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Characteristic properties of subharmonic oscillations and their application in vibration engineering

V.I. Beresnevich*, S.L. Tsyfansky

Research Laboratory "Nonlinear Phenomena of Vibrating Systems", Institute of Mechanics, Riga Technical University, 1 Kalku Street, Riga, LV-1658, Latvia

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Abstract

The results of a theoretical and experimental study of subharmonic vibrations in nonlinear mechanical systems of different structural complexity are presented. Specific nonlinear properties of subharmonic regimes (complex vibration spectrum, essential asymmetry of time response, predominance of low-frequency harmonic components in vibration spectrum, etc.) are studied. It is shown that utilizing these nonlinear effects ensures more effective operation of vibromachines for different technological purposes (vibrocompaction, vibroforming, vibration emptying, etc.). The original procedure of program-simulated actuation of subharmonic vibromachine is proposed.

1. Introduction

For a long time most investigations in the field of applied nonlinear dynamics have been directed at suppression of unfavourable nonlinear resonant vibrations in machines and mechanisms [1–4]. This paper presents other trends in nonlinear dynamics based on the consideration that nonlinear vibration effect is a useful phenomenon, utilization of which makes it possible to develop new original applications in different engineering fields [5–7]. Due to certain characteristic properties of subharmonic oscillations (complex vibration spectrum, essential

*Corresponding author. Tel.: +371-7089469; fax: +371-7089746.
E-mail address: vitalijs.beresnevics@rtu.lv (V.I. Beresnevich).

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asymmetry of time response, predominance of low-frequency harmonic components in vibration spectrum, etc.) [8–12], it has been found expedient to use these nonlinear regimes in vibration engineering. As the result of this research, new high-effective technological vibration machines operating on subharmonic resonance are designed. The dynamics of subharmonic vibromachines and their main salient features are considered in this paper.

2. Subharmonic vibromachines for compaction and forming of granular materials and soils

Vibrocompaction and vibroforming technologies are widely used in engineering practice for production of different structural materials from granular components. A typical example of such material is synthegran—a new composite material made by vibration compaction of crushed granite mixed with epoxy binder. Besides, concrete blocks and plates in many cases are also produced by vibrocompaction and vibroforming [13,14].

In accordance with the experimental data [15,16], the most effective technological regime of vibration compaction and forming is the asymmetric one. This regime is specified by the sufficiently different peak values of vibroacceleration in positive ($\ddot{y}^+$) and negative ($\ddot{y}^-$) directions. As it was shown in many works [13,16], the compressive strength of blocks and plates produced by vibration technology is directly proportional to the asymmetry ratio $\ddot{y}^-/\ddot{y}^+$ of vibration regime. Therefore, exactly the parameter $\ddot{y}^-/\ddot{y}^+$ has been taken as the main criterion for the designing of these vibromachines.

Taking account of the high level of asymmetry of subharmonic oscillatory regimes [5,8–12], it has been found expedient to realize subharmonic vibrations in the vibromachine under design.

2.1. Dynamic model

The dynamic model of a typical subharmonic vibroforming machine is shown in Fig. 1. The working head of this vibromachine is considered as a uniform viscoelastic beam mounted on four elastic supports. Each elastic support consists of a main support k_1 and an additional elastic limiter k_2, installed with initial clearance Δ to the working head. Therefore, the piecewise linear

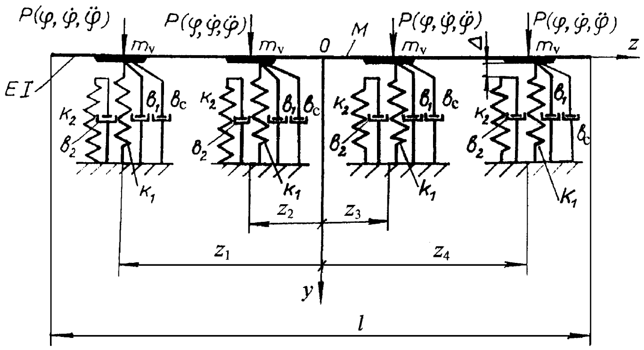

Fig. 1. Dynamic model of the subharmonic vibroforming machine.

elastic characteristic is realized. The influence of compacted mixture (e.g. concrete mix) on vibrations of the working head is taken into account as some additional mass and damping (reduced parameters of concrete mix have been determined experimentally [13]). External excitation forces $P(\varphi, \dot{\varphi}, \ddot{\varphi})$ are generated with the aid of four unbalance vibration exciters rigidly connected with the working head. The shafts of vibroexciters are set in rotation by an induction-driving motor. Synchronization of rotation is achieved with the aid of special synchronizing shafts (these shafts are not shown in Fig. 1).

Taking the origin of the stationary system of coordinates y and z in the centre of gravity of the beam at rest, the differential equation of bending vibrations of each individual span of working head can be stated as follows:

$$\mathrm{EI}\frac{\partial^4 y_s}{\partial z^4} + \lambda\frac{\partial^2 y_s}{\partial t^2} + b\mathrm{EI}\frac{\partial^5 y_s}{\partial t\,\partial z^4} = 0, \tag{1}$$

where EI, b and λ are the rigidity in bending, coefficient of internal friction and distributed mass of the beam (taking account of reduced parameters of concrete mix); $s=1$, 2, 3, 4 and 5 is the ordinal number of beam's span, counting from the left end.

The end boundary conditions are the following:

$$\begin{aligned}&\frac{\partial^2 y_1}{\partial z^2}(-l/2, t) = 0, \quad \frac{\partial^3 y_1}{\partial z^3}(-l/2, t) = 0,\\ &\frac{\partial^2 y_5}{\partial z^2}(l/2, t) = 0, \quad \frac{\partial^3 y_5}{\partial z^3}(l/2, t) = 0.\end{aligned} \tag{2}$$

The interaction of the beam with vibrodrives and elastic supports is mathematically described by the conjugation conditions in points with the coordinates z_s ($s=1$, 2, 3 and 4):

$$y_s(z_s, t) = y_{s+1}(z_s, t), \quad \frac{\partial y_s}{\partial z}(z_s, t) = \frac{\partial y_{s+1}}{\partial z}(z_s, t), \tag{3}$$

$$\frac{\partial^2 y_s}{\partial z^2}(z_s, t) = \frac{\partial^2 y_{s+1}}{\partial z^2}(z_s, t), \tag{4}$$

$$\begin{aligned}\mathrm{EI}\left[\frac{\partial^3 y_s}{\partial z^3}(z_s, t) - \frac{\partial^3 y_{s+1}}{\partial z^3}(z_s, t)\right] &= P(\varphi, \dot{\varphi}, \ddot{\varphi}) - F_r - m_v\frac{\partial^2 y_s}{\partial t^2}(z_s, t)\\ &\quad - (b_1 + b_c + F_d)\frac{\partial y_s}{\partial t}(z_s, t).\end{aligned} \tag{5}$$

Functions F_d and F_r in Eq. (5) can be represented as

$$F_d = \begin{cases} 0, & y_s(z_s, t) \leq \Delta,\\ b_2, & y_s(z_s, t) > \Delta,\end{cases} \tag{6}$$

$$F_r = \begin{cases} k_1 y_s(z_s, t), & y_s(z_s, t) \leq \Delta,\\ (k_1 + k_2)y_s(z_s, t) - k_2\Delta, & y_s(z_s, t) > \Delta.\end{cases} \tag{7}$$

The notation in Eq. (2)–(7) is taken as follows: l is the length of a beam; m_v is the mass of vibrodrive; k_1 and k_2 are the stiffness coefficients of main elastic supports and elastic limiters; b_1

and b_2 are the coefficients of internal friction in main elastic supports and elastic limiters; b_c is the coefficient of additional friction due to the influence of concrete mix; Δ is the clearance between the working head and elastic limiter in the static equilibrium position; and φ is the rotation angle of a shaft of driving motor.

In order to check the possibility of realizing a subharmonic vibration regime in real start-up conditions, it is necessary to take into account the limited power supply of electric driving motor [17,18]. The motion of a shaft of unbalance vibration exciter is described by the following differential equation:

$$\frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} = \frac{1}{I_v}\left(M_t - \vartheta\frac{\mathrm{d}\varphi}{\mathrm{d}t} - M_r\right), \tag{8}$$

where I_v is the moment of inertia of rotating parts of a vibroexciter; M_t is a torque of electric motor; M_r is an antitorque moment; and ϑ is the coefficient of friction in the bearings of an electric motor.

In the case of an induction-driving motor with limited power supply, the external exciting force $P(\varphi, \dot{\varphi}, \ddot{\varphi})$ in Eq. (5) and the antitorque moment M_r in Eq. (8) are determined by the following expressions [13,17]:

$$P(\varphi, \dot{\varphi}, \ddot{\varphi}) = \dot{\varphi}^2 m_u r \cos\varphi + \frac{\mathrm{d}^2\varphi}{\mathrm{d}t^2} m_u r \sin\varphi, \tag{9}$$

$$\begin{aligned} M_r = & -\frac{\partial^2 y_s}{\partial t^2}(z_s, t) m_u r \sin\varphi - m_u g r \sin\varphi \\ & + \left[m_u r \left(\frac{\mathrm{d}\varphi}{\mathrm{d}t}\right)^2 - m_u \frac{\partial^2 y_s}{\partial t^2}(z_s, t)\cos\varphi \right] \frac{d}{2}\vartheta_1, \end{aligned} \tag{10}$$

where m_u and r are the unbalanced mass and the radius of its rotation; d is the reduced diameter of bearings of a vibroexciter; and ϑ_1 is the coefficient of friction in the bearings of a vibroexciter.

The torque M_t of an electric motor is defined by an additional system of differential equations. After an equivalent change of three-phase induction-driving motor into the two-phase one with relatively stationary windings of rotor and stator, the differential equations describing electromagnetic processes in an induction-driving motor are as follows [13,19]:

$$\frac{\mathrm{d}\Psi_{x1}}{\mathrm{d}t} = u_{x1} - \omega_0\alpha'_s\Psi_{x1} + \omega_0\alpha'_s k_r\Psi_{x2} + \omega_0\Psi_{y1}, \tag{11}$$

$$\frac{\mathrm{d}\Psi_{y1}}{\mathrm{d}t} = u_{y1} - \omega_0\alpha'_s\Psi_{y1} + \omega_0\alpha'_s k_r\Psi_{y2} - \omega_0\Psi_{x1}, \tag{12}$$

$$\frac{\mathrm{d}\Psi_{x2}}{\mathrm{d}t} = -\omega_0\alpha'_r\Psi_{x2} + \omega_0\Psi_{y2} - \dot{\varphi}p\Psi_{y2} + \omega_0\alpha'_r k_s\Psi_{x1}, \tag{13}$$

$$\frac{\mathrm{d}\Psi_{y2}}{\mathrm{d}t} = -\omega_0\alpha'_r\Psi_{y2} + \omega_0\alpha'_r k_s\Psi_{y1} + \omega_0\Psi_{y1}, \tag{14}$$

$$M_t = \tfrac{3}{2}p\omega_0\frac{k_r}{x_s\sigma}(\Psi_{x2}\Psi_{y1} - \Psi_{x1}\Psi_{y2}), \tag{15}$$

where Ψ_{x1}, Ψ_{y1}, Ψ_{x2} and Ψ_{y2} are the magnetic-flux linkage of windings; $u_{x1} = \sqrt{2}U_{fn}$ (U_{fn} is the nominal value of phase voltage); $u_{y1} = 0$; p is the number of poles; ω_0 is the frequency of alternating voltage (in standard power network this frequency is equal to $\omega_0 = 100\pi s^{-1}$); $\alpha_s = R_1/X_s$; $\alpha'_s = \alpha_s/\sigma$; $k_s = X_0/X_s$; $\sigma = 1 - k_r k_s$; $\alpha'_r = \alpha_r/\sigma$; $\alpha_r = R_2/X_r$; $k_r = X_0/X_r$; $X_s = X_0 + X_\mu$; $X_r = X_0 + X_v$; R_1, R_2, X_0 , X_μ and X_v are the resistances of electric motor elements (catalogue data [19]).

2.2. Method of analysis

The set of Eqs. (1)–(15) describing the system dynamics is too complex. Therefore, it has been found expedient to simulate dynamic processes in such a system on the specialized analogue–digital computer system (ADCS) designed in Riga Technical University [13,20]. This computer system is predominantly set up for the solution of complex nonlinear dynamic problems and consists of two parts.

Integration of nonlinear differential equations is carried out on the high-speed analogue part of ADCS. The simulation is based on the direct analogy method [21], which establishes a quantitative and qualitative correspondence between mechanical vibrations and current and voltage oscillations in the electrical model of the system to be analysed. A great number of electrical modules being analogous to inertial, elastic and dissipative elements with linear and practically any nonlinear characteristics (piecewise linear, polynomial, relay, etc.) are developed. Units for input of external excitations are also designed by modular principle. In the course of the simulation, the electrical modules must be connected according to the structure of an initial mechanical system.

Partial differential equations (e.g., Eq. (1)) are simulated by their replacement with an equivalent set of finite difference equations [21,22]. Collections of standard circuit diagrams for solution in finite difference formulation of refined differential equations describing flexural, longitudinal and torsional vibrations of bars in view of various nonlinear factors are developed [13,20,21]. The analogue part of ADCS operates in the acoustical frequency range and therefore has very high speed of action, making it possible to obtain a great number of solutions in a comparatively short time.

The digital part of ADCS consists of two types of computing machines: universal personal computer and specialized computer X6-4 with a wired-in program for the calculation of statistic characteristics of random processes. Such structure of the digital part makes it possible to use universal and specialized software for a personal computer in order to control the programming of the analogue part and to process data (scaling, synthesis of regression equations, optimization, etc.). The specialized computer X6-4, however, increases the rate and accuracy of calculations for random characteristics (correlation and autocorrelation functions, distribution density and probability distribution functions, power spectral density, etc.).

The quantitative estimation of accuracy in analogue–digital simulation was carried out by the solution of a great number of test examples [13,21]. The results of test simulation have shown close agreement with exact and numerical solutions for systems with piecewise linear, polynomial and relay elastic-dissipative characteristic under harmonic and polyharmonic excitation.

The results of mathematical simulation of the set of Eqs. (1)–(15) performed by this method are considered further on.

2.3. Vibromachines operating in the absolute stable subharmonic regime

Attainment of a vibration regime with high asymmetry ratio $\ddot{y}^{-}/\ddot{y}^{+}$ is the main criterion for designing vibroforming machines, because this ratio is directly proportional to the compressive strength of produced blocks and plates. Another important property of a technological vibromachine is stability of operation regime to possible external perturbations. From both these points of view, the most preferable is the $\frac{1}{2}$ -order subharmonic regime, which is specified with high values of $\ddot{y}^{-}/\ddot{y}^{+}$ and may be excited (in some ranges of parameters) under arbitrary initial conditions [8,9]. The last property provides absolute stability of operation regime and convenience of its practical realization (excitation under zero or close to zero initial conditions).

By the solution of the set of Eqs. (1)–(15) on the ADCS, the conditions of existence of absolutely stable $\frac{1}{2}$-order subharmonic regime are determined. Fig. 2 shows the domain within which this subharmonic regime may exist. The domain is constructed in the space of non-dimensional parameters $p = m_u r\omega^2/k_1\Delta, k_2/k_1$ and $v = \omega/\omega_1$, where $\omega_1 = \sqrt{4k_1/(4m_v + \lambda l)}$ is the natural frequency of small free vibrations; $\omega = \dot{\varphi}_a$ is the average angular velocity of a shaft of an electric motor in the steady-state conditions. Non-dimensional coordinates $u_1 = u_3 = |z_1|/l = |z_3|/l$ and $u_2 = u_4 = |z_2|/l = |z_4|/l$, indicating the location of elastic supports, have been varied within the limits of $u_1 = 0.05$–0.20 and $u_2 = 0.30$–0.45. It was shown that variation of u_1 and u_2 within such limits could shift the borders of the domain, but not more than by 10%. The domain presented in Fig. 2 has been plotted assuming $u_1 = 0.13$ and $u_2 = 0.38$.

According to the search for an optimum performed by the methodology described in Refs. [13,23], the maximal level of asymmetry ratio $\ddot{y}^{-}/\ddot{y}^{+} = 2.4$ is realized inside this domain under the conditions $p = 6.8$–7.1, $k_2/k_1 = 3.2$–3.5 and $v = 2.4 - 2.5$. The time response $\ddot{y} = f(\tau)$ corresponding to this optimal absolute stable subharmonic regime is shown in Fig. 3 (curve 1).

By the results of experiments [13], implementation of this asymmetric subharmonic vibration regime instead of a routine harmonic one provides high technological efficiency (the rise of compressive strength of compacted concrete blocks and panels on the average of 25–30%). Simultaneously, some other performance data of the vibromachine are improved. Specifically, thanks to resonant tuning of the system, it is possible to realize vibrations of the same intensity

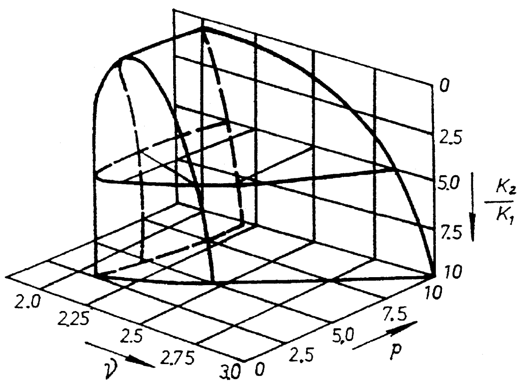

Fig. 2. Domain of existence of absolute stable $\frac{1}{2}$ -order subharmonic regime.

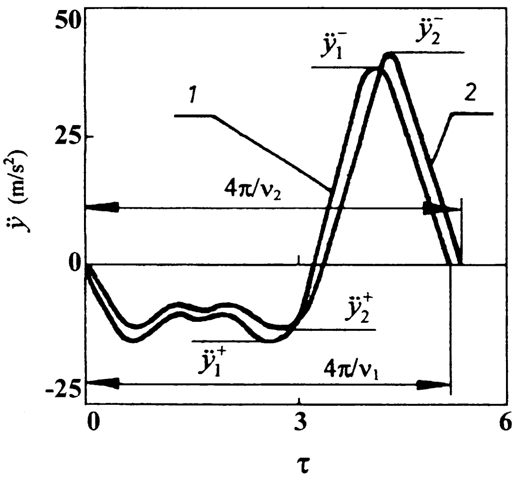

Fig. 3. Time responses $\ddot{y} = f(\tau)$ of two optimal subharmonic regimes of vibrocompaction: 1, absolute stable subharmonic regime; 2, hard-excited subharmonic regime.

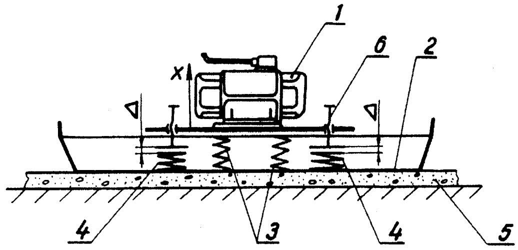

Fig. 4. Dynamic model of the surface subharmonic vibrocompactor: 1, unbalance vibroexciter; 2, working plate; 3, main elastic supports; 4, elastic limiters; 5, compacted material; 6, adjusting screw.

under the rather smaller static moment of unbalanced mass $m_u\, r$. As a consequence, the power consumption in a subharmonic vibromachine is reduced by a half, but operating longevity of vibrodrive's bearings becomes approximately five times greater.

2.4. Surface subharmonic vibrocompactor

Absolute stable subharmonic oscillations are also realized in the surface subharmonic vibrocompactor, intended for compacting concrete or soil surfaces during building of roads, squares, and floors. The dynamic model of this vibrocompactor is shown in Fig. 4. Nonlinearity of elastic characteristics is realized with the aid of additional elastic limiters mounted with initial clearance Δ to the vibrodrive.

Neglecting the viscoelastic properties of concrete mix or soil, it is possible to consider the dynamic model of surface vibrocompactor to be equivalent to the model of vibroforming machine (see Fig. 1, in condition that $\mathrm{EI} = \infty$). Therefore, in designing a subharmonic vibrocompactor the

Table 1
Main performance data of surface vibrocompactors

Property	Vibrocompactor ИВ-91A	Subharmonic vibrocompactor
Type of vibration action on compacted surface	Monoharmonic	Polyharmonic (subharmonic)
Frequency of vibration action (Hz)	46	23 + 46 + 69
Power of electric motor drive (kW)	0.6	0.3
Static moment of unbalanced mass (kg cm)	6.6	3.45
Rotation frequency of unbalanced mass (rev/min)	2800	2800
Operating longevity of drive's bearings (h)	3500–4000	20,000
Maximal thickness of effectively compacted layer of concrete mix (m)	0.3	0.5

earlier determined optimal parameters of a vibroforming machine ($p = 6.8$–7.1, $k_2/k_1 =$ 3.2–3.5 and $\nu = 2.4$–2.5) are valid.

By the application of these results, a subharmonic surface vibrocompactor is designed. The utilization of subharmonic effects makes it possible to transform the vibration motion of the compactor's frame from the symmetric monoharmonic one with the frequency 46 Hz into the asymmetric polyharmonic regime with the frequency components of 23, 46 and 69 Hz. Thanks to this the subharmonic vibrocompactor shows increased technological efficiency (the rise of concrete strength by 20–25%, high quality of the right side of workpiece surface). Besides, electric power consumption is reduced by 50%, but operating longevity of drive bearings is increased 3 times. The main performance data of a subharmonic vibrocompactor (in comparison with parameters of the commercial vibrator ИВ-91A) are presented completely in Table 1.

The initial lot of such vibrocompactors is adopted in more than 10 engineering companies of Latvia, Lithuania and Russia.

2.5. Experimental investigations

The technological efficiency of asymmetric subharmonic regime of vibrocompaction has been checked by experiments held in Riga Technical University [13]. During the experiments, the strength properties of fine concrete specimens produced under the action of the standard harmonic vibration (frequency $\omega = 293\,\mathrm{rad/s}$, acceleration $a = 35\,\mathrm{m/s^2}$) and the asymmetric subharmonic one (frequencies of harmonic components $\omega^{(1/2)} = 146.5, \omega^{(2/2)} = 293$ and $\omega^{(3/2)} = 439.5\,\mathrm{rad/s}$; asymmetry ratio $\ddot{y}^-/\ddot{y}^+ = 2.5$; positive acceleration $\ddot{y}^+ = 15\,\mathrm{m/s^2}$) are compared. Vibrocompaction by standard harmonic technology was carried out on the table vibrator, model 435 [13]. Asymmetric subharmonic regime has been generated on a specially designed vibration-testing machine with nonlinear elastic ties [12]. Measurements of vibration parameters (frequency, amplitude, acceleration) were made by standard instrumentation: vibration meter (model ВШВ-003) complete with piezoelectric transducers, spectrum analyzer (model SBA-101), stroboscopic tachometer (model St-5), double-beam oscilloscope (model C8-11). Fig. 5 shows time responses $\ddot{y} = f(t)$ for standard technology (curve 1) and for subharmonic regime (curve 2) recorded experimentally.

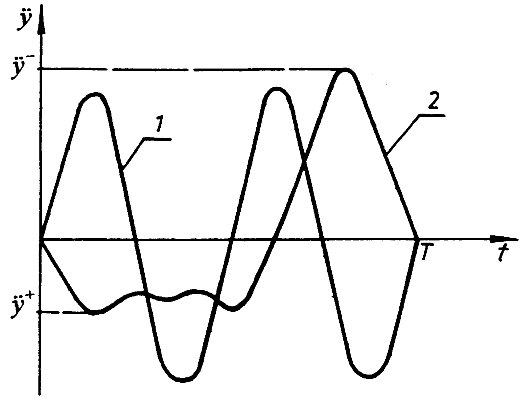

Fig. 5. Time responses $\ddot{y} = f(\tau)$ recordered experimentally: 1, curve for table vibrator (model 435); 2, curve for subharmonic vibration-testing machine.

Table 2
Results of bending and compression tests of concrete specimens

Standard harmonic vibration		Asymmetric subharmonic vibration	
$\sigma_b \times 10^{-5}$ (N/m^2)	$\sigma_c \times 10^{-5}$ (N/m^2)	$\sigma_b \times 10^{-5}$ (N/m^2)	$\sigma_c \times 10^{-5}$ (N/m^2)
64	354	90	456
60	360	87	443
70	312	83	469
69	320	80	452
74	310	77	448
65	380	84	435
67	332	86	471
62	368	85	452
63	357	93	441
71	314	89	468
67	349	81	466
72	308	84	433
70	318	88	449
64	339	86	453
63	370	82	456
64	344	83	446
70	331	90	460
68	328	80	462
66	341	87	447
71	315	85	452
$\sigma_b^m = 67$	$\sigma_c^m = 337.5$	$\sigma_b^m = 85$	$\sigma_c^m = 453$

Bending and compression tests of specimens made from fine concrete were held on special equipment [13]. The results of these tests are presented in Table 2. It is seen that application of asymmetric subharmonic vibration (instead of the standard harmonic one) causes the rise of concrete bending strength σ_b on the average of 25–27% and concrete compression strength σ_c on the average of 34–35% (average values σ_b^m and σ_c^m of bending and compression strength are presented in the last row of the table).

The technological efficiency of asymmetric subharmonic regime of vibrocompaction has been also corroborated with reference to some other materials (synthegran, sawdust, gravel concrete, etc.).

3. Program-simulated actuation of subharmonic vibromachine

Up to here all improvements of technological vibromachines, as stated above, have been achieved by realization of absolutely stable $\frac{1}{2}$-order subharmonic vibrations distinguished with the asymmetry ratio of $\ddot{y}^-/\ddot{y}^+ = 2.4 - 2.5$. But in accordance with the results of simulation, the subharmonic vibrations with appreciably greater asymmetry ratio $\ddot{y}^-/\ddot{y}^+$ may exist in the system under study. However, these sufficiently asymmetric subharmonic regimes can be realized only under special substantially non-zero initial conditions. Therefore, direct start-up of a driving induction electric motor does not allow one to reach the specified subharmonic regimes. This conclusion is confirmed by the results of mathematical simulation.

Fig. 6 shows the domain on the coordinate plane p and ν, within which the $\frac{1}{2}$-order subharmonic regime may exist in the system described by the set of Eqs. (1)–(15). The domain has been plotted for the case of ideal excitation source and on condition that $k_2/k_1 = 3.3$, $u_1 = 0.13$ and $u_2 = 0.38$. The greater frequency of section lining corresponds to the absolutely stable subharmonic vibrations that may be established in the system under any (even sufficiently non-zero) initial conditions. The section lining with smaller frequency distinguishes the part of the complete domain of existence, within which excitation of subharmonic vibrations is possible only under special, substantially non-zero initial values of displacements and (or) velocities.

According to the search for an optimum [13,23], the maximal value of asymmetry ratio $\ddot{y}^-/\ddot{y}^+$ may be reached under the conditions $p = 2.4$–2.7, $k_2/k_1 = 3.2$–3.5, $\nu = 2.33$–2.40, $u_1 = 0.13$ and

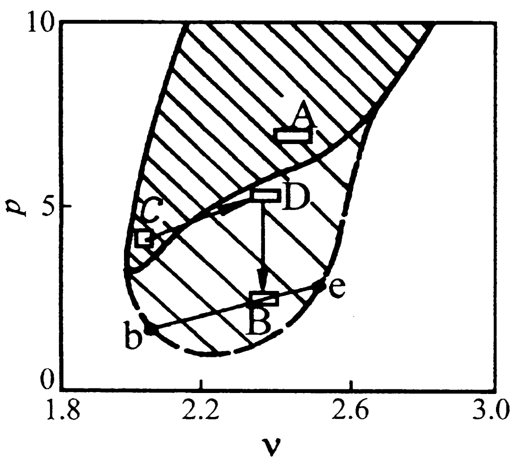

Fig. 6. Domain of existence of $\frac{1}{2}$-order subharmonic regime (for the case $k_2/k_1 = 3.3$, $u_1 = 0.13$ and $u_2 = 0.38$).

$u_2 = 0.38$. In Fig. 6 this set of parameters is labelled as a sub-region B. But the earlier found local optimum within the scope of absolute stable subharmonic vibrations is denoted as a sub-region A. The adjustment of vibromachine on the sub-region B makes it available to increase the asymmetry ratio $\ddot{y}^-/\ddot{y}^+$ of subharmonic regime up to $\ddot{y}^-/\ddot{y}^+ = 3.1 - 3.2$. Thanks to this, the further rise of technological efficiency of the subharmonic vibromachine is expected.

Direct excitation of subharmonic vibrations corresponding to the optimal sub-region B is possible only under sufficiently non-zero initial conditions or with the aid of rather strong additional external disturbances (e.g., action on a system with external impacts). Such a way of actuation (hard excitation) requires the use of special devices, and this certainly complicates the design of a subharmonic vibromachine.

Another approach to the problem decision is based on utilization of the effect of pulling of nonlinear vibrations described in Refs. [24,25]. Specific features of realization of this effect in application to subharmonic vibrations are studied here. It is shown that inside the domain of existence of subharmonic vibrations (see Fig. 6) the frequency ν and amplitude p of excitation force may be continuously changed, and, what is important, the dynamic stability (in small) of the subharmonic regime during such variations is retained. Thanks to this, it is possible to tune the system on subharmonic regime corresponding to the sub-region B on the base of soft excitation: by the pulling of absolute stable subharmonic vibrations into the given region through the smooth variation of frequency ν and amplitude p of external exciting force.

By the use of such pulling, the program-simulated actuation of subharmonic vibromachine is possible. Such actuation is effected in a step-by-step manner, which may be illustrated with the domain of existence (see Fig. 6) and corresponding amplitude–frequency characteristic (AFC) of $\frac{1}{2}$-order subharmonic vibrations (see Fig. 7). The quantities of half-swing of the non-dimensional inertial forces $f_{\text{in}} = F_{\text{in}}/k_1\Delta_0$ (here Δ_0 is the value of clearance Δ before the start-up of vibromachine) are laid off as amplitudes on these AFC.

In accordance with the procedure proposed, the frequency ν and amplitude p of external excitation initially are preset on the sub-region C ($\nu = 2.02$–2.07, $p = 3.8$–4.3). Thanks to this, subharmonic vibrations are excited under zero (or close to zero) initial conditions (section c_1c_2 of the resonance curve, Fig. 7). After that, the frequency ν is smoothly increased up to the required operational value $\nu = 2.33$–2.40 (sub-region D and part d_1d_2 of the AFC). On the finishing stage of

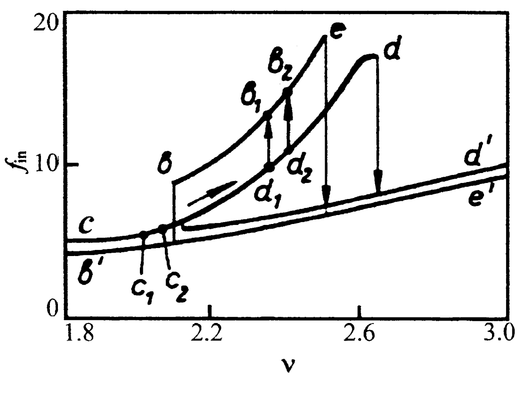

Fig. 7. Amplitude–frequency characteristic of $\frac{1}{2}$-order subharmonic vibrations.

program-simulated actuation, the amplitude p of external excitation is continuously reduced from $p = 5.1$–5.5 to 2.4–2.7 (e.g., by increasing the clearance Δ). As a result, the vibromachine is tuned into the optimal subharmonic regime (sub-region B and part $b_1 b_2$ of the AFC) from zero initial conditions. The time response $\ddot{y} = f(\tau)$ corresponding to this subharmonic regime is shown in Fig. 3 (curve 2).

In accordance with the calculations made, the realization of program-simulated actuation not only stimulates the rise of the asymmetry ratio $\ddot{y}^- / \ddot{y}^+$ of subharmonic regime, but also makes it possible to increase by 3 times the operating longevity of bearings as well as to reduce by 25% the electric power consumption.

4. Vibration emptying of loading hoppers

Loading hoppers are widely used in civil and chemical engineering for storage and further distribution of granular materials (sand, crushed stone, cement, etc.) and concrete mix. Dumping of a hopper is usually carried out spontaneously, by gravity action on the material. But due to *slamming*, the normal process of emptying is often disrupted. The possibility of accelerating the emptying with the aid of vibration is considered here.

Fig. 8 shows the design of a production-type hopper for distribution of granular materials. The body of this hopper is made from standard equal angles 1 and steel sheets 2. The hopper is welded on to the steel frame 3, which is made fast to the massive base 4. Such fastening affords sufficiently high rigidity of all structure, and therefore the possibility of forced displacement of the hopper as a rigid body is ruled out. In this case, vibration emptying may be carried out only by the excitation of elastic flexural vibrations of hopper's wall (e.g., with the aid of unbalance vibration exciter 5).

In order to raise the efficiency of vibration emptying, it is necessary to tune the vibroexciter to resonance. In accordance with the experimental data, the most intensive resonant elastic vibrations of the hopper's walls are realized at the frequency 23 Hz. But production-type unbalance vibration exciters can excite vibrations only in the frequency range of $f = 45$–50 Hz, and in this range an amplitude of elastic vibrations of hopper's walls is extremely small. Therefore, a vibroexciter hard-mounted to hopper's wall is of no due effect.

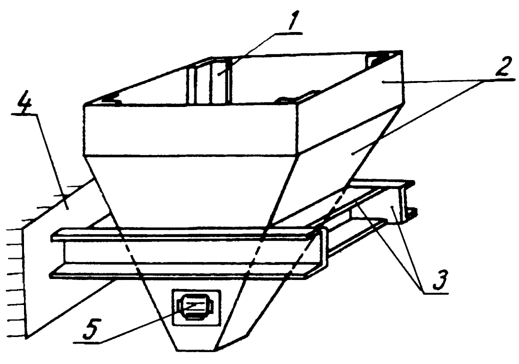

Fig. 8. Hopper for distribution of granular materials: 1, standard equal angles; 2, steel sheets; 3, steel frame; 4, massive base; 5, unbalance vibration exciter.

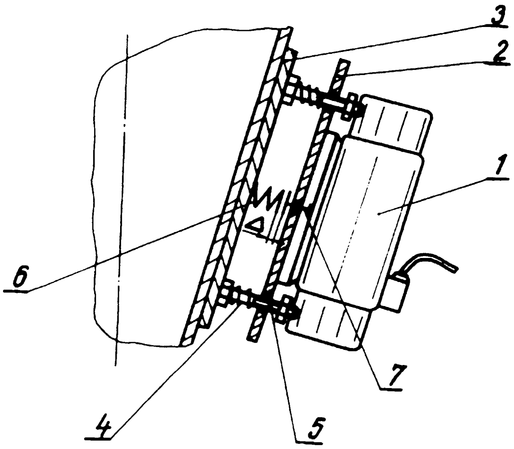

Fig. 9. Subharmonic vibrator for emptying of loading hopper: 1, vibroexciter; 2, rigid steel plate; 3, base; 4, elastic elements; 5, pin; 6, elastic limiter; 7, adjusting screw.

In such conditions it has been found expedient to take advantage of the excitation of subharmonic vibrations. For this purpose, it is necessary to mount a vibroexciter on a hopper's wall through nonlinear elastic ties (instead of reduction gear using for decreasing of rotation frequency of unbalanced mass). A structural model of a subharmonic vibrator for loading hopper's emptying is shown in Fig. 9. Vibroexciter 1 is fastened on rigid steel plate 2 connected with base 3 through linear elastic elements 4. Pins 5 welded to base 3 are intended for the prevention of possible lateral shift of vibroexciter 1. Nonlinearity of an elastic system is realized by mounting on base 3 of elastic limiters 6 (with initial clearance Δ relative to plate 2). Setting of the necessary value of clearance $\varDelta$ is carried out with the aid of adjusting screw 7.

Elastic and mass parameters of subharmonic vibrator are chosen such that generating vibration regime is at subharmonic resonance of $\frac{1}{2}$-order. Such tuning of vibrator may be achieved if its main parameters lie inside the earlier found region (see Fig. 2, $p = 6.8$–7.1, $k_2/k_1 = 3.2$–3.5 and $\nu = 2.4$–2.5).

The results of theoretical study have been adopted in the Civil-Engineering Board of Production Association "Kaliningradrybprom" (Russia) for designing and manufacturing of subharmonic vibrator for emptying of loading hopper. It was shown during the operation of this vibrator that subharmonic vibrations of working head are permanently excited under different states of fullness of the hopper. Presence in spectrum of these vibrations of harmonic component with a frequency of $f^{(1/2)} = 23\,\text{Hz}$ facilitates the excitation of intensive resonant flexural vibrations of hopper's wall. Thanks to this the process of emptying is considerably accelerated.

5. Conclusions

The results presented in this paper show that utilizing specific nonlinear properties of subharmonic regimes (complex vibration spectrum, essential asymmetry of time response, predominance of low-frequency harmonic components in vibration spectrum, etc.) ensures more effective operation of vibromachines for different technological purposes (vibrocompaction, vibroforming, vibration emptying, etc.). Original procedure of program-simulated actuation of subharmonic vibromachine is proposed. The technological efficiency of subharmonic vibration has been verified by experiments with a vibration-testing machine.

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