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Flexural–torsional vibration of simply supported open cross-section steel beams under moving loads

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Summary

The present work deals with linearized modal analysis of the combined flexural-torsional vibration of simply supported steel beams with open monosymmetric cross-sections, acted upon by a load of constant magnitude, traversing its span eccentrically with constant velocity. After thoroughly investigating the free vibrations of the structure, which simulates a commonly used highway bridge, its forced motions under the aforementioned loading type are investigated. Utilizing the capabilities of symbolic computations within modern mathematical software, the effect of the most significant geometrical and cross-sectional beam properties on the free vibration characteristics of the beam are established and presented in tabular and graphical form. Moreover, adopting realistic values of the simplified vehicle model adopted, the effects of eccentricity, load magnitude and corresponding velocity are assessed and interesting conclusions for structural design purposes are drawn. The proposed methodology may serve as a starting point for further in-depth study of the whole scientific subject, in which sophisticated vehicle models, energy dissipation and more complicated bridge models may be used.

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1. Introduction

The linearized as well as nonlinear vibration analysis of beams or beam-like structural elements has been and continues to be a subject of immense importance in engineering science, embracing a wide class of problems. Depending on the assumptions, the type of analysis, the overall beam characteristics and the kind of loading or excitation, a huge number of publications containing a variety of different approaches have been reported in the relevant literature. For 150 years, engineers have been trying to present reliable solutions for such a multi-parameter problem by using two different methods. The first method is to perform tests, and the second is that of pure theoretical investigation. In recent years, transport engineering has experienced serious advances characterized by increasingly higher speeds and weights of vehicles, as a result of which vibrations and dynamic stresses larger than ever before have been developed.

The problem of moving loads was first considered approximately for the case of a girder with negligible mass, compared to the mass of a single moving load of constant magnitude by Stokes [1] and Zimmermann [2]. Afterwards, the case of a moving load with negligible mass compared to the mass of the girder was studied by Krylov [3], Timoshenko [4] and Lowan [5].

The complete problem, including both these parameters, was studied by other investigators such as Steuding [6], Schallemcamp [7], and Bolotin [8]. A very thorough treatise on the dynamic response of several types of railway bridges, traversed by steam locomotives was presented by Inglis [9] using harmonic analysis. Interesting analyses were also presented by Hillerborg [10] using Fourier's analysis and by Biggs et al. [11] using Iglis's technique. The problem of the dynamic response of bridges under moving loads is reviewed in detail by Timoshenko [12], and later on by Kolousek [13]. One should also mention the extended review reported by Fryba [14] in his excellent monograph on this subject. Based on his text Fryba [15,16] studied the effects of the constant speed and damping on the response of a beam.

Vehicle-induced vibrations of bridges and other structures that can be simulated as beams have been extensively investigated by a great number of researchers [17,18], dealing with the effect of various parameters on the 3D motion of these structures, such as vehicle suspension design [19], vehicle weight and speed, damping, matching between bridge and vehicle natural frequencies, deck roughness, various irregularities, etc. [20–26].

Undoubtedly, the whole matter will remain a major topic for further scientific research, since continuing developments in design and material technology enable the realization and construction of lighter and more slender structures, with increased vulnerability to dynamic and especially moving loads.

To the knowledge of the authors however, there seems to be a limited number of works dealing with the combined lateral–torsional vibrations of beams under moving loads [27–29], although simply supported bridges with open monosymmetric—mainly steel—cross-sections with two lanes, are very common in the national road network of many countries, and are quite sensitive to the above type of motions.

The present work examines the combined lateral–flexural motions of steel beams with open cross-sections and only one symmetry axis, acted upon by an eccentrically moving load of constant magnitude and velocity, simulating the passage of a simplified single vehicle load across one lane of a beam-structure, as described earlier.

In the present paper, without restriction of the method's generality, damping is neglected while the whole procedure may without particular difficulties be applied and extended to multi-span beams accounting for energy dissipation. Numerical results in tabular and graphical form reveal the effect of the geometrical and cross-sectional beam properties on the natural frequencies; thereafter the forced motions dealt with are thoroughly discussed and the effect of the loading parameters (i.e. magnitude, eccentricity and velocity) are fully assessed.

2. Mathematical formulation

2.1. Introductory concepts

The equilibrium equations of a simply supported steel beam, with an open cross-section and only one axis of symmetry, as depicted in Fig. 1, are as follows [30]:

$$\begin{aligned} &EJ_y w_S'''' = q_z, \\ &EJ_z \upsilon_S'''' = q_y, \\ &EC_M \vartheta'''' - GJ_d \vartheta'' = m_x. \end{aligned} \tag{1a–c}$$

In the above equations, movements w_S, υ_S, and external forces q_y, q_z act along the main axes of the weight centre, while torsional moment m_x acts around the shear centre.

In addition, the cross-sectional and material properties as well as the displacement components involved are defined by

J_y, J_z: moments of inertia with respect to principal axes y and z, respectively,
J_d: Saint–Venant torsional moment of inertia,
E, G: elasticity and shear moduli,
C_M: warping coefficient with respect to the shear centre M,
ϑ: rotation of the cross-section.

Note that m_x includes the possible external torsional moment and, also, that produced by the loads q_y, q_z. After the deformation of the beam, the loads q_y, q_z produce a new torsional moment and so the final one $\bar{m}_x$ includes the above-mentioned m_x, and also the torsional moment caused by the change of the distance between the shear centre and the weight centre after the cross-section's turn by angle ϑ.

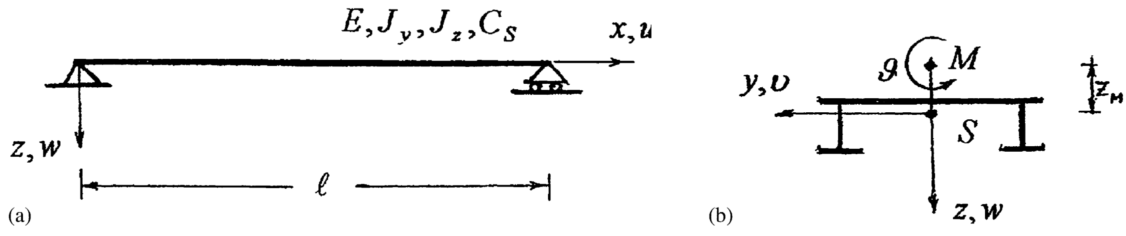

Fig. 1. Geometry and sign convention of a simply supported steel beam with open monosymmetric cross-section.

So, the final torsional moment acting becomes

$$\bar{m}_x = m_x + z_M E J_z \upsilon_S'''' + z_M \vartheta E J_y w_S'''' \tag{2}$$

while the coordinates of the new position of the weight centre become

$$\begin{aligned} \upsilon &= \upsilon_S + z_M \vartheta, \\ w &= w_S. \end{aligned} \tag{3a, b}$$

Eqs. (1), because of (2) and (3), become

$$\begin{aligned} &EJ_y w'''' = q_z, \\ &EJ_z \upsilon'''' + EJ_z z_M \vartheta'''' = q_y, \\ &EC_S \vartheta'''' + EJ_z z_M \upsilon'''' - GJ_d \vartheta'' = m_x. \end{aligned} \tag{4a–c}$$

2.2. Free vibration analysis

Now consider the free undamped torsional–lateral vibration of a simply supported steel beam, with an open cross-section and only one symmetry axis, as depicted in Fig. 1, where also the geometry of the structure and the adopted sign convention are also shown. Using Eq. (4), one can easily find the following differential equations governing the motion under consideration

$$\begin{aligned} &EJ_y w'''' + m\ddot{w} = 0, \\ &EJ_z \upsilon'''' + EJ_z z_M \vartheta'''' + m\ddot{\upsilon} = 0, \\ &EC_S \vartheta'''' + EJ_z z_M \upsilon'''' - GJ_d \vartheta'' + \Theta_M \ddot{\vartheta} = 0 \end{aligned} \tag{5a–c}$$

where the prime denotes differentiation with respect to x, while the dot with respect to time t. In these equations, the following new symbols are used:

z_M: distance between gravity centre S and shear centre M,
Θ_M: polar moment of inertia of the mass of the cross-section,
C_S: warping coefficient with respect to the centre of gravity S,
υ, w: deformations of S along y and z axes, respectively.

Evidently, the vertical eigenvibrations are independent of the horizontal and flexural ones, which are in fact coupled, and hence, applying modal analysis, one may write that

$$\begin{aligned} w(x,t) &= \bar{w}(x)(A_k \sin \omega_k t + B_k \cos \omega_k t), \\ \vartheta(x,t) &= \bar{\vartheta}(x)(A_\sigma \sin \omega_\sigma t + B_\sigma \cos \omega_\sigma t), \\ \upsilon(x,t) &= \bar{\upsilon}(x)(A_\sigma \sin \omega_\sigma t + B_\sigma \cos \omega_\sigma t), \end{aligned} \tag{6a–c}$$

where $\bar{w}(x)$, $\bar{\vartheta}(x)$, $\bar{\upsilon}(x)$ are the corresponding shape functions and ω_k, ω_σ the circular frequencies of the vertical motion and the coupled lateral–torsional motion respectively. In the sequel, the in-plane vertical dynamic deflection $w(x,t)$ can be expressed as

$$w(x,t)=\sum_n \sin\frac{n\pi x}{\ell}(A_{kn}\sin\omega_{kn}t+B_{kn}\cos\omega_{kn}t),$$
$$\omega_{kn}^2=\frac{n^4\pi^4EJ_y}{m\ell^4},\quad n=1,\ 2,\ldots \tag{7}$$

while combining the coupled equations (5b) and (5c) one gets

$$\bar{\upsilon}(x)=\frac{1}{mz_M\omega_\sigma^2}\left(-EC_S\bar{\vartheta}''''+EJ_zz_M^2\bar{\vartheta}''''+GJ_d\bar{\vartheta}''+\Theta_M\omega_\sigma^2\bar{\vartheta}\right) \tag{8}$$

Since $C_M=C_S-z_M^2J_z$ and after cumbersome elaboration, the following differential equation of eighth order with respect to the shape function of the rotation $\bar{\vartheta}(x,t)$ is reached:

$$\alpha\bar{\vartheta}^{(8)}+\beta\bar{\vartheta}^{(6)}+\gamma\bar{\vartheta}^{(4)}+\delta\bar{\vartheta}^{(2)}+\varepsilon\bar{\vartheta}=0, \tag{9}$$

where

$$\begin{aligned}\alpha&=-E^2J_zC_M,\\ \beta&=EGJ_zJ_d\\ \gamma&=EJ_z\Theta_M\omega_\sigma^2+EC_Mm\omega_\sigma^2+EJ_zz_M^2m\omega_\sigma^2,\\ \delta&=-GJ_dm\omega_\sigma^2,\\ \varepsilon&=-\Theta_Mm\omega_\sigma^4;\end{aligned} \tag{10}$$

the characteristic polynomial equation used for the solution of Eq. (9) is

$$\alpha\rho^8+\beta\rho^6+\gamma\rho^4+\delta\rho^2+\varepsilon=0 \tag{11}$$

while the boundary conditions valid for a simply supported beam are

$$\begin{aligned}&\vartheta(0)=\vartheta(\ell)=\vartheta''(0)=\vartheta''(\ell)=0,\\ &\upsilon(0)=\upsilon(\ell)=\upsilon''(0)=\upsilon''(\ell)=0.\end{aligned} \tag{12}$$

The general solution of Eq. (9), yielding the expression of the shape function associated with the cross-sectional rotation, can be written in series form as follows:

$$\bar{\vartheta}(x)=\sum_i k_i e^{a_ix}[\sin(b_ix)+\cos(b_ix)]+\sum_\lambda k_\lambda e^{r_\lambda x},\quad i+\lambda=8, \tag{13}$$

where $\alpha_i\pm jb_i$ and $\pm r_\lambda$ are the complex conjugate and real roots of Eq. (11) respectively, while k_i, k_λ are eight coefficients to be determined. Substituting expression (13) into Eq. (9) and taking into account the aforementioned boundary conditions one obtains a linear homogeneous system with respect to coefficients k_q $(q=1,\ldots,8)$. This can be achieved utilizing advanced symbolic manipulations and modern mathematical software [31], despite the rather simplified type of analysis.

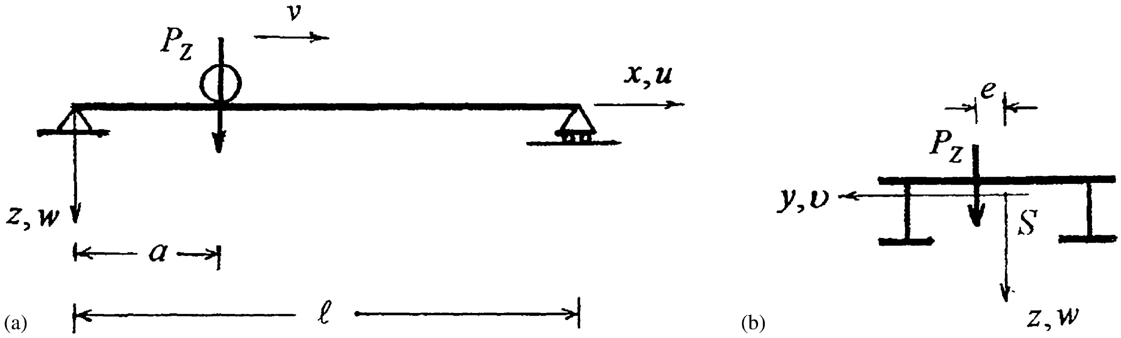

Fig. 2. The beam of Fig. 1 traversed by a constant force P_z with constant velocity υ and eccentricity e.

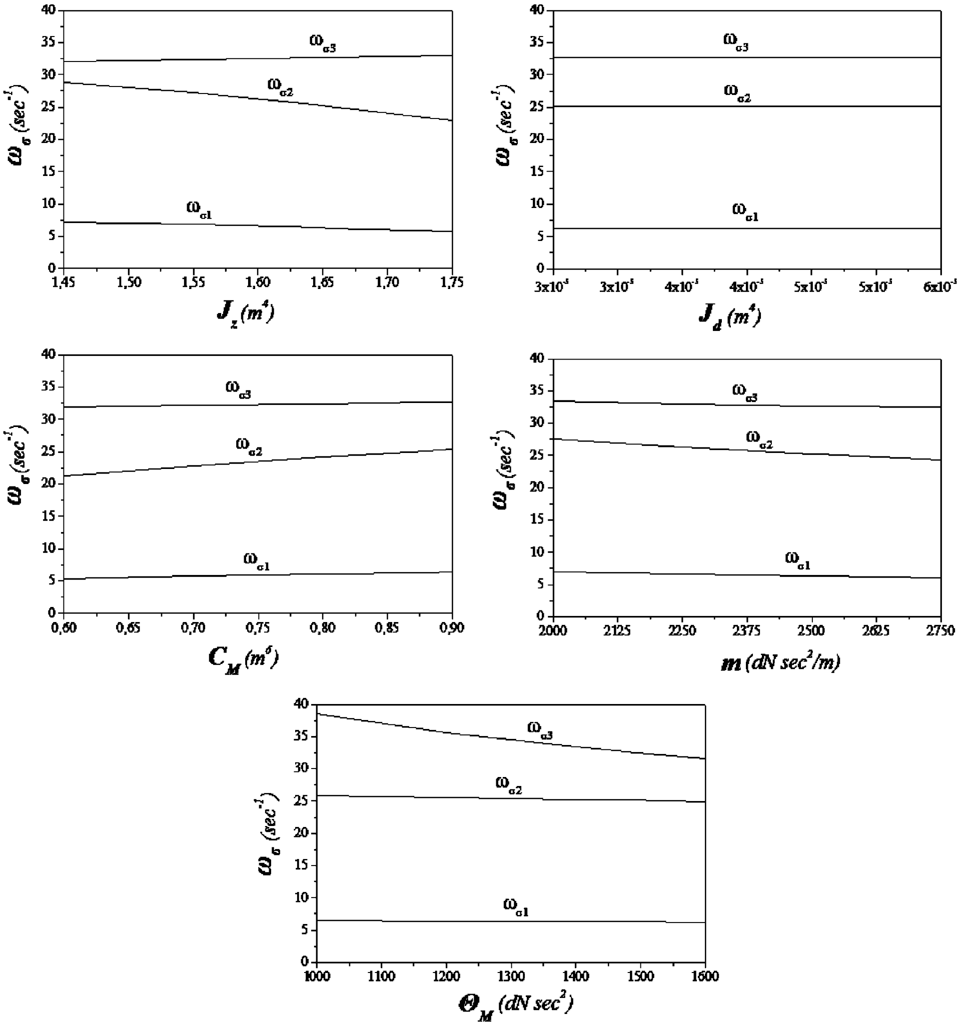

Fig. 3. Variation of the three first out-of-plane flexural/torsional circular frequencies $\omega_{\sigma i}, i = 1, \ldots, 3$ due to the change of various cross-sectional properties of the beam.

Thus, one may write

$$\bar{\vartheta}(x,t)=\sum_n \Psi_n(A_{\sigma n}\sin\omega_{\sigma n}t+B_{\sigma n}\cos\omega_{\sigma n}t),$$
$$\bar{\upsilon}(x,t)=\sum_n Z_n(A_{\sigma n}\sin\omega_{\sigma n}t+B_{\sigma n}\cos\omega_{\sigma n}t), \tag{14}$$

where $\Psi_n=\bar{\vartheta}(x)$ and $Z_n=\bar{\upsilon}(x)$ are the shape functions of the rotation and lateral deflection respectively, to be analytically computed or at least properly numerically approximated, a task performed also symbolically. Using also the notation $X_n(x)=\bar{w}(x)$, it can be easily proven that the orthogonality conditions governing the free motions dealt with are given by the following relations:

$$\int_0^\ell X_nX_m\,\mathrm{d}x=0\quad\text{for } n\neq m,$$
$$m\int_0^\ell Z_nZ_m\,\mathrm{d}x+\Theta_M\int_0^\ell \Psi_n\Psi_m\mathrm{d}x=0\quad\text{for } n\neq m. \tag{15}$$

2.3. Forced vibration analysis

If the simply supported steel beam under consideration is traversed by a moving load P_z of constant magnitude and constant velocity υ acting eccentrically, as schematically depicted in Fig. 2, the corresponding undamped forced vibrations are governed by the following set of

Table 1
Maximum midspan dynamic deformations of the beam considered for all combinations of the values of the chosen loading parameters

P_z (kp)	e (m)	υ (m/s)	max $w(\frac{\ell}{2},t)$ (m)	max $\upsilon(\frac{\ell}{2},t)$ (m)	max $\theta(\frac{\ell}{2},t)$ (rad)
1500	1	20	0.00098	−0.00031	0.00021
		25	0.00113	−0.00036	0.00025
		30	0.00124	−0.00040	0.00028
		35	0.00133	−0.00044	0.00030
		40	0.00138	−0.00046	0.00031
	2	20	0.00098	−0.00062	0.00043
		25	0.00113	−0.00072	0.00050
		30	0.00124	−0.00081	0.00055
		35	0.00133	−0.00088	0.00060
		40	0.00138	−0.00093	0.00062
50000	1	10	0.02958	−0.01040	0.00734
		15	0.03148	−0.01110	0.00779
		20	0.03251	−0.01028	0.00714
		25	0.03765	−0.01202	0.00825
	2	10	0.02958	−0.02079	0.01468
		15	0.03148	−0.02221	0.01559
		20	0.03251	−0.02057	0.01429
		25	0.03765	−0.02405	0.01560

differential equations of motion:

$$\begin{aligned} &EJ_y w'''' + m\ddot{w} = P_z \delta(x-\alpha), \\ &EJ_z \upsilon'''' + EJ_z z_M \vartheta'''' + m\ddot{\upsilon} = 0, \\ &EC_S \vartheta'''' + EJ_z z_M \upsilon'''' - GJ_d \vartheta'' + \Theta_M \ddot{\vartheta} = eP_z \delta(x-\alpha), \end{aligned} \tag{16}$$

where $\alpha = \upsilon t$ (is the load position) and δ the Dirac function.

Seeking a solution in modal form, prescribed by

$$\begin{aligned} w(x,t) &= \sum_n X_n(x) T_n(t), \quad X_n(x) = \sin\frac{n\pi x}{\ell}, \\ \vartheta(x,t) &= \sum_n \Psi_n(x) \Phi_n(t), \\ \upsilon(x,t) &= \sum_n Z_n(x) \Phi_n(t), \end{aligned} \tag{17}$$

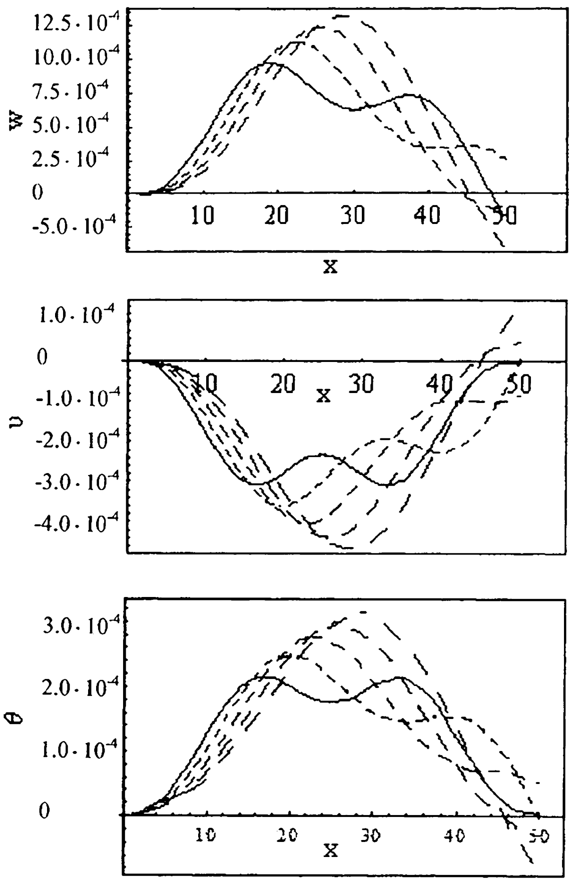

Fig. 4. Dynamic influence lines of the beam's midspan deformations w, υ, and ϑ for $P = 1500$ kp, $e = 1.00$ m and five values of the load velocity υ as indicated: —, $\upsilon = 20$ m/s; ——, $\upsilon = 25$ m/s; - - -, $\upsilon = 20$ m/s; - -, $\upsilon = 35$ m/s; - -, $\upsilon = 40$ m/s.

the amplitudes $T_n(t)$ and $\Phi_n(t)$, based on the preceding free vibration analysis, are found by solving the following uncoupled differential equations:

$$\ddot{T}_n(t) + \omega_{kn}^2 T_n(t) = \frac{2P}{m\ell}\sin\frac{n\pi\upsilon t}{\ell},$$
$$\ddot{\Phi}_n(t) + \omega_{\sigma n}^2 \Phi(t) = \frac{eP_z}{m\int_0^\ell Z_n^2\,\mathrm{d}x + \Theta_M\int_0^\ell \Psi_n^2\,\mathrm{d}x}\Psi_n(\upsilon t) \tag{18a, b}$$

which according to Duhamel yield purely analytical expressions, outlined below

$$T_n(t) = \frac{2P_z}{m\ell\omega_{kn}}\int_0^t \sin\frac{n\pi\upsilon\tau}{\ell}\sin\omega_{kn}(t-\tau)\,\mathrm{d}\tau,$$
$$\Phi_n(t) = \frac{eP_z}{\omega_{\sigma n}\left(m\int_0^\ell Z_n^2\mathrm{d}x + \Theta_M\int_0^\ell \Psi_n^2\mathrm{d}x\right)}\int_0^t \Psi_n(\upsilon\tau)\sin\omega_{\sigma n}(t-\tau)\,\mathrm{d}\tau. \tag{19a, b}$$

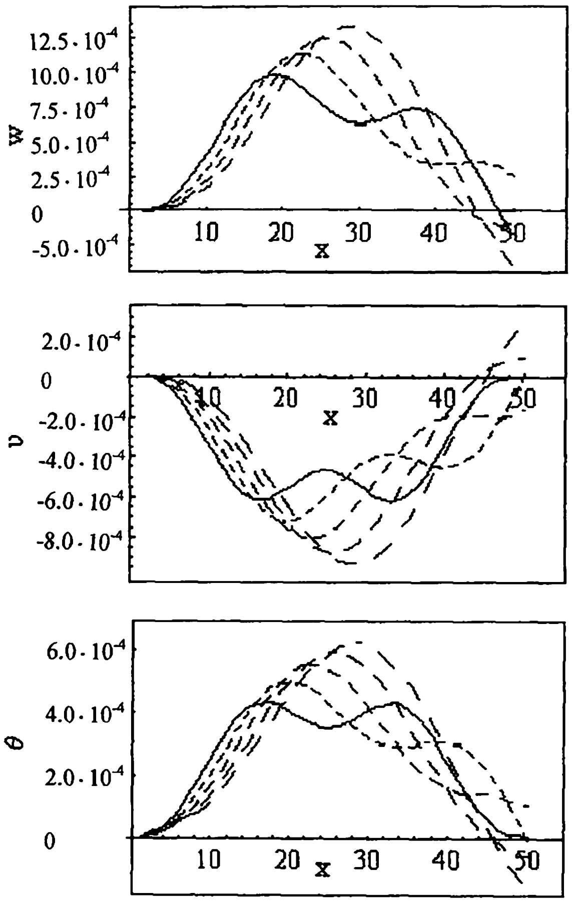

Fig. 5. As in Fig. 4 for $e = 2.00\,\mathrm{m}$.

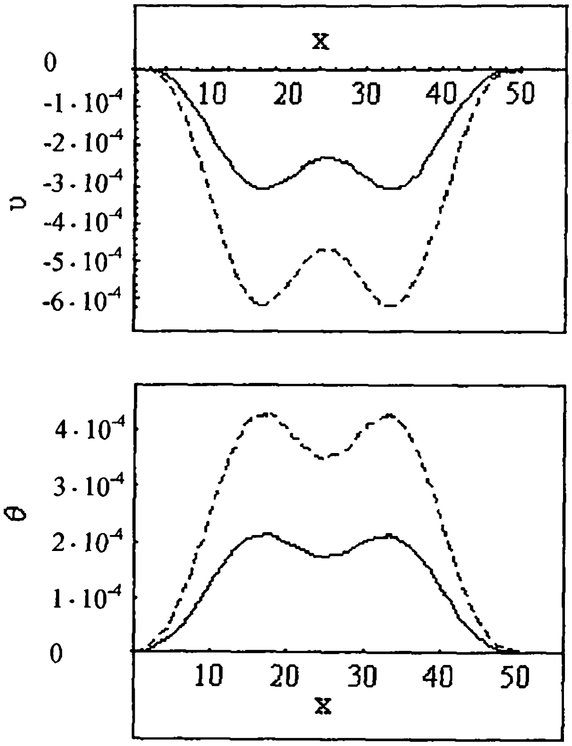

Fig. 6. Dynamic influence lines of the beam's midspan deformations υ and ϑ for $P = 1500\,\text{kp}$, $\upsilon = 20\,\text{m/s}$ and $e = 1.00\,\text{m}$ (continuous line), 2.00 m (dashed line).

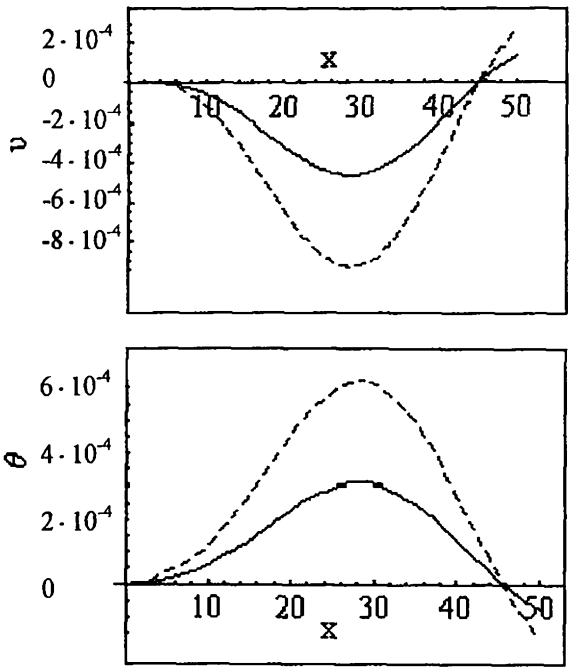

Fig. 7. As in Fig. 6 for $\upsilon = 40\,\text{m/s}$.

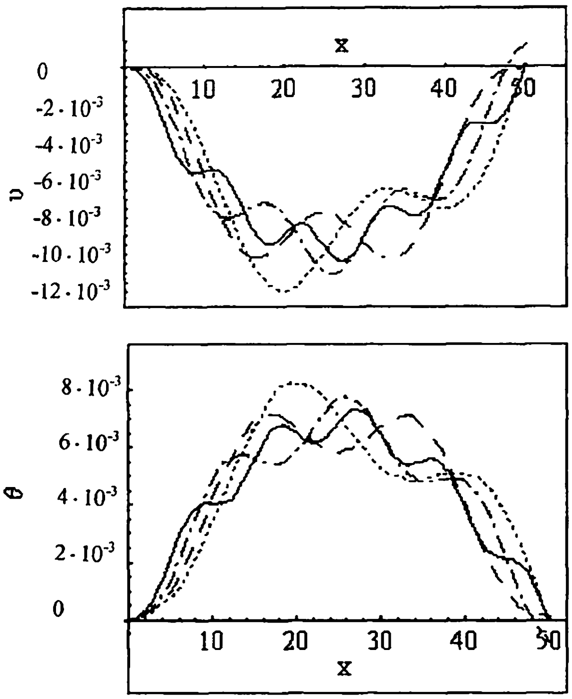

Fig. 8. Dynamic influence lines of the beam's midspan deformations υ and ϑ for $P = 50000\,\mathrm{kp}$, $e = 1.00\,\mathrm{m}$ and four values of the load velocity υ as indicated: —, $\upsilon = 10\,\mathrm{m/s}$; ——, $\upsilon = 15\,\mathrm{m/s}$; - -, $\upsilon = 20\,\mathrm{m/s}$; ⋯⋯, $\upsilon = 25\,\mathrm{m/s}$.

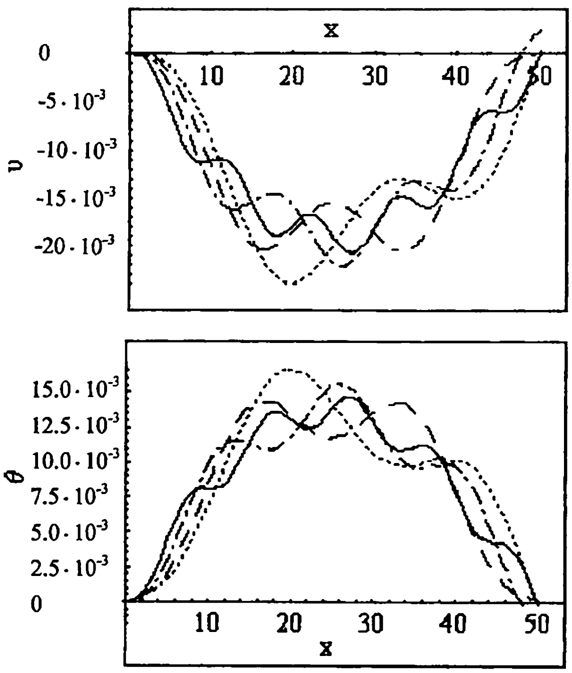

Fig. 9. As in Fig. 8 for $e = 2.00\,\mathrm{m}$.

At this point it should be noted that the same problem of forced vibration, if the moving load is replaced by a two mass–spring vehicle model is quite complicated, since there exist no analytical solutions and hence the employment of sophisticated numerical algorithms is required. Theoretical aspects and solution techniques on the subject can be found in a recent publication by the first two authors [27], but nevertheless the whole scientific matter remains open for further in depth investigation, which will possibly include sophisticated vehicle models, damping and more complicated multi-span bridge simulations.

3. Numerical results and discussion

3.1. Free vibrations—parametric study

Aiming to establish the influence of the most significant geometrical as well as cross-sectional properties on the free lateral–torsional vibration characteristics of a steel beam representing a commonly used double-lane simply supported highway bridge, the numerical results presented in this section correspond to a structure possessing the following properties: $\ell = 50\,\mathrm{m}$, $J_y = 0.277\,\mathrm{m}^4$, and $z_M = 1.376\,\mathrm{m}$ (distance between gravity centre S and shear center M. The

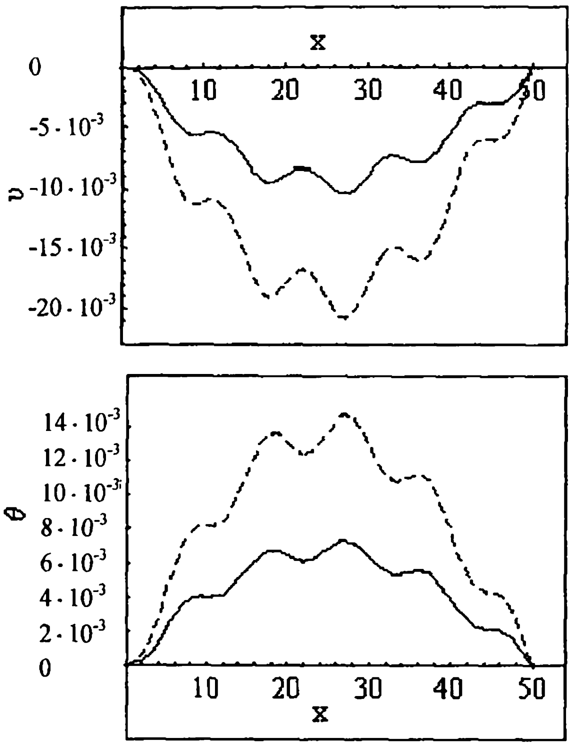

Fig. 10. Dynamic influence lines of the beam's midspan deformations υ and ϑ for $P = 50000\,kp$, $\upsilon = 10\,\mathrm{m/s}$ and $e = 1.00\,\mathrm{m}$ (continuous line), 2.00 m (dashed line).

remaining five (5) parameters, i.e. J_z, J_d, C_M, m and Θ_M are kept constant (and equal to certain basic values) in groups of four, with the fifth one varying, and the three first torsional—out of plane lateral circular frequencies $\omega_{\sigma i}$ $(i = 1, 2, 3)$ are computed.

The basic values mentioned earlier are :

$$J_z = 1.654\,\mathrm{m}^4,\ J_d = 6 \times 10^{-5}\,\mathrm{m}^4,\ C_M = 0.88\,\mathrm{m}^6,\ m = 2512\,\mathrm{kp\,m/s^2},\ \text{and}\ \Theta_M = 1476.6\,\mathrm{kp\,s^2}.$$

The results obtained are presented in graphical form throughout Fig. 3, for all five possible combinations. From these plots it can be readily perceived that only the variation of J_z and C_M has a considerable effect on $\omega_{\sigma i}$. More specifically, as J_z increases both $\omega_{\sigma 1}$ and $\omega_{\sigma 2}$ decrease to about 9.50%, while the effect on $\omega_{\sigma 3}$ is negligible. Moreover, the increase of C_M is associated with increase of all $\omega_{\sigma i}$, more pronounced on $\omega_{\sigma 1}$ and $\omega_{\sigma 2}$ ($\cong$5–7%) and less on $\omega_{\sigma 3}$ ($\sim$1%). On the other hand, if m is varied from 2000 to 2740 kp/m all $\omega_{\sigma i}$ decrease from 1% to 5%, starting from the higher mode. Finally, as Θ_M increases, $\omega_{\sigma 1}$ and $\omega_{\sigma 2}$ both decrease to about 2%, while $\omega_{\sigma 3}$ decreases by 7%, while the effect of J_d on all three frequencies can be is insignificant.

3.2. Forced vibrations

In order to investigate the effect of eccentricity, moving load magnitude and velocity on the forced vibration of the beam, the case of the structure dealt with in the free vibration analysis section, containing all constant and basic parameter values is considered. Additionally, two specific values of the moving load are adopted, a "light" one $P_z = 1500$ kp (representing a family

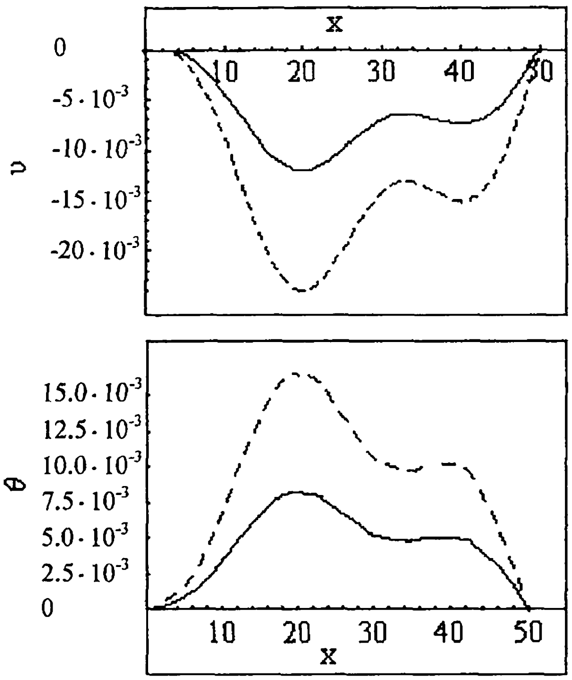

Fig. 11. As in Fig. 10 for $\upsilon = 25\,\mathrm{m/s}$.

automobile) and a "heavy" one $P_z = 50000$ kp. The corresponding values of the load's velocity used in this study are $\upsilon = 20$, 25, 30, 35 and 40 m/s for the light one and 10, 15, 20 and 25 m/s for the heavy one, while both loads are considered crossing the span of the beam through one lane, at eccentricities $e = 1.00$ or 2.00 m.

Thereafter, the maximum midspan dynamic flexural (in-plane as well as out-of-plane) deflections and the cross-section rotation are evaluated for all combinations of the foregoing parameters chosen, and their values are comparatively presented in Table 1. Additionally, the corresponding influence lines for characteristic loading combinations are presented. So, in Fig. 4 the influence lines of w, υ, and ϑ are presented for $P_z = 1500$ kp and $e = 1.00$ m, while in Fig. 5 the same influence lines are presented but for $e = 2.00$ m. The influence lines of υ, and ϑ are presented for nine characteristic loading combinations in Figs. 6–12.

In these plots x is the position within the span of the load that moves with the lowest velocity. From the above results, in both tabular and graphical form, one may comprehensively discuss the individual or combined (coupling) effect of the magnitudes of P_z, υ and e on the dynamic behaviour of the beam.

In doing this, it is evident that the increase of P_z affects both rotation and flexural deflections, since the corresponding time functions (amplitudes), as given in relations (19) are proportional to the loading. This is not the case for the eccentricity e, the variation of which affects only the

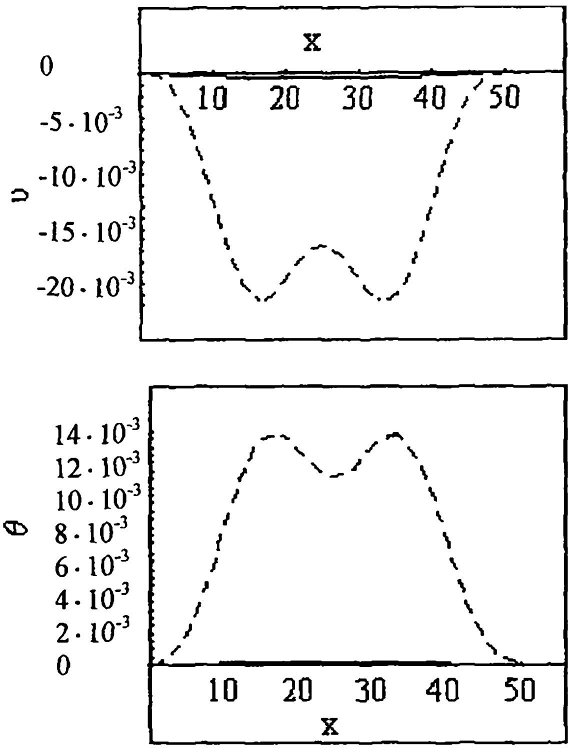

Fig. 12. Dynamic influence lines of the beam's midspan deformations υ and ϑ for two values of the moving load ($P = 1500$ kp: continuous line—$P = 50000$ kp: dashed line) with $e = 1.00$ m and $\upsilon = 20$ m/s.

coupled vibrations, i.e out-of-plane flexural and rotation, since its value appears only in $\Phi_n(t)$, as given in Eq. (19b). Moreover, the increase of the load's velocity results in an increase of all deformations, and their maximum values occur for different load positions, depending on the specific value of υ. Finally, the worst combination of the parameters involved, associated with the absolute maximum deformations, is the one of a heavy load traversing the beam with moderate velocity and large eccentricity.

4. Conclusions

In this paper the combined flexural–torsional vibrations of simply supported two-lane steel bridges with open monosymmetric cross-sections, traversed by a single vehicle across one lane are studied, performing a linearized modal analysis on a beam with similar cross-section acted upon eccentrically by a constant moving load with constant velocity. Based on the realistic examples investigated, the following can be drawn:

(1) The coupled out-of-plane lateral–torsional free motions of the beam are significantly affected primarily by the change of the strong-axis moment of inertia of the cross-section, and secondarily by the change of the warping coefficient; the effect of the remaining cross-sectional and geometrical properties on the free vibration characteristics of the beam can be considered as negligible
(2) The corresponding forced vibrations are strongly affected by the magnitude of the moving load, a fact valid for all deformations, while the eccentricity affects only the coupled motions. The velocity of the moving load also has a considerable effect on the dynamic deflections and rotations, with the worst combination of loading parameters being a heavy load traversing the beam with moderate speed and large eccentricity.

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