11} \left( \frac{\partial^2 W'}{\partial x'^2} \right)^2 + 2D_{12} \frac{\partial^2 W'}{\partial x'^2} \frac{\partial^2 W'}{\partial y'^2} + D_{22} \left( \frac{\partial^2 W'}{\partial y'^2} \right)^2 + 4D_{66} \left( \frac{\partial^2 W'}{\partial x' \partial y'} \right)^2 + 4 \left[ D_{16} \left( \frac{\partial^2 W'}{\partial x'^2} \right) + D_{26} \left( \frac{\partial^2 W'}{\partial y'^2} \right) \right] \left( \frac{\partial^2 W'}{\partial x' \partial y'} \right) \right\} dx' dy', \tag{2}$$

where the well-established Lekhnitskii’s notation [10] for the flexural rigidities  $D_{ij}$  of the plate has been used, and the second term is

$$L[W'] = p_0 \int \int W' dx' dy', \tag{3}$$

the work done by the distributed load.

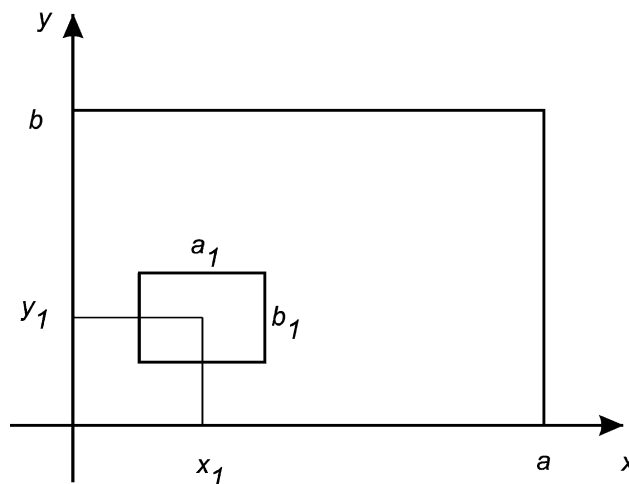


Fig. 1. Double connected plate under study.

Table 1

Values of  $W(x,y)$  (Eq. (6)) and  $M_x$  and  $M_y$  (Eq. (7)) in the case of a rectangular isotropic plate and aspect ratio 2/3 for two different sizes of cutouts when they are displaced along the horizontal middle line and along the diagonal of the plate

Cut out size	Cut out position	Plate point		$W \times 10^3$	$M_x \times 10^2$	$M_y \times 10^2$
		$x/a$	$y/b$			
$a_1/a=0.1$ $b_1/b=0.1$	$x_1/a=0.50$ $y_1/b=0.50$ Fig. 2(a)	0.25	0.75	0.8313	1.4590	2.1644
		0.50	0.75	1.1318	1.6903	2.5973
		0.75	0.25	0.8313	1.4590	2.1644
		0.75	0.75	0.8313	1.4590	2.1644
	$x_1/a=0.25$ $y_1/b=0.50$ Fig. 2(b)	0.25	0.75	0.8373	1.5331	2.0103
		0.50	0.50	1.5442	2.1926	3.6980
		0.75	0.25	0.8151	1.4751	2.1683
		0.75	0.75	0.8151	1.4751	2.1683
	$x_1/a=0.05$ $y_1/b=0.50$ Fig. 2(c)	0.25	0.75	0.8123	1.4727	2.1532
		0.50	0.50	1.5232	2.2442	3.6203
		0.75	0.25	0.8125	1.4837	2.1664
		0.75	0.75	0.8125	1.4837	2.1664
$x_1/a=0.25$ $y_1/b=0.25$ Fig. 2(d)	0.25	0.75	0.8156	1.4662	2.1114	
	0.50	0.50	1.5361	2.2033	3.6125	
	0.75	0.25	0.8158	1.4667	2.1675	
	0.75	0.75	0.8148	1.4825	2.1680	
$x_1/a=0.05$ $y_1/b=0.05$ Fig. 2(e)	0.25	0.75	0.8284	1.5191	2.1793	
	0.50	0.50	1.5456	2.2248	3.6478	
	0.75	0.25	0.8194	1.4829	2.1834	
	0.75	0.75	0.8188	1.4842	2.1779	
$a_1/a=0.3$ $b_1/b=0.3$	$x_1/a=0.50$ $y_1/b=0.50$ Fig. 2(a)	0.25	0.75	0.9383	1.2818	2.1382
		0.50	0.75	1.3535	1.6401	1.0773
		0.75	0.25	0.9383	1.2818	2.1382
		0.75	0.75	0.9383	1.2813	2.1382
	$x_1/a=0.25$ $y_1/b=0.50$ Fig. 2(b)	0.25	0.75	0.9558	1.4666	0.8521
		0.50	0.50	1.6706	1.4618	0.4012
		0.75	0.25	0.8231	1.4027	2.1560
		0.75	0.75	0.8231	1.4027	2.1560
	$x_1/a=0.15$ $y_1/b=0.50$ Fig. 2(c)	0.25	0.75	0.9052	1.6116	1.0783
		0.50	0.50	1.5569	1.9731	3.7192
		0.75	0.25	0.8091	1.4462	2.1456
		0.75	0.75	0.8091	1.4462	2.1456
$x_1/a=0.25$ $y_1/b=0.25$ Fig. 2(d)	0.25	0.75	0.8601	1.4230	1.7470	
	0.50	0.50	1.6586	2.0610	3.7433	
	0.75	0.25	0.8408	1.3859	2.2043	
	0.75	0.75	0.8320	1.4598	2.1712	
$x_1/a=0.15$ $y_1/b=0.15$ Fig. 2(e)	0.25	0.75	0.9044	1.6598	2.1286	
	0.50	0.50	1.6721	2.2404	3.8514	
	0.75	0.25	0.8559	1.4742	2.2918	
	0.75	0.75	0.8489	1.4798	2.2204	

Table 2

Values of  $W(x,y)$  (Eq. (6)) and  $M_x$  and  $M_y$  (Eq. (7)) in the case of a square isotropic plate for two different sizes of cutouts when they are displaced along the horizontal middle line and along the diagonal of the plate

Cut out size	Cut out position	Plate point		$W \times 10^3$	$M_x \times 10^2$	$M_y \times 10^2$
		$x/a$	$y/b$			
$a_1/a=0.1$ $b_1/b=0.1$	$x_1/a=0.50$ $y_1/b=0.50$ Fig. 2(a)	0.25	0.75	2.1685	2.9006	2.9006
		0.50	0.75	3.0124	3.7116	3.7292
		0.75	0.25	2.1685	2.9006	2.9006
		0.75	0.75	2.1685	2.9006	2.9006
	$x_1/a=0.25$ $y_1/b=0.50$ Fig. 2(b)	0.25	0.75	2.1744	3.0740	2.8199
		0.50	0.50	4.1038	4.6997	4.9024
		0.75	0.25	2.1326	2.9171	2.9342
		0.75	0.75	2.1326	2.9171	2.9342
	$x_1/a=0.05$ $y_1/b=0.50$ Fig. 2(c)	0.25	0.75	2.1302	2.9349	2.9245
		0.50	0.50	4.0553	4.8247	4.8002
		0.75	0.25	2.1288	2.9418	2.9410
		0.75	0.75	2.1288	2.9418	2.9410
	$x_1/a=0.25$ $y_1/b=0.25$ Fig. 2(d)	0.25	0.75	2.1375	2.9345	2.8959
		0.50	0.50	4.0857	4.7777	4.7777
		0.75	0.25	2.1375	2.8959	2.9345
		0.75	0.75	2.1345	2.9413	2.9413
$x_1/a=0.05$ $y_1/b=0.05$ Fig. 2(e)	0.25	0.75	2.1618	2.9894	2.9491	
	0.50	0.50	4.1273	4.8430	4.8430	
	0.75	0.25	2.1618	2.9491	2.9894	
	0.75	0.75	2.1525	2.9569	2.9569	
$a_1/a=0.3$ $b_1/b=0.3$	$x_1/a=0.50$ $y_1/b=0.50$ Fig. 2(a)	0.25	0.75	2.3673	2.6349	2.6349
		0.50	0.75	3.3985	3.8116	1.9775
		0.75	0.25	2.3673	2.6349	2.6349
		0.75	0.75	2.3673	2.6349	2.6349
	$x_1/a=0.25$ $y_1/b=0.50$ Fig. 2(b)	0.25	0.75	2.4227	3.2156	1.5266
		0.50	0.50	4.4843	2.7940	5.3372
		0.75	0.25	2.1470	2.6878	2.8562
		0.75	0.75	2.1470	2.6878	2.8562
	$x_1/a=0.15$ $y_1/b=0.50$ Fig. 2(c)	0.25	0.75	2.3719	3.5088	1.5995
		0.50	0.50	4.2236	4.0218	4.9902
		0.75	0.25	2.1255	2.7973	2.8861
		0.75	0.75	2.1255	2.7973	2.8861
	$x_1/a=0.25$ $y_1/b=0.25$ Fig. 2(d)	0.25	0.75	2.2040	2.9206	2.6223
		0.50	0.50	4.3591	4.7271	4.7271
		0.75	0.25	2.2040	2.6223	2.9206
		0.75	0.75	2.1692	2.9101	2.9101
$x_1/a=0.15$ $y_1/b=0.15$ Fig. 2(e)	0.25	0.75	2.3124	3.2137	2.9206	
	0.50	0.50	4.4873	5.0475	5.0475	
	0.75	0.25	2.3124	2.9206	3.2137	
	0.75	0.75	2.2506	2.9974	2.9974	

Table 3

Values of  $W(x,y)$  (Eq. (6)) and  $M_x$  and  $M_y$  (Eq. (7)) in the case of a rectangular isotropic plate of aspect ratio 3/2 for two different sizes of cutouts when they are displaced along the horizontal middle line and along the diagonal of the plate

Cut out size	Cut out position	Plate point		$W \times 10^3$	$M_x \times 10^2$	$M_y \times 10^2$
		$x/a$	$y/b$			
$a_1/a=0.1$ $b_1/b=0.1$	$x_1/a=0.50$ $y_1/b=0.50$ Fig. 2(a)	0.25	0.75	4.2089	4.8700	3.2828
		0.50	0.75	5.8888	6.4738	4.3711
		0.75	0.25	4.2089	4.8700	3.2828
		0.75	0.75	4.2089	4.8700	3.2828
	$x_1/a=0.25$ $y_1/b=0.50$ Fig. 2(b)	0.25	0.75	4.1982	5.1091	3.2678
		0.50	0.50	7.8440	7.8647	5.0832
		0.75	0.25	4.1291	4.8294	3.3276
		0.75	0.75	4.1291	4.8294	3.3276
	$x_1/a=0.05$ $y_1/b=0.50$ Fig. 2(c)	0.25	0.75	4.1178	4.8807	3.3171
		0.50	0.50	7.7152	8.1348	4.9888
		0.75	0.25	4.1124	4.8708	3.3351
		0.75	0.75	4.1124	4.8708	3.3351
$x_1/a=0.25$ $y_1/b=0.25$ Fig. 2(d)	0.25	0.75	4.1300	4.8769	3.3002	
	0.50	0.50	7.7768	8.1282	4.9574	
	0.75	0.25	4.1293	4.7508	3.2991	
	0.75	0.75	4.1250	4.8780	3.3356	
$x_1/a=0.05$ $y_1/b=0.05$ Fig. 2(e)	0.25	0.75	4.1485	4.9127	3.3366	
	0.50	0.50	7.8248	8.2076	5.0059	
	0.75	0.25	4.1941	4.9034	3.4180	
	0.75	0.75	4.1455	4.9004	3.3394	
$a_1/a=0.3$ $b_1/b=0.3$	$x_1/a=0.50$ $y_1/b=0.50$ Fig. 2(a)	0.25	0.75	4.7503	4.8109	2.8842
		0.50	0.75	6.7995	7.2806	2.7703
		0.75	0.25	4.7503	4.8109	2.8842
		0.75	0.75	4.7503	4.8109	2.8842
	$x_1/a=0.25$ $y_1/b=0.50$ Fig. 2(b)	0.25	0.75	4.8465	5.9706	2.0445
		0.50	0.50	9.2975	4.0422	6.0240
		0.75	0.25	4.3167	4.4501	3.2194
		0.75	0.75	4.3167	4.4501	3.2194
	$x_1/a=0.15$ $y_1/b=0.50$ Fig. 2(c)	0.25	0.75	4.7538	6.2261	1.8549
		0.50	0.50	8.6157	6.2170	5.5216
		0.75	0.25	4.2437	4.6016	3.2702
		0.75	0.75	4.2437	4.6016	3.2702
$x_1/a=0.25$ $y_1/b=0.25$ Fig. 2(d)	0.25	0.75	4.2567	4.9597	3.1183	
	0.50	0.50	8.3970	8.4225	4.6374	
	0.75	0.25	4.3546	3.9308	3.2019	
	0.75	0.75	4.2122	4.8853	3.2847	
$x_1/a=0.15$ $y_1/b=0.15$ Fig. 2(e)	0.25	0.75	4.3333	5.1565	3.3170	
	0.50	0.50	8.4654	8.6658	5.0409	
	0.75	0.25	4.5785	4.7895	3.7346	
	0.75	0.75	4.2977	4.9959	3.3296	

The integrals in expressions (2) and (3) extend over the actual area of the double connected plate under study, i.e. the whole original plate surface minus the area of the cutout.

Taking the lengths of the sides of the rectangular plate to be  $a$  and  $b$  in the  $x$  and  $y$  directions, respectively, and introducing the non-dimensional variables

$$W = W'/a; \quad x = x'/a; \quad y = y'/b \quad \text{and} \quad r = b/a, \tag{4}$$

Eqs. (2) and (3) above can be recast in a non-dimensional form. One gets for the functional for the whole system of Fig. 1,

$$J_{nd} = \frac{2J}{rD_{11}} = \int \int \left\{ \left( \frac{\partial^2 W}{\partial x^2} \right)^2 + \frac{2d_{12}}{r^2} \frac{\partial^2 W}{\partial x^2} \frac{\partial^2 W}{\partial y^2} + \frac{d_{22}}{r^4} \left( \frac{\partial^2 W}{\partial y^2} \right)^2 + \frac{4d_{66}}{r^2} \left( \frac{\partial^2 W}{\partial x \partial y} \right)^2 + 4 \left[ \frac{d_{16}}{r} \left( \frac{\partial^2 W}{\partial x^2} \right) + \frac{d_{26}}{r^3} \left( \frac{\partial^2 W}{\partial y^2} \right) \right] \left( \frac{\partial^2 W}{\partial x \partial y} \right) \right\} dx dy - 2 \frac{p_0 a^3}{D_{11}} \int \int W dx dy, \tag{5}$$

where as usual,  $d_{ij} = D_{ij}/D_{11}$  for  $(i,j) = (1,2,6)$ .

Expressing the non-dimensional displacement amplitude  $W(x,y)$  in terms of a double Fourier series,

$$W(x,y) \cong W_a(x,y) = \frac{W'(x',y')}{[p_0 a^4/D_{11}]} = \sum_{n=1}^N \sum_{m=1}^M b_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b}, \tag{6}$$

the (distributed) flexural moments in the  $x$  and  $y$  directions are given by

$$M_x = \frac{M'_x(x',y')}{p_0 a^2} = \sum_{n=1}^N \sum_{m=1}^M \pi^2 \left[ m^2 + \frac{n^2 d_{12}}{r^2} \right] b_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} - \sum_{n=1}^N \sum_{m=1}^M 2d_{16} \frac{\pi^2 mn}{r} b_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b}, \tag{7a}$$

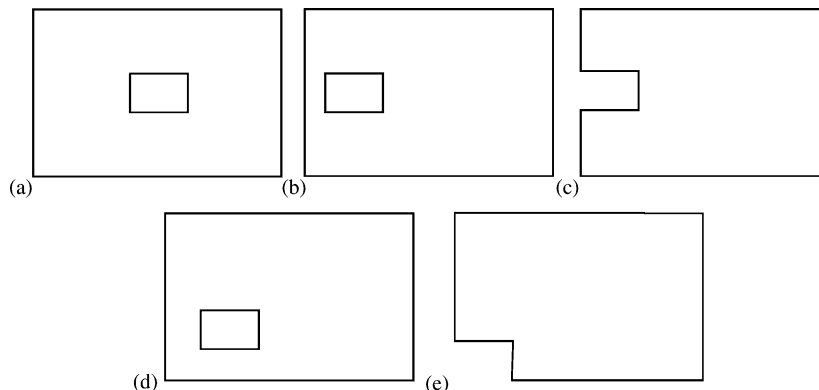


Fig. 2. Cutout along the plate middle horizontal axis ((a), (b) and (c)) and along its diagonal ((d) and (e)).

$$M_y = \frac{M'_y(x', y')}{p_0 a^2} = \sum_{n=1}^N \sum_{m=1}^M \pi^2 \left[ m^2 d_{12} + \frac{d_{22}^2 n^2}{r^2} \right] b_{mn} \sin \frac{m\pi x}{a} \sin \frac{n\pi y}{b} - \sum_{n=1}^N \sum_{m=1}^M 2d_{26} \frac{\pi^2 mn}{r} b_{mn} \cos \frac{m\pi x}{a} \cos \frac{n\pi y}{b}. \tag{7b}$$

Table 4

Influence of the anisotropy parameters on the values of  $W(x,y)$  (Eq. (6)) and  $M_x$  and  $M_y$  (Eq. (7)) in the case of a square plate for two different positions of the cutout when the plate is isotropic, orthotropic and general anisotropic

Cut out position	Isotropy case	Plate point		$W \times 10^3$	$M_x \times 10^2$	$M_y \times 10^2$
		$x/a$	$y/b$			
$x_1/a=0.1$ $y_1/b=0.5$	<i>Isotropic plate</i>	0.25	0.75	2.1882	3.0577	2.6596
		0.50	0.75	2.9394	3.4646	3.8413
		0.75	0.25	2.1213	2.9081	2.9240
		0.75	0.50	2.9233	3.9303	3.5801
		0.75	0.75	2.1213	2.9081	2.9240
	<i>Orthotropic plate</i>	0.25	0.75	2.1144	2.9068	1.6937
		0.50	0.75	2.8639	3.4243	2.2730
		0.75	0.25	2.0738	2.8702	1.7823
		0.75	0.50	2.8265	3.7576	2.1351
		0.75	0.75	2.0738	2.8702	1.7823
	<i>Anisotropic plate</i>	0.25	0.75	2.1039	3.1145	2.3018
		0.50	0.75	3.4117	3.9044	2.3198
		0.75	0.25	2.1147	3.3305	2.7371
		0.75	0.50	3.3716	4.2212	2.4072
		0.75	0.75	2.7876	3.0390	1.1459
$x_1/a=0.5$ $y_1/b=0.5$	<i>Isotropic plate</i>	0.25	0.75	2.2760	2.7828	2.7828
		0.50	0.75	3.2230	3.9276	2.9970
		0.75	0.25	2.2760	2.7828	2.7828
		0.75	0.50	3.2230	2.9970	3.9276
		0.75	0.75	2.2760	2.7828	2.7828
	<i>Orthotropic plate</i>	0.25	0.75	2.1020	2.5766	1.5981
		0.50	0.75	2.9781	3.8430	2.1080
		0.75	0.25	2.1020	2.5766	1.5981
		0.75	0.50	2.9511	3.0978	2.1939
		0.75	0.75	2.1020	2.5766	1.5981
	<i>Anisotropic plate</i>	0.25	0.75	2.1411	3.2461	2.7572
		0.50	0.75	3.5642	4.5134	2.3001
		0.75	0.25	2.1411	3.2461	2.7572
		0.75	0.50	3.5052	3.4209	2.4974
		0.75	0.75	2.8236	2.4657	0.6990

The size of the cutout is  $a_1/a=0.2=b_1/b$ .

Table 5

Influence of the anisotropy parameters on the values of  $W(x,y)$  (Eq. (6)) and  $M_x$  and  $M_y$  (Eq. (7)) in the case of a square plate for three different sizes of the cutout when the plate is isotropic, orthotropic and general anisotropic

Cut out size	Isotropy case	Plate point		$W \times 10^3$	$M_x \times 10^2$	$M_y \times 10^2$
		$x/a$	$y/b$			
$a_1/a=0.1$ $b_1/b=0.1$	<i>Isotropic plate</i>	0.25	0.75	2.1685	2.9006	2.9006
		0.50	0.75	3.0124	3.7116	3.7292
		0.75	0.25	2.1685	2.9006	2.9006
		0.75	0.50	3.0124	3.7292	3.7116
	<i>Orthotropic plate</i>	0.25	0.75	2.0948	2.8155	1.7550
		0.50	0.75	2.9138	3.7108	2.3345
		0.75	0.25	2.0948	2.8155	1.7550
		0.75	0.50	2.8777	3.6392	2.2203
	<i>Anisotropic plate</i>	0.25	0.75	2.1147	3.3126	2.7377
		0.50	0.75	3.4656	4.2808	2.5468
		0.75	0.25	2.1147	3.3126	2.7377
		0.75	0.50	3.3956	4.0359	2.4552
$a_1/a=0.2$ $b_1/b=0.2$	<i>Isotropic plate</i>	0.25	0.75	2.2760	2.7828	2.7828
		0.50	0.75	3.2230	3.9276	2.9970
		0.75	0.25	2.2760	2.7828	2.7828
		0.75	0.50	3.2230	2.9970	3.9276
	<i>Orthotropic plate</i>	0.25	0.75	2.1020	2.5766	1.5981
		0.50	0.75	2.9781	3.8430	2.1080
		0.75	0.25	2.1020	2.5766	1.5981
		0.75	0.50	2.9511	3.0978	2.1939
	<i>Anisotropic plate</i>	0.25	0.75	2.1411	3.2461	2.7572
		0.50	0.75	3.5642	4.5134	2.3001
		0.75	0.25	2.1411	3.2461	2.7572
		0.75	0.50	3.5052	3.4209	2.4974
$a_1/a=0.3$ $b_1/b=0.3$	<i>Isotropic plate</i>	0.25	0.75	2.3673	2.6349	2.6349
		0.50	0.75	3.3985	3.8116	1.9775
		0.75	0.25	2.3673	2.6349	2.6349
		0.75	0.50	3.3985	1.9775	3.8116
	<i>Orthotropic plate</i>	0.25	0.75	2.0689	2.3035	1.4021
		0.50	0.75	2.9534	3.3967	1.6029
		0.75	0.25	2.0689	2.3035	1.4021
		0.75	0.50	2.9295	2.1108	1.7886
	<i>Anisotropic plate</i>	0.25	0.75	2.1450	3.1841	2.7774
		0.50	0.50	3.5805	4.1135	1.8030
		0.75	0.25	2.1450	3.1841	2.7774
		0.75	0.50	3.5244	2.3536	2.0575

The position of the center of the cutout is  $x_1/a=0.5=y_1/b$ .



In the present work, expressions (6) and (7) for non-dimensional displacement amplitude and flexural moments have been normalized by the distributed load.

Minimizing the governing functional with respect to the  $b_{mn}s$ , expression (5) yields (for  $M=N$ ) an inhomogeneous, linear system of equations in the  $b_{mn}s$ . Solving the system the values for the  $b_{mn}s$  are obtained.

The present study is concerned with the determination of non-dimensional displacement amplitude (6) and flexural moments (7) in points of the loaded plate in the case of plates of varying degrees of anisotropy with a single cutout of equal aspect ratio.

### 3. Numerical results

All calculations were performed for simply supported rectangular plates of uniform thickness and for three different types of constitutive relations, i.e. isotropic, orthotropic and general anisotropic plates. For simplicity, in all cases the cutout has been chosen to be of the same aspect ratio as the original entire plate.

For an isotropic plate, with Poisson coefficient  $\mu=0.3$ , Tables 1–3 depict values of the displacement amplitude and flexural moments, each with a different value of the aspect ratio  $b/a$ : 2/3, 1 (square plate) and 3/2. In each Table, in turn, two different values for the size of the cutout has been taken as they are placed along the middle horizontal line of the plate and along one of its diagonals (Fig. 2).

Tables 4 and 5 show the influence of the degree of anisotropy in the case of a square plate. In Table 4 for two different positions of the cutoff, while in Table 5 three different sizes of the cutoff placed at the center of the plate are depicted.

In both Tables 4 and 5 results are presented for an orthotropic plate, where  $\mu_2 = 0.3$ ;  $D_2/D_1 = 1/2$  and  $D_k/D_1 = 1/2$ . The general anisotropic plate had elastic parameters  $D_{12}/D_{11} = 0.3$ ;  $D_{22}/D_{11} = D_{66}/D_{11} = 1/2$  and  $D_{16}/D_{11} = D_{26}/D_{11} = 1/3$ .

For the double Fourier series, Eq. (6), satisfactory convergence is achieved for  $N=M=20$ . As usual, special care has been taken to manipulate such a large system of equations and 80 bits floating point variables (IEEE-standard temporary reals) have been used to satisfy accuracy requirements.

It is worth noting that computations are very stable and all values uniformly converge as the number of terms in the Fourier series is increased. Typically, the values for the frequency coefficients differ by less than 0.5% when  $M$  and  $N$  are increased from 20 to 30.

As a general conclusion one may say that the mathematical model seems to be quite realistic and accurate, within the realm of the classical theory of vibrating plates.

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