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Robust finite element model updating using Taguchi method

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Abstract

A novel model updating technique is presented such that the parameters in an analytical finite element (FE) model can be updated in a robust way in presence of random errors in measured data and systematic errors in the analytical model. For efficient and robust updating, Taguchi method is applied to the optimization of the objective function, which is defined by the difference between measured and analytical vibration data. As reference data for updating, both cases of using frequency as well as modal data are discussed. To demonstrate the effectiveness of the proposed methods, FE models of truss structure and cantilever beam are used for numerical simulations of model updating in presence of random and systematic errors.

1. Introduction

In many engineering problems, mathematical model accuracy has been an essential part for design and analysis. As a way of improving dynamic models, model updating has been widely used for correcting analytical finite element (FE) models using experimental data.

Most common model updating method is based on eigensensitivity method [1,2]. Here, the parameters in analytical FE model are updated iteratively using pseudo-inverse of sensitivity matrix. However, the updated parameters may be non-unique or unreliable if the sensitivity matrix is under-determined or ill-conditioned. So, the updated results are subject to various noises in measured data because of the under-determined or ill-conditioned equations to identify the physical parameters. On the other hand, a large number of frequency response function (FRF)

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data can be used as reference data for updating. However, lots of data from FRFs are redundant and available data are believed to be equivalent to that of modal data such as natural frequencies and mode shapes [2]. In order to solve the problem of under-determined or ill-conditioned equations in model updating, regularization methods have been tried [3] but minimum norm constraints in regularization method may result in spreading physical parameter changes rather than locating parameter errors in an analytical FE model and correcting them. Reducing the number of parameter to be updated can be another option in order to ensure the over-determined equations for updating [4,5]. However, there exists difficult problem of selecting updating parameters which might have errors.

Recently, genetic algorithms have been used in an effort to reach the global minimum in optimization problem of model updating especially in application of damage identification [6–10]. Using genetic algorithm, the problem of non-unique solutions seems to be effectively released. However, the genetic algorithm is very slow in execution since the method employed is based on stochastic search. On the other hand, neural network can also be used for model updating [11]. However, the updated results are dependent on the training cases. Recently, Chang et al. have used orthogonal array as an efficient training method for neural network based updating [12].

If reference data are corrupted by measurement noises, then the updated parameters can be affected. In addition, the updated parameters are subject to systematic errors such as discretization errors in an analytical FE model. Therefore, a robust model updating technique should be used for reliable parameter updating in presence of various errors. On the other hand, Taguchi method has been widely used for robust design and quality engineering in industry [13–16]. In this study, as an optimization method for objective function, which is defined by difference between experimental and analytical data, the concept of Taguchi method is applied to model updating. The proposed method can be efficient in a computational point of view since orthogonal arrays are used for screening the main effect of parameters with least number of runs of simulations. Moreover, the updated results are robust against various noises since parameters are updated such that the so-called signal to noise (SN) ratio can be maximized. As reference data for model updating, both cases of frequency data and modal data are discussed for the comparison of each method. The method using measured FRFs as reference data has advantages because errors from modal parameter extraction can be eliminated. However, the selection of reference frequency points for updating may not be easy because the magnitude of FRF at selected frequency points should be monotonic and smooth according to parameter changes for reliable updating.

For the demonstration of the proposed method, both truss structure and cantilever beam are taken as numerical examples. Random noises in simulated experimental data as well as systematic errors in analytical model are included in the simulation for the verification of robustness.

2. Formulation of objective function

Model updating is to adjust parameters of existing model such that difference between analytical and experimental data can be minimized. Thus, the updating problem can be formulated into an objective function, which is defined by the difference between analytical and experimental data, to be minimized. Recently, as a method for optimization of the objective function, genetic algorithm has been used in an attempt to obtain global minimum in presence of many local minima. The method has advantages because it is less sensitive to the initial values of the updating parameters. Furthermore, no modal sensitivities have to be calculated. However, the optimization process using genetic algorithm is slow in nature because of its stochastic approach. In this work, as an alternative way of optimization of the objective function, Taguchi method, which has been widely used in robust design in industry, is used. The optimization method employed in this work has advantages over genetic algorithm in that it can be computationally efficient due to screening main effects of parameters using orthogonal arrays rather than using stochastic approach. In addition, the updating process can be more straightforward since it maximizes the SN ratio, an alternative form of objective function, at each iteration.

2.1. Objective function based on frequency data

The objective function can be defined by the difference between measured FRFs and FRFs from an analytical model. However, the direct difference of FRFs may result in bad performance because the objective function is dominated by the FRF resonant peaks. Therefore, it is necessary to balance the effect of frequency points whether they are selected from near resonance or anti-resonance. In practice, the difference between the logarithm magnitude of FRFs is considered as objective function, J_1, as [10]

$$J_1 = \sum_j \sum_k \left(\frac{\log|H_j^m(\omega_k)| - \log|H_j^a(\omega_k)|}{\omega_k} \right)^2, \tag{1}$$

where $H_j^m(\omega_k)$ and $H_j^a(\omega_k)$ are represent measured and analytical FRFs at frequency ω_k, respectively. Here, difference between FRFs are divided by frequency, ω_k, in order to reduce the gain in high frequencies because the FRFs of higher frequencies are likely to be erroneous due to the discretization effect in analytical FE model.

The use of measured FRF data as reference data for updating has advantages because the errors from modal parameter extraction can be eliminated. However, it should be noted that there is problem of selection of frequency points. In order to update parameters using Taguchi method, the objective function should be monotonously changed according to the change of parameters. For stable frequency points, non-monotonous regions of FRF such as frequency points close to resonances should be avoided. If a parameter, say A, is increasing ($A_0 < A_1 < A_2$), then corresponding FRF should be increasing or decreasing accordingly at selected frequency points. Therefore, as seen in Fig. 1, the frequency points, such as ω_1, which are likely to have non-monotonous characteristics of FRF, should be avoided. Similar frequency selection method from monotonous regions of FRF has been discussed by Chang and Park [17].

2.2. Objective function based on modal data

Modal data such as natural frequencies and mode shapes are believed to be essence form of a large number of frequency domain data. Therefore, there is little loss of information using modal data unlike a limited number of selected frequency domain data for updating. The objective function using modal data, J_2, is defined by difference between modal data of analytical model

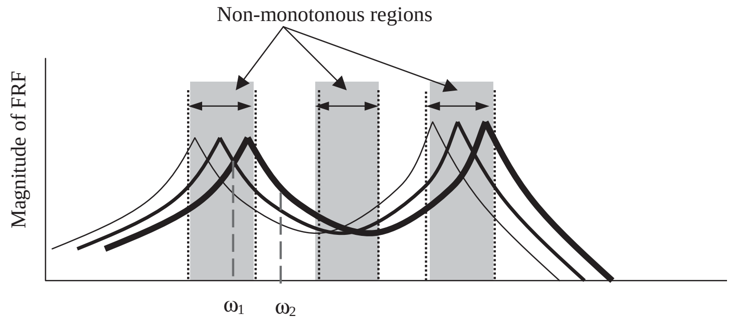


Fig. 1. Frequency response functions according to parameter value changes: —— A_0; —— A_1; and —— A_2.

and experimental data as [6–9]

$$J_2 = W_\omega J_\omega + W_\varphi J_\varphi, \tag{2}$$

where J_ω and J_φ are objective functions related with natural frequency data and mode shape data, respectively. Here, W_ω and W_φ are weighting factor for each objective function, J_ω and J_φ, respectively. The each objective function, if first n modes are used, can be written as

$$J_\omega = \sum_{r=1}^{n} \left(\frac{\omega_r^m - \omega_r^a}{\omega_r^m} \right)^2 \quad \text{and} \quad J_\phi = \sum_{r=1}^{n} (MSF\{\phi\}_r^m - \{\phi\}_r^a)^{\mathrm{T}} (MSF\{\phi\}_r^m - \{\phi\}_r^a), \tag{3}$$

where ω_r and $\{\phi\}_r$ are natural frequency and mode shape vector, respectively. Here, superscripts m, a and T represent measured data, analytical data and transpose of a vector, respectively. It should be noted that modal scale factor, MSF, should be used in order to compare the two analytical and experimental mode shape directly as [2]

$$MSF = \frac{(\{\phi\}_r^a)^{\mathrm{T}} (\{\phi\}_r^m)}{(\{\phi\}_r^m)^{\mathrm{T}} (\{\phi\}_r^m)}. \tag{4}$$

The modal scale factor will also solve the problem that the measured and analytical mode shapes could be 180° out of phase. In order to pair analytical data and experimental data properly, modal assurance criterion (MAC) should be used as [2]

$$MAC_{jk} = \frac{|(\{\phi\}_j^m)^{\mathrm{T}} (\{\phi\}_k^a)|^2}{((\{\phi\}_j^m)^{\mathrm{T}} (\{\phi\}_j^m))((\{\phi\}_k^a)^{\mathrm{T}} (\{\phi\}_k^a))}. \tag{5}$$

The MAC has its value between 0 and 1 according to the closeness between eigenvectors of analytical and experimental modes. If the modes pair in order, then the MAC matrix will have values close to 1 on the diagonal and close to 0 elsewhere.

It has been known that proper weightings, W_φ and W_ω, of objective function in Eq. (2) can improve results significantly. Therefore, relative weights of natural frequencies and mode shapes should be chosen carefully. The effect of weighting values, W_φ and W_ω, on updated results has been discussed in Ref. [6–9]. Because the proposed method uses the same objective function, the conclusions in the references can be applied to the proposed method. Since, natural frequencies are more accurately measured than mode shapes, the weighting value on natural frequencies, W_ω, is set to be higher than the weighting on mode shapes, W_φ. However, if too much weighting on natural frequency, W_ω, and too little weighting on mode shape, W_φ, were used in objective function, the information from mode shapes might disappear. Considering these facts, $W_\omega = 10$ and $W_\varphi = 1$ are used throughout simulations.

The updated results are also influenced by the number of modes included in objective function [7]. A sufficient number of modes should be included in the objective function in order to identify the parameters concerned. However, it should be noted that using high modes may not be reliable in many cases because those are subjected to various errors not only from the measurement but also from the discretization effect in analytical FE model.

2.3. Signal to noise (SN) ratio

In order to increase robustness of design against noises, Taguchi used the concept of SN ratio [13]. According to Taguchi method, the loss function, which is equivalent to objective function, J, can be divided into three characteristics: (1) nominal-the-best; (2) smaller-the-better; (3) larger-the-better [13–16]. In this work, the optimization of objective function defined in Eqs. (1) and (2) can be classified into smaller-the-better process. Then, SN ratio can be defined by

$$SN = -10 \log\left(\frac{1}{n} J\right), \tag{6}$$

where n is the number of modes or number of frequency points. Then, the minimization of J becomes maximization of SN ratio which is measured in decibels (dB).

3. Model updating using Taguchi method

The robust design method using Taguchi method uses a mathematical tool called orthogonal array to study a large number of decision variables with a small number of experiments [13–16]. It also uses a new measure of quality, so-called signal to noise (SN) ratio, for robust design against noises [13–16]. In this study, these two concepts are effectively applied to model updating.

3.1. Orthogonal array

In order to maximize the SN ratio by adjusting parameters, the main effects of each parameter should be evaluated. To investigate whole effects of each parameter requires a lot of computational efforts especially when a large number of parameters are required to be updated. Therefore, the orthogonal array is used in order to reduce the number of simulations efficiently for the investigation of main effect of each parameter [13–16]. For example, if whole effect of p parameters with 3 levels were investigated using full factorial design, then the total number of simulations to investigate every effect of parameters would be 3^p. Here, the number of simulations increases exponentially with increase of the number of parameters. However, if orthogonal array (OA) is used for screening main effect of p parameters with 3 levels, only $2p + 1$ or slightly greater number of runs of simulation are required [14]. Therefore, OA is sometimes called fractional factorial design because it screens factors, or parameters, in an efficient way

Table 1
$OA(9,4,3,2)$ orthogonal array

Run number	Parameters				SN ratio
	1 (A)	2 (B)	3 (C)	4 (D)	
1	0	0	0	0	SN_1
2	0	1	1	2	SN_2
3	0	2	2	1	SN_3
4	1	0	1	1	SN_4
5	1	1	2	0	SN_5
6	1	2	0	2	SN_6
7	2	0	2	2	SN_7
8	2	1	0	1	SN_8
9	2	2	1	0	SN_9

rather than simulate all possible combination of cases. The orthogonal array is extensively discussed in reference [18] and can be downloaded from website [19].

The OA can be represented by notation, $OA(N,p,s,t)$ where N, p, s and t represent the number of experiments, factors (parameters), levels and strength, respectively [18]. Here, the strength means number of columns, which can be seen equal number of times in OA. Note that different notation, $L_N(s^p)$, is also frequently used for orthogonal array of $OA(N,p,s,t)$ [13–16]. In this study, orthogonal array with 3-level factors is used for optimization of parameters [15,16] because it can cover wider range of parameter variation than using 2-level factors. On the other hand, if the number of updating parameters is large, then the use of OA with 2-level factors can be advantageous because the number of simulations for screening parameters can be reduced [20]. It should be also noted that a very large OA can be easily constructed in order to deal with large number of parameters [18].

Without loss of generality, the orthogonal array of $OA(9,4,3,2)$ is used for the explanation of the proposed method. Table 1 shows the $OA(9,4,3,2)$, which requires 9 runs of simulations to investigate the main effect of up to 4 factors with 3 levels. Note that each column or row of OA in Table 1 consists of '0', '1' and '2', which represent levels of a factor. Parameters to be updated are assigned to the columns in OA whereas rows represent the parameter setting method for efficient main effect screening of each parameter concerned. If the number of parameters is less than the number of columns in OA, not assigned columns can be left to be empty. Therefore, it is required to select the orthogonal array with equal or more number of columns than number of parameters to be updated. If OA with much larger size than necessary is used, then the number of simulations increases accordingly. Therefore, the smallest possible orthogonal array is recommended to be chosen if interactions between parameters can be neglected. For example, if the number of parameters is assumed to be 10, then the orthogonal array of $OA(27,13,3,2)$ is recommended to be used rather than $OA(81,40,3,2)$. Even though the updated results are likely to be the same by using either of two OAs, the use of $OA(27,13,3,2)$ for screening the main effects of 10 parameters has advantages because computational efforts can be significantly reduced from 81 runs of simulations to 27.

3.2. Parameter updating using SN ratio

Without loss of generality, a design problem with 4 design variables, A, B, C and D, is considered, as an illustrative example, to be optimized simultaneously using Taguchi method. It should be noted that the design variables correspond to parameters of analytical FE model to be updated. Here, $OA(9,4,3,2)$ shown in Table 1 is used for the explanation of updating process employed in this work. Each design parameter can be assigned to columns in $OA(9,4,3,2)$ in arbitrary manner. In this study, for the sake of convenience, the parameters are assigned from the first column to next columns in sequence such that A, B, C and D are assigned to 1st, 2nd, 3rd and 4th columns, respectively. In orthogonal array as seen in Table 1, '1' represents the current level of parameters or starting level whereas '0' and '2' represents the decreased and increased level, respectively, by predefined level intervals of each parameter, Δ_i^k, $i = A$, B, C, and D. Here, superscript, k, represents iteration index in order to account for iterative process of updating. The main effect on SN ratio of decreased or increased value of parameters, which are represented by '0' and '2', respectively, are compared with the main effect of current level of parameters, which are represented by '1'. Then, the parameters are adjusted according to the screened main effect of each parameter in order to maximize SN ratio. This procedure continues iteratively until the SN ratio no longer increases.

In order to screen the main effect of each parameter concerned, total 9 runs of simulation associated with rows in $OA(9,4,3,2)$ should be performed. For example, if the first and second rows in $OA(9,4,3,2)$ are considered, then corresponding each SN ratios, SN_1^k and SN_2^k, can be calculated from setting levels of each parameter according to the row vectors of OA as

$$SN_1^k = SN(A_0^k B_0^k C_0^k D_0^k) \quad \text{and} \quad SN_2^k = SN(A_0^k B_1^k C_1^k D_2^k), \tag{7}$$

where $SN_j^k, j = 1,2$, are SN ratios calculated by Eq. (6), where subscript j in SN ratio represents run number of simulations related with rows in OA. Here, the levels '0' and '2' of each parameter, A_i^k, B_i^k, C_i^k, and D_i^k, $i = 0,2$, can be obtained from the current level '1' of parameters, A_1^k, B_1^k, C_1^k, and D_1^k, as

$$\begin{aligned} &A_0^k = A_1^k - \Delta_A^k, \quad B_0^k = B_1^k - \Delta_B^k, \quad C_0^k = C_1^k - \Delta_C^k, \quad D_0^k = D_1^k - \Delta_D^k, \\ &A_2^k = A_1^k + \Delta_A^k, \quad B_2^k = B_1^k + \Delta_B^k, \quad C_2^k = C_1^k + \Delta_C^k \quad \text{and} \quad D_2^k = D_1^k + \Delta_D^k. \end{aligned} \tag{8}$$

Here superscript, k, represents the iteration number to account for iteration process. Similar to SN_1^k and SN_2^k, other SN ratios, SN_j^k, $j = 1,2,\ldots,9$, can be obtained according to the OA in Table 1.

Then, the average SN ratios for levels A_0^k, A_1^k and A_2^k of parameter A, $SN(A_0^k)$, $SN(A_1^k)$ and $SN(A_2^k)$ can be obtained from the corresponding columns of OA as

$$\begin{aligned} &SN(A_0^k) = \frac{SN_1^k + SN_2^k + SN_3^k}{3}, \quad SN(A_1^k) = \frac{SN_4^k + SN_5^k + SN_6^k}{3} \\ &\text{and} \quad SN(A_2^k) = \frac{SN_7^k + SN_8^k + SN_9^k}{3}. \end{aligned} \tag{9}$$

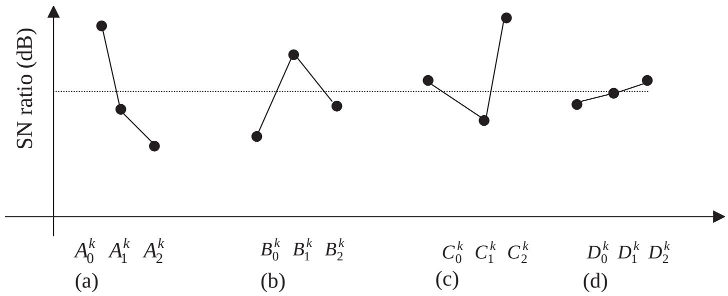


Fig. 2. Plots of factor effects: (a) decreasing; (b) level interval adjustment; (c) increasing; and (d) ignored.

Similarly, the average SN ratio for levels B_0^k, B_1^k and B_2^k of parameter B, $SN(B_0^k)$, $SN(B_1^k)$ and $SN(B_2^k)$, can be obtained from

$$SN(B_0^k) = \frac{SN_1^k + SN_4^k + SN_7^k}{3}, \quad SN(B_1^k) = \frac{SN_2^k + SN_5^k + SN_8^k}{3}$$
$$\text{and} \quad SN(B_2^k) = \frac{SN_3^k + SN_6^k + SN_9^k}{3}. \tag{10}$$

Then, the main effects of parameter, A, at levels, A_0^k, A_1^k and A_2^k are given by $(SN(A_0^k) - SN_{ave}^k)$, $(SN(A_1^k) - SN_{ave}^k)$ and $(SN(A_2^k) - SN_{ave}^k)$, respectively. Here, SN_{ave}^k is defined by

$$SN_{ave}^k = \frac{(SN_1^k + SN_2^k + \cdot\cdot + SN_9^k)}{9}. \tag{11}$$

Similarly, the main effects of parameter at levels, B_0^k, B_1^k and B_2^k, are given by $(SN(B_0^k) - SN_{ave}^k)$, $(SN(B_1^k) - SN_{ave}^k)$ and $(SN(B_2^k) - SN_{ave}^k)$, respectively. Other main effects of parameters concerned can be obtained from OA in a similar manner. Since orthogonal arrays are perfectly balanced for each parameter, other effects are averaged out in order to calculate the main effect of specific parameters [13–15]. Note that the total 9 runs of simulation are needed to evaluate the main effects of each parameter concerned in case of using $OA(9, 4, 3, 2)$. The main effects of each parameter on SN ratio can be effectively visualized in plots of factor effects as in Fig. 2.

If the main effects are to be evaluated for each parameter at iteration, k, then, analysis of variance (ANOVA) should be performed in order to determine if individual parameter is significant by comparing its variation with the overall variation [13–15]. The variance of each parameter, $V^k(i)$, $i = A$, B, C and D, can be written from

$$V^k(i) = (SN(i_0^k) - SN_{ave}^k)^2 + (SN(i_1^k) - SN_{ave}^k)^2 + (SN(i_2^k) - SN_{ave}^k)^2, \quad i = A,\ B,\ C \text{ and } D. \tag{12}$$

There are two methods for ANOVA. The one method is to use the unassigned column in OA as an error column and only parameters, whose variance is greater than that of the error column, are selected for updating [14,15]. The parameters whose effect is similar to or even smaller than that of error column can be considered as insignificant parameters. The parameters with small variances are not used for updating since the results are subject to various noises. The other method is to use Pareto ANOVA, which quantifies the importance of each parameter and prioritizes activities for improvement [14]. In this study, for the sake of simplicity, Pareto ANOVA is used to select important parameters for updating and the parameters with small effect are not used for updating. For this purpose, only significant factors or parameters, which cumulatively contribute to 95% of total summation variance of each factor, are selected. This is a quick and easy method, which does not require ANOVA table or F-test [14]. For example, when the main effect of parameters are shown in plots of factor effects as Fig. 2, the main effect of parameter, D, whose variance of main effects is small compared with that of other parameters, is ignored.

The model updating process employed in this work is to adjust the parameters in analytical FE model for the maximization of SN ratio. For illustrative example, the main effects of each parameter is assumed to be represented by the plots of factor effects shown in Fig. 2. Then, the parameters are updated in such a manner to maximize SN ratio. If the main effect of SN ratios of a parameter are decreasing or increasing according to the parameter change as shown in Fig. 2(a) or (c), then, parameters are updated to be decreased or increased at the next step, $k+1$, to maximize the SN ratio, as

$$A_1^{k+1} = A_1^k - \kappa \Delta_A^k \quad \text{and} \quad C_1^{k+1} = C_1^k + \kappa \Delta_C^k \quad \text{with } \Delta_A^{k+1} = \Delta_A^k \quad \text{and} \quad \Delta_C^{k+1} = \Delta_C^k, \tag{13}$$

where superscript k represents iteration index. Here, κ is a constant related with amount of parameter changes at the next step. In most cases, the value of κ is set to one [15,16]. However, in order to check the SN values between levels, the value of κ is set to 0.5 in this study. The use of κ with the value of less than 1 might be advantageous because global minimum is more likely to be sought by searching optimal parameters between levels. However, it should be noted that the use of κ close to 0 should be avoided because updating process is likely to be very slow. On the other hand, if a parameter approaches to the optimum value, the SN ratio of the parameter is maximum at current level such as B_1^k in Fig. 2(b). Then the parameter is updated to remain the same value and level interval is reduced to half as [16]

$$B_1^{k+1} = B_1^k \quad \text{and} \quad \Delta_B^{k+1} = \frac{\Delta_B^k}{2}. \tag{14}$$

Here, by reducing the level intervals using Eq. (14), precise optimum value can be found. In case of small main effect such as parameter, D, in Fig. 2(d), $D_1^{k+1} = D_1^k$ is used instead of increasing parameters at next step because the main effect of parameter, D, is ignored due to the small variance of the parameter. After updating current level '1' at iteration step, $k+1$, using Eqs. (13) and (14), other two levels, '0' and '2', at $k+1$, are calculated using Eq. (8). Then, the same updating process is repeated until the optimum values are sought where the SN ratio no longer increases.

4. Numerical example

For the verification of the proposed scheme for model updating, FE models of truss structure and cantilever beam are used for numerical simulations. For reference data, the so-called structural model with 10 times finer element than analytical model is used to generate simulated experimental data. The structural model can be considered to be more close to real structure [21,22]. By using structural model, the effect of discretization errors on updated results, which is inherent in analytical FE model, can be investigated. For the verification of robustness of the proposed updating method, random errors in experimental data as well as systematic error in analytical model are considered.

4.1. Cantilever beam

For demonstration of the proposed method, cantilever beam with 10 elements shown in Fig. 3(a) is considered as analytical model for parameter updating. For the simulation of measured vibration data such as FRF and modal data, a structural model with fine mesh shown in Fig. 3(b) is used to generate vibration data. The following parameters are used for both models: modulus of elasticity $E = 2.06 \times 10^{11}$ N/m^2; cross sectional area $AR = 0.02 \times 0.02$ m^2; length of cantilever beam $l = 0.8$ m. For systematic error in analytical model, parameter error is included throughout the simulations such that the density, $\rho = 7895$ kg/m^3, is used for analytical model whereas the density of structural model is assumed to be $\rho = 7973.95$ kg/m^3 which is 1% higher than that of analytical model.

In this work, for the updating parameters of analytical FE model, the 10 non-dimensional parameters, β_i, are considered, which are defined by ratio of modulus of elasticity, E, of each element, i, in analytical FE model

$$\beta_i = \frac{E_i^d}{E_i^o}, \quad i = 1, 2, \ldots, 10, \tag{15}$$

where superscripts, d and o, represent damaged and original parameters, respectively. In this work, the optimum values of β_i are sought in an iterative way in order to maximize SN ratio. Then, by investigation of updated values of β_i, parameter errors in analytical model can be identified and corrected. As an illustrative example, damaged structural model of cantilever beam shown in Fig. 3(b), which has stiffness reduction of 50% and 30% in 21–30th and 61–70th elements, respectively, is considered for generation of reference data for updating parameters in analytical model. So, the updating problem in cantilever beam is to update each parameter in analytical model, whose value are $0 < \beta_i \leqslant 1$ for physical consideration, in order to identify the damages in structure, which are represented by parameter errors in analytical model.

4.1.1. Model updating using FRF

For an illustrative example of model updating using FRFs, 3 FRFs, whose measurement points are at 2, 6 and 10 with excitation point at free end (at 10) in Fig. 3(b), are assumed to be available

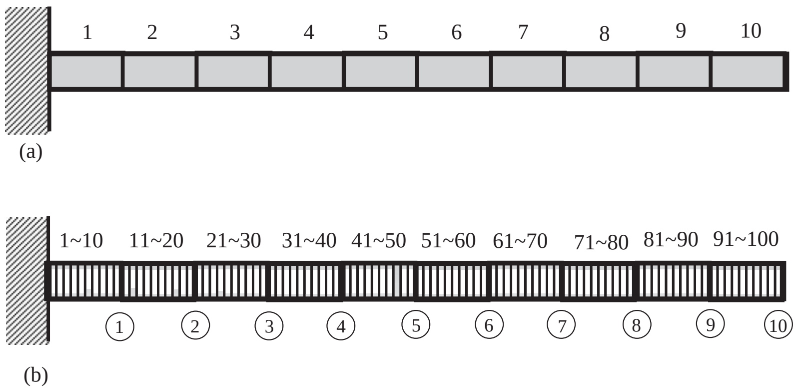


Fig. 3. Cantilever beam: (a) analytical model; and (b) structural model.

as reference data. Each measured FRF, which is obtained from structural model, is contaminated with random noise such that

$$H'_2(\omega_k) = H_2(\omega_k)(1+\sigma), \quad H'_6(\omega_k) = H_6(\omega_k)(1+\sigma) \quad \text{and}$$
$$H'_{10}(\omega_k) = H_{10}(\omega_k)(1+\sigma), \tag{16}$$

where superscript, $'$, indicates noise polluted experimental FRFs and subscripts, 2, 6 and 10, in FRFs represent the measurement point in Fig. 3(b). Here, for the test of robustness against measurement noise, 5% uniform distributed random noise, σ, is considered in this study. The initial values of updating parameters in analytical model are set to 1, that is, $\beta_i^1 = 1$, $i = 1, 2, \ldots, 10$. As seen in Fig. 4, there are differences between the FRFs of initial analytical model and simulated measured data. The differences are to be corrected by updating analytical FE model. For reference data, 6 frequency points (30, 170, 500, 950, 1500, 2300 Hz) are carefully selected from measured FRFs such that frequency points from non-monotonic regions can be avoided.

In order to investigate the main effects of 10 parameters, $OA(27, 13, 3, 2)$, which is downloaded from website [19], is used. Here, 10 parameters are assigned to $OA(27, 13, 3, 2)$ in sequential manner and last 3 columns are left to be empty. The initial level intervals, $\Delta^1_{\beta_i}$, $i = 1, 2, \ldots, 10$, are set to 0.05, which correspond to 5% of initial parameters. Here, initial level intervals should be set between 0 and 1 for physical consideration. If too small initial level intervals were used, the main effects of parameters might be subject to the noises and convergence tends to be slow. On the other hand, initial level intervals with large value should be avoided because too much deviation of analytical FRFs from the experimental data may result in non-monotonous characteristics of FRFs at selected frequency points. Note that the level intervals are apt to be smaller value because the level intervals are decreased by half using Eq. (14) when the parameters approach to the optimum values. So, after dozens of iterations, the level intervals can be very small and the updating process might be slown down. However, the problem can be effectively released by re-setting level intervals, for example, $\Delta^k_{\beta_i} = 0.01$, $i = 1, 2, \ldots, 10$, at every 50 iteration ($k = 50$, 100, 150, etc).

Fig. 5(a) shows the updated results of parameters after 200 iterations. Here, the selection of stable frequencies out of a large number of frequency points might result in loss of information. Therefore, the updated results may not be exact enough to identify damages or parameter errors in terms of location as well as severity as seen in Fig. 5(a). Nevertheless, as seen in Fig. 4, the differences between updated analytical and experimental FRFs are reduced significantly by maximization of SN ratio as shown in Fig. 5(b). The increased SN ratio means closeness between analytical and experimental data. The updated results can be significantly improved if more number of FRFs or frequency points are used as reference data. On the other hand, the use of frequency domain criteria proposed by Zang et al. [23] can be useful because a lot of frequency data may be incorporated into objective function without selection of stable frequency points. The use of different form of frequency data is beyond the scope of this paper.

4.1.2. *Model updating using modal data*

For the objective function using modal data, modal data from structural model of cantilever beam are used as reference data for updating 10 parameters in analytical FE model. The analytical model has both rotational and translational degrees of freedom. In practice, mode shapes

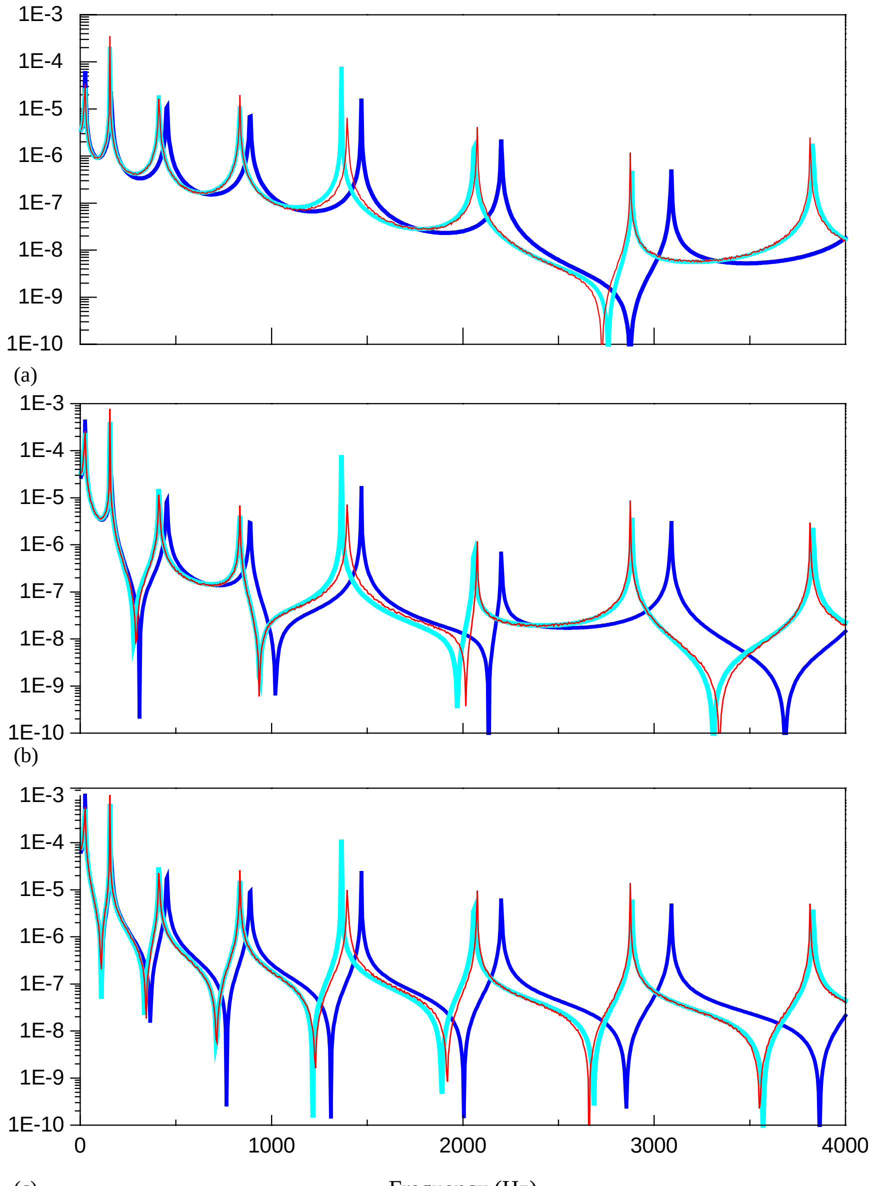
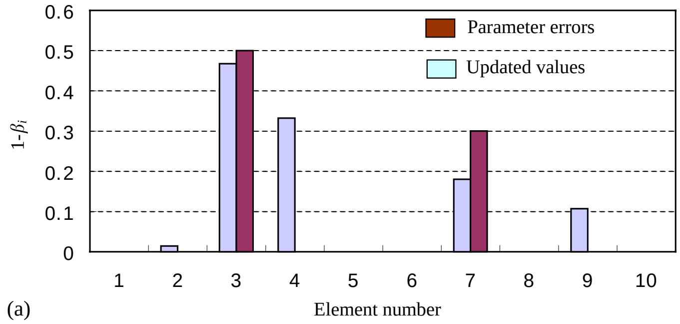


Fig. 4. Magnitude plots of FRFs of cantilever beam: (a) $H_2(\omega)$; (b) $H_6(\omega)$; (c) $H_{10}(\omega)$; —— initial analytical model; —— updated analytical model; and —— structural model.

associated with rotational degree of freedom are not directly available from experimental data. In this study, mode shape data from translational degrees of freedom at 10 measured points in structural model shown in Fig. 3(b) are used as reference data for the objective function in Eq. (2).

Since the structural model is more close to real structure, discretization effect of analytical model can be understood by comparison of two modal data from structural and analytical model. Table 2 shows the eigenvalues and MAC of exactly the two same models with different number of



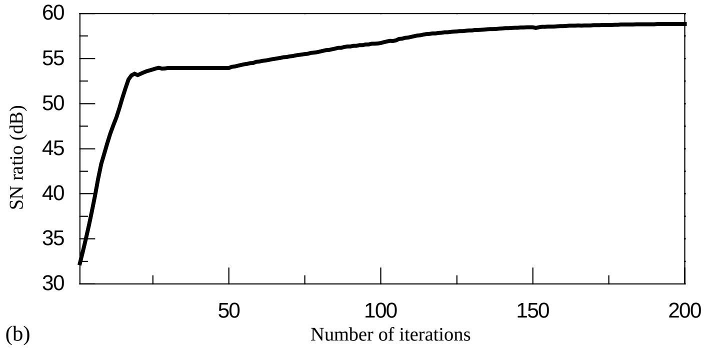


Fig. 5. Updated results of cantilever beam using FRF: (a) updated parameters; and (b) SN ratio plot.

elements. As seen in Table 2, the discretization errors are not severe at first 10 modes using analytical model with 10 elements. However, it should be noted that special care should be taken when using higher modes because the updated results are subject to discretization errors. Considering the discretization effect, first 8 modes are used as reference data for updating 10 parameters of analytical model. For the demonstration of robustness against random noise, 1% and 5% random noises are added to natural frequencies and mode shapes of simulated experimental data obtained from structural model, respectively.

In order to investigate the main effect of each parameter, orthogonal array, $OA(27, 13, 3, 2)$, which requires 27 runs of simulations at each iteration, is used. The initial parameters are set to be $\beta_i^1 = 1$, $i = 1, 2, \ldots, 10$, with initial level intervals to be $\Delta_{\beta_i}^1 = 0.3$, $i = 1, 2, \ldots, 10$. Here, the initial level intervals can be chosen from wider range of values unlike the case of using FRFs as reference data. However, it should be noted that initial level intervals close to 1 should be avoided since the pairing of natural frequencies and mode shapes using Eq. (5) can be unreliable due to the large variance of parameters.

Table 3 shows the initial and updated modal data, such as eigenvalues and mode shapes (MAC), of analytical model. As seen in Table 3, the differences between analytical and experimental modal data can be reduced significantly by updating the analytical model. By updating the analytical model, parameter errors in analytical FE model can be located and corrected. As seen in Fig. 6, damages in cantilever beam can be identified with reasonable accuracy even in presence of random, parameter and discretization errors.

Table 2
Discretization effect of FE model of cantilever beam (Undamaged with no other errors) ($\rho = 7973.95\ \mathrm{kg/m^3}$)

Modes	10 elements analytical model (eigenvalue, Hz)	100 elements structural model (eigenvalue, Hz)	Errors (%)	MAC (diagonal)
1	25.66	25.66	0.00	1.000
2	160.8	160.80	0.00	1.000
3	450.35	450.24	0.02	1.000
4	883.13	882.29	0.10	1.000
5	1460.2	1458.5	0.25	1.000
6	2190.2	2178.7	0.54	1.000
7	3073.4	3043.0	1.00	1.000
8	4117.4	4051.3	1.63	1.000
9	5323.8	5203.7	2.31	1.000
10	6618.3	6500.2	1.82	0.996
11	8808.6	7940.7	10.93	0.996
12	10642	9525.3	11.72	0.982
13	12900	11254	14.63	0.980
14	15570	13127	18.61	0.972
15	18710	15143	23.56	0.961
16	22377	17304	29.32	0.945
17	26558	19609	35.44	0.924
18	31005	22059	40.55	0.895
19	34915	24652	41.63	0.854
20	43696	27390	59.53	0.292

Fig. 6(b) shows the SN ratio plot of updating process. Here, SN ratio seems to be reached at maximum value after 20 iterations. However, since level intervals are changed into smaller value using Eq. (14), the level interval can be very small after dozens of iterations. This small level intervals might result in local minimum solutions. On the other hand, the possible local minimum solutions can be avoided effectively by level interval re-setting, for example, $\Delta_{\beta_i}^k = 0.01$, $i = 1, 2, \ldots, 10$, at every 50 iteration ($k = 50$, 100, 150, etc.) as seen in SN ratio plot of Fig. 6(b).

4.2. Truss structure

For the demonstration of practical applicability, analytical FE models of plane truss structure with 36 elements and 72 elements, which is shown in Fig. 7(a) and (b), are considered for model updating. The FE models have 3 degree of freedom at each node, which results in total 90 and 198 degrees of freedom for FE models with 36 and 72 elements, respectively. In truss application, modal data from the structural model with 360 elements are used as reference data. Note that the modal data are essence form of a large number of FRF data points and data loss from the selection of frequencies can be avoided. For simulations, the following parameters are used for analytical FE model and structural model: modulus of elasticity $E = 0.75 \times 10^{11}\ \mathrm{N/m^2}$; second moment of area of each member $I = 0.0756\ \mathrm{m^4}$; cross sectional area of member $Ar = 0.004\ \mathrm{m^2}$. For the simulation of incomplete measurement, only translational degrees of freedom at 6

Table 3
Modal data of initial and updated analytical FE model of cantilever beam (using first 8 modes as reference data)

Modes	Reference data (Hz)	Analytical model (initial eigenvalue)	Initial MAC (diagonal)	Updated model (eigenvalue)	Updated MAC (diagonal)
1	23.62	25.79 (9.15)*	1.000	23.46 (−0.68)*	1.000
2	155.10	161.6 (4.19)*	1.000	154.45 (−0.42)*	1.000
3	411.75	452.6 (9.92)*	0.996	410.51 (−0.30)*	1.000
4	834.06	887.53 (6.41)*	0.995	832.77 (−0.15)*	1.000
5	1396.5	1469.5 (5.23)*	0.996	1391.5 (−0.36)*	1.000
6	2074.2	2201.4 (6.13)*	0.992	2069.8 (−0.21)*	1.000
7	2875.5	3088.7 (7.41)*	0.990	2889.3 (0.48)*	1.000
8	3816.0	4138.0 (8.44)*	0.982	3863.9 (1.26)*	1.000
9	4911.4	5350.4 (8.94)*	0.985	4987.8 (1.56)*	1.000
10	6120.5	6651.3 (8.65)*	0.993	6182.5 (1.00)*	0.998
11	7567.4	8852.5 (16.98)*	0.935	8353.3 (10.39)*	0.990
12	9060.4	10695 (18.04)*	0.963	10127 (11.77)*	0.980
13	10524	12965 (23.19)*	0.936	11991 (13.77)*	0.980
14	12413	15648 (26.06)*	0.883	14606 (17.67)*	0.973
15	14389	18803 (30.68)*	0.898	17581 (22.88)*	0.908
16	16363	22488 (37.43)*	0.884	21566 (31.80)*	0.899
17	18574	26690 (43.70)*	0.855	25159 (35.45)*	0.951
18	20873	31160 (49.28)*	0.777	28561 (36.83)*	0.824
19	23239	35089 (50.99)*	0.762	33168 (42.73)*	0.794
20	25821	43914 (70.07)*	0.308	43183 (67.24)*	0.308

( )*: errors (%).

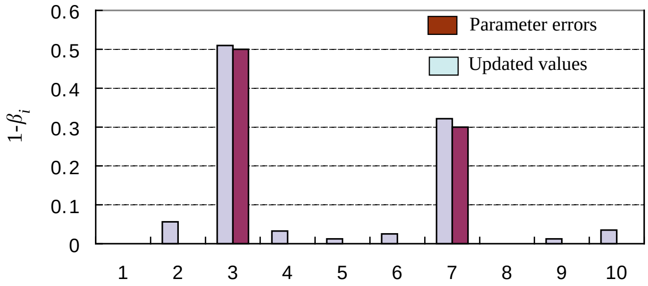
measurement points are assumed to be available for mode shape data as reference data throughout simulation as shown in Fig. 7(c).

In this study, stiffness modelling errors in analytical model, without loss of generality, are considered such that modal data are generated as reference data from structural FE model which has different modulus of elasticity at 2nd, 5th and 10th members ($\beta_2 = 1.1$, $\beta_5 = 0.85$ and $\beta_{10} = 1.2$). Note that 2nd and 10th members of truss have increased values of modulus of elasticity whereas 5th member has decreased value. Therefore, the total effect on eigenvalues of structural model is mixed up and differences in eigenvalues between structural and initial analytical model appear to be small as seen in Tables 5 and 6. Therefore, the eigenvectors must be incorporated in objective function in order to locate parameter errors in analytical model.

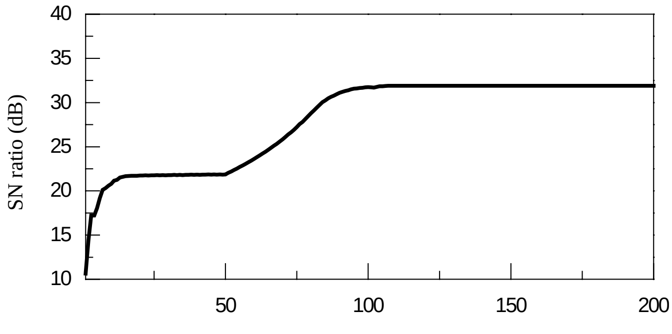
For consideration of systematic parameter error in analytical model, the density of each member for structural model is assumed to be $\rho = 2828\ \text{kg/m}^3$ which is 1% higher than $\rho = 2700\ \text{kg/m}^3$ of analytical model. For the demonstration of robustness against random noise, 1% and 5% random noises are added to natural frequencies and mode shapes of simulated experimental data from structural model, respectively.

4.2.1. Model updating using analytical model with coarse elements

Prior to updating analytical FE model, discretization effect, which is inherent in FE model, needs to be evaluated. Table 4 shows the discretization effect of analytical models with 36 and 72 elements. In this section, first 8 modes of truss structure are considered as reference data for



(a)



(b)

Fig. 6. Updated results of cantilever beam using modal data: (a) updated parameters; and (b) SN ratio plot.

updating 12 parameters of FE model with 36 elements. Then, the eigenvalue errors from discretization effect are up to about 4% as seen in Table 4. Therefore, the updated parameters might be subject to discretization errors because the parameters in analytical model are updated such that the both errors from parameters and discretization in FE model are minimized simultaneously.

In order to update 12 parameters of analytical model with 36 elements shown in Fig. 7(a), $OA(27, 13, 3, 2)$ is used for the screening the main effect of each parameter. Here, $OA(27, 13, 3, 2)$ can handle 13 parameters with 27 runs of simulations for screening parameters. In this example, 12 parameters are assigned to columns of $OA(27, 13, 3, 2)$ in sequential manner and the last column of the OA is left to be empty. The initial parameters of analytical model are set to be $\beta_i^1 = 1, i = 1, 2, \ldots, 12$, with initial level intervals to be $\Delta^1_{\beta_i} = 0.1, i = 1, 2, \ldots, 12$. Here, as a scheme for avoiding possible local minimum solutions, level intervals are set to $\Delta^k_{\beta_i} = 0.01, i = 1, 2, \ldots, 12$, at every 50 iteration ($k = 50$, 100, 150, etc.).

Table 5 shows the initial and updated eigenvalues and MAC (eigenvectors). Here, it should be noted that the accuracy of updated eigenvalues and eigenvectors can be varied according to the weightings used in objective function in Eq. (2). If higher weighting on eigenvalues (eigenvectors) is used, then the updated eigenvalues (eigenvectors) are more accurate. However, the update eigenvectors (eigenvalues) of analytical model may not be close to those of measured mode shapes (natural frequencies). In this study, $W_\omega = 10$ and $W_\varphi = 1$ are used for weighting values in Eq. (2) as discussed in Section 2.2. As seen in Table 5, updated diagonal MAC is almost 1 for the first 8 modes. In addition, the updated eigenvalues of first 8 modes have less than 1% errors compared with reference data, which are almost similar levels to random noises added in reference

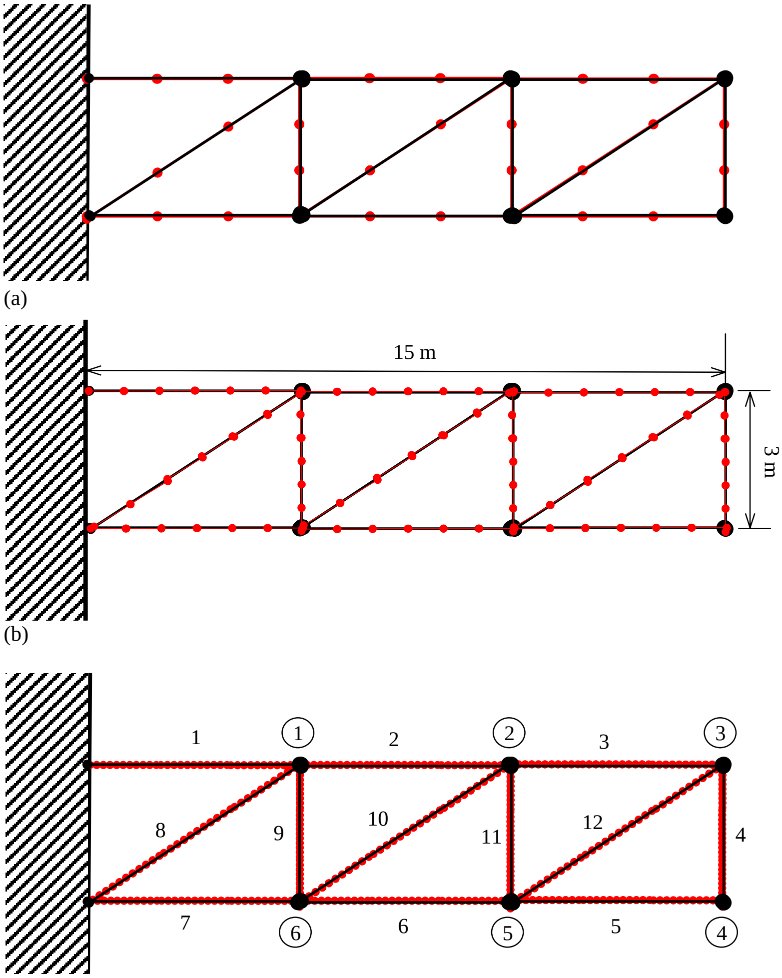


Fig. 7. Truss structure: (a) coarse analytical model; (b) fine analytical model; (c) structural model; ● measured nodal point; and • non-measured nodal point.

eigenvalue data. Thus, the updated first 8 modal data of analytical model with 36 elements seem to be close to the reference data. However, higher modes (especially mode shapes) of updated analytical model, which are not included in objective function, are not close enough to that of simulated measured data as seen in Table 5. Furthermore, as seen in Fig. 8, the parameter errors of analytical model are difficult to be located and corrected sufficiently due to the various errors especially from discretization errors. If the discretization errors are large, updating parameters will compensate the errors by changing its values. Therefore, the updated parameters are likely to lose their physical meanings in case where the discretization errors are significant [22]. Nonetheless, the SN ratio has increased as a result of parameter updating as seen in Fig. 8. The increase in SN ratio means the closeness between analytical and experimental modal data used in objective function.

Table 4
Discretization effect of truss structure (with same parameters and no other errors) ($\rho = 7973.95\ \mathrm{kg/m^3}$)

Modes	360 elements structural model (eigenvalue, Hz)	36 elements analytical model (eigenvalue, Hz)	MAC 36 elements (diagonal)	72 elements analytical model (eigenvalue, Hz)	MAC 72 elements (diagonal)
1	46.15	46.16 (0.02)*	1.000	46.15 (0.00)*	1.000
2	81.65	81.72 (0.09)*	1.000	81.67 (0.02)*	1.000
3	234.87	236.71 (0.78)*	1.000	235.32 (0.19)*	1.000
4	253.82	254.74 (0.36)*	1.000	254.05 (0.09)*	1.000
5	378.05	385.79 (2.05)*	1.000	379.98 (0.51)*	1.000
6	432.08	446.93 (3.44)*	0.975	435.75 (0.85)*	0.999
7	442.69	460.13 (3.94)*	0.997	446.9 (0.95)*	1.00
8	466.15	481.53 (3.30)*	0.965	470.23 (0.88)*	0.997
9	482.83	500.51 (3.66)*	0.975	487.12 (0.89)*	0.998
10	501.87	514.95 (2.61)*	0.981	504.8 (0.58)*	0.999
11	511.13	531.86 (4.06)*	0.523	516.56 (1.06)*	0.896
12	512.86	534.23 (4.17)*	0.300	517.49 (0.90)*	0.984
13	544.66	569.16 (4.50)*	0.984	550.54 (1.08)*	0.999
14	632.06	647.0 (2.36)*	0.984	636.01 (0.62)*	0.998
15	660.95	688.54 (4.17)*	0.783	668.83 (1.19)*	0.985
16	692.55	713.36 (3.00)*	0.835	697.4 (0.70)*	0.992
17	761.98	804.13 (5.53)*	0.975	773.88 (1.56)*	0.997
18	869.13	901.66 (3.74)*	0.810	884.59 (1.78)*	0.898
19	884.9	968.75 (9.48)*	0.025	911.92 (3.05)*	0.457
20	887.04	977.32 (10.18)*	0.447	914.6 (3.11)*	0.768

( )*: errors (%).

4.2.2. Model updating of analytical model with fine elements

If the discretization effects are significant, updating parameters are likely to compensate the errors from discretization. Here, the compensation for the discretization errors will distort the physical meanings of the updating parameters [21]. Since the disrectization errors, if not negligible, can affect the updated parameters, proper treatment of the errors is needed. The compensation method of discretization errors using parameters, especially using modulus of elasticity, has been discussed to minimize those errors prior to updating [21]. However, it can have limitations because the discretization errors are not directly related with parameter errors and cannot be compensated fully by adjusting parameters [22]. The discretization errors are related with number of elements used in analytical FE model. Therefore, the errors from discretization effect can be significantly reduced by using FE model with fine mesh. In this study, as a way to reduce the discretization errors, fine analytical model with 72 elements shown in Fig. 7(b) is used to update 12 parameters, which correspond to modulus of elasticity in each truss member. Then, eigenvalue errors from discretization effect can be limited less than 1% for first 8 modes concerned as seen in Table 4. Then, the updated parameters can locate parameter errors and correct them with reasonable accuracy as seen in Fig. 9(a). Note that, as seen in Fig. 9(b), SN ratio is maximized in spite of various errors including random and systematic errors as well as incomplete measurement.

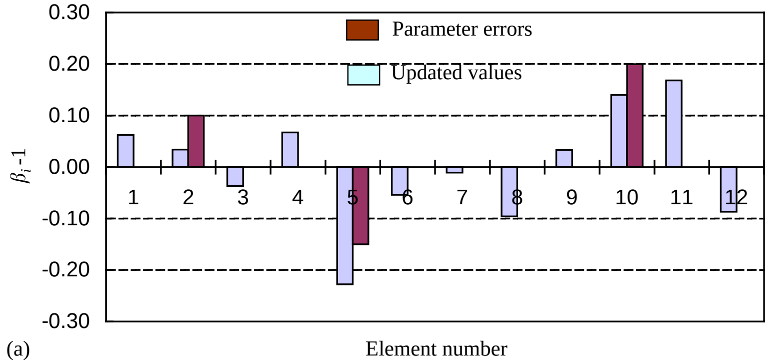
Table 5
Modal data of initial and updated coarse analytical model of truss structure (using first 8 modes as reference data)

Modes	Reference data (Hz)	Analytical model (initial eigenvalue)	Initial MAC (diagonal)	Updated model (eigenvalue)	Updated MAC (diagonal)
1	46.09	46.39 (0.64)*	1.000	46.34 (0.53)*	1.000
2	83.27	82.13 (−1.36)*	0.999	81.92 (−1.62)*	0.999
3	234.93	237.89 (1.26)*	0.997	233.23 (−0.72)*	0.999
4	257.36	256.01 (−0.52)*	0.999	254.65 (−1.05)*	1.000
5	378.17	387.72 (2.53)*	0.995	380.54 (0.63)*	0.999
6	426.29	449.16 (5.36)*	0.838	430.16 (0.91)*	0.997
7	446.03	462.43 (3.68)*	0.947	448.35 (0.52)*	0.996
8	482.55	483.93 (0.29)*	0.948	485.73 (0.66)*	0.998
9	488.22	503.01 (3.03)*	0.119	503.87 (3.21)*	0.369
10	501.34	517.51 (3.23)*	0.551	507.93 (1.31)*	0.981
11	509.4	534.51 (4.93)*	0.000	528.48 (3.75)*	0.945
12	518.7	536.9 (3.51)*	0.015	539.46 (4.00)*	0.376
13	543.41	572.0 (5.26)*	0.947	527.05 (5.27)*	0.948
14	615.86	650.23 (5.58)*	0.951	630.06 (2.31)*	0.991
15	668.89	691.98 (3.45)*	0.766	686.06 (2.58)*	0.939
16	703.24	716.92 (1.95)*	0.882	717.94 (2.09)*	0.942
17	761.21	808.14 (6.17)*	0.953	795.99 (4.57)*	0.969
18	862.37	906.15 (5.08)*	0.128	899.77 (4.34)*	0.095
19	879.78	973.58 (10.66)*	0.199	959.47 (9.06)*	0.149
20	884.05	982.20 (11.10)*	0.238	984.55 (11.37)*	0.454

( )*: errors (%).

Table 6 shows the initial and updated modal data using first 8 modes as reference data for updating. By updating parameters, the initial errors of eigenvalues and eigenvectors are reduced to acceptable levels. In order to investigate closeness between analytical and experimental eigenvectors, MAC is used. As seen in Table 6, the diagonal values of updated MACs are more close to 1 than those of initial MACs. The MAC value close to 1 means eigenvector of an analytical model is close to that of reference (experimental) model at a specific mode. When compared with Table 5 of coarse analytical model, the updated eigenvalues and eigenvectors of fine analytical model are more close to reference data because discretization errors can be reduced to an acceptable level. Note that even higher modes, which are not included in objective function, become close to those of reference after updating of parameters. For example, 9th and 12th updated MACs, which represent correlation between updated and reference eigenvectors, become 0.984 and 0.999, respectively as seen Table 5. On the other hand, in case of using coarse analytical model, 9th and 12th updated MACs were 0.369 and 0.376, respectively, as seen in Table 5.

It should be noted that the SN ratio plots in Figs. 8 and 9 may not be always monotonous increasing even though the scheme employed in this work maximizes the SN ratio at each iteration. The problem is mainly caused by interactions among the parameters. The interactions can be classified according the degree of interaction as seen in Fig. 10 [15]. If the interaction is large as in case of Fig. 10(c), then the updating of one parameter affects the updating of the other



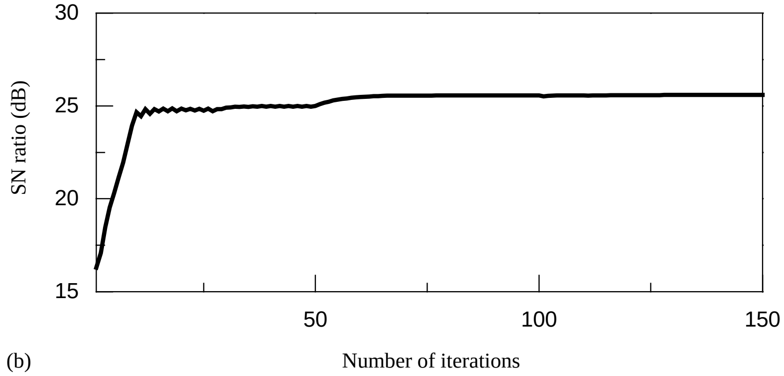


Fig. 8. Updated results of truss structure using coarse analytical FE model: (a) updated parameters; and (b) SN ratio plot.

parameters. As a result, SN ratio may not increase monotonously. To consider the whole interactions among parameters concerned can be very complicated process [15]. The severe interactions shown in Fig. 10(c) are related with overshooting of parameters from the optimum value in model updating application [20]. In this study, a level interval adjusts its value into smaller value when the updated parameter becomes close to the optimum value as explained in Fig. 2(b). In addition, the overshooting from the target value can be adjusted by the increasing or decreasing the parameter using Eq. (13) to maximize the SN ratio. Therefore, optimum parameters can be reached by iterative way even in case of non-monotonous increase in SN ratio due to interactions.

5. Concluding remarks

Taguchi method, which has been widely used in robust design and quality engineering application, is effectively applied to FE model updating by optimizing the objective function. The updated results of two numerical examples of cantilever beam and truss structure indicate that the

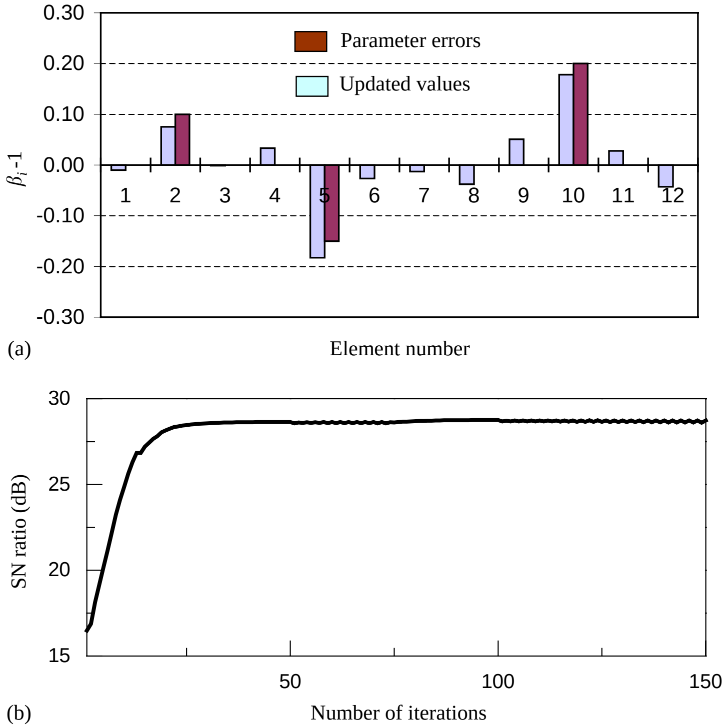


Fig. 9. Updated results of truss structure using fine analytical FE model: (a) updated parameters; and (b) SN ratio plot.

proposed method is robust against various errors including random and systematic errors. Moreover, the method can be computationally efficient due to the use of orthogonal array rather than relying on stochastic search.

For reference data, frequency data as well as modal data can be used for model updating using Taguchi method. However, the model updating using frequency data might be more difficult because selected frequency points should have monotonous characteristics of FRF. In addition, there can be some loss of information in the process of selecting frequency points. Insufficient information is very likely to lead to local minimum solutions of the objective function. Therefore, modal data, which are believed to be essence form of a large number of FRF data, are recommended to be used for reference data of updating using Taguchi method.

If the effect of discretization errors, which are inherent in analytical FE model, is significant, the updated results are likely to be affected. Therefore, the discretization errors should be investigated prior to updating. However, the discretization effect on updated results can be reduced to an acceptable level by using fine analytical FE model.

The proposed method can be extended to a case where a large number of parameters need to be updated. However, it should be noted that updated results may not reach global minimum solutions if the information in objective functions is not sufficient for updating parameters

Table 6
Modal data of initial and updated fine analytical model of truss structure (using first 8 modes as reference data)

Modes	Reference data (Hz)	Analytical model (initial eigenvalue)	Initial MAC (diagonal)	Updated model (eigenvalue)	Updated MAC (diagonal)
1	46.09	46.38 (0.63)*	1.000	46.31 (0.47)*	1.000
2	83.27	82.08 (−1.43)*	0.999	82.4 (−1.04)*	0.999
3	234.93	236.49 (0.66)*	0.997	234.48 (−0.19)*	0.999
4	257.36	255.31 (−0.80)*	0.999	256.52 (−0.33)*	1.000
5	378.17	381.87 (0.98)*	0.995	380.63 (0.65)*	0.999
6	426.29	437.92 (2.73)*	0.860	427.15 (0.20)*	0.999
7	446.03	449.13 (0.70)*	0.952	445.29 (−0.17)*	0.999
8	482.55	472.58 (−2.07)*	0.913	480.75 (−0.37)*	1.000
9	488.22	489.55 (0.27)*	0.125	494.02 (1.19)*	0.984
10	501.34	507.31 (1.19)*	0.508	502.48 (0.23)*	0.998
11	509.4	519.13 (1.91)*	0.116	514.5 (1.00)*	0.998
12	518.7	520.07 (0.26)*	0.165	524.7 (1.16)*	0.999
13	543.41	553.39 (1.84)*	0.968	552.29 (1.63)*	0.999
14	615.86	639.18 (3.79)*	0.978	622.81 (1.13)*	0.999
15	668.89	672.17 (0.49)*	0.951	671.94 (0.46)*	0.999
16	703.24	700.88 (−0.34)*	0.994	706.99 (0.53)*	0.998
17	761.21	777.74 (2.17)*	0.983	776.31 (1.98)*	0.998
18	862.37	889.00 (3.09)*	0.191	887.75 (2.94)*	0.253
19	879.78	916.47 (4.17)*	0.377	892.91 (1.49)*	0.606
20	884.05	919.17 (3.97)*	0.077	915.4 (3.55)*	0.901

( )*: errors (%).

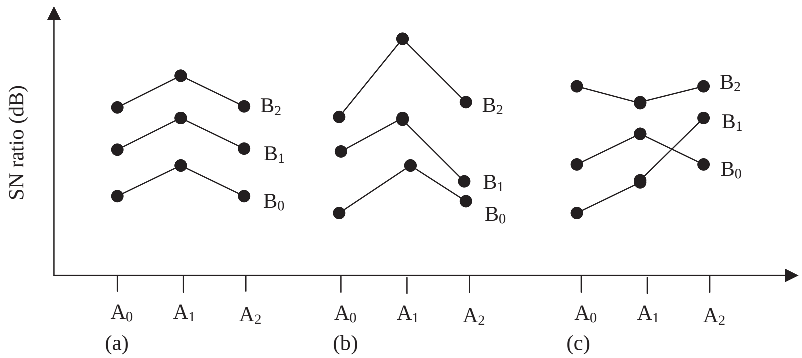


Fig. 10. Interaction between parameters: (a) no interaction; (b) synergistic interaction; and (c) antisynergistic interaction.

concerned due to incomplete measurement coordinates or limited number of modes used for reference data. Furthermore, the convergence is likely to be slow if a lot of parameters have strong interactions one another.

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