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Prediction of source characteristics of engine exhaust manifolds

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Abstract

Engine exhaust noise source is often described in frequency domain by the impedance and strength of an equivalent acoustic one-port source at a reference plane downstream of the exhaust manifold. These parameters can be measured by experimental methods, however, it is also desirable to develop methods for their prediction. A pre-requisite for the prediction of the equivalent one-port acoustic source parameters of an engine manifold is a linear mathematical model of the breathing noise generation mechanism at the valves. The present paper proposes new mathematical models for this source mechanism and the prediction of the one-port source characteristics of engine exhaust noise. The analysis is based on basic fluid dynamic equations for inviscid one-dimensional flow and encompasses both linear time-invariant, linear time-variant and non-linear one-port source models. A systematic procedure is presented for the calculation of one-port source characteristics of engine manifolds, with an application to a 4 cylinder engine exhaust manifold.

The theory of the paper is applicable, with almost no modification, also to the prediction of one-port source characteristics of engine intake noise and the pressure fluctuations associated with discharge and suction processes in compressors.

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1. Introduction

Source characterization of engine exhaust noise, which is a task that is required to be accomplished for making possible the prediction of radiated tailpipe noise or the insertion loss of an exhaust muffler, is often based on the equivalent network approach, in which, in analogy with linear four terminal electrical networks, the source is modelled as a one-port element. This enables the exhaust noise source to be described in frequency domain by the impedance and strength of an acoustic one-port source at a reference plane, usually called the source plane, which is typically taken at downstream of the manifold or the converter. These parameters can be measured by experimental methods such as the two-load method, however, it is also desirable to develop methods for their prediction, but this is a more difficult problem, as it calls for a mathematical model of the sound generation mechanism. To the author's knowledge, no one has presented until now an analytical one-port model for engine exhaust noise source characterization, although, as can be expected, the number of publications on prediction of tailpipe noise radiation has been rather extensive. However, most of this work is concerned with the numerical solution of the nonlinear gas dynamics equations in time-domain for the whole or part of the exhaust line and is outside the scope of the present study. The prediction of the source characteristics of engine manifolds in the spirit of the equivalent linear network theory appears to have received less attention. Åbom, et al. [1] have studied the possibility of prediction of the source impedance of a 4-cylinder engine manifold by making numerical conjectures about the values of terminal impedances at the valves. Desmons and Kergomard [2], have considered the prediction of the tailpipe sound pressure level of a 4-cylinder engine by assuming the source impedance at an exhaust valve is infinite and that the volume velocity injection at a steady speed can be represented by a periodic rectangular pulse. In a study of the influence of the manifold geometry on exhaust noise, Torregrosa et al. [3] have considered an exhaust system consisting of a 4-cylinder engine manifold and a straight pipe by both linear and nonlinear wave analysis, and in the linear formulation they also have assumed the source impedance to be infinite at the exhaust valves. The theory that will be presented in the present paper aims to improve this type of approach, by providing a mathematical model for the source characteristics at the valves.

One-port acoustic source characterization is based on the Thévenin theorem on equivalent linear networks, which is usually carried over to duct acoustics by assuming fundamental mode propagation and the voltage–pressure and current–volume velocity analogy between the electrical and acoustic variables. This assumes tacitly that the actual noise sources and the duct system that transmits the noise generated, can be modelled as a two-terminal network of passive linear elements and independent pure sources. The Thévenin theorem states that, any such two-terminal network is equivalent to a voltage source in series with the network in which all sources are set to zero; the voltage source having the instantaneous value of the voltage appearing at the open-circuit terminals of the original network. The proof of this theorem (and its dual, the Norton theorem) can be found in textbooks on linear networks. A direct application of the Thévenin theorem to acoustic source characterization is thus confronted with the problem of finding the acoustic counterparts of electrical concepts such as open-circuit voltage and setting sources to zero. In particular, the latter is a challenging concept, as it apparently implies that the engine must be switched off. No attempt is made in this paper to translate the Thévenin theorem into acoustic terms, rather, the equivalent acoustic one-port source concept follows naturally in the course of the present analysis.

A pre-requisite for the prediction of the equivalent one-port acoustic source parameters of an engine manifold is a mathematical model of the breathing noise generation mechanism at the valves. The dominant exhaust noise generation mechanism is the blow-down pulse of burnt hot gas mass. It is the main purpose of this paper to present a simple linear time-invariant one-port model of this source mechanism and describe its use in acoustic source characterization of engine exhaust manifolds, with an application to a four cylinder engine. The analysis, which is based on the basic fluid dynamic equations for inviscid one-dimensional flow, also encompasses the formulation of linear time-variant and nonlinear one-port source models.

2. Formulation of linear time-invariant one-port source for exhaust noise

2.1. General considerations

The exhaust valve–port model adopted in the present analysis consists of a uniform straight pipe of cross-sectional area S. The valve and port diameters are assumed to be compact, and the plane $x=0_-$ is assumed to represent the valve surface when the valve is at its closed position, or an open end (that expands into the cylinder over the blockage of the valve) when the valve is open, where x denotes the pipe axis and the subscript minus '$-$' denotes just upstream of $x=0$. The exhaust gas mass injection into the pipe is modelled as uniformly distributed external flow through a circumferential area that extends down a very short length of the pipe at $x=0$, at the rate of $\mu(x,t)$ per unit volume of the pipe. The valve action and $\mu(x,t)$ are assumed to be periodic in time, t, of period $T=1/\mathrm{FCF}$, where FCF denotes the firing cycle frequency, since a steady running engine is considered. The fluid motion in this region of the pipe is governed by the equations of conservation of mass, momentum and energy. Assuming one-dimensional flow, conservation of mass requires that

$$\frac{\partial \rho}{\partial t}+\frac{\partial \rho v}{\partial x}=\mu(x,t), \tag{1}$$

where ρ and v denote, respectively, the fluid density and the particle velocity in the x direction. For inviscid flow that satisfies Eq. (1), the conservation equation for momentum in the x direction can be expressed as

$$\rho\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}\right)+\frac{\partial p}{\partial x}=0, \tag{2}$$

where p denotes the fluid pressure. The energy equation is

$$\frac{\partial e\rho}{\partial t}+\frac{\partial v(p+e\rho)}{\partial x}=h^{\circ}\mu(x,t). \tag{3}$$

Here, e denotes the total (internal plus kinetic) specific energy of the fluid in the pipe and h^{o} denotes the specific stagnation enthalpy of the exhaust gas mass injection into the pipe. Assuming that the exhaust gas behaves as a perfect gas and using Eqs. (2) and (3), Eq. (1) can be expressed as

$$\frac{\partial p}{\partial t}+v\frac{\partial p}{\partial x}+\gamma p\frac{\partial v}{\partial x}=(\gamma-1)(h^{\circ}-v^2/2)\,\mu(x,t). \tag{4}$$

Here, γ is the ratio of specific heat coefficients. In deriving this equation, the temperature dependence of the specific heat coefficients is neglected. This approximation is usually made, as it gives results within engineering accuracy for practical exhaust gas conditions.

The exhaust gas mass injection into the port is assumed to be concentrated in the close vicinity of $x=0$, and that it can be expressed as $\mu(x,t)=m(t)\delta(x)/S$, where $\delta(x)$ denotes the Dirac function at $x=0$ and $m(t)$ is the rate of exhaust gas mass injection into the port. Hence, Eq. (4) becomes

$$S\left\{\frac{\partial p}{\partial t}+v\frac{\partial p}{\partial x}+\gamma p\frac{\partial v}{\partial x}\right\}=(\gamma-1)m\{h^{\circ}-v^2/2\}\delta(x), \tag{5}$$

and, recasting Eq. (2) using the relationship $c^2=\gamma p/\rho$, where c is the speed of sound, one obtains the second equation that governs the fluid pressure and particle velocity:

$$\gamma p\left(\frac{\partial v}{\partial t}+v\frac{\partial v}{\partial x}\right)+c^2\frac{\partial p}{\partial x}=0. \tag{6}$$

In the foregoing equations, h^{o} and m are required as input quantities. For the valve flow, it is usual to assume that the stagnation enthalpy is conserved. Then, h^{o} may be taken as the specific stagnation enthalpy of the gas in the cylinder and, since the kinetic energy of the gas in the cylinder is in general small compared to its total energy, it can be approximated by the specific enthalpy of the gas in the cylinder, $h(t)$, say. Strictly speaking, both m and h^{o} are influenced by the wave motion in the exhaust line, however, as a first approximation, this effect can be neglected if the wave amplitudes are in the linear range. To this approximation, m and h can be determined by thermodynamic cycle simulation using the time-averaged operational conditions at exhaust ports, without paying attention to the gas dynamics of the exhaust line. At this point, it is noteworthy that, since the exhaust conditions at the ports are in general load-dependent, m and h, and, therefore, the source characteristics, will be load-dependent.

These considerations require slight modification if there is back-flow at an exhaust port: For exhaust gas back-flow, m represents the rate of mass injection into the cylinder and, consequently, h^{o} should be evaluated as the specific stagnation enthalpy of the exhaust gas inflow into the cylinder. The latter can be approximated by the specific enthalpy of the gas in the exhaust port, again on grounds of having negligibly small kinetic energy component. Thus, for a period corresponding to a back-flow, h denotes the specific enthalpy of the gas in the exhaust port.

Upon integrating Eqs. (5) and (6) across the region $x=0$, one obtains, respectively,

$$S\{<v>[p]+\gamma\langle p\rangle[v]\}=(\gamma-1)m\{h-\langle v^2/2\rangle\}, \tag{7}$$

$$\gamma\langle p\rangle[v^2/2]+c_0^2[p]=0. \tag{8}$$

Here, the Heaviside function $H_{1/2}(x)$, where $H_{1/2}(x<0)=0$, $H_{1/2}(0)=1/2$, $H_{1/2}(x>0)=1$, is used in defining the jumps across the source plane, $c_0^2=c_0^2(t)$ denotes the speed of sound squared in the region $x=0$ and the brackets $\langle\;\rangle$ and [ ] are used exclusively to denote the mean values and jumps across the source discontinuity plane, e.g.

$$\langle p\rangle = (p_+ + p_-)/2, \quad [p] = p_+ - p_-,$$
$$\langle v\rangle = (v_+ + v_-)/2, \quad [v] = v_+ - v_-, \tag{9}$$

where the subscripts minus '−' and plus '+', respectively, refer to just upstream and just downstream of $x = 0$. The fluid pressure and the particle velocity can be expressed as

$$p(t,x) = \bar{p} + p'(t,x), \quad v(t,x) = \bar{v} + v'(t,x). \tag{10}$$

Here, over bar denotes a time-averaged mean part and prime denotes fluctuations having zero time average (this notation is used similarly for the other fluctuating quantities as well). The fluctuating parts p' and v' are assumed to be due to acoustic wave motion that is generated by periodic exhaust gas mass injection into the port. The equations governing the jumps in p' and v' across the region $x = 0$ can be expressed as

$$S\{\langle\bar{v}\rangle[p'] + \langle v'\rangle[\bar{p}] + \gamma\langle p'\rangle[\bar{v}] + \gamma\langle\bar{p}\rangle[v']\}$$
$$= (\gamma-1)\{(mh)' - \bar{m}\langle\bar{v}v'\rangle - m'\langle\bar{v}^2/2\rangle - (m'\langle\bar{v}v'\rangle)')\} - \varepsilon'_1, \tag{11}$$

$$\gamma\langle\bar{p}\rangle[\bar{v}v'] + \gamma[\bar{v}^2/2]\langle p'\rangle + \overline{c_0^2}[p'] + (c_0^2)'[\bar{p}] + ((c_0^2)'[p'])' = -\varepsilon'_2, \tag{12}$$

where the nonlinear terms are given by

$$\varepsilon'_1 = (\gamma-1)m\langle v'^2/2\rangle + S\{\langle v'\rangle[p'] + \gamma\langle p'\rangle[v']\}, \tag{13}$$

$$\varepsilon'_2 = \gamma\{\langle\bar{p}\rangle[v'^2/2] + \langle p'\rangle[\bar{v}v' + v'^2/2]\}. \tag{14}$$

Eqs. (11) and (12), which follow upon application of the decompositions of Eq. (10) to Eqs. (7) and (8) and assuming that the mean quantities satisfy these equations identically, constitute the basic equations from which acoustic one-port source models are derived in the present analysis. In implementing these equations, it is further assumed that the mean pressure in the vicinity of the discontinuity plane is constant, p_o, say, and that the mean flow just upstream of this plane is zero; that is,

$$[\bar{p}] = 0, \ \ \langle\bar{p}\rangle = p_o, \ \ [\bar{v}] = v_o, \ \ \langle\bar{v}\rangle = v_o/2, \tag{15}$$

where v_o denotes the mean flow velocity just downstream of the source discontinuity plane. In this model, no axial flow upstream of the source discontinuity, except that associated with the wave motion, is assumed to be present to any discernible extent. Hence, Eqs. (11) and (12) can be expressed as

$$S\{v_o[p']/2 + \gamma v_o\langle p'\rangle + \gamma p_o[v']\}$$
$$= (\gamma-1)\{(mh)' - \bar{m}v_o v'_+/2 - v_o^2 m'/4 - v_o(m'v'_+)'/2\} - \varepsilon'_1, \tag{16}$$

$$\gamma p_o v_o v'_+ + \gamma\langle p'\rangle v_o^2/2 + \overline{c_o^2}[p'] + ((c_o^2)'[p'])' = -\varepsilon'_2, \tag{17}$$

respectively. Temporal variation of the speed of sound squared is caused by the temperature fluctuation associated with the generated wave motion, as well as the temporal variation of the port ambient temperature. The present analysis assumes that the effect of the former is negligible and, therefore, $c_0^2(t)$ is determined solely by the port ambient temperature.

2.2. Linear time-invariant one-port source model

A linear one-port source model for engine exhaust noise can be developed by neglecting the nonlinear terms in the foregoing jump equations. The resulting model is time-variant because of the terms involving products of fluctuations in Eqs. (16) and (17). The formulation of linear one-port source models in which these terms are considered will be taken up in Section 4. The present section will develop a time-invariant one-port source model for exhaust noise.

Upon eliminating the time-variant and nonlinear terms, Eqs. (16) and (17) simplify to

$$S\{v_o[p']/2 + \gamma v_o\langle p'\rangle + \gamma p_o[v']\} = (\gamma - 1)\{(mh)' - (\bar{m}v_o v'_+ + m' v_o^2/2)/2\}, \tag{18}$$

$$\gamma p_o v_o v'_+ + \gamma\langle p'\rangle v_o^2/2 + \overline{c_o^2}[p'] = 0, \tag{19}$$

respectively. It should be noted that, since these equations are linear in the fluctuating parts, the primed quantities can be interpreted as the complex Fourier coefficients of the constituent harmonics.

Upon defining the upstream acoustic impedance as $z_- = p'_-/v'_-$, Eqs. (18) and (19) can be combined in the form of a Thévenin type equivalent one-port source

$$p_S = Z_S v'_+ + p'_+, \tag{20}$$

where

$$p_S = \frac{1}{\beta}\left\{\frac{2(mh)'}{Sv_o} - \frac{m' v_o}{2S}\right\}, \tag{21}$$

$$Z_S = \frac{1}{\beta}\left\{\frac{2\gamma p_o}{v_o(\gamma - 1)} + \frac{\bar{m}}{S} + 4\alpha S\gamma p_o v_o\right\}, \tag{22}$$

$$\alpha = \frac{1 - 2\gamma p_o/(\gamma - 1)z_- v_o}{4\overline{c_o^2} - \gamma v_o^2}, \tag{23}$$

$$\beta = \frac{\gamma + 1}{\gamma - 1} + (4c_o^2 + \gamma v_o^2)\alpha. \tag{24}$$

The parameters p_S and Z_S are called the source pressure strength and the source impedance, respectively. Note that, these parameters, and the upstream acoustic impedance, are evaluated on per harmonic basis. It is also notable that, the source impedance is determined by the mean operating conditions of an engine.

For most engines, the port Mach number, M_o, which is defined by $M_o = v_o/\bar{c}_o$, where $\bar{c}_o = \sqrt{\overline{c_o^2}}$, is less than 0.3 and, therefore, the condition $M_o^2 \ll 1$ may be assumed to be valid with less than 10% error. For such low-port Mach numbers, upon applying the approximation[1] $\bar{m} \approx$

[1]Just downstream of the discontinuity, the fluid density can be partitioned into acoustic and non-acoustic parts. The approximation here entails the assumption that $\bar{m} \approx \bar{\rho}_o v_o S$, where the time-averaged non-acosutic density is determined by $\bar{\rho}_o = \gamma p_o/c_o^2$.

$S\gamma M_o^2 p_o/v_o$, Eqs. (21) and (22) can be expressed approximately as

$$p_S \approx \frac{1}{Sv_o}\left(1 - \frac{p_o}{v_o z_-}\right)^{-1} \frac{\gamma - 1}{\gamma}[(mh)' - v_o^2 m'/4], \tag{25}$$

$$Z_S \approx \frac{p_o}{v_o}\left(1 - \frac{p_o}{v_o z_-}\right)^{-1}\left(1 - \frac{\gamma M_o^2 p_o}{v_o z_-}\right) \tag{26}$$

respectively. From Eq. (26), it follows that, if the upstream acoustic impedance is large enough for the condition $z_- > p_o/v_o$ to be satisfied, then the source impedance is

$$Z_S \approx \frac{p_o}{v_o}\left(1 - \frac{p_o}{v_o z_-}\right)^{-1}, \tag{27}$$

and, if the condition $z_- \gg p_o/v_o$ is satisfied, it is simply

$$Z_S \approx \frac{p_o}{v_o}, \tag{28}$$

approximately. This result can be expressed also as $\varsigma_S \approx 1/\gamma M_o$. Here, ς_S denotes the normalized source impedance, which is defined by $\varsigma_S = Z_S/z_o$, where $z_o = \gamma p_o/\bar{c}_o$. Thus, according to the present source model, the normalized source impedance is inversely proportional to the port Mach number.

Eq. (25) can also be simplified further for low-port Mach numbers:

$$p_S \approx \frac{\gamma - 1}{\gamma}\frac{(mh)'}{Sv_o}\left(1 - \frac{p_o}{v_o z_-}\right)^{-1}, \tag{29}$$

since, for a perfect gas, $(mh)' = (mc_o^2/(\gamma - 1))' \cong m'\bar{c}_o^2/(\gamma - 1)$, and, consequently, $v_o^2 m'/4 \approx M_o^2(\gamma - 1)(mh)'/4 \ll (mh)'$, the condition $M_o^2 \ll 1$ being applicable. Furthermore, if the condition $z_- \gg p_o/v_o$ is true, the source pressure strength is given by

$$p_S \approx \frac{\gamma - 1}{\gamma}\frac{(mh)'}{Sv_o}, \tag{30}$$

approximately. This result shows that, when the upstream acoustic impedance is large enough, the source strength is proportional to the fluctuations of the stagnation enthalpy (recall that h^o has been approximated by h), and is inversely proportional to the port Mach number.

Implementation of the foregoing results requires knowledge of the upstream acoustic impedance z_-. During the period in which a valve is closed, the valve surface determines the boundary condition. On the other hand, as a first approximation, the acoustic conditions at upstream of the discontinuity may be assumed to be approximately akin to that imposed by the valve surface also during the period when a valve is open, since the maximum valve displacements are in general very small compared to the wavelengths at the frequencies of interest. Obviously, if the valve surface is modelled as a hard surface of infinite impedance, Eqs. (21) and (22) will reduce to equations (30) and (28), respectively, for the case of $M_o^2 \ll 1$. Therefore, for low-port Mach numbers, the condition $z_- \gg p_o/v_o$ can be stated to be the criterion for the upstream acoustic impedance to be considered infinitely large. This condition is assumed to be valid in the rest of the present analysis, for simplicity.

Typical *mh* injection from a 4-stroke spark-ignition engine cylinder is shown in Fig. 1. This characteristic was determined by thermodynamic cycle simulation of an engine having 0.1 m stroke, 0.1 m bore and a compression ratio of 8, running at a steady crank-shaft speed of 3000 rev/min, being fuelled with C_4H_{10} at equivalence ratio of unity and about 4% burnt gas content. Exhaust valve opening and closing angles were, respectively, 145° and 375° after the top dead centre. The Fourier magnitude spectrum of the *mh* time-history of Fig. 1 is given in Fig. 2 for the first 50 harmonics of the firing cycle frequency (FCF).

Eqs. (28) and (30) delineate the analytical time-invariant one-port source model that is presented in this section for the prediction of acoustic source impedance and strength at an exhaust valve. In previous work, the values used for these parameters were based on presumptions which, as it now transpires, are not completely consistent with the present theory. The source of engine exhaust noise is often conjectured to be a pure volume velocity, or mass velocity, source with infinite impedance. To the approximations underlying Eqn. (26), the source impedance can be infinite if $z_- = p_o/v_o$ or if $v_o = 0$, but these cases, for which the source pressure strength also becomes infinitely large, are not feasible physically for a steady running engine exhaust. Thus, it is clear that, infinite source impedance is not allowed in the present theory. Conversely, using a finite value for the source impedance at the exhaust ports when the mean flow velocity is assumed to be zero, is not consistent with the present theory. On the other hand, according to the present theory,

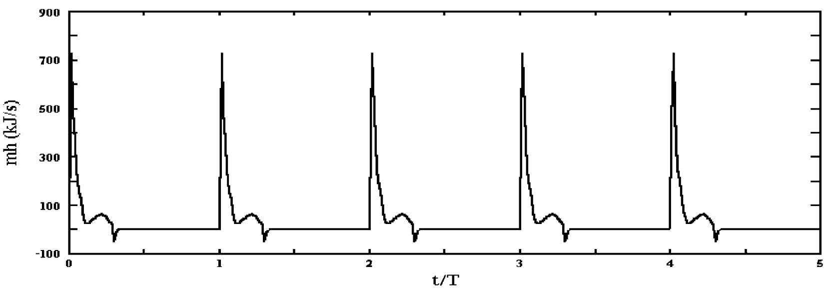


Fig. 1. Exhaust gas $m(t)h(t)$ injection from a cylinder of a 4-stroke spark-ignition engine.

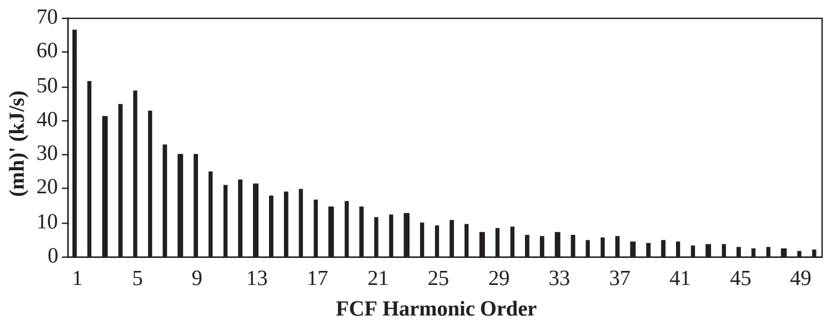


Fig. 2. Fourier spectrum of the $m(t)h(t)$ injection of Fig. 1.

the source strength is governed by the fluctuating stagnation enthalpy, and not by the fluctuating mass velocity alone.[2]

3. Equivalent source characterization of engine manifolds

One-port source characterization of engine exhaust noise is usually required for the calculation of radiated tailpipe sound pressure level, or the insertion loss of a muffler. Assuming that the one-port source characteristics p_S and Z_S at the valves are determined, the first step of typical calculations is the determination of the acoustic impedance at a reference plane downstream of the manifold, as seen from the tailpipe end. This step comprises the usual duct acoustics wave transfer calculations. In the second step of the computations, the one-port sources at the valves are assumed to be linear and all 'moved' to the reference plane to obtain an equivalent one-port source of the manifold at that plane. Then, the sound pressure level at the reference plane and, hence, at any other plane downstream of the manifold, can be computed using the parameters of the equivalent one-port source computed in the second step and the acoustic impedance computed in the first step. Described in this section are the basic calculations involved in the second step. This type of calculations are described also in Refs. [2,3,5], however, the present formulation takes into account the time variance of the source (see Section 4.2) and the mean flow and temperature effects.

For source characterization, it is usually satisfactory to model engine manifolds as an assembly of pipes forming a 'tree' structure. The 'tree' is formed by a repeating pattern of a number of pipes carrying joining flows connecting at junctions with a single outlet pipe, the exhaust pipe constituting the final outlet pipe. The reference plane for equivalent source calculations is usually taken at the inlet of the exhaust pipe, or at the outlet of the converter, if there is one. The object of the calculations is to determine the source strength and impedance of the acoustic one-port source, imagined to be located at the selected reference plane, that produces the same acoustic field downstream of the reference plane as the actual sources at the exhaust valves. This can be achieved by repeated application of elementary operations of transferring one-port sources along pipes and across a junction. These operations are described separately in the following sections. The analysis assumes, for simplicity of the presentation, isentropic sound wave propagation and neglects the effects local temporal variations in the ambient temperature.

3.1. Moving one-port sources along pipes

Let plane 1 be the original source plane, and plane 2 the plane at which the equivalent one-port source characteristics are required. In Thévenin form, the source relationships in planes 1 and 2 can be expressed as

$$p_{S1} = Z_{S1}v'_1 + p'_1, \quad p_{S2} = Z_{S2}v'_2 + p'_2, \tag{31}$$

[2]Here, it may be of interest to note the contention of Doak [4] on the fluctuating stagnation enthalpy being the basic acoustic field. The present analysis captures the effect of stagnation enthalpy fluctuation on the source strength. As can be deduced from Eq. (25), if the specific enthalpy fluctuation is negligible, then the source strength is governed by the mass velocity fluctuation.

where the subscripts '1' and '2' refer to plane 1 and plane 2, respectively. Since time-invariant one-dimensional wave motion is assumed, the transfer of acoustic motion between the planes 1 and 2 can be expressed in frequency domain by the two-port relationship

$$\begin{bmatrix} p'_1 \\ v'_1 \end{bmatrix} = \begin{bmatrix} A & B \\ C & D \end{bmatrix} \begin{bmatrix} p'_2 \\ v'_2 \end{bmatrix}. \tag{32}$$

Eqs. (31) and (32) can now be manipulated to show that

$$p_{S2} = \frac{p_{S1}}{A + Z_{S1}C}, \quad Z_{S2} = \frac{B + Z_{S1}D}{A + Z_{S1}C}. \tag{33}$$

These relations, which define the equivalent one-port source at plane 2, in terms of the parameters of a known one-port source at plane 1, are general as no conjecture has been made about the form of Eq. (32). For the simplest case, the two-port for isentropic propagation in a pipe carrying a uniform mean flow can be used, however, it is also possible to use two-ports that take into account the effects ambient temperature gradients, visco-thermal losses, or even components such as a converter or muffler.

For isentropic sound propagation in a uniform pipe having axially uniform speed of sound c_0 and carrying a uniform mean flow of Mach number M_0, Eq. (33) can be expressed as (harmonic wave motion of radian frequency ω and $\exp(-\mathrm{i}\omega t)$ time dependence being assumed)

$$p_{S2} = \frac{2p_{S1}\mathrm{e}^{\mathrm{i}K^+L}}{\varsigma_{S1} + 1 - (\varsigma_{S1} - 1)\mathrm{e}^{\mathrm{i}K^+L}\mathrm{e}^{-\mathrm{i}K^-L}}, \tag{34}$$

$$\varsigma_{S2} = \frac{\varsigma_{S1} + 1 + (\varsigma_{S1} - 1)\mathrm{e}^{\mathrm{i}K^+L}\mathrm{e}^{-\mathrm{i}K^-L}}{\varsigma_{S1} + 1 - (\varsigma_{S1} - 1)\mathrm{e}^{\mathrm{i}K^+L}\mathrm{e}^{-\mathrm{i}K^-L}}, \tag{35}$$

where i denotes the unit imaginary number, L is the length of the pipe between the planes 1 and 2, and

$$K^{\pm} = \frac{\pm\omega}{c_o(1 \pm M_o)} \tag{36}$$

Strictly speaking, these expression are valid for a pipe having a constant mean temperature along its length, however, they can be used effectively when there exists an appreciable axial mean temperature gradient, by dividing the pipe into a number of segments with constant temperatures so that the actual temperature gradient is approximated in a step-wise manner. If the mean temperature gradient is constant along the pipe, Eqs. (34) and (35) can be used with good accuracy without segmentation, provided that $K^{\pm}L$ is replaced by $K^{\pm}L\beta^{\pm}$, where $\beta^{\pm} = 1 + \tau(1 \pm 2M_o)/4(1 \pm M_o)$, τ is the ratio mean temperature drop between planes 1 and 2 to the mean temperature at plane 1, and both here and in Eq. (36), M_{o} now denotes the Mach number of the mean flow velocity at plane 1 [6].

3.2. *Moving one-port sources at a junction*

A junction is modelled as a compact region, the input planes of which have known one-port sources, and the single output plane is the reference plane of the equivalent source. Let plane 1 be the output plane, and planes 2,3,… the input planes. In Thévenin form, the source relationships in these planes are

$$p_{Si} = Z_{Si}v'_i + p'_i, \quad i = 2, 3, \ldots \tag{37}$$

Assuming quasi-static conditions in the junction, and that the wave motion is isentropic, the conservation of mass fluctuations gives

$$(z_{o1}v'_1 + M_{o1}p'_1)s_1 = \sum_{i=2,3,\ldots} (z_{oi}v'_i + M_{oi}p'_i)s_i. \tag{38}$$

Here, M_{oi} denotes the mean flow Mach number at plane i, $s_i = S_i/c_{oi}$, where S_i denotes the cross-sectional area of the pipe connecting at plane i, and z_{oi} and c_{oi} denote, respectively, the characteristic acoustic impedance and the speed of sound at plane i. Neglecting losses, the corresponding energy equation can be expressed as the equality of the fluctuating part of the stagnation enthalpy at all planes

$$(p'_1 + z_{o1}M_{o1}v'_1)/\rho_{o1} = (p'_i + z_{oi}M_{oi}v'_i)/\rho_{oi}, \quad i = 2, 3, \ldots, \tag{39}$$

where $\rho_{oi} = z_{oi}/c_{oi}$. Solving Eqs. (38) and (39) for the equivalent source at plane 1 gives

$$p_{S1} = \left\{ \frac{\sum_{i=2,3,\ldots} (a_i + M_{oi})s_i p_{Si}}{M_{o1}s_1\rho_{o1} + \sum_{i=2,3,\ldots} a_i s_i \rho_{oi}} \right\} \rho_{o1}, \tag{40}$$

$$Z_{S1} = \left\{ \frac{s_1\rho_{o1} + M_{o1} \sum_{i=2,3,\ldots} a_i s_i \rho_{oi}}{M_{o1}s_1\rho_{o1} + \sum_{i=2,3,\ldots} a_i s_i \rho_{oi}} \right\} z_{o1}. \tag{41}$$

Here,

$$a_i = \frac{z_{oi} - M_{oi}Z_{Si}}{Z_{Si} - z_{oi}M_{oi}}, \quad i = 2, 3, \ldots. \tag{42}$$

Thus, given the one-port source parameters at the input planes of a junction, the equivalent source at its output plane can be determined by using the foregoing equations.

3.3. Source parameters of engine manifolds

The calculation of the equivalent one-port source parameters of an engine manifold starts from the ports. Having determined the one-port source characteristics at the valves using Eqs. (28) and (30), these are moved to the input planes of the first set of junctions. Next, the sources at the input planes of these junctions are transferred to their respective output planes. Repeating this process up to the reference plane gives the equivalent source characteristics at that plane. These calculations using the present source theory are implemented in ADEM, a software that has been developed by the author for acoustic analysis of mufflers and flow ducts.

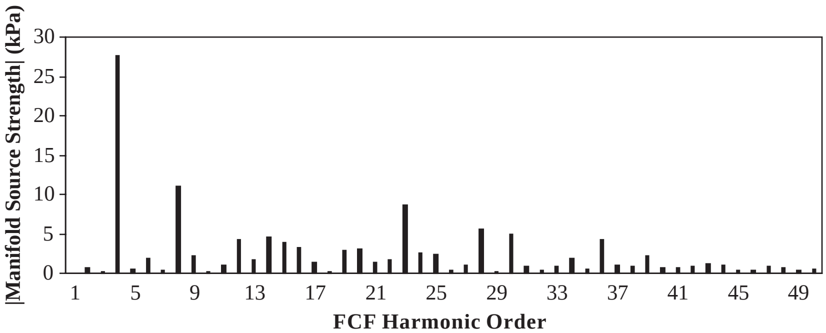


Fig. 3. Fourier magnitude spectrum of the source pressure strength of a 4-cylinder engine manifold having the configuration and dimensions of Fig. 6.

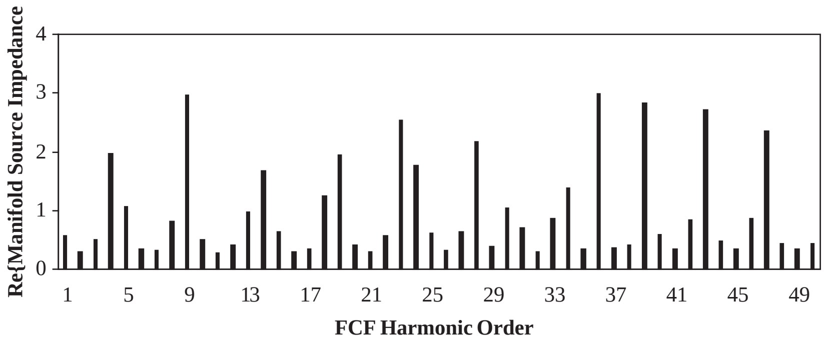


Fig. 4. Fourier spectrum of the real part of the source impedance of a 4-cylinder engine manifold having the configuration and dimensions of Fig. 6.

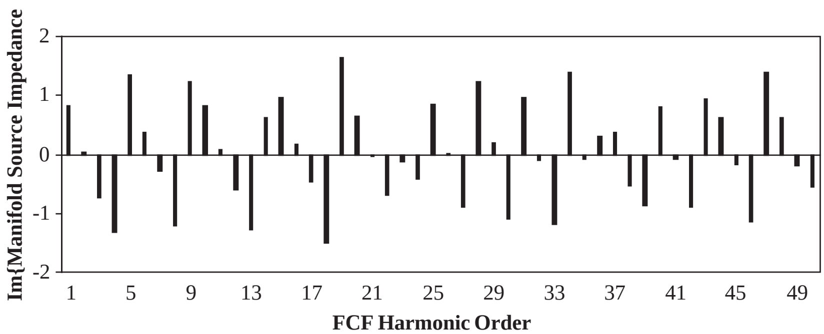
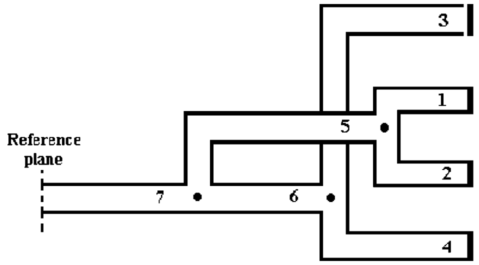


Fig. 5. Fourier spectrum of the imaginary part of the source impedance of a 4-cylinder engine manifold having the configuration and dimensions of Fig. 6.



Pipe	Length (m)	Diameter (m)	Temperature (C)
1	0.12	0.043	850
2	0.16	0.043	850
3	0.265	0.043	850
4	0.2	0.043	850
5	0.5	0.043	700
6	0.47	0.043	700
7	0.55	0.055	600

Fig. 6. A typical 4-cylinder spark-ignition engine exhaust manifold configuration.

Shown in Figs. 3–5 are the source characteristics of a typical four cylinder engine manifold that have been calculated by this process. Eqs. (28) and (30) were used in modelling the source characteristics at the valves and cylinder $(mh)'$ injections were assumed to be as in Fig. 2, with the phasing of the Fourier spectra according to the firing order, which is assumed to be 3-2-4-1 (see Fig. 6), taken into account. The port Mach numbers, and time-averaged port temperatures were 0.1 and 850 °C, respectively. For the fuel used in the cycle simulation, this corresponds to a speed of sound of about 650 m/s, a ratio of specific heat coefficients of 1.286, which is assumed to be constant throughout the manifold and, from Eq. (28), a port normalized source impedance of 7.776, approximately.

Shown in Fig. 6 are the geometry of this manifold, which is taken from Ref. [1], and the position of the reference plane. The equivalent one-port source impedance of the manifold has the real and imaginary parts shown in Figs. 4 and 5, respectively, and the pressure strength magnitude spectrum shown in Fig. 3. The dominant harmonics of the latter correspond to the first few multiples of the fundamental firing frequency, which is equal to 4 times the FCF. In Ref. [1], the source impedance of this manifold was calculated by making conjectures about the values of the terminal impedance at the exhaust valves. The characteristics of Figs. 4 and 5 are in fairly good agreement with the results of Ref. [1] for normalized terminal impedance of 10 at the four valves.

4. Advanced one-port source models for exhaust noise

This section will present one-port source models for exhaust noise generation, which capture the typical effects of linear time-variant and nonlinear terms in Eqs. (16) and (17). For simplicity, these models are built over the linear time-invariant model described by Eqs. (28) and (30). Therefore, it is expedient to give first the forms of Eqs. (16) and (17) from which this simple time-invariant model can be deduced directly. These are

$$S\{v_o[p']/2 + \gamma v_o\langle p'\rangle + \gamma p_o v'_+\} = (\gamma - 1)(mh)', \tag{43}$$

$$\gamma p_o v_o v'_+ + \overline{c_o^2}[p'] = 0. \tag{44}$$

Indeed, elimination of p'_- gives

$$\frac{\gamma-1}{\gamma}\frac{(mh)'}{Sv_o} = \frac{p_o}{v_o}\{1+\tfrac{1}{2}(\gamma-1)M_o^2\}v'_+ + p'_+ \approx \frac{p_o}{v_o}v'_+ + p', \tag{45}$$

since the condition $M_o^2 \ll 1$ is applicable. This shows that Eqs. (43) and (44), which are subsequently referred to as the basic time-invariant model, yield the Thévenin form of the time-invariant one-port source relationship that correspond to Eqs. (28) and (30). The remaining linear terms in Eqs. (18) and (19) are neglected in building the linear time-variant and non-linear extensions of the basic time-invariant one-port source model.

4.1. A time-variant one-port source model

To show the effect of the linear time-variant terms, the following extension of the basic time-invariant model is considered:

$$S\{v_o[p']/2 + \gamma v_o\langle p'\rangle + \gamma p_o v'_+\} = (\gamma-1)\{(mh)' + v_o(m'v'_+)'/2\}, \tag{46}$$

$$S\{\gamma p_o v_o v'_+ + \overline{c_o^2}[p'] + ((c_o^2)'[p'])'\} = 0, \tag{47}$$

This includes all linear time-variant terms.

For a steady running engine, the fluctuating terms in Eqs. (46) and (47) will be periodic and can be represented by their complex Fourier series

$$\{p'_+, p'_-, v'_+, (c_o^2)', (mh)', m'\} = \sum_{\substack{k=-\infty \\ k\neq 0}}^{\infty}\{p_k, q_k, v_k, c_k, h_k, m_k\}\exp(\mathrm{i}2\pi kt/T). \tag{48}$$

Upon substituting these series in Eqs. (46) and (47) and equating the coefficients of the harmonics of the like order, one obtains

$$S\left\{\frac{\gamma+1}{\gamma-1}p_k + q_k + \frac{2\gamma p_o v_k}{v_o}\right\} = \frac{2h_k}{v_o} + \sum_{\substack{n=1 \\ n\neq k}}^{\infty} v_n m_{k-n} + \sum_{n=1}^{\infty} v_n^* m_{k+n}, \tag{49}$$

$$\gamma p_o v_o v_k + \overline{c_o^2}(p_k - q_k) + \sum_{\substack{n=1 \\ n\neq k}}^{\infty}(p_n - q_n)c_{k-n} + \sum_{n=1}^{\infty}(p_n^* - q_n^*)c_{k+n} = 0, \tag{50}$$

where $k = 1,2,\ldots$, an asterisk (*) denotes the conjugate of a complex quantity and the summations are the convolution summations that represent the linear time-variant terms.

Eqs. (49) and (50) are manipulated more easily using matrix algebra. To this end, it is convenient to define the following vectors:

$$\mathbf{P} = \begin{bmatrix} p_1 \\ p_2 \\ \vdots \\ p_K \\ p_1^* \\ p_2^* \\ \vdots \\ p_K^* \end{bmatrix}, \quad \mathbf{Q} = \begin{bmatrix} q_1 \\ q_2 \\ \vdots \\ q_K \\ q_1^* \\ q_2^* \\ \vdots \\ q_K^* \end{bmatrix}, \quad \mathbf{V} = \begin{bmatrix} v_1 \\ v_2 \\ \vdots \\ v_K \\ v_1^* \\ v_2^* \\ \vdots \\ v_K^* \end{bmatrix}, \quad \mathbf{h} = \begin{bmatrix} h_1 \\ h_2 \\ \vdots \\ h_K \\ h_1^* \\ h_2^* \\ \vdots \\ h_K^* \end{bmatrix}, \tag{51}$$

Then, writing Eq. (49) for $k = 1,2,\ldots,K$, truncating the convolution summations at K terms, and combining the resulting set of equations with the complex conjugate of the same set of equations, one obtains

$$S\left\{\frac{\gamma+1}{\gamma-1}\mathbf{P} + \mathbf{Q}\right\} + \left\{\frac{2S\gamma p_o}{(\gamma-1)v_o}\mathbf{E} - \mathbf{M}\right\}\mathbf{V} = \frac{2}{v_o}\mathbf{h}. \tag{52}$$

Here, $\mathbf{E}$ denotes a real unit matrix of dimension $2K$ and

$$\mathbf{M} = \begin{bmatrix} \mathbf{M}_1 & \mathbf{M}_2 \\ \mathbf{M}_2^* & \mathbf{M}_1^* \end{bmatrix}, \tag{53}$$

where

$$\mathbf{M}_1 = \begin{bmatrix} 0 & \boldsymbol{m}_{1-2} & \cdots & \boldsymbol{m}_{1-K} \\ \boldsymbol{m}_{2-1} & 0 & \cdots & \boldsymbol{m}_{2-K} \\ \vdots & \vdots & \cdots & \vdots \\ \boldsymbol{m}_{K-1} & \boldsymbol{m}_{K-2} & \cdots & 0 \end{bmatrix}, \quad \mathbf{M}_2 = \begin{bmatrix} \boldsymbol{m}_{1+1} & \boldsymbol{m}_{1+2} & \cdots & \boldsymbol{m}_{1+K} \\ \boldsymbol{m}_{2+1} & \boldsymbol{m}_{2+2} & \cdots & \boldsymbol{m}_{2+K} \\ \vdots & \vdots & \cdots & \vdots \\ \boldsymbol{m}_{K+1} & \boldsymbol{m}_{K+2} & \cdots & \boldsymbol{m}_{K+K} \end{bmatrix} \tag{54}$$

Similarly, writing Eq. (50) for $k = 1, 2, \ldots, K$, truncating the convolution summations at K terms, and combining the resulting set of equations with the complex conjugate of the same set of equations,

$$\gamma p_o v_o \mathbf{V} + \{\overline{c_o^2}\mathbf{E} + \mathbf{C}\}\mathbf{P} = \{\overline{c_o^2}\mathbf{E} + \mathbf{C}\}\mathbf{Q}, \tag{55}$$

where matrix $\mathbf{C}$ has the same form as matrix $\mathbf{M}$, but with the terms $\boldsymbol{m}_k$ replaced by c_k. Upon elimination of vector $\mathbf{Q}$ from Eqs. (52) and (55), one obtains the one-port source equation

$$\mathbf{P}_S = \mathbf{Z}_S\mathbf{V} + \mathbf{P}, \tag{56}$$

where the source impedance matrix, $\mathbf{Z}_S$, and the source pressure strength vector, $\mathbf{P}_S$, are given by

$$\mathbf{P}_S = \frac{\gamma - 1}{\gamma S v_o}\mathbf{h}, \tag{57}$$

$$\mathbf{Z}_S = \frac{p_o}{v_o}\mathbf{E} - \frac{\gamma - 1}{2\gamma S}\mathbf{M}. \tag{58}$$

Thus, the time-variant one-port source is described by Eq. (56). This model embodies a number of interesting features. Firstly, it is seen that the source pressure strength of this model is the same as that of the basic time-invariant model; that is, time-variance has no effect on the source pressure strength. Secondly, time-variance modifies the concept of source impedance to that of source impedance matrix, $\mathbf{Z}_S$. If the source impedance matrix is to be computed to the accuracy of K harmonics of FCF, the Fourier spectrum of $m(t)$ must be available to the accuracy of $2K$ harmonics, and some elements of the real part of $\mathbf{Z}_S$ may be negative. Finally, and peculiarly, the time-variance of the source impedance matrix is determined by the time-variance of exhaust gas mass injection, and the time-variance of the speed of sound squared at an exhaust port, that is, matrix $\mathbf{C}$, has no effect.

For the engine cylinder having the exhaust gas mass injection of Fig. 1, the contribution of the time-variant term to the source impedance matrix is almost indiscernible. The order of magnitude of this term may not be as small as this for all engines, but, it is clear that, it can, in general, be neglected as a first approximation for most engines.

4.2. Moving time-variant one-port sources along a manifold

If the sound wave propagation is assumed to be time-invariant, the time-variant one-port sources can be moved along the pipes and junctions of a manifold as described in Section 3. The same equations will be valid for every harmonic of the FCF, but now these equations must be expressed collectively for the harmonics considered in the source spectrum so as to conform with the time-variant one-port source equation, Eq. (56). Hence, it can be shown that, the working equations for moving a time-variant one-port source along an isentropic uniform pipe carrying a uniform mean flow can be expressed as

$$\mathbf{P}_{S2} = 2(\mathbf{E} - \mathbf{G})^{-1}\mathbf{T}^{+}(\varsigma_{S1} + \mathbf{E})^{-1}\mathbf{P}_{S1}, \tag{59}$$

$$\varsigma_{S2} = (\mathbf{E} + \mathbf{G})(\mathbf{E} - \mathbf{G})^{-1}, \tag{60}$$

which are the counterparts of Eqs. (34) and (35), respectively. Here, ς_{Si} ($=\mathbf{Z}_{Si}/z_{oi}$) denotes the normalized source impedance matrix of time-variant source at plane i, and matrix $\mathbf{G}$ is given by

$$\mathbf{G} = \mathbf{T}^{+}(\varsigma_{S1} + \mathbf{E})^{-1}(\varsigma_{S1} - \mathbf{E})\mathbf{T}^{-*}, \tag{61}$$

where $\mathbf{T}^{\pm}$ denotes the diagonal matrix

$$\mathbf{T}^{\pm} = \exp\lfloor \mathrm{i}K_1^{\pm}L\ iK_2^{\pm}L \ldots \mathrm{i}K_K^{\pm}L - \mathrm{i}K_1^{\pm}L - \mathrm{i}K_2^{\pm}L \ldots - \mathrm{i}K_K^{\pm}L\rfloor \tag{62}$$

and $K_k^{\pm}$ is given by Eq. (36) with $\omega = 2\pi k(\mathrm{FCF})$, $k = 1,2,\ldots,K$.

Similarly, the working equations for moving time-variant one-port sources across a junction can be shown to be

$$\mathbf{P}_{S1} = \rho_{o1}\left(M_{o1}s_1\rho_{o1}\mathbf{E} + \sum_{i=2,3,\ldots}\mathbf{a}_i s_i \rho_{oi}\right)^{-1}\sum_{i=2,3,\ldots}(\mathbf{a}_i + M_{oi}\mathbf{E})s_i\mathbf{P}_{Si}, \tag{63}$$

$$\mathbf{Z}_{S1} = z_{o1}\left(M_{o1}s_1\rho_{o1}\mathbf{E} + \sum_{i=2,3,\ldots}\mathbf{a}_i s_i\rho_{oi}\right)^{-1}\left(s_1\rho_{o1}E + M_{o1}\sum_{i=2,3,\ldots}\mathbf{a}_i s_i\rho_{oi}\right), \tag{64}$$

where

$$\mathbf{a}_i = (z_{oi}\mathbf{E} - M_{oi}\mathbf{Z}_{Si})(Z_{Si} - z_{oi}M_{oi}E)^{-1}. \tag{65}$$

Clearly, Eqs. (63) and (64) are the counterparts of Eqs. (40) and (41), respectively.

The foregoing equations have been applied to compute the equivalent source characteristics of the manifold shown in Fig. 6, however, this part of the results is not reproduced here, because, as can be expected from the above discussed relative order of magnitudes of the terms of the source impedance matrix, no deviation worthy of reporting exists from the time-invariant source characteristics shown in Figs. 3–5.

4.3. A nonlinear one-port source model

The nonlinear one-port source model that will be presented in this section follows from the basic time-invariant model by addition of the non-linear term in Eq. (17) to Eq. (44). The nonlinear term in Eq. (16) is not taken into account on the grounds that it will in general be small compared to the total enthalpy fluctuations at the ports. Thus, the equations governing the present nonlinear one-port source model of exhaust noise generation are

$$S\{v_o[p']/2 + \gamma v_o\langle p'\rangle + \gamma p_o v'_+\} = (\gamma - 1)(mh)', \tag{66}$$

$$\gamma p_o v_o v'_+ + c_o^2[p'] = -\gamma\{p_o[v'^2/2] + \langle p'\rangle v_o v'_+\}, \tag{67}$$

where the third-order nonlinear term in ε'_2 has been neglected. Upon elimination of p'_-, these equations can be written in the form

$$\begin{aligned}\frac{p_o}{v_o}\left[\left\{1 - \tfrac{1}{4}(\gamma - 1)M_o\left(\frac{v'_+}{\bar{c}_o}\right)\right\}v'_+\right]' + \left[\left\{1 - \tfrac{1}{2}M_o\left(\frac{v'_+}{\bar{c}_o}\right)p'_+\right\}\right]' \\ = \frac{\gamma - 1}{Sv_o\gamma}\left[(mh)'\left[1 - \tfrac{1}{2}\gamma M_o\left(\frac{v'_+}{\bar{c}_o}\right)\right]\right]'.\end{aligned} \tag{68}$$

This result reveals that the effect of nonlinearity can be neglected as long as the Mach number of the particle velocity amplitude remains of the same order as the Mach number of the mean flow velocity at the exhaust port (the condition $M_o^2 \ll 1$ being assumed to be applicable). Thus, according to the present theory, the critical parameter for marked nonlinear effects on exhaust noise generation, is the amplitude of the particle velocity generated at the ports, and not the pressure amplitudes. It is not the purpose of the present paper to dwell on the use of Eq. (68) for developing a nonlinear model of an engine exhaust manifold source. In fact, it appears that it would be difficult to implement Eq. (68) in any practically useful manner. Indeed, if nonlinearity is important at the ports, its effects should also be prevalent in the manifold pipes and junctions, which means that the acoustic modelling of the manifold should also take the nonlinear effects into account. This is a task of almost similar complexity as solving the nonlinear gas dynamic equations, however, since the Mach number of the particle velocity amplitude at the exhaust ports is not likely to exceed that of the mean flow velocity in most engines, the undertaking of such a task should not be warranted in many cases.

5. Conclusion

A new mathematical model has been presented for the exhaust noise generation mechanism in a steady running engine. This model assumes that the gas injection at an exhaust valve is concentrated in a discontinuity plane and generates plane wave motion and uniform mean flow downstream of this plane. It has been shown that the acoustic impedance upstream of the discontinuity plane can be assumed to be infinitely large during the cycle if it is much larger than p_o/v_o. The exact boundary condition is difficult to specify with certainty due to the time-variance of the valve-lift, however, the assumption that this infinite impedance condition is satisfied in practice is not expected to incur substantial modelling inaccuracy at the frequencies of interest, as the valve-lift is usually small.

Linear time-invariant, linear time-variant and nonlinear models have been presented for one-port source characterization of engine exhaust noise generation mechanism. The impedance and strength of the linear time-invariant model have terms that are inversely proportional to the Mach number of the mean flow generated at the port. These are the dominant terms, as port mean flow Mach numbers are in general low for most engines. The linear time-invariant source model based on Eqs. (28) and (30) was pursued for simplicity and to provide insight, however, the parent model, Eqs. (21)–(24), can be used for more accurate computations.

The presence of a definite mean flow at a port, that is, *an operating engine*, is required for the proposed source model to be meaningful. Therefore, the source impedance may not be given values independently of the port mean flow, and since this rules out the case of infinite source impedance, exhaust noise source mechanism may not be modelled as a pure volume velocity source according to the present theory.

The rate of exhaust gas mass injection and the specific stagnation enthalpy of exhaust gas flow are required as input for the present theory, the product of which being the predominant parameter that determines the source pressure strength. These parameters can be predicted fairly accurately by thermodynamic cycle simulation. To first approximation, cycle simulation can be carried out by neglecting the effects of pressure and temperature fluctuations at exhaust and intake ports. Thermodynamic cycle analysis can then be refined, if required, in an iterative scheme in which the present theory is used, together with a heat transfer model, to predict the cyclic back-pressure and temperature variations at the ports. Since the latter are in general dependent on the load, the source characteristics will also be load-dependent. Thus, the present theory can be used to estimate the likely effects of load dependence on the source characteristics.

When the time-variant terms are taken into account, the source impedance becomes a matrix of size equal to the number of harmonics considered in its representation. In general, the use of a time-variant one-port source model is appropriate if the rate of gas mass injection has substantial fluctuation, and only the dominant harmonics need to be considered in the representation of the source impedance matrix.

Systematic procedures have been described for the calculation of one-port source characteristics of an engine manifold for both time-invariant and time-variant one-port source models. These are strictly applicable if the wave propagation in a manifold is time-invariant. Clearly, if the port sources are time-variant, the equivalent source at the reference plane will also be time-variant, even if the wave propagation in the manifold is modelled as being time-invariant.

The present theory is applicable, with almost no modification, to engine intake noise source characterization, or to the discharge and suction noise source characterization of compressors.

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