



ELSEVIER

Available online at [www.sciencedirect.com](http://www.sciencedirect.com)

SCIENCE @ DIRECT®

Journal of Sound and Vibration 280 (2005) 699–718

JOURNAL OF  
SOUND AND  
VIBRATION

[www.elsevier.com/locate/jsvi](http://www.elsevier.com/locate/jsvi)

# Fault diagnosis of rotating machinery using an intelligent order tracking system

Mingsian Bai\*, Jiamin Huang, Minghong Hong, Fucheng Su

*Department of Mechanical Engineering, National Chiao-Tung University, 1001 Ta-Hsueh Road, Hsin-Chu 300, Taiwan, Republic of China*

Received 16 July 2003; accepted 15 December 2003

---

## Abstract

This research focuses on the development of an intelligent diagnostic system for rotating machinery. The system is composed of a signal processing module and a state inference module. In the signal processing module, the recursive least square (RLS) algorithm and the Kalman filter are exploited to extract the order amplitudes of vibration signals, followed by fault classification using the fuzzy state inference module. The RLS algorithm and Kalman filter provide advantages in order tracking over conventional Fourier-based techniques in that they are insensitive to smearing problems arising from closely spaced orders or crossing orders. On the basis of thus obtained order features, the potential fault types are then deduced with the aid of a state inference engine. Human diagnostic rules are fuzzified for various common faults, including the single fault and double fault situations. This system is implemented on the platform of a floating point digital signal processor, where a photo switch and an accelerometer supply the shaft speed and acceleration signals, respectively. Experiments were carried out for a rotor kit and a practical four-cylinder engine to show the effectiveness of the proposed system in tracking the rotating order with precise inference.

© 2003 Elsevier Ltd. All rights reserved.

---

## 1. Introduction

Rotating machinery represent an important class of mechanical systems. To name a few, compressors, pumps, fans, motors, generators and engines all fall into this category. Many kinds of rotating machines such as those found in modern semiconductor industry are very costly to maintain and any malfunction of which could cause tremendous loss and hazard to the industry. It is then highly desirable to establish an on-line monitoring and diagnostic mechanism that is capable of detecting anomalies and identifying fault types of machines at the very early stage. This

---

\*Corresponding author. Fax: +886-3-5720634.

*E-mail address:* [msbai@mail.nctu.edu.tw](mailto:msbai@mail.nctu.edu.tw) (M. Bai).

is also the widespread notion of preventive maintenance (PM) that prevents industries from unnecessary maintenance cost and even disastrous results.

Faults commonly found in rotating machinery such as unbalance, misalignment, looseness, bearing fault, gear fault, resonance, etc. [1] except resonance, could often be related to rotating speed. These faults may produce excessive vibration at the shaft speed as well as its multiples. The conditions of rotating machine can thus be monitored by measuring the vibration signals at certain locations, e.g., bearing mounts. The conventional method of rotor diagnosis is to examine the vibration spectrum at a fixed speed. However, the information thus obtained is only partial because some faults do not respond significantly at the fixed operation speed. As a more comprehensive approach to the diagnosis of rotor faults, the *order tracking* technique that exploits vibration signals, supplemented with information of shaft speed, serves as a useful tool for diagnosis of rotating machinery. Conventional methods of order tracking are primarily based on the Fourier analysis, where narrow-band spectra of the frequency-varying signals are calculated by using proper time-windowing [2]. Re-sampling process is generally required in the fast Fourier transform (FFT)-based methods to compromise between time and frequency resolution for various shaft speeds. However, conventional FFT methods suffer from a number of shortcomings. In particular, smearing problem may arise in two situations. First, orders can be closely spaced, particularly at low shaft speed. Second, undesirable beating may occur due to crossing of speed-dependent orders and speed-independent resonance. In addition, conventional methods are ineffective for the applications involving multiple independent shaft speeds. For example, the speed of crank-shaft of an engine and the speed of cooling fans are independent. If one calculates the orders based on either speed, the signal related to other speed would appear as uncorrelated noise and adversely degrade the tracking accuracy.

In order to avoid the aforementioned problems encountered in conventional method, model-based methods have been proposed. A representative example is the Vold–Kalman method [2–4]. In the method, the order tracking problem is formulated in terms of state space models. Order amplitudes are calculated by using the least-squares approach. The method is based on a fixed sampling rate and generally implemented as a post-processing scheme. More accurate interpretation of vibration signature can be drawn by converting frequency spectra into the so-called order spectra, with order defined as the frequency normalized by the shaft speed. With the order estimation of the vibration signals, the system faults can be accordingly detected and analyzed.

In this paper, a high resolution, adaptive, on-line and intelligent order tracking system is presented. The system is composed of a signal processing module and a state inference module. In the signal processing module, the recursive least square (RLS) algorithm and the Kalman filter are exploited to extract the order amplitudes of vibration signals. The RLS algorithm and Kalman filter provide advantages in order estimation over conventional Fourier-based techniques in that they are insensitive to smearing problems arising from closely spaced orders or crossing orders. The proposed method is also capable of tracking the orders due to multiple independent shaft speeds. These two algorithms are adaptive in nature, making them well suited for varying speed situations. In the state inference module, an intelligent fault classification procedure is conducted using fuzzy logic, in an attempt to mimic an experienced engineer. Human diagnostic rules are fuzzified for various common faults, including the single fault and double fault situations. Previous work by Mechefske [5] and Pantelelis et al. [6] using fuzzy logic and neural networks for fault diagnosis is also along the same line in the present paper.

This system is implemented on the platform of a floating point digital signal processor (DSP), where a photo switch and an accelerometer supply the shaft speed and acceleration signals, respectively. Experiments were carried out for a rotor kit and a practical four-cylinder engine to show the effectiveness of the proposed system in tracking the rotating order with high resolution and precise inference. The experimental results and possible extension of this work will be discussed in the paper.

## 2. Adaptive order feature extraction

### 2.1. RLS order tracking algorithm

The vibration signal generated by rotating machinery can generally be represented as the superposition of sinusoids. For instance, the signal  $x(t)$  containing  $k$  orders generated by a rotating shaft can be written as

$$x(t) = A_1 \cos[\theta(t) + \phi_1] + A_2 \cos[2\theta(t) + \phi_2] + \dots + A_k \cos[k\theta(t) + \phi_k], \quad (1)$$

where  $A_k$  and  $\phi_k$  denote the amplitude and phase, respectively, of the  $k$ th order, and  $\theta(t)$  is the angular displacement. By the trigonometric identity, Eq. (1) can be expanded as

$$x(t) = A_{1I} \cos[\theta(t)] - A_{1Q} \sin[\theta(t)] + A_{2I} \cos[2\theta(t)] - A_{2Q} \sin[2\theta(t)] \\ + \dots + A_{kI} \cos[k\theta(t)] - A_{kQ} \sin[k\theta(t)], \quad (2)$$

where  $A_{kI}$  and  $A_{kQ}$  denote the in-phase and quadrature components, respectively. Here

$$A_{kI} = A_k \cos \phi_k, \quad A_{kQ} = A_k \sin \phi_k. \quad (3)$$

The amplitude of the  $k$ th order can be written as

$$|A_k| = \sqrt{A_{kI}^2 + A_{kQ}^2}. \quad (4)$$

In light of the special structure of the signal described in Eq. (2), the order tracking problem can be recast into a parameter identification problem. After discretization,

$$e(n) = x(n) - \mathbf{w}^T(n)\mathbf{u}(n), \quad (5)$$

where  $n$  is the discrete-time index,

$$\mathbf{u}^T(n) = [\cos[\theta(n)] \quad -\sin[\theta(n)] \quad \cos[2\theta(n)] \quad -\sin[2\theta(n)] \quad \dots \quad \cos[k\theta(n)] \quad -\sin[k\theta(n)]] \quad (6)$$

is the regressor,

$$\mathbf{w}^T(n) = [A_{1I}(n) \quad A_{1Q}(n) \quad A_{2I}(n) \quad A_{2Q}(n) \quad \dots \quad A_{kI}(n) \quad A_{kQ}(n)] \quad (7)$$

is the parameter vector, and  $x(n)$  and  $e(n)$  are the measurement and estimation error, respectively. The solution of  $\hat{\mathbf{w}}(n)$  can be found by the method of least-squares with the following cost function  $\zeta(n)$ :

$$\zeta(n) = \sum_{i=1}^n \lambda^{n-i} |e(i)|^2, \quad (8)$$

where the forgetting factor  $0 < \lambda \leq 1$  weighs exponentially the estimation error from the present to the past. The RLS method is such a algorithm that calculates the optimal parameters  $\hat{\mathbf{w}}(n)$  in recursive way [7,8]:

$$\mathbf{k}(n) = \frac{\lambda^{-1} \mathbf{P}(n-1) \mathbf{u}(n)}{1 + \lambda^{-1} \mathbf{u}^H(n) \mathbf{P}(n-1) \mathbf{u}(n)}, \quad (9)$$

$$e(n) = x(n) - \hat{\mathbf{w}}^H(n-1) \mathbf{u}(n), \quad (10)$$

$$\hat{\mathbf{w}}(n) = \hat{\mathbf{w}}(n-1) + \mathbf{k}(n) e^*(n), \quad (11)$$

$$\mathbf{P}(n) = \lambda^{-1} \mathbf{P}(n-1) - \lambda^{-1} \mathbf{k}(n) \mathbf{u}^H(n) \mathbf{P}(n-1). \quad (12)$$

In the procedure above, the matrix  $\mathbf{P}(n)$  is the inverse of auto-correlation matrix of input vector  $\mathbf{u}$ ,  $e(n)$  is the a priori estimation error, and  $\mathbf{k}(n)$  is the gain vector. To initialize the RLS algorithm, the initial conditions are generally taken to be:  $\hat{\mathbf{w}}(0) = \mathbf{0}_{M \times 1}$  ( $M$  is the number of parameters),  $\mathbf{P}(0) = \delta^{-1} \mathbf{I}$  ( $\mathbf{I}$  is a  $M \times M$  identity matrix), and  $\delta$  is a small positive constant.

The forgetting factor  $\lambda$  is extremely crucial to the convergence of the RLS algorithm. Small values of  $\lambda$  enable fast tracking of signals, but could diverge easily during adaptation, whereas large values ensure stable adaptation, but the convergence performance is poor. In the present work, the following dynamic scheme is employed to adjust  $\lambda$  in response to the status of convergence:

$$\lambda = \begin{cases} \frac{2(1-f)}{E^3} |e(n)|^3 - \frac{3(1-f)}{E^2} |e(n)|^2 + 1 & \text{if } 0 \leq |e(n)| \leq E, \\ f & \text{if } |e(n)| < E, \end{cases} \quad (13)$$

where  $f$  and  $E$  are two user-specified constants. The factor  $\lambda$  varies between  $f$  and unity. The constant  $f$  is generally chosen to be close to unity for stability of the algorithm, e.g.,  $f = 0.995$ . On the other hand, one-half of the maximum signal amplitude would be a good choice of the constant  $E$ . Using this dynamic scheme, tracking would be fast (small  $\lambda$ ) in the early stage when error is large, then the adaptation process is slowed down (large  $\lambda$ ) as the error decreases until the threshold  $E$ , as depicted in Fig. 1. A sample result of the order amplitude of a rotor obtained using the RLS algorithm with dynamic forgetting factor is shown in Fig. 2(a)–(d), where (a) is the 1st order, (b) is the 2nd order, (c) is the 3rd order, and (d) is the 4th order.

## 2.2. Order tracking by recursive Kalman filtering

There are two key equations in the order tracking method based on the Kalman filter [7,9]. The first equation is called the *process equation*:

$$\mathbf{x}(n+1) = \mathbf{F}(n+1, n) \mathbf{x}(n) + \mathbf{v}_1(n), \quad (14)$$

where  $\mathbf{F}(n+1, n)$  is a known  $M \times M$  *state transition matrix* that relates the state of the system at times  $n+1$  and  $n$ . The  $M \times 1$  vector  $\mathbf{v}_1(n)$  represents a zero-mean and white process noise. The second equation is called the *measurement equation*:

$$\mathbf{y}(n) = \mathbf{C}(n) \mathbf{x}(n) + \mathbf{v}_2(n), \quad (15)$$

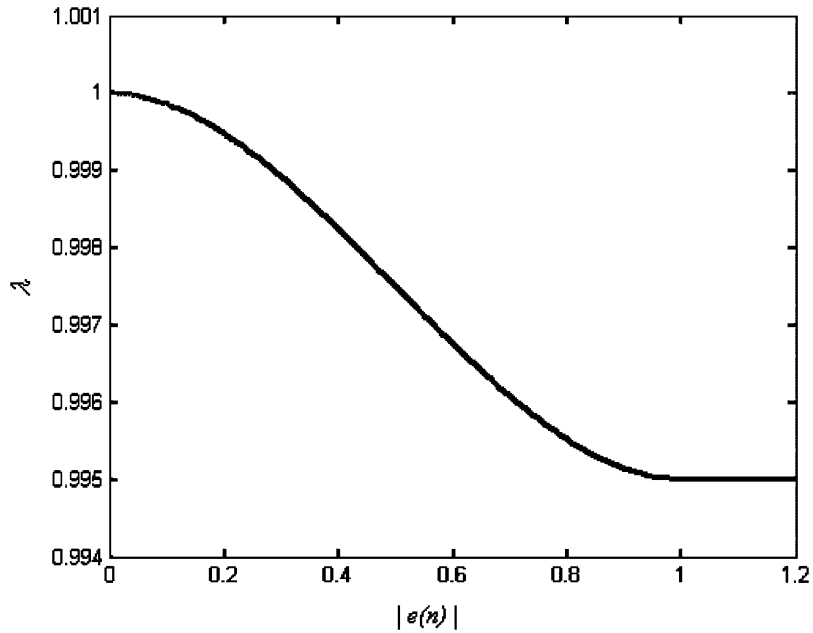


Fig. 1. Dynamic forgetting factor in the RLS algorithm adjusted according to the estimation error.

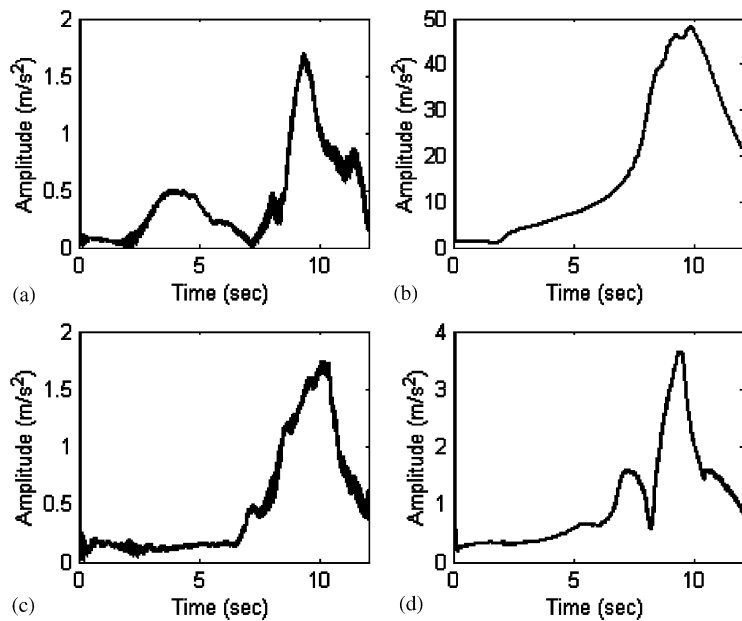


Fig. 2. Order amplitudes estimated in an engine run up test, using the RLS algorithm with dynamic forgetting factor. (a) The 1st order; (b) the 2nd order; (c) the 3rd order; (d) the 4th order.

where  $\mathbf{C}(n)$  is the  $N \times M$  measurement matrix. The  $N \times 1$  vector  $\mathbf{v}_2(n)$  is also a zero-mean and white measurement noise. For our order tracking problem, the measurement equation can be written explicitly as

$$y(n) - \sum_{i=-N}^N A_i(n) \exp[j\theta_i(n)] = v_2(n), \tag{16}$$

where  $j = \sqrt{-1}$ ,  $A_i(n)$  is the complex order amplitude, and  $A_i(n) \exp[j\theta_i(n)]$  is the  $i$ th amplitude modulated complex order and  $\theta_i(n)$  is the angular displacement. It is assumed in this paper that the order amplitude  $A_i(n)$  follows a second order autoregressive (AR) model:

$$H(z)A_i(z) = V_1(z), \tag{17}$$

where

$$H(z) = \frac{1}{1 - 2z^{-1} + 1z^{-2}} \tag{18}$$

and  $V_1(z)$  is the  $z$  transform of  $v_1(n)$ . Note that this model has a double pole at  $z = 1$ , which means that it is capable of tracking a ramp input. Consequently, the state transition matrix can be written as

$$\mathbf{F}(n + 1, n) = \begin{bmatrix} \mathbf{P} & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \ddots & \mathbf{0} & \dots & \mathbf{0} \\ \mathbf{0} & \mathbf{0} & \mathbf{P} & \mathbf{0} & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \ddots & \mathbf{0} \\ \mathbf{0} & \dots & \mathbf{0} & \mathbf{0} & \mathbf{P} \end{bmatrix}, \tag{19}$$

where matrix

$$\mathbf{P} = \begin{bmatrix} 2 & -1 \\ 1 & 0 \end{bmatrix} \tag{20}$$

and  $\mathbf{0}$  is  $2 \times 2$  zero matrix. The measurement matrix can be written as

$$\mathbf{C}(n) = [\exp[j\theta_k(n)] \ 0 \ \dots \ \exp[j\theta_0(n)] \ 0 \ \dots \ \exp[j\theta_{-k}(n)] \ 0]. \tag{21}$$

The aforementioned parameter identification problem can be recursively solved by using the following algorithm [7,9]:

$$\mathbf{G}(n) = \mathbf{F}(n + 1, n)\mathbf{K}(n, n - 1)\mathbf{C}^H(n)[\mathbf{C}(n)\mathbf{K}(n, n - 1)\mathbf{C}^H(n) + \mathbf{Q}_2(n)]^{-1}, \tag{22}$$

$$\alpha(n) = y(n) - \mathbf{C}(n)\hat{\mathbf{x}}(n | y_{n-1}), \tag{23}$$

$$\hat{\mathbf{x}}(n + 1 | y_n) = \mathbf{F}(n + 1, n)\hat{\mathbf{x}}(n | y_{n-1}) + \mathbf{G}(n)\alpha(n), \tag{24}$$

$$\mathbf{K}(n) = \mathbf{K}(n, n - 1) - \mathbf{F}(n, n + 1)\mathbf{G}(n)\mathbf{C}(n)\mathbf{K}(n, n - 1). \tag{25}$$

$$\mathbf{K}(n + 1, n) = \mathbf{F}(n + 1, n)\mathbf{K}(n)\mathbf{F}^H(n + 1, n) + \mathbf{Q}_1(n), \tag{26}$$

To initialize the recursive Kalman filtering process, the initial conditions are generally taken to be  $\hat{\mathbf{x}}(1 | y_0) = \mathbf{0}_{(4k+2) \times 1}$  ( $4k + 2$  is the number of parameters,  $A_i(n)$ ),  $\mathbf{K}(1, 0) = \mathbf{I}$  ( $\mathbf{I}$  is a  $(4k + 2) \times (4k + 2)$  identity matrix).

Like RLS algorithm, prior information of the number of order is required in applying this method. Technically, this can be obtained by a preliminary scan by using conventional order tracking methods. If this can be done, the proposed technique should provide results with improved accuracy.

Although the Kalman filter algorithm is more computationally intensive than the RLS method, the convergence of the former algorithm is generally better in terms of speed and stability than the latter one. An approach to reducing the computation loading of the Kalman filter algorithm is to down sample the input signal. In Fig. 3, a sample result of engine test suggests the down sampling action up to 12.5% did not seem to produce any significant effect on the estimation of the order amplitude. Owing to its superior performance, only Kalman filter method is utilized to carry out order tracking in the following presentation.

### 3. Fuzzy-based intelligent diagnostic interface

With the features extracted the RLS or Kalman filter algorithms, an intelligent inference procedure based on fuzzy logic can then be used to classify the machine faults. Fuzzy logic seeks to reach a decision following linguistic rules as an experienced human dealing with fault diagnosis problems [10–13]. Membership functions and fuzzy rules must be established before the condition of machine is diagnosed. In this paper, a triangular and a  $\pi$  membership functions are used, as shown in Fig. 4. The definition of the  $\pi$  membership function is given as follows:

$$\pi(x, a, b) = \frac{1}{1 + ((x - a)/b)^2}, \tag{27}$$

where  $a$  is the average and  $b = n \times \text{S.D.}$  of the selected feature of each order. To characterize the parameters of the membership functions, statistical data analysis is required to find the mean value ( $a$ ) and standard deviation (S.D.) of each order feature. The averages of each order  $\pm h$  times the standard deviation are taken as the upper and lower limits of the fuzzy membership functions [5]. The root-mean-square (rms) values of order amplitudes are selected as the input feature to the fuzzy logic inference module:

$$x_{2k} = \sqrt{\frac{1}{H} \left( \sum_{h=1}^H A_{2k}(h) \right)}, \tag{28}$$

where  $x_{2k}$  is the rms amplitude of the  $2k$ th order and  $A_{2k}$  is the  $2k$ th order amplitude, and  $H$  is the number of sample.

Next, the fuzzy rules are established. Generically, a linguistic variable  $x$  in a *universe of discourse*  $U$  is characterized by  $T(x) = \{T_x^1, T_x^2, \dots, T_x^k\}$  and  $M(x) = \{M_x^1, M_x^2, \dots, M_x^k\}$ , where  $T(x)$  is the term set of  $x$ . For example,  $T(x)$  may be taken as {good, fair, alarm} in our experiment. That is, the set of linguistic values of  $x$  with each value  $T_x^i$  being a fuzzy number with

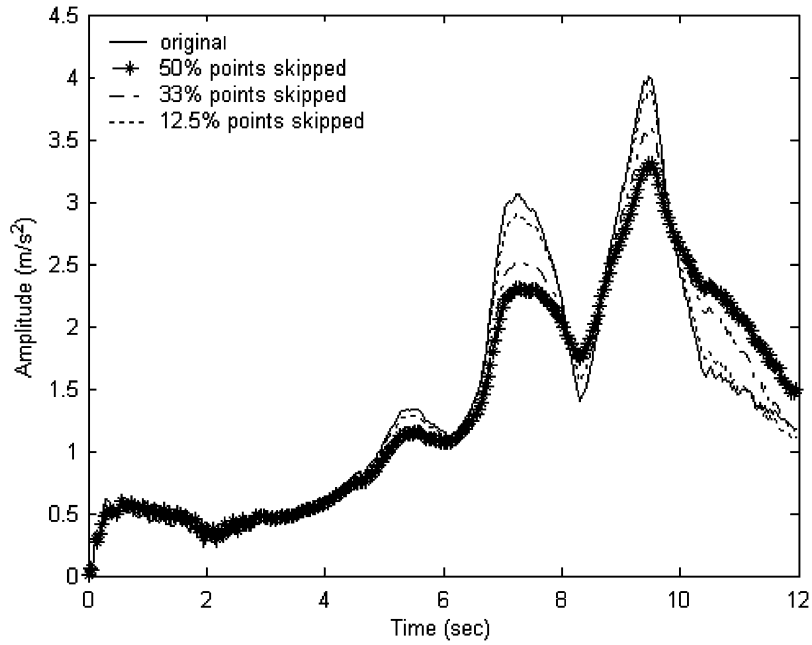


Fig. 3. The effect of down sampling on the estimation of the 4th order amplitude in an engine run up test, using the Kalman filter algorithm.

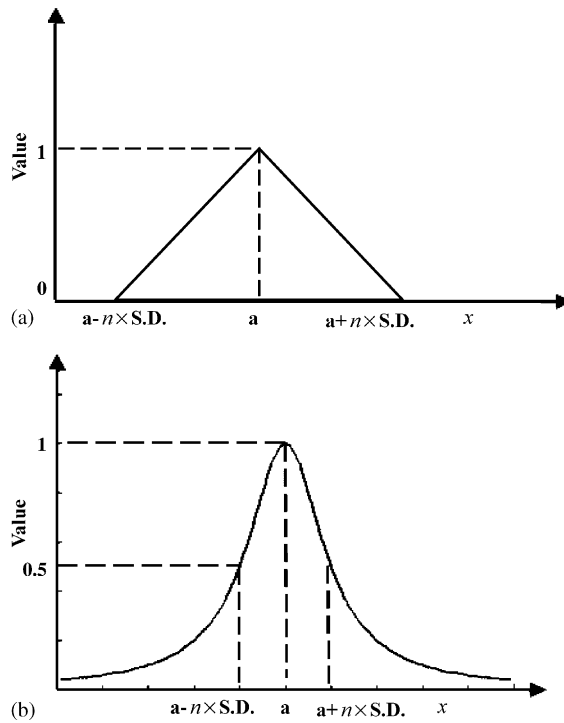


Fig. 4. Fuzzy membership functions: (a) Triangular membership function; (b)  $\pi$  membership function.



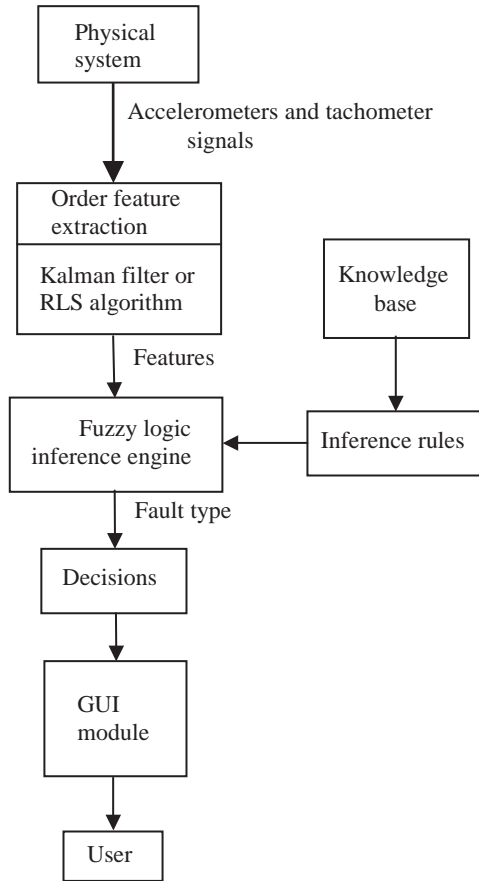


Fig. 5. The overall framework of the intelligent order tracking system, including the order feature extraction module and the fuzzy logic inference module.

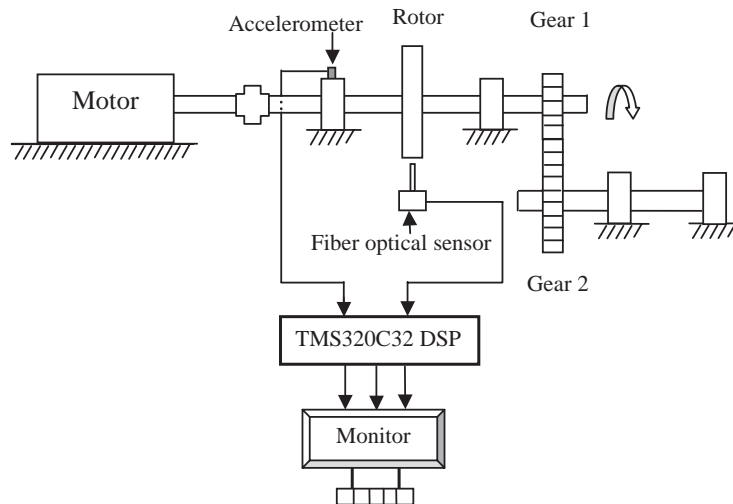


Fig. 6. Experimental arrangement of the rotor kit.

membership function  $M_x^i$  are defined on  $U$ . The fuzzy rules take the following forms:

$$\text{R1: IF } x_1 \text{ is } T_{x_1}^1 \text{ and } x_2 \text{ is } T_{x_2}^1 \text{ THEN } y \text{ is } T_y^1.$$

$$\text{R2: IF } x_1 \text{ is } T_{x_1}^2 \text{ and } x_2 \text{ is } T_{x_2}^2 \text{ THEN } y \text{ is } T_y^2, \text{ etc.}$$

Then the firing strength of rules R1 and R2 are defined as  $p_1$  and  $p_2$ , respectively. For example,  $p_1$  is defined as

$$p_1 = M_{x_1}^1(x_1) \wedge M_{x_2}^1(x_2) = \min(M_{x_1}^1(x_1), M_{x_2}^1(x_2)). \tag{29}$$

According to the calculated firing strength of fuzzy rules, the fault with maximal probability is found, and the inference cycle is completed. The fuzzy-based order tracking system is schematically summarized in the flow chart of Fig. 5.

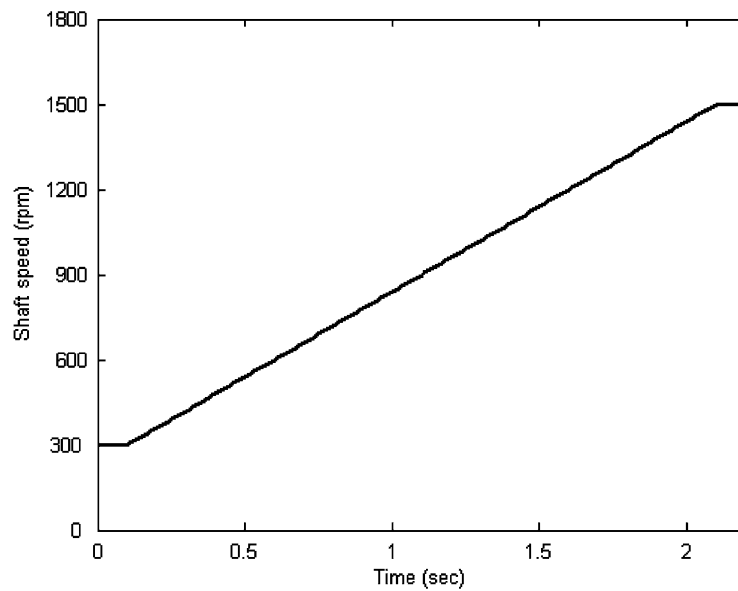


Fig. 7. Shaft speed schedule of the rotor run up test.

Table 1  
Fuzzy rules of single-fault for the rotor system

IF						THEN				
1X	2X	3X	4X	5X	9X	Norm	Mis	Los	Unbl	REB
Good	Good	Good	Good	Good	Good	X				
Alm2	Alm2	Alm1	Good	Good	Good		X			
Alm2	Alm2	Alm2	Alm2	Alm2	Good			X		
Alm1	Good	Good	Good	Good	Good				X	
Alm1	Alm1	Alm1	Fair	Fair	Alm1					X

Note: Good, good; Fair, fair; Alm1, alarm1 signifies “minor fault condition”; Alm2, alarm2 signifies “major fault condition”; Norm, normal; Mis, misalignment; Los, looseness; Unbl, unbalance; REB, rolling element bearing fault.

### 4. Experimental investigations

In order to verify the proposed intelligent order tracking system, experimental investigations were undertaken for a rotor kit and a car engine.

#### 4.1. Rotor kit

As a preliminary verification of the proposed system, a rotor kit was employed in experiments (Fig. 6). An AC servomotor was used to drive the rotor stand. Located at the middle of the axle

Table 2  
Fuzzy rules of double-fault for the rotor system

IF						THEN				
1X	2X	3X	4X	5X	9X	Norm	Mis	Los	Unbl	REB
Good	Good	Good	Good	Good	Good	X				
Alm2	Alm2	Alm2	Alm2	Alm2	Fair		X	X		
Alm2	Alm2	Alm1	Good	Good	Good		X		X	
Alm2	Alm2	Alm2	Fair	Fair	Alm2		X			X
Alm2	Alm2	Alm2	Alm2	Alm2	Alm1			X	X	
Alm2	Alm2	Alm2	Alm2	Alm2	Alm2			X		X
Alm2	Alm1	Alm1	Alm1	Alm1	Alm2				X	X

Note: Good, good; Fair, fair; Alm1, alarm1 signifies “minor fault condition”; Alm2, alarm2 signifies “major fault condition”; Norm, normal; Mis, misalignment; Los, looseness; Unbl, unbalance; REB, rolling element bearing fault.

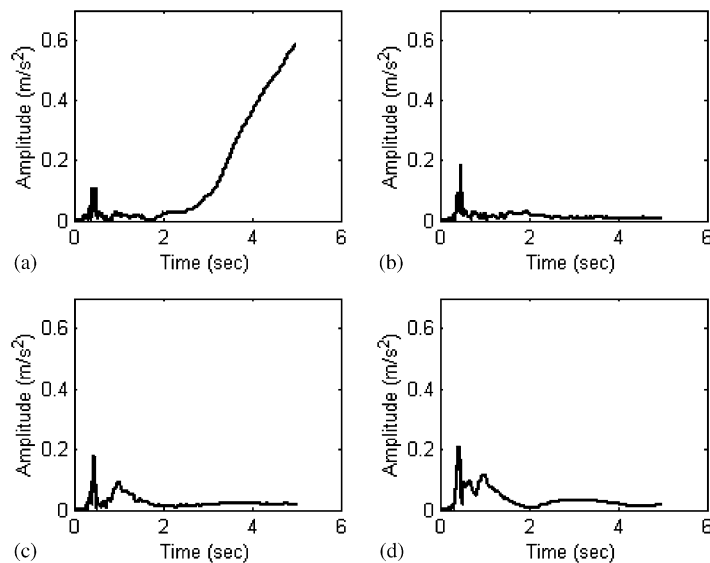


Fig. 8. Order amplitudes estimated in a rotor run up test, using the Kalman filter algorithm under the unbalance condition. (a) The 1st order; (b) the 2nd order; (c) the 3rd order; (d) the 4th order.

was an aluminum circular disc with weight stud to produce the unbalance fault. An accelerometer was mounted on bearing block. A tachometer was used to measure the rotor speed. The shaft speed and acceleration signals were sampled by using a digital signal processor (TMS320C32), and transferred to a Pentium 4 computer to perform order tracking and fault inference.

During a run-up test, the shaft speed is recorded and shown in Fig. 7. With reference to this speed information, the order amplitude was calculated using the Kalman filter order tracking technique. The rms extracted order amplitudes served as the input features to the fuzzy logic inference module. During the training stage, the membership function corresponding to each fault condition was also determined with the aid of the statistical analysis of the experimental data. In this work, our five rotor conditions were chosen: normal (Norm), unbalance (Unbl), looseness (Los), misalignment (Mis) and rolling element bearing damage (REB). The fuzzy rules are based on the following linguistic descriptions of rotor faults:

- Normal:* all harmonics have low amplitudes.
- Unbalance:* the 1st harmonic has very high amplitude.
- Looseness:* the 1st to 5th harmonics have high amplitudes.
- Misalignment:* the 1st and 2nd harmonics have high amplitudes.
- Ball bearing fault:* the 9th harmonic has very high amplitude.

Through these fault conditions, we further combine two single fault conditions into double-faults conditions. Thus, there are 12 conditions in a total, as summarized Tables 1 and 2 for the single fault and double fault conditions, respectively.

Figs. 8(a)–(d) show the amplitudes of the first four orders estimated using the Kalman filter algorithms under the unbalance condition. As expected, the first order has significantly higher

Table 3

Faults	<i>n</i>									
	2	3	4	6	8	10	12	14	16	18
<i>(a) Probability of inference using the triangular membership function under the unbalance condition</i>										
Norm	0	0	0	0	0	0	0.010	0.132	0.241	0.325
Unbl	<b>0.258</b>	<b>0.505</b>	<b>0.629</b>	<b>0.753</b>	<b>0.814</b>	<b>0.852</b>	<b>0.876</b>	<b>0.894</b>	<b>0.907</b>	<b>0.918</b>
Los	0	0	0	0.057	0.251	0.401	0.501	0.572	0.626	0.667
Mis	0	0	0	0	0	0.122	0.269	0.373	0.451	0.510
REB	0	0	0	0	0	0	0	0	0.075	0.178
<i>(b) Probability of inference using the π membership function with n = 4</i>										
Test faults	Confidence level of faults									
	Norm	Unbl	Los	Mis	REB					
Norm	<b>0.881</b>	0.023	0.053	0.102	0.032					
Unbl	0.095	<b>0.875</b>	0.321	0.172	0.068					
Los	0.151	0.123	<b>0.872</b>	0.165	0.052					
Mis	0	0	0.002	<b>0.905</b>	0.357					
REB	0	0	0.003	0.396	<b>0.882</b>					

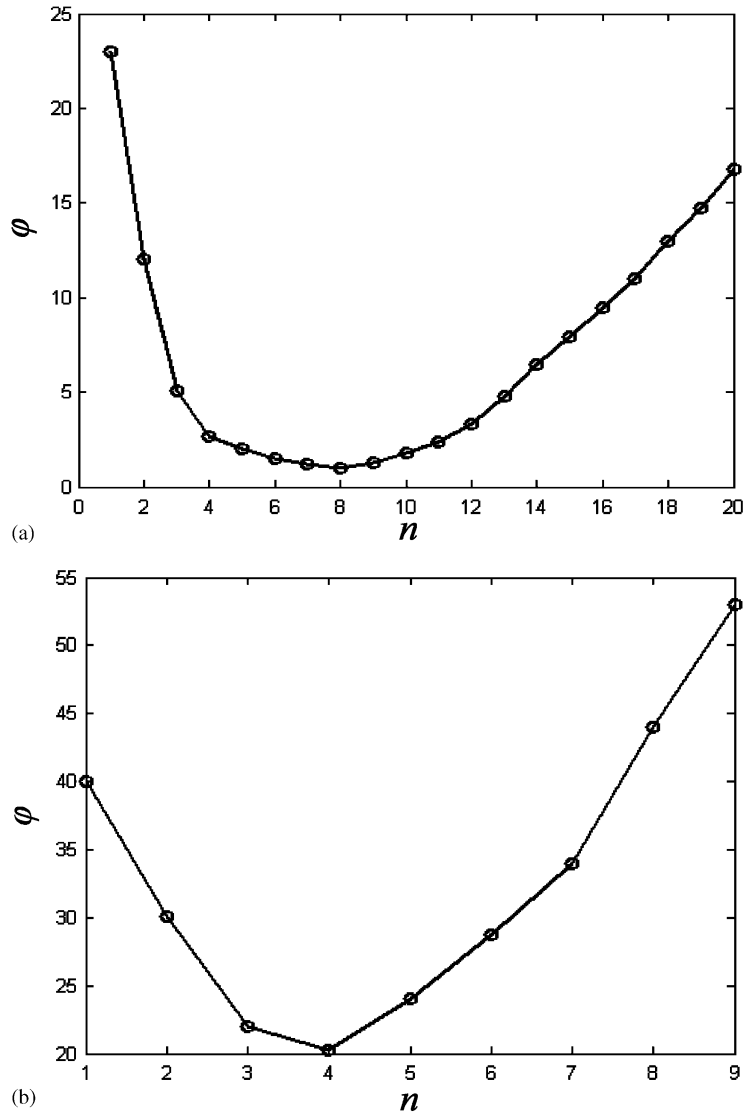


Fig. 9. Performance of fuzzy inference versus the parameter  $n$ : (a) Triangular membership function; (b)  $\pi$  membership function.

amplitude than the others. With the fuzzy inference, the performance indicated as probabilities of inference is shown in Table 3 for two kinds of membership functions. From Table 3(a), it can be seen that the probability of correct inference increases with increasing  $n$  when the triangular membership function is used. However, the probability of false classification also increases with increasing  $n$ . This suggests that an optimal choice of  $n$  exists in selecting the membership function. To assess the inference error versus  $n$ , the following performance index is calculated using the

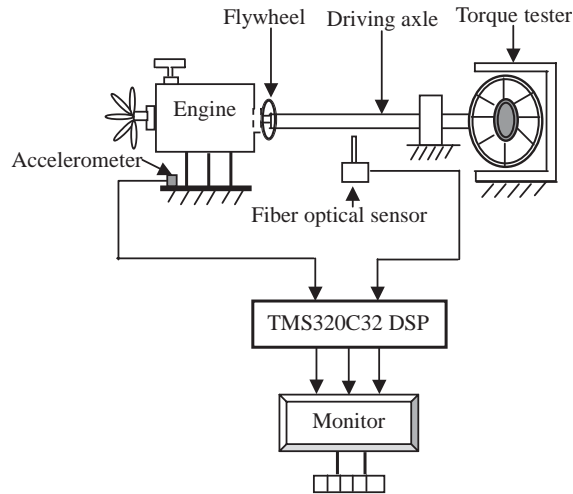


Fig. 10. Experimental arrangement of a Renault 1400 cc, four cylinder, four stroke car engine.

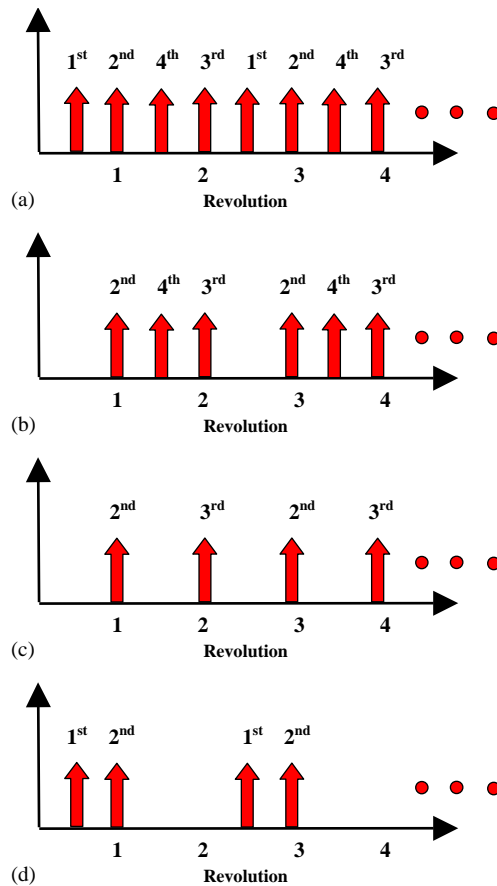


Fig. 11. Firing sequence of the car engine. (a) Normal condition; (b) the 1st plug fire missing; (c) the 1st and 4th plugs fire missing; (d) the 3rd and 4th plugs fire missing.

triangular membership function:

$$\varphi = \sum_{i=1}^m \sum_{j=1}^k (p_{ij} - \hat{p}_{ij})^2, \quad (30)$$

where  $p_{ij}$  is the probability of the  $i$ th test data that is inferred to the  $j$ th fault.  $\hat{p}_{ij}$  is the true probability of the  $i$ th test data pertaining to the  $j$ th fault. The parameters  $k$  and  $m$  are the numbers of fault and test pattern, respectively. This performance index versus  $n$  is shown in Fig. 9(a). There is indeed an optimal choice of  $n$  for the triangular membership function ( $n = 8$  in this case).

Similar performance analysis is applied to the  $\pi$  membership function. The performance index versus  $n$  plotted in Fig. 9(b) reveals that the optimal choice for the  $\pi$  membership function is  $n = 4$ . Thus, in Table 3(b), we calculate the probability of inference using the  $\pi$  membership function with  $n = 4$ . Overall, the probability of correct inference is significantly higher than the triangular membership function with  $n = 5$ . This is because the  $\pi$  membership function appears more capable of highlighting the similarity and discounting the dissimilarity of data features than the triangular membership function.

#### 4.2. Car engine

To verify the practicality of the present order tracking system, a Renault 1400 cc, four-cylinder and four-stroke engine was employed in the experimental investigation (Fig. 10). Fig. 11 depicts the firing sequences corresponding to the normal condition and the other three faulty conditions. The arrows signify which cylinders (indicated as number) that are ignited. The firing sequence under the normal condition is 1–2–4–3, where two power strokes arise within one revolution. The other three faulty conditions were selected as the 1st plug fire missing, the 1st and 4th plugs fire

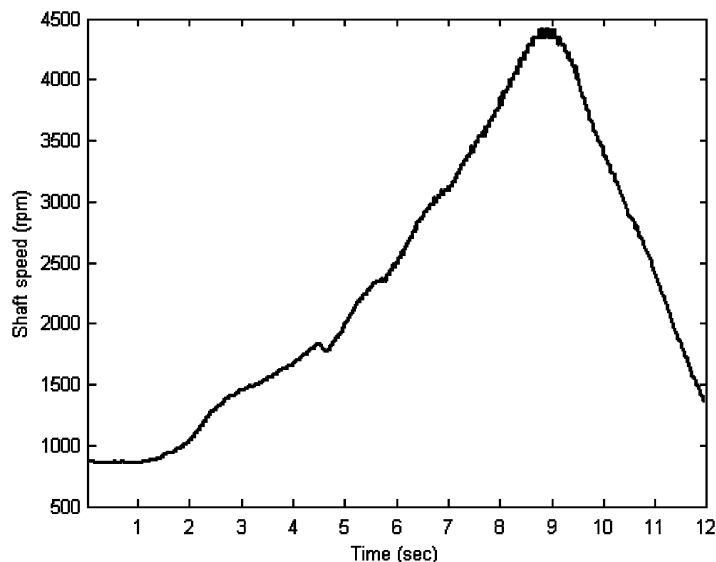


Fig. 12. Shaft speed schedule of the engine run up test.

missing, and the 2nd and 3rd plugs fire missing, respectively. These faults are created simply by disconnecting the spark plugs and the ignition system.

An engine run-up test under the normal condition was then conducted according to the schedule depicted in Fig. 12. The order amplitudes estimated by using the Kalman filter method is shown as a contour plot in Fig. 13(a). Clearly visible are the first four orders. Peaks corresponding to very dark areas seem to align as vertical lines, which suggests that resonance may exist. To see

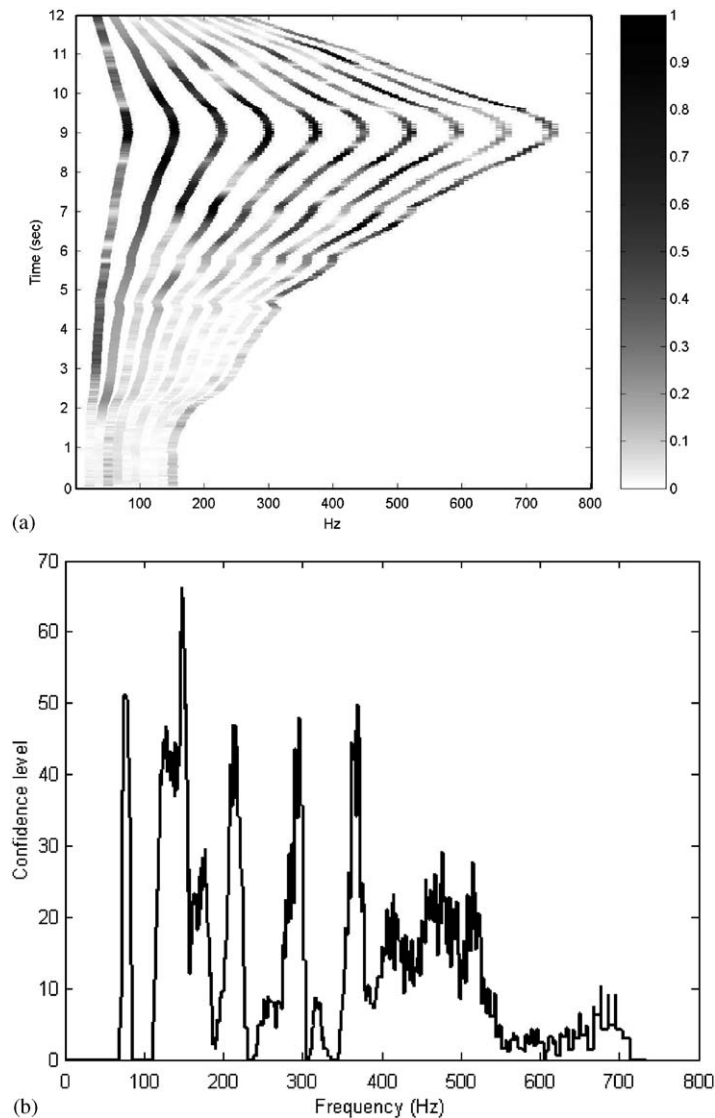


Fig. 13. Engine run up test with resonance analysis. (a) Contour plot of the order amplitudes estimated using the Kalman filter algorithm under the normal condition; (b) order amplitudes normalized with respect to its maximum, followed by superposition of all time slices of these normalized order onto the frequency axis. The most pronounced resonance frequencies are located approximately at 77, 125 and 148 Hz.



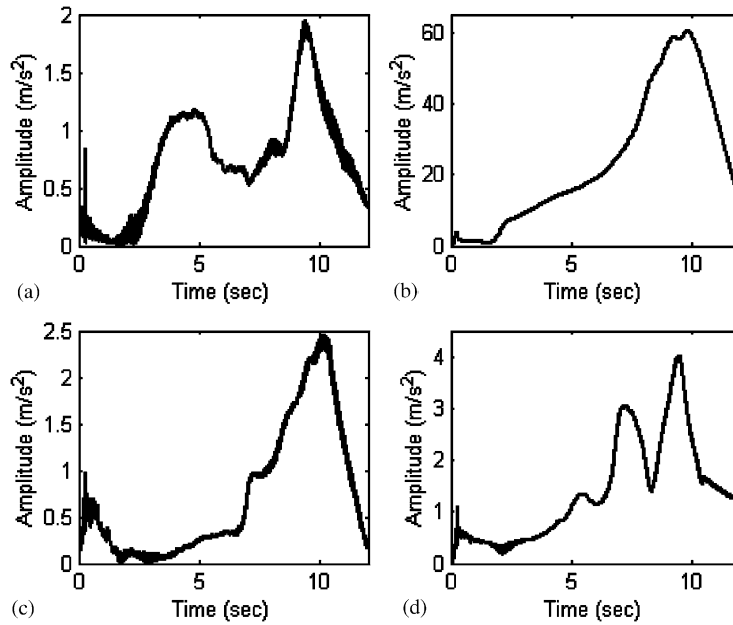


Fig. 14. Order amplitudes estimated in an engine run up test, using the Kalman filter algorithm under the normal condition. (a) The 1st order; (b) the 2nd order; (c) the 3rd order; (d) the 4th order.

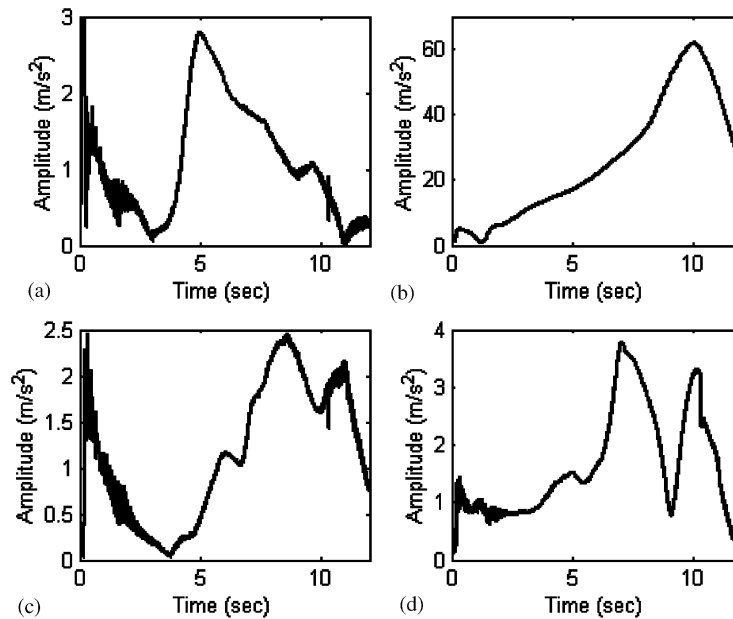


Fig. 15. Order amplitudes estimated in an engine run up test, using the Kalman filter algorithm under the 1st plug fire missing condition. (a) The 1st order; (b) the 2nd order; (c) the 3rd order; (d) the 4th order.

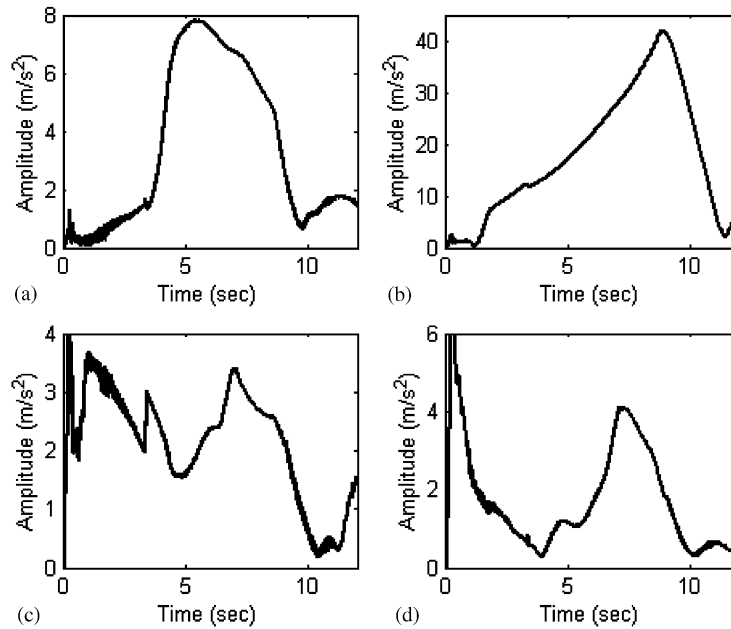


Fig. 16. Order amplitudes estimated in an engine run up test, using the Kalman filter algorithm under the 1st and 4th plugs fire missing condition. (a) The 1st order; (b) the 2nd order; (c) the 3rd order; (d) the 4th order.

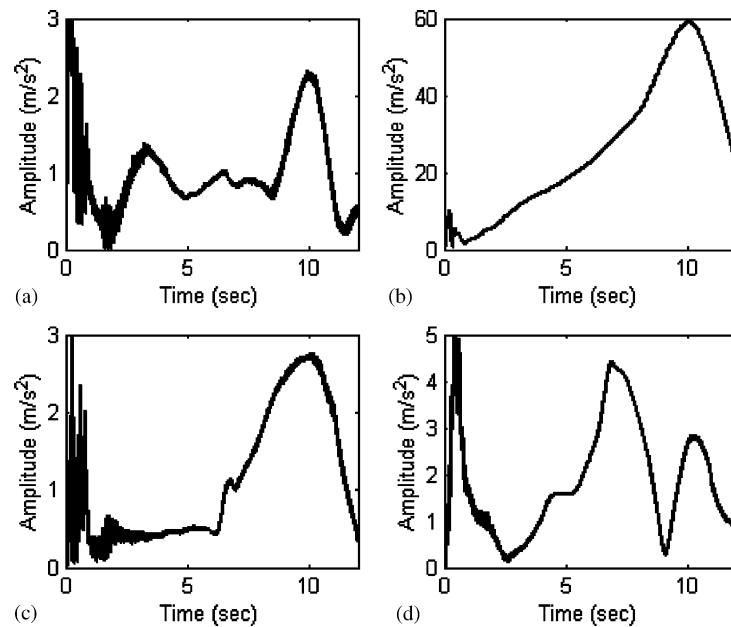


Fig. 17. Order amplitudes estimated in an engine run up test, using the Kalman filter algorithm under the 3rd and 4th plugs fire missing condition. (a) The 1st order; (b) the 2nd order; (c) the 3rd order; (d) the 4th order.

Table 4  
Fuzzy rules for the engine system

IF					THEN			
1X	2X	3X	4X	6X	Norm	1PFM	14PFM	34PFM
Reg	Reg	Reg	Reg	Reg	X			
SL	Reg	SL	SL	Reg		X		
Fair	Small	Fair	Fair	Reg			X	
Reg	Reg	Reg	Fair	Fair				X

Note: Reg, regular; Fair, fair; Small, too small; SL, somewhat large; Norm, normal; 1PFM, 1st plug fire missing; 14PFM, 1st, 4th plugs fire missing; 34PFM, 3rd, 4th plugs fire missing.

Table 5  
Probability of inference using the  $\pi$  membership function with  $n = 4$

	Norm	1PFM	14PFM	34PFM
Norm	<b>0.941</b>	0.297	0.198	0.240
1PFM	0.023	<b>0.945</b>	0.008	0.254
14PFM	0.009	0.030	<b>0.936</b>	0.015
34PFM	0.013	0.015	0.004	<b>0.939</b>

Note: Norm, normal; 1PFM, 1st plug fire missing; 14PFM, 1st, 4th plugs fire missing; 34PFM, 3rd, 4th plugs fire missing.

this better, each order is normalized with respect to its maximum, followed by superposition of all time slices of these normalized order onto the frequency axis. Fig. 13(b) shows the result obtained using such procedure. In this figure, the most pronounced resonance frequencies are located approximately at 77, 125 and 148 Hz.

Next, the effect of plug fire missing is investigated. The order amplitudes estimated by using the Kalman filter method for the normal condition and three faulty conditions are shown in Figs. 14–17, where (a) is the 1st order, (b) is the 2nd order, (c) is the 3rd order, and (d) is the 4th order. From these results, the following observations are noted. Under the normal condition, we see that the second orders tend to have large amplitudes due to the fact that each revolution contains two power strokes. The 1st and 4th plugs fire missing leads to large amplitudes in the first, third and fourth orders, and small amplitude in second order. The 3rd and 4th plugs fire missing leads to large amplitudes in the even multiple orders. On the basis of these observations, the fuzzy logic rules were established and summarized in Table 4. The test results of inference, indicated as probabilities, using the  $\pi$  membership function are summarized in Table 5. Probabilities of correct inference are all greater than 0.9. This justifies the effectiveness of the developed intelligent order tracking system when applied to practical machines such as a car engine.

### 5. Conclusions

An intelligent diagnostic system for rotating machinery has been presented in this paper. This system is capable of identifying the features potentially related to faults with higher resolution

than the conventional Fourier-based methods. The RLS algorithm and the Kalman filter method are exploited to extract the order features embedded in the vibration signals. Some technical refinements such as dynamic forgetting factors and AR process model are employed to make these adaptive filtering methods better suited for the present order tracking problems. Instead of using conventional human judgment on the order tracking results, an automated intelligent inference module based on fuzzy logic is utilized. This system is implemented on the platform of a DSP, where a photo switch and an accelerometer supply the shaft speed and acceleration signals, respectively. From experiments carried out for a rotor kit and a four-cylinder engine, the proposed system proved effective in tracking the rotating order with precise inference.

The future perspective of this work is to extend the present system to other machines with more variety of faults and signals. In addition, more sophisticated methods in the areas of speech and pattern recognition are being sought, in an attempt to enhance the feature extraction and intelligent inference.

### Acknowledgements

The work was supported by the National Science Council of Republic of China under the Grant NSC 91-2212-E009-032.

### References

- [1] J.E. Berry, Proven method for specifying both 6 spectral alarm bands as well as narrowband alarm envelopes using today's predictive maintenance software system, Bandid Technical Literature, Technical Associates of Charlotte, Inc. Charlotte, NC, 1993.
- [2] H. Vold, J. Leuridan, High resolution order tracking at extreme slew rates, using Kalman filters, The Society of Automotive Engineers No. 931288, 1993, pp. 219–226.
- [3] H. Vold, M. Mains, J. Blough, Theoretical foundations for high performance order tracking with the Vold–Kalman tracking filter, The Society of Automotive Engineers No. 972007, 1997, pp. 1083–1088.
- [4] M.C. Pan, Y.Y. Lin, Dynamic signal processing of rotary machines using Vold–Kalman filtering order tracking, *Proceedings of IEEE/ASME International Conference on Advanced Manufacturing Technologies and Education in the 21st Century*, Vol. C177, Chia-Yi, Taiwan, 2002.
- [5] C.K. Mechefske, Objective machinery fault diagnosis using fuzzy logic, *Mechanical Systems and Signal Processing* 12 (1998) 855–862.
- [6] G. Nikos, T. Pantelelis, A.E. Kanarachos, N. Gotzias, Neural networks and simple methods for the fault diagnosis of naval turbochargers, *Mathematics and Computers in Simulation* 51 (2000) 387–397.
- [7] S. Haykin, *Adaptive Filter Theory*, Prentice-Hall, Englewood Cliffs, NJ, 1986.
- [8] M.R. Bai, J. Jeng, C. Chen, Adaptive order tracking technique using recursive least-square algorithm, *American Society of Mechanical Engineers, Journal of Vibrations and Acoustics* 124 (2002) 502–511.
- [9] M.R. Bai, J. Jeng, C. Chen, Adaptive order tracking technique using recursive Kalman filtering, *American Society of Mechanical Engineers, Journal of Vibrations and Acoustics* 125 (2002) 202–210.
- [10] C.T. Lin, *Neural Fuzzy Systems*, Prentice-Hall, Englewood Cliffs, NJ, 1996.
- [11] H.J. Zimmermann, U. Thole, On the suitability of minimum and product operators for the intersection of fuzzy sets, *Fuzzy Sets and Systems* 2 (1992) 173–186.
- [12] H.P. Chen, T.M. Parng, A new approach of multi-stage fuzzy logic inference, *Fuzzy Sets and Systems* 78 (1996) 51–72.
- [13] C.T. Lin, F.B. Duh, D.J. Liu, A neural network for word information processing, *Fuzzy Sets and Systems* 127 (2002) 37–48.