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Journal of Sound and Vibration 280 (2005) 449–454

JOURNAL OF SOUND AND VIBRATION

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Short Communication

Streamline representation for structural intensity fields

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Received 15 October 2003; accepted 12 February 2004

1. Introduction

The concept of field lines has been used in the studies of various field theories to improve their understanding [1–4]. A line in a vector field is drawn so that its direction at any point is the same as the direction of the field at that point. Similar to a contour plot for representing a potential field, the magnitude as well as the direction of the energy flow in acoustic intensity fields can be better visualized by the flow lines as well as their relative spacings between the lines [1]. The same premise is investigated in this paper. This concept is also very useful for visualizing the structural intensity fields determined by numerical methods at discrete points in space. A picture of the streamlines may help us to understand and obtain more information on vibrational energy flow than the previous vector representation. In this work, the structural intensities are calculated for a rectangular plate with/without slots by the finite element method, and the two-dimensional steady energy flow patterns are presented by the streamline visualization.

2. Theoretical background

The structural intensity is the vibrational power flow per unit cross-sectional area of a dynamically loaded elastic body. The net power flow or active intensity in a two-dimensional

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2004.02.008

structure with a steady-state vibration is given as [5]

$$\begin{aligned} I_x(\omega) &= -\tfrac{1}{2}\,\mathrm{Re}(\tilde{\sigma}_{xk}(\omega)\tilde{v}_k^*(\omega)), \\ I_y(\omega) &= -\tfrac{1}{2}\,\mathrm{Re}(\tilde{\sigma}_{yk}(\omega)\tilde{v}_k^*(\omega)), \quad k = x, y, z, \end{aligned} \tag{1}$$

where $\tilde{\sigma}_{xk}(\omega)$, $\tilde{\sigma}_{yk}(\omega)$ is the complex stress and $\tilde{v}_k^*(\omega)$ is the complex conjugate of the velocity.

The structural intensity in a flat plate element can be expressed in the form of power flow per unit width. The x and y components of structural intensities can be expressed as

$$\begin{cases} I_x = -(\omega/2)\mathrm{Im}[\tilde{N}_x\tilde{u}^* + \tilde{N}_{xy}\tilde{v}^* + \tilde{Q}_x\tilde{w}^* + \tilde{M}_x\tilde{\theta}_y^* - \tilde{M}_{xy}\tilde{\theta}_x^*], \\ I_y = -(\omega/2)\mathrm{Im}[\tilde{N}_v\tilde{v}^* + \tilde{N}_{yx}\tilde{u}^* + \tilde{Q}_y\tilde{w}^* - \tilde{M}_y\tilde{\theta}_x^* + \tilde{M}_{yx}\tilde{\theta}_y^*], \end{cases} \tag{2}$$

where $\tilde{N}_x$,$\tilde{N}_y$ and $\tilde{N}_{xy} = \tilde{N}_{yx}$ are complex membrane forces per unit width of the plate, $\tilde{M}_x$,$\tilde{M}_y$ and$\tilde{M}_{xy} = \tilde{M}_{yx}$ are complex bending and twisting moments per unit width of the plate, $\tilde{Q}_x$ and $\tilde{Q}_y$ are complex transverses shear forces per unit width of the plate, $\tilde{u}^*$,$\tilde{v}^*$ and $\tilde{w}^*$ are complex conjugates of translational displacements in x, y and z directions, $\tilde{\theta}_x^*$and $\tilde{\theta}_y^*$ are complex conjugates of rotational displacement about the x and y directions.

The streamline technique displays the flow as lines everywhere parallel to the velocity field. The relative spacing of the lines indicates the speed of the flow. The structural intensity streamline can be mathematically expressed as

$$\mathrm{d}\mathbf{r} \times \mathbf{I}(\mathbf{r}, t) = 0. \tag{3}$$

where $\mathbf{r}$ is the energy flow particle position. For the steady state energy flows, the cross product can be written as

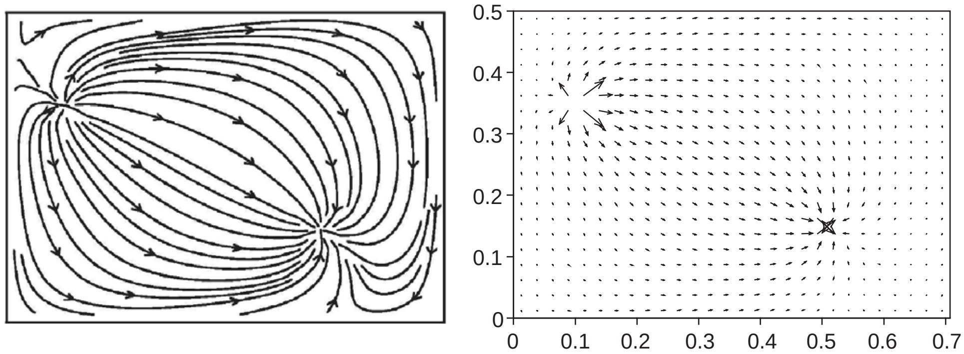
$$\begin{vmatrix} \mathbf{i} & \mathbf{j} & \mathbf{k} \\ I_x & I_y & I_z \\ \mathrm{d}x & \mathrm{d}y & \mathrm{d}z \end{vmatrix} = 0. \tag{4}$$

Thus, for the two-dimensional plate structures, the differential equation describing a streamline is

$$\frac{\mathrm{d}x}{I_x} = \frac{\mathrm{d}y}{I_y}. \tag{5}$$

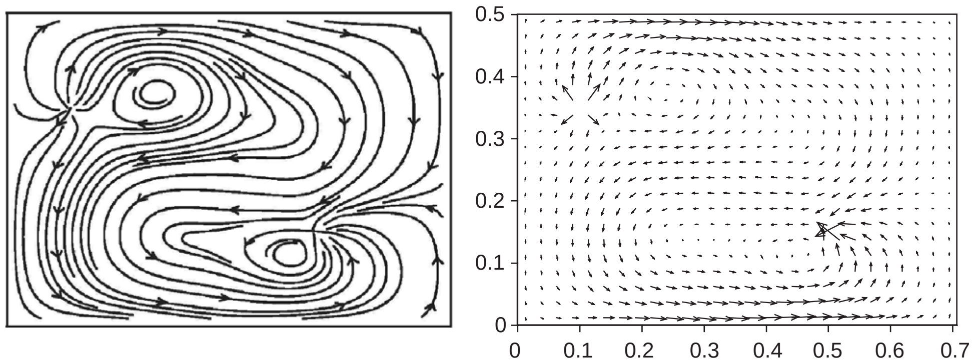
3. Results and discussion

The structural intensity fields of a rectangular aluminium plate are calculated and represented by vectors and streamlines shown in Figs. 1–4. The model and its excitation conditions are the same as those in the published paper [6], where a thin plate with length 0.707 m, width 0.5 m and thickness 3 mm is simply supported on its short edges. Figs. 1(a) and 2(a) depict the vibrational energy streamlines due to a point force excitation (with amplitude of 1 N, applied at $x = 0.101$ m



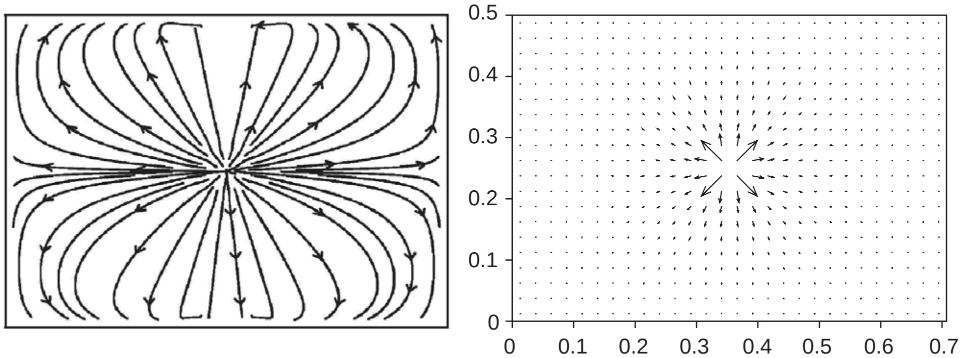
(a) (b)

Fig. 1. Streamline (a) and vector (b) representation for a structural intensity field under point force excitation and dashpot dissipation near the first bending frequency.



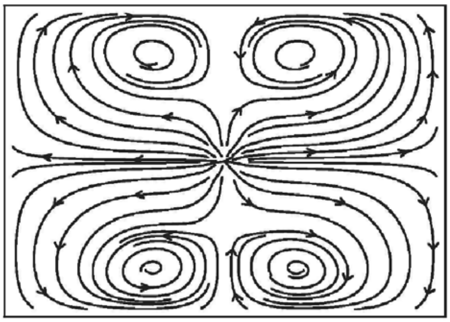
(a) (b)

Fig. 2. Streamline (a) and vector (b) representation for a structural intensity field under point force excitation and dashpot dissipation near the third bending frequency.

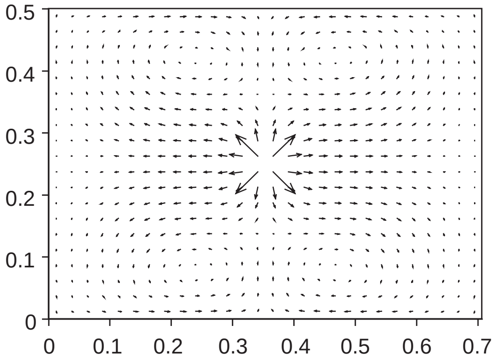


(a) (b)

Fig. 3. Streamline (a) and vector (b) representation for a structural intensity field under central point force excitation and structural damping dissipation near the first bending frequency.



(a)

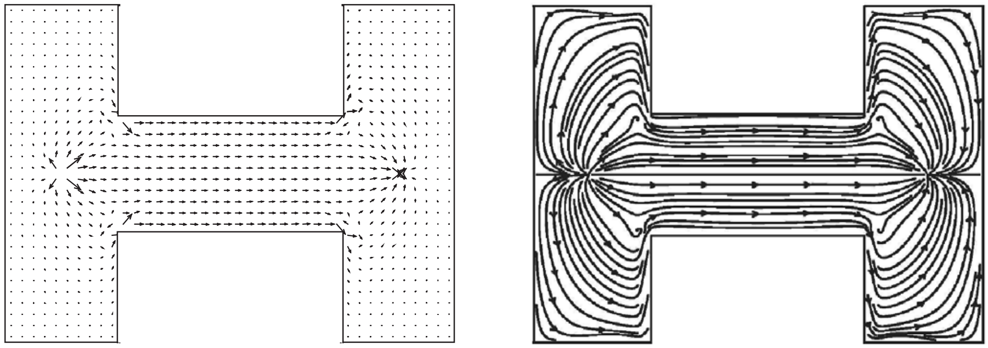


(b)

Fig. 4. Streamline (a) and vector (b) representation for a structural intensity field under central point force excitation and structural damping dissipation near the third bending frequency.

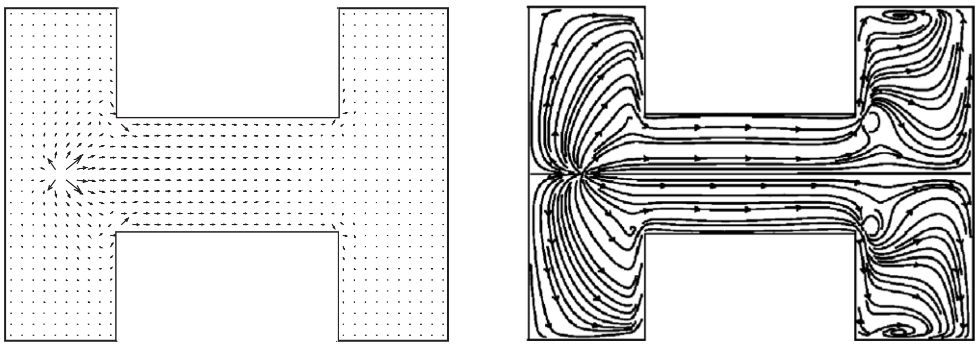
and $y = 0.35\,\mathrm{m}$) as well as a dashpot dissipation (with a damping rate of $2000\,\mathrm{N\,s/m}$, attached at $x = 0.505\,\mathrm{m}$ and $y = 0.15\,\mathrm{m}$) under the excitation frequencies corresponding to the first and third bending modes of the plate, respectively. Figs. 3(a) and 4(a) show those due to a central point force excitation and the structural damping dissipation with a damping ratio of 0.005. The patterns of streamlines are compared with the vector plots as shown in the (b) series of Figs. 1–4. From these patterns, it is easier to examine the energy flow transmission path, especially to visually follow the generation process of energy vortices as shown in Figs. 2 and 4.

In order to further investigate the effects of the geometry and edge on the energy flow path, a rectangular steel plate of 2.0 m length, 1.5 m width and 10 mm thickness with two symmetrical slots of 1.0 m length and 0.5 m width is presented as the second example. In the first case, the plate is assumed to be without any structural damping, while a dashpot of 100 N s/m is attached on the opposite position to the excitation force, the magnitude of excitation force is 10 N, two transverse end edges are simply supported while the others are free, and the excitation frequency is close to its fundamental frequency. The structural intensity vectors and streamlines are shown in Fig. 5(a) and (b), respectively. It can be observed that the locations of the source and the sink can be identified both by vector and streamline representations; however, the energy flow path cannot be shown obviously by vector patterns in the regions of relatively small magnitudes. This is just the advantage of the streamline representation. Fig. 6 shows the second case with the dashpot removed, while the structural damping is considered. It can be observed how the energy is dissipated by structural damping from the right side of the streamline pattern. Figs. 7 and 8 correspond to the first two cases where all the edges of the slotted plate are simply supported. From the vector plots in Fig. 7, it is difficult to identify the energy flow transmission paths due to the presence of complex energy vortices, however, that can be distinctly indicated via streamline representations. Moreover, only the streamline representation can indicate the energy flow and dissipation process for the cases involving structural damping as shown in Figs. 6 and 8.



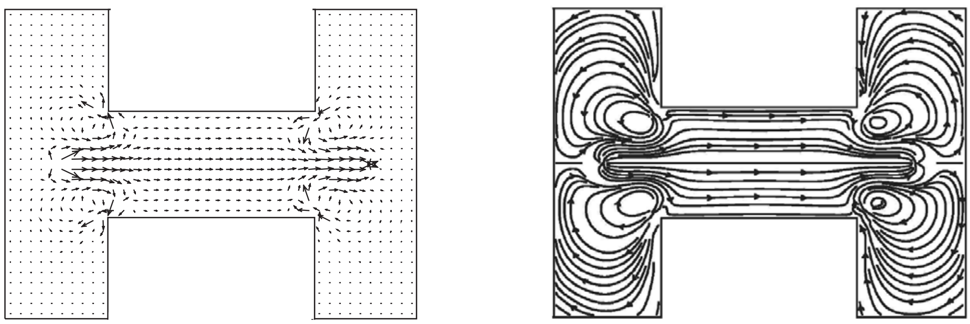
(a) (b)

Fig. 5. Structural intensities of the slotted plate with 2-edge simply supported and dashpot attached: (a) vector representation; (b) streamline representation.



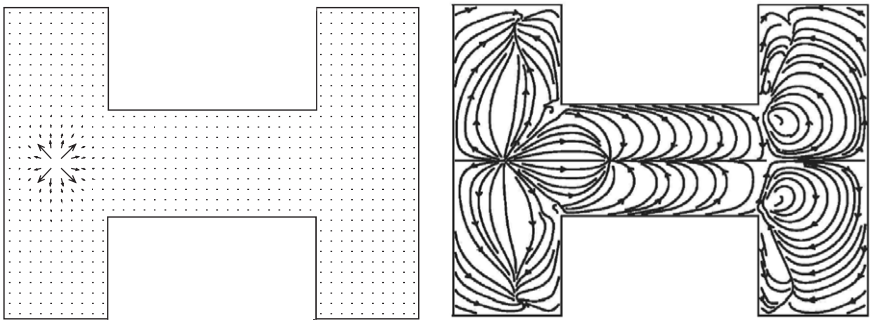
(a) (b)

Fig. 6. Structural intensities of the slotted plate with 2-edge simply supported and structural damping considered. (a) vector representation; (b) streamline representation.



(a) (b)

Fig. 7. Structural intensities of the slotted plate with all-edge simply supported and dashpot attached. (a) vector representation; (b) streamline representation.



(a) (b)

Fig. 8. Structural intensities of the slotted plate with all-edge simply supported and structural damping considered: (a) vector representation; (b) streamline representation.

4. Conclusions

The streamline is introduced to visualize the vibrational energy flow paths in plate structures. It has been shown to be an effective representation of two-dimensional steady energy flow fields. The picture of the streamlines plays an important part in the analysis and understanding of complex energy flow shapes, especially suitable for showing the vortex as well as the energy dissipation by structural damping. However, it is important to know that the streamline is calculated based on the discrete vectors, so it needs denser point distributions to obtain a better flow curve. Although the streamline cannot indicate the relative magnitudes of structural intensities, it can provide a better means of visualizing the dominant power flow paths.

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