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Moving load response of micropolar elastic half-space with voids

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Abstract

The steady state response of a semi-infinite micropolar elastic medium with voids at the free surface is determined. The analytic expressions of the displacement components, force stress, couple stress and volume fraction field are obtained by the use of Fourier transform technique and the numerical results are illustrated graphically for magnesium crystal-like material. Special cases have been deduced.

1. Introduction

The theory of linear elastic materials with voids is one of the most important generalizations of the classical theory of elasticity. This theory has practical utility in investigating various types of geological, biological and synthetic porous materials for which the elastic theory is inadequate. This theory is concerned with elastic materials consisting of a distribution of small pores (voids), in which the void volume is included among the kinematic variables and in the limiting case of vanishing this volume, the theory reduces to the classical theory of elasticity.

A non-linear theory of elastic materials with voids was developed by Nunziato and Cowin [1]. Later Cowin and Nunziato [2] developed a theory of linear elastic materials with voids, for the mathematical study of the mechanical behavior of porous solids. They considered several applications of the linear theory by investigating the response of the materials to homogeneous deformations, pure bending of a beam and small amplitudes acoustic waves. Iesan [3] derived the basic equations of micropolar elastic materials with voids. Different authors [4–9] have discussed

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different type of problems in the theory of elastic materials with voids. Marin [10–13] discussed different type of problems in micropolar theory of elastic solid with voids. Scarpetta [14] studied the fundamental solutions in micropolar elasticity with voids.

The dynamical response to moving loads is an interesting subject in various technological and geophysical circumstances and some recent investigations are concerned with this problem. For instance, it is of great interest in solid dynamics where ground motions and stresses can be produced by blast waves (surface pressure waves due to explosions), or by supersonic aircraft. This type of investigation occur in many branches of engineering, for e.g., in bridges and railways, beams subjected to pressure waves and piping systems subjected to two-phase flow. Other applications are encountered within the context of contact mechanics like, the problem of high-velocity rocket sleds sliding over steel guide rails. Most of the moving load problems solved so far involve potential functions. However, the eigenvalue approach has the advantage that the solutions of equations are found in the coupled form directly in the matrix notations, whereas the potential functions approach requires decoupling of equations. Various authors [15–22] have discussed the problems of moving load in the theory of elastic solids. Kumar and Gogna [23] and Kumar and Deswal [24] studied the steady state response to moving loads in micropolar theory of elasticity.

The purpose of the present paper is to determine the normal displacement, normal force stress, tangential couple stress and volume fraction field in a micropolar elastic solid with voids due to moving point load by applying integral transform technique. Numerical calculations are used to invert the Fourier transform. Application of the paper may be found in mechanics viz. in designing highways and airport runaways.

2. Formulation and solution of the problem

We consider a normal point load of magnitude F moving over the free surface of micropolar half-space with voids. The rectangular Cartesian co-ordinate are introduced having origin on the surface $z = 0$ and z-axis pointing vertically into the medium. Let us consider a pressure pulse $F(x + Ut)$ which is moving with a constant speed in the negative x direction for an infinite long time so that a steady state prevails in the neighborhood of the loading as seen by an observer moving with the load.

Following Iesan [3], the field equations and stress–strain relations in micropolar elastic solid with voids without body forces and body couples can be written as

$$(\lambda + \mu)\nabla(\nabla \cdot \mathbf{u}) + (\mu + K)\nabla^2\mathbf{u} + K(\nabla \times \boldsymbol{\phi}) + \beta^*\nabla\psi = \rho\frac{\partial^2\mathbf{u}}{\partial t^2}, \tag{1}$$

$$(\alpha + \beta + \gamma)\nabla(\nabla \cdot \boldsymbol{\phi}) - \gamma\nabla \times (\nabla \times \boldsymbol{\phi}) + K(\nabla \times \mathbf{u}) - 2K\boldsymbol{\phi} = \rho j\frac{\partial^2\boldsymbol{\phi}}{\partial t^2}, \tag{2}$$

$$\alpha^*\nabla^2\psi - \varsigma^*\psi - \omega^*\frac{\partial\psi}{\partial t} - \beta^*\nabla \cdot \mathbf{u} = \rho\zeta^*\frac{\partial^2\psi}{\partial t^2}, \tag{3}$$

$$t_{ij} = \lambda u_{r,r} + \mu(u_{i,j} + u_{j,i}) + K(u_{j,i} - \varepsilon_{ijr}\phi_r) + \beta^*\psi\delta_{ij}, \tag{4}$$

$$m_{ij} = \alpha\phi_{r,r}\delta_{ij} + \beta\phi_{i,j} + \gamma\phi_{j,i}, \tag{5}$$

where $\alpha, \beta, \gamma, \lambda, \mu, K$ are the material constants, j is the microinertia, ρ is the density of solid, $\mathbf{u}$ is the displacement vector, $\boldsymbol{\phi}$ is the microinertia, ψ is the volume fraction field, t_{ij} is the force stress, m_{ij} is the couple stress and $\alpha^*, \beta^*, \varsigma^*, \omega^*$ and ζ^* are the material constant due to the presence of voids.

For two-dimensional problem, we assume

$$\mathbf{u} = (u_1, 0, u_3), \qquad \boldsymbol{\phi} = (0, \phi_2, 0). \tag{6}$$

Following Fung [25], a Galilean transformation

$$x^* = x + Ut, \qquad z^* = z, \qquad t^* = t \tag{7}$$

is introduced. The boundary conditions would be independent of t^* and assuming the dimensionless variables defined by

$$\begin{aligned} &x' = \frac{\omega}{c_1}x^*, \qquad z' = \frac{\omega}{c_1}z^*, \qquad \phi_2' = \frac{\omega^2}{c_1^2}j\phi_2, \qquad u_1' = \frac{\omega}{c_1}u_1, \\ &u_3' = \frac{\omega}{c_1}u_3, \qquad t_{ij}' = \frac{t_{ij}}{\lambda}, \qquad m_{ij}' = \frac{\omega}{c_1\lambda}m_{ij}, \qquad \psi' = \frac{\omega^2}{c_1^2}j\psi, \end{aligned} \tag{8}$$

where

$$c_1^2 = \frac{\lambda + 2\mu + K}{\rho} \quad \text{and} \quad \omega^2 = \frac{K}{\rho j}.$$

In Eqs. (1)–(3) we get (after suppressing the primes)

$$(\lambda + \mu)\frac{\partial}{\partial x}\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}\right) + (\mu + K)\nabla^2 u_1 - \frac{Kc_1^2}{\omega^2 j}\frac{\partial \phi_2}{\partial z} + \frac{\beta^* c_1^2}{\omega^2 j}\frac{\partial \psi}{\partial x} = \rho U^2\frac{\partial^2 u_1}{\partial x^2}, \tag{9}$$

$$(\lambda + \mu)\frac{\partial}{\partial z}\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}\right) + (\mu + K)\nabla^2 u_3 - \frac{Kc_1^2}{\omega^2 j}\frac{\partial \phi_2}{\partial x} + \frac{\beta^* c_1^2}{\omega^2 j}\frac{\partial \psi}{\partial z} = \rho U^2\frac{\partial^2 u_3}{\partial x^2}, \tag{10}$$

$$\frac{\gamma}{j}\nabla^2\phi_2 + K\left(\frac{\partial u_1}{\partial z} - \frac{\partial u_3}{\partial x}\right) - \frac{2Kc_1^2}{\omega^2 j}\phi_2 = \rho U^2\frac{\partial^2 \phi_2}{\partial x^2}, \tag{11}$$

$$\frac{\alpha^*}{j}\nabla^2\psi - \frac{\varsigma^* c_1^2}{\omega^2 j}\psi - \frac{\omega^* U c_1}{\omega j}\frac{\partial \psi}{\partial x} - \beta^*\left(\frac{\partial u_1}{\partial x} + \frac{\partial u_3}{\partial z}\right) = \frac{\rho\zeta^* U^2}{j}\frac{\partial^2 \psi}{\partial x^2}. \tag{12}$$

Applying Fourier transform defined by

$$\tilde{f}(\xi, z) = \int_{-\infty}^{\infty} f(x, z)e^{i\xi x}\,dx \tag{13}$$

in Eqs. (9)–(12) we obtain (where primes denotes differentiation with respect to z)

$$\tilde{u}_1'' = a_{11}\xi^2\tilde{u}_1 + i\xi a_{13}\tilde{u}_3' + a_{14}\tilde{\phi}_2' + i\xi a_{12}\tilde{\psi}, \tag{14}$$

$$\tilde{u}_3'' = \xi^2 a_{21}\tilde{u}_3 + i\xi a_{23}\tilde{u}_1' + i\xi a_{22}\tilde{\phi}_2 + a_{24}\tilde{\psi}, \tag{15}$$

$$\tilde{\phi}_2'' = (\xi^2 a_{32} - a_{33})\tilde{\phi}_2 - a_{31}\tilde{u}_1' + \mathrm{i}\xi a_{31}\tilde{u}_3, \tag{16}$$

$$\tilde{\psi}'' = (\xi^2 a_{42} - a_{43})\tilde{\psi} + a_{41}\tilde{u}_1 - a_{41}\tilde{u}_3', \tag{17}$$

where

$$a_{11} = \frac{(\lambda + 2\mu + K) - \rho U^2}{\mu + K}, \quad a_{12} = \frac{\beta^* c_1}{\omega j(\mu + K)}, \quad a_{21} = \frac{(\mu + K) - \rho U^2}{\lambda + 2\mu + K},$$
$$a_{22} = \frac{K c_1^2}{\omega^2 j(\lambda + 2\mu + K)}, \quad a_{13} = \frac{\lambda + \mu}{\mu + K}, \quad a_{33} = \frac{2K c_1^2}{\omega^2 j}, \quad a_{32} = \frac{\gamma - \rho U^2 j}{\gamma},$$
$$a_{14} = \frac{K c_1^2}{\omega^2 j(\mu + K)}, \quad a_{23} = \frac{\lambda + \mu}{\lambda + 2\mu + K}, \quad a_{24} = -\frac{\beta^* c_1^2}{\omega^2 j(\lambda + 2\mu + K)},$$
$$a_{31} = \frac{Kj}{\gamma}, \quad a_{42} = \frac{\alpha^* - \rho U^2 \zeta^*}{\alpha^*}, \quad a_{41} = -\frac{\beta^* j}{\alpha^*}, \quad a_{43} = \frac{\mathrm{i}\xi\omega\omega^* U c_1 - \varsigma^* c_1^2}{\alpha^* \omega^2}, \tag{18}$$

The set of equations (14)–(17) may be written as

$$\frac{\mathrm{d}}{\mathrm{d}z} W(\xi, z) = A(\xi) W(\xi, z), \tag{19}$$

where

$$W = \begin{pmatrix} V \\ V' \end{pmatrix}, \quad A = \begin{pmatrix} O & I \\ A_1 & A_2 \end{pmatrix}, \quad V = \begin{pmatrix} \tilde{u}_1 \\ \tilde{u}_3 \\ \tilde{\phi}_2 \\ \tilde{\psi} \end{pmatrix}, \tag{20}$$

$$A_1 = \begin{pmatrix} \xi^2 a_{11} & 0 & 0 & \mathrm{i}\xi a_{12} \\ 0 & \xi^2 a_{21} & \mathrm{i}\xi a_{22} & 0 \\ 0 & \mathrm{i}\xi a_{31} & \xi^2 a_{32} - a_{33} & 0 \\ \mathrm{i}\xi a_{41} & 0 & 0 & \xi^2 a_{42} - a_{43} \end{pmatrix}, \tag{21}$$

$$A_2 = \begin{pmatrix} 0 & \mathrm{i}\xi a_{13} & a_{14} & 0 \\ \mathrm{i}\xi a_{23} & 0 & 0 & a_{24} \\ -a_{31} & 0 & 0 & 0 \\ 0 & -a_{41} & 0 & 0 \end{pmatrix} \tag{22}$$

and O and I are zero and identity matrices of the order 4.

To solve Eq. (19), we assume

$$W(\xi, z) = X(\xi)\mathrm{e}^{qz} \tag{23}$$

which leads to an eigenvalue problem. The characteristic equation corresponding to matrix A is given by

$$|A - qI| = 0. \tag{24}$$

This gives

$$q^8 + \lambda_1 q^6 + \lambda_2 q^4 + \lambda_3 q^2 + \lambda_4 = 0, \tag{25}$$

where

$$\begin{aligned}
\lambda_1 &= \xi^2(a_{13}a_{23} - a_{42} - a_{32} - a_{11} - a_{21}) - a_{24}a_{41} + a_{14}a_{31} + a_{43} + a_{33},\\
\lambda_2 &= \xi^4\eta_1 + \xi^2\eta_2 + \eta_3,\\
\lambda_3 &= \xi^6\eta_4 + \xi^4\eta_5 + \xi^2\eta_6,\\
\lambda_4 &= \xi^8\eta_7 + \xi^6\eta_8 + \xi^4\eta_9,\\
\eta_1 &= a_{11}(a_{42} + a_{21} + a_{32}) + (a_{32} + a_{42})(a_{21} - a_{13}a_{23}) + a_{32}a_{42},\\
\eta_2 &= (a_{13}a_{23} - a_{11} - a_{21})(a_{33} + a_{43}) - a_{14}a_{31}(a_{21} - a_{23}) - a_{41}a_{24}(a_{11} - a_{13})\\
&\quad - a_{31}(a_{14}a_{42} + a_{22}a_{13}) - a_{41}(a_{12}a_{23} + a_{24}a_{32}) + a_{31}a_{22} + a_{41}a_{12} - a_{32}a_{43} - a_{33}a_{42},\\
\eta_3 &= (a_{33} + a_{31}a_{14})(a_{43} + a_{41}a_{24}),\\
\eta_4 &= a_{32}(a_{23}a_{13}a_{42} - a_{21}a_{42} - a_{11}a_{42} - a_{11}a_{21}),\\
\eta_5 &= a_{31}a_{42}(a_{13}a_{22} + a_{14}a_{21} - a_{14}a_{23}) + a_{23}a_{32}(a_{12}a_{41} - a_{13}a_{43}) + a_{41}a_{32}a_{24}(a_{11} - a_{13})\\
&\quad + (a_{11} + a_{21})(a_{32}a_{43} + a_{33}a_{42}) - a_{23}a_{13}a_{33}a_{42} - a_{21}(a_{11}a_{42} + a_{12}a_{41} - a_{11}a_{33})\\
&\quad - a_{31}a_{22}(a_{11} + a_{42}) - a_{12}a_{41}a_{32},\\
\eta_6 &= (a_{22}a_{31} - a_{23}a_{33})(a_{12}a_{41} - a_{13}a_{43}) - a_{14}a_{31}(a_{43}a_{21} + a_{43}a_{23} - a_{41}a_{24})\\
&\quad - a_{33}a_{41}a_{24}(a_{11} - a_{13}) - a_{21}a_{43}(a_{33} - a_{11}) + a_{33}(a_{11}a_{43} + a_{12}a_{41}) + a_{31}a_{22}a_{43},\\
\eta_7 &= a_{11}a_{21}a_{42}a_{32}, \qquad \eta_8 = a_{11}a_{31}a_{22}a_{42} + a_{41}a_{12}a_{21}a_{32} - a_{11}a_{21}(a_{42}a_{33} + a_{43}a_{32}),\\
\eta_9 &= (a_{11}a_{43} - a_{41}a_{12})(a_{21}a_{33} - a_{31}a_{22}),
\end{aligned} \tag{26}$$

The eigenvalues of matrix A are characteristic roots of Eq. (25). The vectors $X(\xi)$ corresponds to the eigenvalues q_s can be determined by solving the homogeneous equation

$$[A - qI]X(\xi) = 0. \tag{27}$$

The set of eigenvectors $X_s(\xi)$, $s = 1, 2, \ldots, 8$ are obtained as

$$X_s(\xi) = \begin{pmatrix} X_{s1}(\xi) \\ X_{s2}(\xi) \end{pmatrix}, \tag{28}$$

where

$$X_{s1}(\xi) = \begin{pmatrix} \xi \\ \mathrm{i}b_s q_s \\ \xi a_s q_s \\ \mathrm{i}d_s \end{pmatrix}, \qquad X_{s2}(\xi) = \begin{pmatrix} \xi q_s \\ \mathrm{i}b_s q_s^2 \\ \xi a_s q_s^2 \\ \mathrm{i}d_s q_s \end{pmatrix}, \qquad q = q_s, \quad s = 1, 2, 3, 4, \tag{29}$$

$$X_{j1}(\xi)=\begin{pmatrix}\xi\\ -\mathrm{i}b_jq_j\\ -\xi a_jq_j\\ \mathrm{i}d_j\end{pmatrix},\qquad X_{j2}(\xi)=\begin{pmatrix}-\xi q_j\\ \mathrm{i}b_jq_j^2\\ \xi a_jq_j^2\\ -\mathrm{i}d_jq_j\end{pmatrix},\qquad j=s+3,\ \ q=-q_s \tag{30}$$

and

$$\begin{aligned}a_s=&\,a_{12}\left[q_s^2a_{24}-q_s^2a_{24}\left(a_{11}+\frac{a_{12}a_{23}}{a_{24}}+a_{32}+a_{33}+a_{31}a_{14}\right)+\xi^4a_{32}(a_{11}a_{24}+a_{12}a_{23})\right.\\&\left.+\ \xi^2(a_{31}a_{12}a_{22}-a_{33}a_{11}a_{24}-a_{33}a_{12}a_{23})\right]/\nabla,\\ b_s=&\,a_{12}a_{31}[\xi^2(a_{11}a_{24}+a_{12}a_{23}-a_{12}a_{21})+q_s^2(a_{24}a_{31}+a_{12}-a_{24})]/\nabla,\\ d_s=&\,\frac{a_{41}(q_s^2b_s-\xi^2)}{\xi^2a_{42}-a_{43}-q_s^2},\\ \nabla=&\,-q_s^4(a_{24}a_{31}+a_{12})+q_s^2[(a_{24}a_{31}+a_{12})(\xi^2a_{32}-a_{33})-a_{31}a_{14}a_{24}+\xi^2a_{12}a_{21}]\\&+\ \xi^2a_{12}a_{21}(a_{33}-\xi^2a_{32})-\xi^2a_{31}a_{12}a_{22}.\end{aligned} \tag{31}$$

The solution of Eq. (23) is given by

$$W(\xi,z)=\sum_{s=1}^{4}[B_sX_s(\xi)\exp(q_sz)+B_{s+4}X_{s+4}(\xi)\exp(-q_sz)]. \tag{32}$$

3. Boundary conditions

For a concentrated line load, we take $F(x+Ut)=F\delta(x^*)$, where $\delta(x^*)$ is Dirac delta function and F is the magnitude of force applied, therefore in moving co-ordinates the boundary conditions at the free surface $z=0$ are

$$t_{33}=-F\delta(x^*),\quad t_{31}=m_{32}=\frac{\partial\psi}{\partial z}=0. \tag{33}$$

Applying Fourier transform defined by Eq. (13) in the boundary conditions (33) and using Eqs. (4)–(8) and (32), we obtain the expressions for displacement components, force stress, couple stress and volume fraction field for micropolar elastic solid with voids as

$$\tilde{u}_3=\frac{F\mathrm{i}}{\Delta}[a_1q_1\Delta_1'\mathrm{e}^{-q_1z}-a_2q_2\Delta_2'\mathrm{e}^{-q_2z}+a_3q_3\Delta_3'\mathrm{e}^{-q_3z}-a_4q_4\Delta_4'\mathrm{e}^{-q_4z}], \tag{34}$$

$$\tilde{t}_{33}=-\frac{F}{\Delta}[r_1\Delta_1'\mathrm{e}^{-q_1z}-r_2\Delta_2'\mathrm{e}^{-q_2z}+r_3\Delta_3'\mathrm{e}^{-q_3z}-r_4\Delta_4'\mathrm{e}^{-q_4z}], \tag{35}$$

$$\tilde{m}_{32}=-\frac{F\gamma\xi}{j\lambda\Delta}[b_1q_1^2\Delta_1'\mathrm{e}^{-q_1z}-b_2q_2^2\Delta_2'\mathrm{e}^{-q_2z}+b_3q_3^2\Delta_3'\mathrm{e}^{-q_3z}-b_4q_4^2\Delta_4'\mathrm{e}^{-q_4z}], \tag{36}$$

$$\tilde{\psi} = -\frac{Fi}{\Delta}[d_1\Delta_1' e^{-q_1 z} - d_2\Delta_2' e^{-q_2 z} + d_3\Delta_3' e^{-q_3 z} - d_4\Delta_4' e^{-q_4 z}], \tag{37}$$

where

$$\begin{aligned}
&\Delta = q_3q_4\tau_1f_1 - q_2q_4\tau_2f_2 + q_2q_3\tau_3f_3 - q_1q_4\tau_4f_4 + q_1q_3\tau_5f_5 - q_1q_2\tau_6f_6,\\
&\tau_{1,2,3} = r_1t_{2,3,4} - r_{2,3,4}t_1, \qquad \tau_{4,5,6} = r_{3,4,4}t_{2,2,3} - r_{2,2,3}t_{3,4,4},\\
&f_{1,2,3} = b_{3,2,2}q_{3,2,2}d_{4,4,3} - b_{4,4,3}q_{4,4,3}d_{3,2,2}, \qquad f_{4,5,6} = b_1q_1d_{4,3,2} - b_{4,3,2}q_{4,3,2}d_1,\\
&\Delta_1' = t_2q_3q_4f_1 - t_3q_2q_4f_2 + t_4q_2q_3f_3, \qquad \Delta_2' = t_1q_3q_4f_1 - t_3q_1q_4f_4 + t_4q_1q_3f_5,\\
&\Delta_3' = t_1q_2q_4f_2 - t_2q_1q_4f_4 + t_4q_1q_2f_6, \qquad \Delta_4' = t_1q_2q_3f_3 - t_2q_1q_3f_5 + t_3q_1q_2f_6,\\
&r_n = i\left[-\xi^2 + \left(\frac{\lambda+2\mu+K}{\lambda}\right)b_nq_n^2 + \frac{\beta^* c_1^2}{\omega^2 j\lambda}d_n\right],\\
&t_n = -\xi q_n\left[\mu b_n + (\mu+K) - \frac{Kc_1^2}{\omega^2 j}a_n\right], \quad n = 1,2,3,4.
\end{aligned} \tag{38}$$

4. Particular case

Neglecting the material constants due to the presence of voids (i.e., $\alpha^* = \beta^* = \zeta^* = \varsigma^* = \omega^* = 0$), we obtain the expressions for normal displacement, force stress and couple stress for a micropolar elastic solid as

$$\tilde{u}_3 = -\frac{Fi\xi}{\Delta_0}[b_1'\Delta_{10}' e^{-p_1 z} - b_2'\Delta_{20}' e^{-p_2 z} + b_3'\Delta_{30}' e^{-p_3 z}], \tag{39}$$

$$\tilde{t}_{33} = -\frac{Fi\xi}{\Delta_0}[s_1'\Delta_{10}' e^{-p_1 z} - s_2'\Delta_{20}' e^{-p_2 z} + s_3'\Delta_{30}' e^{-p_3 z}], \tag{40}$$

$$\tilde{m}_{32} = \frac{F\gamma}{j\lambda\Delta_0}[a_1'p_1\Delta_{10}' e^{-p_1 z} - a_2'p_2\Delta_{20}' e^{-p_2 z} + a_3'p_3\Delta_{30}' e^{-p_3 z}], \tag{41}$$

where

$$\begin{aligned}
&\Delta_0 = s_1'\Delta_{10}' - s_2'\Delta_{20}' + s_3'\Delta_{30}', \qquad \Delta_{10}' = r_2'a_3'p_3 - r_3'a_2'p_2,\\
&\Delta_{20}' = r_1'a_3'p_3 - r_3'a_1'p_1, \qquad \Delta_{30}' = r_1'a_2'p_2 - r_2'a_1'p_1,\\
&s_\Theta' = i\xi\left[p_\Theta - \left(\frac{\lambda+2\mu+K}{\lambda}\right)b_\Theta' p_\Theta\right], \qquad r_\Theta' = \mu\xi^2 b_\Theta' + (\mu+K)p_\Theta^2 - \frac{Kc_1^2}{\omega^2 j}a_\Theta',\\
&d_\Theta' = -\frac{p_\Theta^4 - p_\Theta^2\xi^2(a_{13}a_{23} - a_{21} - a_{11}) - \xi^4 a_{11}a_{21}}{\nabla_0},\\
&b_\Theta' = \frac{\xi^2 a_{11}a_{22} - p_\Theta^2(a_{22} + a_{14}a_{23})}{\nabla_0}, \qquad \nabla_0 = \xi^2(a_{13}a_{22} + a_{14}a_{21}) - p_\Theta^2 a_{14}, \quad \Theta = 1,2,3.
\end{aligned} \tag{42}$$

The eigenvalues $\pm p_\Theta$ ($\Theta = 1,2,3$) for a micropolar elastic solid are given by the equation

$$p^6 + \lambda_1' p^4 + \lambda_2' p^2 + \lambda_3' = 0, \tag{43}$$

where

$$\lambda_1' = \xi^2(a_{13}a_{23} - a_{11} - a_{21} - a_{32}) + a_{14}a_{31} - a_{33},$$
$$\lambda_2' = \xi^4[a_{11}a_{21} + a_{32}(a_{11} + a_{21} - a_{13}a_{23})] + \xi^2[-a_{31}(a_{22}a_{13} + a_{21}a_{14} + a_{14}a_{23}) + a_{33}(a_{11} + a_{21} - a_{13}a_{23})] - a_{22}a_{31}$$

and

$$\lambda_3' = -\xi^4 a_{11}[a_{21}(\xi^2 a_{32} + a_{33}) - a_{22}a_{31}]. \tag{44}$$

Inversion of the transform: To obtain the solution of the problem in the physical domain, we must invert the transform in Eqs. (34)–(37) and (39)–(41). These expressions are functions of z and the parameter of Fourier transform ξ, hence are of the form $\tilde{f}(\xi, z)$. To get the function $f(x, z)$ in the physical domain, we invert the Fourier transform using

$$f(x, z) = \frac{1}{2\pi}\int_{-\infty}^{\infty} \tilde{f}(\xi, z)\mathrm{e}^{-\mathrm{i}\xi x}\,\mathrm{d}\xi \tag{45}$$

$$= \frac{1}{2\pi}\int_{-\infty}^{\infty} [\mathrm{Cos}(\xi x)f_e - \mathrm{i}\,\mathrm{Sin}(\xi x)f_o]\,\mathrm{d}\xi, \tag{46}$$

where f_e and f_o are, respectively, even and odd parts of the function $\tilde{f}(\xi, z)$. The method for evaluating this integral is described by Press et al. [26] which involves the use of Rhomberg's integration with adaptive step size. This also uses the results from successive refinements of the extended trapezoidal rule followed by extrapolation of the results to the limit when the step size tends to zero.

5. Numerical results and discussions

We take magnesium crystal-like material [27] as micropolar elastic solid with voids,

$$\rho = 1.74\ \mathrm{gm/cm^3}, \quad \lambda = 9.4 \times 10^{11}\ \mathrm{dyn/cm^2}, \quad \mu = 4.0 \times 10^{11}\ \mathrm{dyn/cm^2},$$

$$K = 1.0 \times 10^{11}\ \mathrm{dyn/cm^2}, \quad \gamma = 0.779 \times 10^{-4}\ \mathrm{dyn}, \quad j = 0.2 \times 10^{-15}\ \mathrm{cm^2},$$

$$\alpha^* = 3.668 \times 10^{-4}\ \mathrm{dyn}, \quad \beta^* = 1.13849 \times 10^{11}\ \mathrm{dyn/cm^2}, \quad \varsigma^* = 1.475 \times 10^{11}\ \mathrm{dyn/cm^2},$$

$$\omega^* = 0.0787 \times 10^{-2}\ \mathrm{dyn\ s/cm^2}, \quad \zeta^* = 1.753 \times 10^{-15}\ \mathrm{cm^2}.$$

The variations of normal displacement $U_3 = (u_3/F)$, normal force stress $T_{33} = (t_{33}/F)$, tangential couple stress $M_{32} = (m_{32}/F)$ for micropolar elastic solid with voids (MWV) and micropolar elastic solid (MES) have been studied and the variations of these components with distance x have been shown by (a) solid line (——) for MWV and (b) dashed line (- - - -) for MES. These variations are shown in Figs. 1–4. The computations are carried out for $U < c_1$ and $z = 1.0$ in the range $0 \leqslant x \leqslant 10.0$.

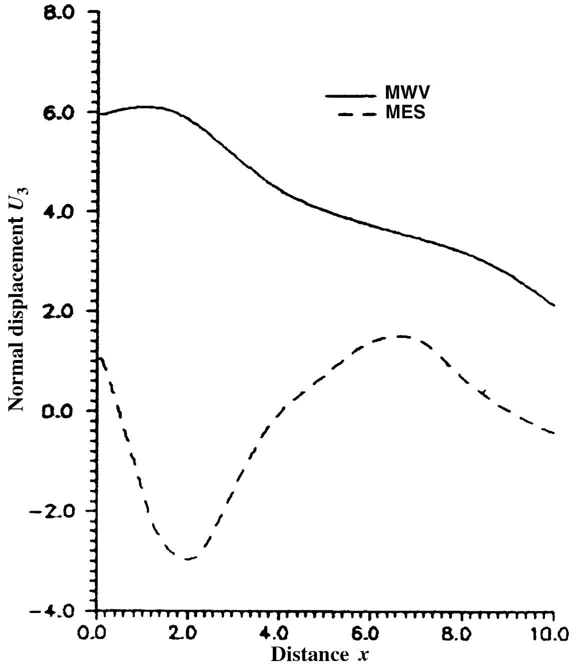

Fig. 1. Variation of normal displacement $U_3 = (u_3/F)$ with distance x.

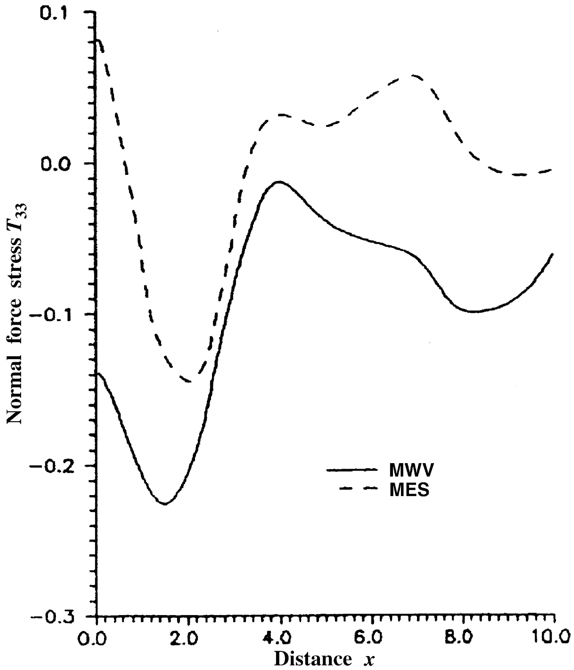

Fig. 2. Variation of normal force stress $T_{33} = (t_{33}/F)$ with distance x.

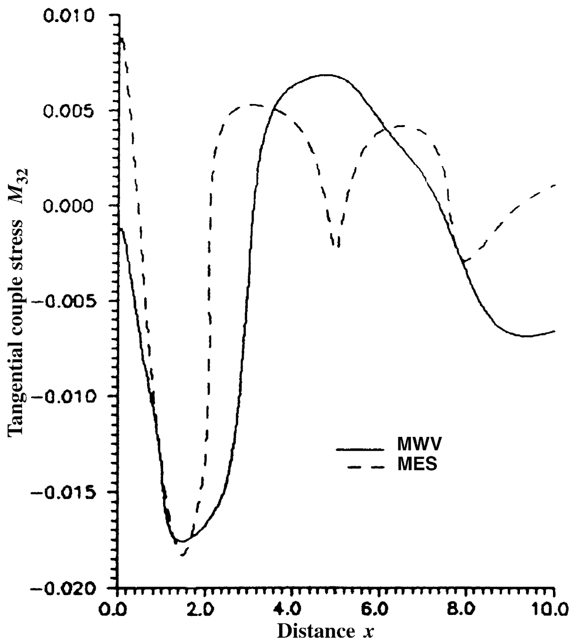

Fig. 3. Variation of tangential couple stress $M_{32} = (m_{32}/F)$ with distance x.

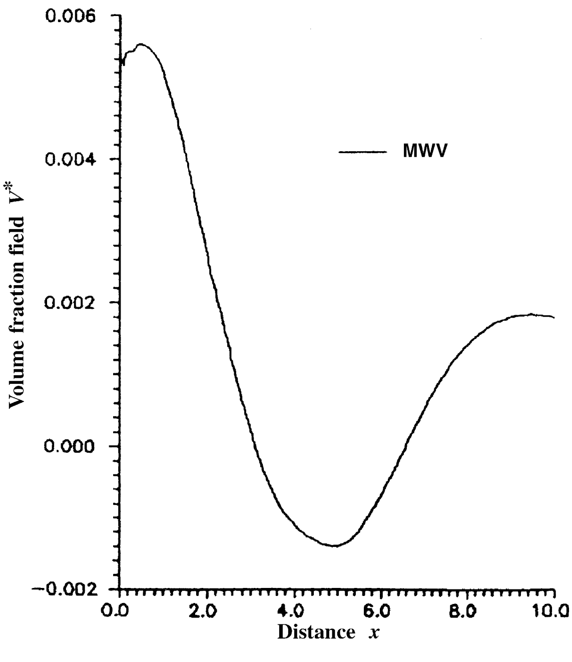

Fig. 4. Variation of volume fraction field $V^* = (v/F)$ with distance x.

6. Discussions for various cases

The values of normal displacement for MWV decreases sharply with increase in distance x. The values of normal displacement for MES are very small as compared to the values for MWV and hence the values for MES have been magnified by multiplying the original values by 100. These variations of normal displacement are shown in Fig. 1. The variations of normal force stress are similar in nature for MWV and MES with difference in their magnitudes. The values of normal force stress for both the solids initially decreases and then oscillates with increase in distance x. However, the decrease is more sharp for MES. These variations are shown in Fig. 2.

Initially, the values of tangential couple stress in both the cases decreases sharply and then oscillates with increase in distance. The oscillations which converges to zero forms more crests and troughs for MES. The values of tangential couple stress for MES have been magnified by multiplying the original values by 10. The variations of tangential couple stress are shown in Fig. 3.

The volume fraction field has a sharp fall in values for $0 \leqslant x \leqslant 5.0$ and then the values starts oscillating and converges to zero. The variation of volume fraction field for MWV is shown in Fig. 4.

7. Conclusion

The presence of voids plays a significant role on all the quantities. The values of normal displacement and tangential couple stress increases while normal force stress decreases due to the presence of voids. The values of all the quantities converges to zero with increase in distance x.

References

[1] J.W. Nunziato, S.C. Cowin, A non-linear theory of elastic materials with voids, *Archive for Rational Mechanics and Analysis* 72 (1979) 175–201.

[2] S.C. Cowin, J.W. Nunziato, Linear elastic materials with voids, *Journal of Elasticity* 13 (1983) 125–147.

[3] D. Iesan, Shock waves in micropolar elastic materials with voids, *Analele Stiintifice ale Universitu atti "Al. I. Cuza" din Iași* 31 (1985) 177–186.

[4] S.C. Cowin, The stress around a hole in a linear elastic material with void, *Quarterly Journal of Mechanics and Applied Mathematics* 37 (1984) 441–465.

[5] P. Puri, S.C. Cowin, Plane waves in linear elastic materials with voids, *Journal of Elasticity* 15 (1985) 167–183.

[6] D. Ieasn, A theory of thermoelastic materials with voids, *Acta Mechanica* 60 (1986) 67–89.

[7] D.S. Chandrasekharaiah, Rayleigh–Lamb waves in an elastic plate with voids, *Journal of Applied Mechanics* 54 (1987) 509–512.

[8] D.S. Chandrasekharaiah, Complete solution in the theory of elastic materials with voids II, *Quarterly Journal of Mechanics and Applied Mathematics* 42 (1989) 41–54.

[9] R.S. Dhaliwal, Wang Jun, A domain of influence theorem in the linear theory of elastic materials with voids, *International Journal of Engineering Science* 32 (1994) 1823–1828.

[10] M. Marin, The mixed problem in elasto static of micropolar materials with voids, *Analele Stiintifice ale Universitu atti "Ovidius Constanta Seria: Matematică* 3 (1995) 106–117.

[11] M. Marin, Some basic theorems in elastostatics of micropolar materials with voids, *Journal of Computational and Applied Mathematics* 70 (1996) 115–126.

[12] M. Marin, Generalized solutions in elasticity of micropolar bodies with voids, *Revista de la Academia Canaria de Ciencias* 8 (1996) 101–106.
[13] M. Marin, A temporally evolutionary equation in elasticity of micropolar bodies with voids, *"Politehnica" University of Bucharest Scientific Bulletin Series A. Applied Mathematics and Physics* 60 (1998) 3–12.
[14] E. Scarpetta, On the fundamental solutions in micropolar elasticity with voids, *Acta Mechanica* 82 (1990) 151–158.
[15] R.G. Payton, An application of the dynamic Betti–Rayleigh reciprocal theorem to moving point loads in elastic media, *Quarterly of Applied Mathematics* 21 (1964) 299–339.
[16] G. Eason, The stresses produced in a semi infinite solid by moving surface force, *International Journal of Engineering Science* 2 (1965) 581–609.
[17] D.C. Gakenheimer, J. Miklowitz, Transient excitation of elastic half space by a point load traveling on the surface, *Journal of Applied Mechanics* 36 (3) (1969) *Transactions of the American Society of Mechanical Engineers*, *Series E* 91 (1969) 505–515.
[18] T.C. Kennedy, G. Hermann, Moving loads on fluid–solid interface, *Journal of Applied Mechanics* 40 (1973) 137–142.
[19] M.R. Halpern, P. Christiano, Steady state harmonic response of a rigid plate bearing on a liquid saturated poroelastic half space, *Earthquake Engineering and Structural Dynamics* 14 (1986) 439–454.
[20] S. Nath, P.R. Sengupta, Steady state response to moving loads in an elastic solid media, *Indian Journal of Pure and Applied Mathematics* 30 (1999) 317–327.
[21] R. Katz, The dynamical response of a rotating shaft subject to an axially moving and rotating load, *Journal of Sound and Vibration* 246 (5) (2001) 757–775.
[22] A. Verruijt, C.C. Cordova, Moving loads on an elastic half plane with hysteretic damping, *Journal of Applied Mechanics* 68 (2001) 915–922.
[23] R. Kumar, M.L. Gogna, Steady state response to moving loads in micropolar elastic medium with stretch, *International Journal of Engineering Science* 30 (1992) 811–820.
[24] R. Kumar, S. Deswal, Steady state response of a micropolar generalized thermoelastic half space to the moving mechanical/thermal loads, *Proceedings of the Indian Academy of Sciences–Mathematical Sciences* 110 (4) (2000) 449–465.
[25] Y.C. Fung, *Foundations of Solid Mechanics*, Prentice-Hall, New Delhi, India, 1968.
[26] W.H. Press, S.A. Teukolsky, W.T. Vellerling, B.P. Flannery, *Numerical Recipes*, Cambridge University Press, Cambridge, 1986.
[27] A.C. Eringen, Plane waves in non-local micropolar elasticity, *International Journal of Engineering Science* 22 (1984) 1113–1121.