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Journal of Sound and Vibration 280 (2005) 1125–1131
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Short Communication

K-space identification of apparent structural behaviour

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Received 27 May 2003; accepted 23 February 2004
Available online 12 October 2004

1. Introduction

In the mid-high-frequency domain classical normal modes synthesis is not relevant due to high modal density and/or high modal overlaps (cf. Refs. [1,2]). Several mid-high-frequency methods use the dispersion curve, assumed to be known, and free waves characteristics as inputs. However, analytical models providing these frequency-dependent wavenumbers are limited to the low-frequency domain, as demonstrated by Graff [3], among others. Therefore, it seems relevant to propose identification tools of such a frequency equation mainly for complex structures in the medium- and high-frequency domain. The latter has been achieved in one-dimensional structures. For example, McDaniel et al. [4] uses a wave approach to identify the loss factor, thanks to a complex wavenumber. The wave approach allows estimation of this loss factor over a densely discretized frequency bandwidth. In contrast, identification of propagation constants is a difficult task for 2-D structures. Only a very few contributions can be found in the available literature. For instance, Ferguson et al. [5] recently proposed a method dealing with a windowed field of the normal displacement of a plate. The latter makes it possible to identify a unique dominant wavenumber at the considered area of the plate.

The present contribution sets the bases of a new method, named Inhomogeneous wave correlation method (IWC), that uses the complete wave field of a vibrating plane structure, in order to identify the complete θ-dependent dispersion curve. It should be noticed that the proposed procedure works with either a virtual or an experimental field. The outline of the method is given in Section 2 together with its implementation. Section 3 regroups two levels of validation with full

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0022-460X/$ - see front matter
doi:10.1016/j.jsv.2004.02.044

experimental data. The first aims to roughly check the method on an isotropic damped plate. The second considers a ribbed panel in order to validate more precise capabilities of the method. In both cases, dispersion curves are extracted, proving the effectiveness of the method.

2. Inhomogeneous wave correlation method (IWC)

2.1. Outline of IWC method

The new method proposed here is based on several ideas:

(i) Whereas Ferguson et al. [5] use a continuous 2-D spatial Fourier transform, which compares the field with a plane propagative wave, the new method includes the loss factor. The field is thus compared with an inhomogeneous wave (i.e. a damped plane wave).
(ii) Extending the continuous Fourier transform, which deals with an arbitrary grid data, the IWC method employs similar arbitrary distributed data points, with the ability to use measured coherence signals (if available).
(iii) Previous work [5] windowed the field to avoid the effect of near field due to sources and boundary conditions. But the near field corresponds mainly to imaginary wave-numbers. The introduction of loss factor identification (see idea *i*) offers the ability of recognizing near field from far field: near field corresponding to a high apparent loss factor (imaginary part of the wavenumber greater than its real part). The algorithm using the IWC method may eliminate the identified wavenumbers with high apparent loss factors. This allows the use of the vibrational field of the whole surface S of the structure.
(iv) In the mid-frequency domain under consideration, the modal density and overlap are high. Hence, with regard to the complete field used by the present method (see idea (iii)), one can assume that there is a wave in each direction θ within the plate.

It should be noticed that the input field can be either experimental or numerical. In this last situation, a *virtual prototyping* process can thus be achieved.

2.2. Development of the IWC method

The method uses a harmonic field $\widehat{w}(x, y)$ (where the ω-dependence is comprised in the hat $\widehat{\ }$), either from a harmonic excitation or from a temporal Fourier transform

$$w(x, y, t) = \int_0^{+\infty} \widehat{w}(x, y) e^{i\omega t} \, d\omega. \tag{1}$$

Idea (i) introduces the use of an inhomogeneous wave, noted $\widehat{o}_{k,\gamma,\theta}$ (wave with heading θ, γ wave attenuation[1] and apparent wavelength $2\pi/k$). It is defined as follows:

$$\widehat{o}_{k,\gamma,\theta}(x, y) = e^{-ik(\theta)(1+i\gamma(\theta))(x\cos(\theta)+y\sin(\theta))}. \tag{2}$$

[1]This wave attenuation is a way of introducing damping. Lyon and DeJong [1] formulates the link between this wave attenuation and the classical damping loss factor as: $\gamma = \eta c_\varphi / 2c_g$, c_φ and c_g being the phase and group velocities, respectively.

Then, the correlation between this inhomogeneous wave and the complete wave field is calculated, like a modal assurance criterion [6], by the formulae

$$\mathrm{IWC}(k,\gamma,\theta) = \frac{|\int\int_S \widehat{w}.\widehat{o}^*_{k,\gamma,\theta}\,\mathrm{d}x\,\mathrm{d}y|}{\sqrt{\int\int_S |\widehat{w}|^2\,\mathrm{d}x\,\mathrm{d}y.\int\int_S |\widehat{o}_{k,\gamma,\theta}|^2\,\mathrm{d}x\,\mathrm{d}y}}, \tag{3}$$

where * denotes the complex conjugate. The identification of a complex wave number for a given direction θ leads to maximization (with a fixed θ) of the function $(k,\gamma)\longrightarrow\mathrm{IWC}(k,\gamma,\theta)$.

2.3. Numerical implementation of IWC method

This section describes how to apply the IWC method in practice. First it is assumed that the wave field $\widehat{w}$ is known on arbitrary data points $(x_i, y_i)_{i\in\mathbb{N}_n}$. The integrations over the complete surface S in Eq. (3) are replaced by a finite weighted sum

$$\int\int_S \mathrm{d}x\,\mathrm{d}y \quad \longleftrightarrow \quad \sum_n \rho_i S_i, \tag{4}$$

where ρ_i is the coherence of measurement data at point M_i ($\rho_i = 1$ if the coherence is not available), and S_i is an estimation of the surface around point M_i (according to idea (ii)).

The algorithm first puts angle θ into a discrete set of values (θ_j). For each of these angles, the maximum of IWC is located at a value (k_j, γ_j). Thus, the method creates two functions $\theta \to k(\theta)$ and $\theta \to \gamma(\theta)$ defined on a set of discrete values (θ_j). Finally, the triplet $(\theta_{j_0}, k_{j_0}, \gamma_{j_0})$ is removed from the list if γ_{j_0} is greater than 1 (according to idea (iii)).

3. Validations

3.1. Steel panel with a bonded porous layer

As a first step in validating the method, a rectangular steel panel has been tested. The panel was covered with a soft porous sheet, as shown in Fig. 1.

This structure was freely suspended and excited by a point force using an electrodynamic shaker Brüel & Kjær 4809. The normal velocity of the panel was measured by a scanning Laser vibrometer (Ometron VPI+). The phase reference was obtained by a force transducer Brüel & Kjær 8001. Both signals were sampled with a Hewlett Packard Paragon 35654A. The acquired data of the transfer functions directly provided a field $(\widehat{w}(x_i, y_i))_{i\in\mathbb{N}_n}$.

Results of the IWC with measured data are given in Fig. 2. It is clearly shown in Fig. 2a that the wavenumber k does not depend on the angle of propagation θ. The apparent flexural motion of the steel plate covered with the porous material exhibits an isotropic behaviour. The frequency dependence of the identified apparent wavenumbers, Fig. 2b, shows a parabolic evolution as is the case in the classical Kirchoff–Love plate theory.

Fig. 1. Steel panel with porous materials.

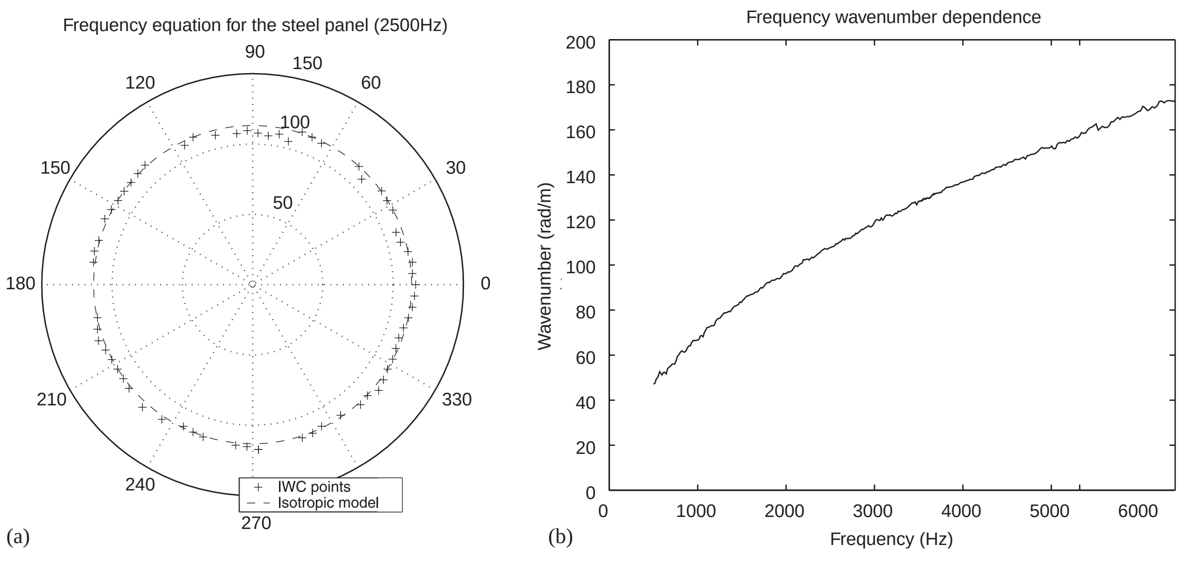

Fig. 2. IWC results with a steel plate: (a) apparent hormonic wavenumber, (b) frequency dependence of apparent wavenumber.

3.2. Ribbed panel case study

It is expected that the method can identify anisotropic dispersion curves. So a ribbed aluminium panel was tested with the same previously described experimental setup (see Fig. 3).

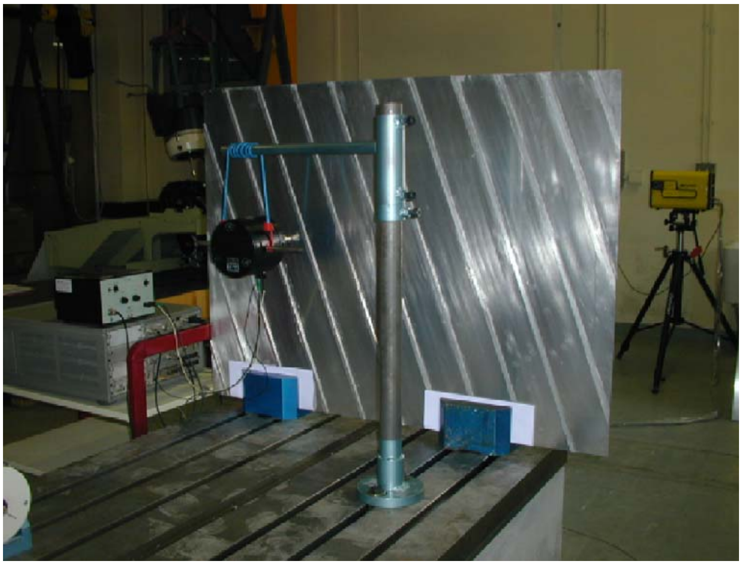

Fig. 3. Experimental set-up for the partially clamped ribbed panel.

The test was done in the *structural orthotropy* frequency domain (as designated by Orrenius and Finnveden [7]). Results are given in Fig. 4a. As one can see, a good overview of the θ-dependence wavenumber k is obtain, and an elliptic orthotropic model can be recognized.

Idea (iii) gives another interesting result of the IWC method. Since the near-field provided by singularities is automatically eliminated, results are dependent neither on sources (nature and location) nor on boundary conditions. In order to verify this feature, the previous ribbed panel was tested with a different support (one side was partially clamped). The results of this test were compared to the previous ones in Fig. 4b. Even if the exact position of each identified wavenumber k is dependent on the boundary conditions, both the global aspect of the dispersion curve and the identified orthotropic model remain similar. Other tests have demonstrated that the identification depends neither on the plate's geometry nor on the source location.

4. Conclusion

The inhomogeneous wave correlation method is a new method that seeks to identify a complete θ-dependent dispersion equation. This method deals with measurement data from an arbitrary mesh of a 2-D structure. Its feasibility on isotropic and anisotropic panels has been shown, and the results appear to be independent of geometry, boundary conditions and sources.

Further developments of the method are in progress. Indeed, IWC can lead to a θ-dependent wave attenuation factor, the later should be compared to loss factor estimation through the power injection method [8], or reverberation time measurements [9]. More fundamentally, the equivalence between several damping representations in the context of wave-mode duality should be theoretically investigated. This aspect deserves careful thought, and will be addressed in depth

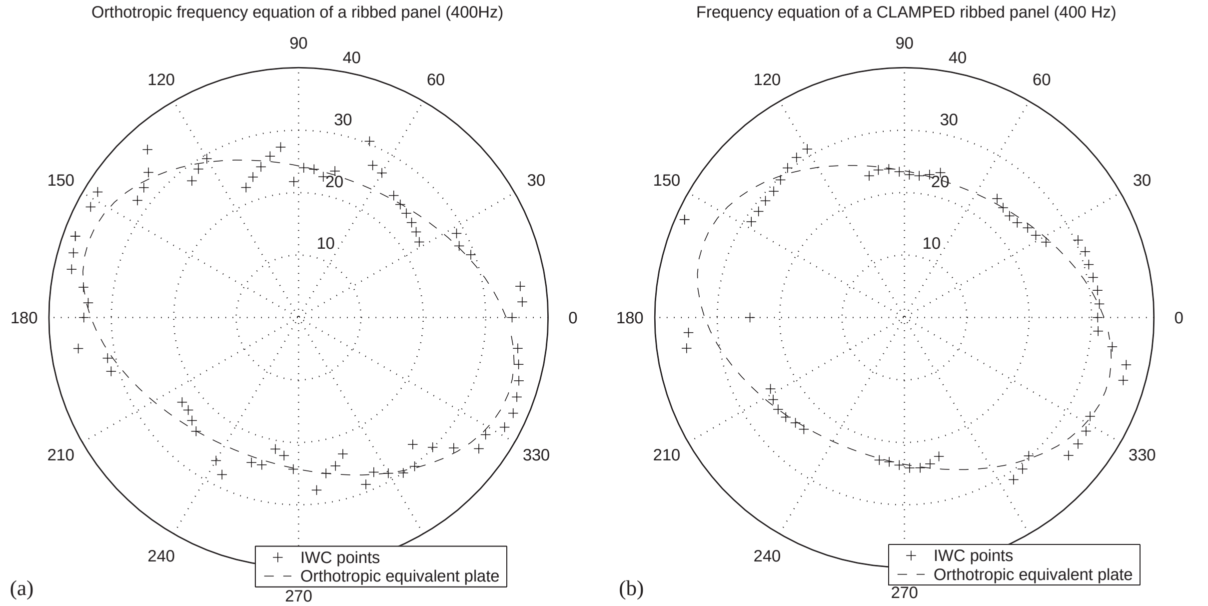

Fig. 4. IWC results with a ribbed panel: (a) freely suspended, (b) partially clamped.

in future paper. Ultimately, applications of the method to complex structures (sandwich honeycomb built-up structures, different ribbed panels, etc.) can provide some insights into their behaviour up to high frequencies.

Acknowledgements

The authors gratefully acknowledge the French Education Ministry for its support for the investigations presented herein through Grant No 00-2-10606. The authors also sincerely thank the Région Rhônes-Alpes, whose support made it possible to acquire the scanning vibrometer used in this work during the IMPACT project. Thanks also to reviewer for valuable comments.

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