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# Bounded nonlinear stochastic control based on the probability distribution for the sdof oscillator

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**Abstract**

In the response control of building structures subjected to strong excitations, the controller saturation problem needs be considered in the design stage. The nonlinearity of saturated controllers, however, makes the exact probabilistic evaluation of control effectiveness difficult. In this study, a stochastic nonlinear control algorithm with bounded control force is proposed and the closed-loop system's joint probability density function (PDF) governed by the reduced Fokker–Planck equation is derived. Based on the derived joint PDF, the bound of the control force that restricts the first-passage failure probability of the displacement response to a prescribed value is determined. Numerical analyses results show that the proposed controller reduces the maximum required control force considerably compared to the linear controller for the same displacement reduction, and the proposed approximation of the failure probability is considerably accurate.


## 1. Introduction

Many active control methods have been developed and implemented in the field of civil engineering in order to suppress excessive vibrations induced by earthquake or wind loads [1,2]. In design of active control of structures, two important problems need to be considered due to the probabilistic characteristics of such dynamic loads. The first problem is the *controller saturation*.

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Since the magnitude of the dynamic loads cannot be known a priori, the control law may require excessive control forces greater than the capacity of the control device in a real situation. Consequently, the controller saturation problem needs to be considered in the design stage. The second problem is the exact evaluation of control effectiveness taking into account the stochastic nature of dynamic loads on civil structures. The nonlinearity of saturated controllers, which is required to solve the first problem, makes the second one more difficult. This is due to the reason that the joint probability density function (PDF) of the stationary response of the nonlinear system under a white noise excitation can be exactly estimated by solving the reduced Fokker–Planck equation (RFPE), while the solution of the equation is known only for a limited number of cases [3–5].

The most widely known control algorithm with bounded control force is the bang–bang control algorithm [6]. However, the abrupt change of the control force from the upper bound to the lower bound or vice versa makes its actual implementation difficult. Wu and Soong [7] and Cai et al. [8] proposed modified bang–bang type controllers to prevent the rapid change of the control force considering actuator dynamics. But aforementioned researches do not address the stochastic nature of the excitation in structural control.

Spencer et al. [9] and May and Beck [10] proposed controller optimizations based on the first-passage failure probability for sdof and mdof structures, respectively, considering parameter uncertainty of the controlled system. But the controller in their studies was restricted to the linear one. Zhu and his co-workers [11–17] and Dimentberg et al. [18] applied the stochastic dynamic programming principle to a variety of controlled systems simplified by the stochastic averaging method. Resulting controllers were nonlinear in their general forms. In particular, bang–bang type controllers were obtained in the minimization of the first-passage failure probability [16–18]. But these types of controllers may exhibit the abrupt control force change in the actual implementation. In addition, the stochastic responses are predicted only for the stochastically averaged system.

In this paper, the control of the sdof oscillator subjected to the Gaussian white noise excitations is studied and a probability-based nonlinear control algorithm with bounded control force is proposed. The exact solution of RFPE, i.e., the exact joint PDF of the closed-loop system is derived. Using this joint PDF, bounds of the control force that restrict the first-passage failure probability of the displacement response to a prescribed value are calculated. Numerical examples are presented for the verification of the proposed controller and the accuracy of the approximated first-passage failure probability.

## 2. Bounded nonlinear control

This study focuses on a linear sdof structure subjected to multiple white noise excitations of which mass-normalized equation of motion is represented by

$$\ddot{x} + 2\xi_0\omega_0\dot{x} + \omega_0^2 x = \mathbf{b_1 w} + b_2 u, \tag{1}$$

where $\omega_0$, $\xi_0$ are the natural frequency and the damping ratio of the structure, respectively, and $x$, $\mathbf{w}$, $u$, $\mathbf{b}_1$ and $b_2$ are the displacement, the excitation vector, the control force, a constant vector, and a coefficient, respectively.

In general, the structural response control is achieved by changing dominant natural frequencies or damping ratios of the structure. Particularly in civil structures, the latter is more acceptable than the former, because changing natural frequencies may increase acceleration response and requires relatively large control force. For comparison, velocity and displacement feedback controllers are designed for an sdof structure with a mass of 1 kg, natural frequency of 0.5 Hz and a damping ratio of 2%. Control gains for both controllers are scaled to have the same maximum control force for the El Centro earthquake. The displacement time histories of the sdof system are presented in Fig 1. It is clear from the result that the velocity feedback is much more effective in reducing displacement response than the displacement feedback. This implies that the increase of damping ratio has a greater effect in reducing responses for the given sdof structure.

In the velocity feedback control, the control force is assumed to have the following form in order to guarantee the stability of the closed-loop system:

$$u = -f_0(H)\dot{x}, \tag{2}$$

where $f_0$ is the feedback gain and $H$ is the non-negative total energy of the structure represented by

$$H = \tfrac{1}{2}\left(\dot{x}^2 + \omega_0^2 x^2\right). \tag{3}$$

From the definition of the total energy presented in Eq. (3), the absolute value of the velocity is bounded as follows:

$$|\dot{x}| = \sqrt{2H - \omega_0^2 x^2} \leqslant \sqrt{2H}, \tag{4}$$

where the equality is satisfied for the zero displacement. As a result, the upper bound of the control force magnitude is given by

$$|u| \leqslant f_0(H)\sqrt{2H}. \tag{5}$$

Therefore, the nonlinear control gain function, $f_0(H)$, is proposed in this study as follows so that the control force in Eq. (2) does not exceed a given maximum control force limit:

$$f_0(H) = \frac{u_{\max}}{\sqrt{2H}}, \tag{6}$$

where $u_{\max}$ is a design variable representing the upper bound of the control force magnitude. As the increased damping ratio of the closed-loop system with a nonlinear gain of Eq. (6) is also a

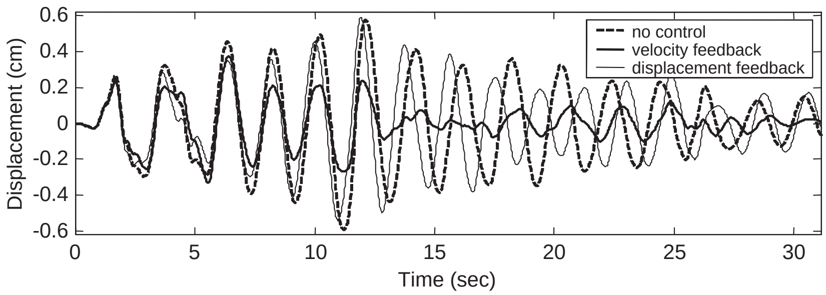


Fig. 1. Comparison of the velocity and the displacement feedback controls.

function of the total energy, authors name the proposed control algorithm *Energy Damping Control*. This new control algorithm generates the positive or negative maximum control forces whenever the displacement becomes zero. This means that the controller exerts its full capacity whenever the velocity reaches its maximum amplitude for harmonic response, even for low-level vibration.

The abrupt change of the control force direction due to the sudden change of the velocity sign, which may cause instability for the control system with time delay effect, is prevented by introducing an energy function that represents the level of structural response. This function is given by

$$V(H) = \frac{H^p}{H^p + \alpha^{2p}} \quad (p>0, \quad \alpha>0), \tag{7}$$

where $p$ and $\alpha$ are design parameters related to the function shape. Multiplication of $V(H)$ to the original gain, Eq. (6), yields the following modified gain:

$$f(H) = V(H)f_0(H) = \frac{u_{\max}}{\sqrt{2}} \frac{H^{p-1/2}}{H^p + \alpha^{2p}} \tag{8}$$

and corresponding control force

$$u = -f(H)\dot{x}. \tag{9}$$

$V(H)$ has a non-negative value smaller than 1.0 and approaches to 0.0 and 1.0 as the total energy goes to zero and infinity, respectively. Therefore, the control force settles down or exerts its full capacity for low or high level of structural response, respectively. Some examples of $V(H)$ for several values of $p$ and $\alpha$ are plotted in Fig. 2. It can be observed that the range of $H$ corresponding to the flat range of $V(H)$ close to 1.0 increases for smaller $\alpha$ and larger $p$. Also, increasing the value of $p$ makes the separation of two flat ranges of $H$ clearer, one close to 0.0 and the other close to 1.0. But the inflection point does not change by changing the value of $p$.

Modified control gains, $f(H)$, for different values of parameter $p$ and $\alpha$ are plotted in Fig. 3. It can be observed that, for smaller $\alpha$ and larger $p$, the peak of $f(H)$ increases. Since a large control gain is unfavorable for the stability if uncertainty exists in the plant to be controlled, the choice of proper $p$ and $\alpha$ is important.

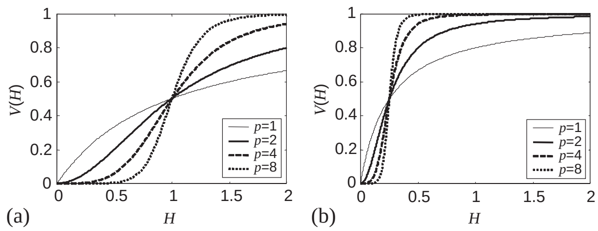


Fig. 2. Shape of $V(H)$. (a) $\alpha = 0.5$; (b) $\alpha = 0.25$.

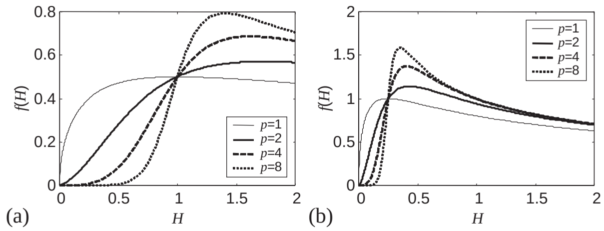


Fig. 3. Shape of $f(H)$. (a) $\alpha = 0.5$; (b) $\alpha = 0.25$.

## 3. Joint PDF of the closed-loop system

The closed-loop system for the equation of motion, Eq. (1), is represented by

$$\ddot{x} + h(x, \dot{x}) = \mathbf{b}_1 \mathbf{w}, \tag{10}$$

where $\mathbf{w}$ is assumed to be a vector of stationary Gaussian white noise processes and $h(x, \dot{x})$ is given by

$$h(x, \dot{x}) = \left\{ 2\xi_0\omega_0 + b_2 f(H) \right\} \dot{x} + \omega_0^2 x. \tag{11}$$

Then, the joint PDF of this closed-loop system is governed by the following RFPE [19]:

$$0 = \dot{x}\frac{\partial P(x, \dot{x})}{\partial x} - \frac{\partial [h(x, \dot{x}) P(x, \dot{x})]}{\partial \dot{x}} - \pi \mathbf{b}_1 \mathbf{S}_\mathbf{w} \mathbf{b}_1^\mathrm{T} \frac{\partial^2 P(x, \dot{x})}{\partial \dot{x}^2}, \tag{12}$$

where $P(x, \dot{x})$ and $\mathbf{S}_\mathbf{w}$ are the joint PDF and the power spectral density (PSD) matrix of $\mathbf{w}$, respectively. Since the coefficient of the velocity term in Eq. (11) is a function of the total energy, $H$, the closed-loop system belongs to the class of generalized stationary potential and its analytical solution is given by [3]

$$P(x, \dot{x}) = C\mathrm{e}^{-\varphi(H)}, \tag{13}$$

where $C$ is a normalization coefficient and $\phi(H)$ is a function of the total energy of which the derivative is represented by

$$\varphi'(H) = \frac{1}{\pi \mathbf{b}_1 \mathbf{S}_\mathbf{w} \mathbf{b}_1^\mathrm{T}} \left[ 2\xi_0\omega_0 + \frac{b_2 u_{\max}}{\sqrt{2}} \frac{H^{p-1/2}}{H^p + \alpha^{2p}} \right]. \tag{14}$$

To obtain the joint PDF presented in the Eq. (13), the integration of the derivative in Eq. (14) is required. Since the analytical integration of Eq. (14) for the general value of $p$ is very difficult, it is induced from the integration results for the $p$'s of 1, 2, 3, and 4 and given by

$$\begin{aligned} \varphi(H) = \frac{\sqrt{2}\alpha b_2 u_{\max}}{\pi p \mathbf{b}_1 \mathbf{S}_\mathbf{w} \mathbf{b}_1^\mathrm{T}} \Bigg[ & \frac{\sqrt{2} p \xi_0 \omega_0}{\alpha b_2 u_{\max}} H + \frac{p}{\alpha}\sqrt{H} \\ & + \sum_{i=1}^{n} \cos\left(\frac{2i-1}{2n}\pi\right) \ \ln \sqrt{H - 2\cos\left(\frac{2i-1}{2n}\pi\right) \alpha\sqrt{H} + \alpha^2} \\ & - \sum_{i=1}^{n} \sin\left(\frac{2i-1}{2n}\pi\right) \ \arctan\left(\frac{\sqrt{H}}{\alpha} \csc\left(\frac{2i-1}{2n}\pi\right) - \cot\left(\frac{2i-1}{2n}\pi\right)\right)\Bigg] \end{aligned} \tag{15}$$

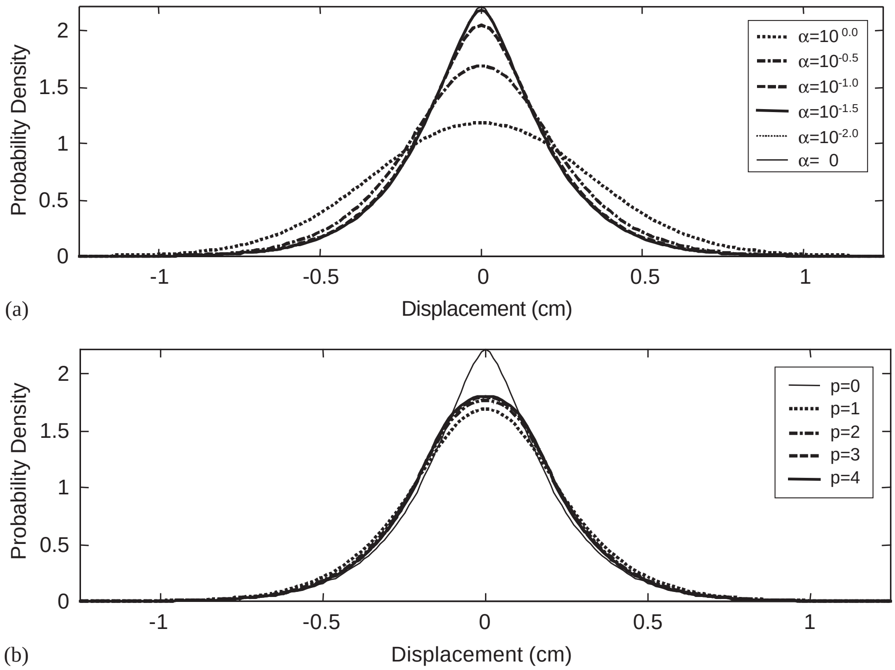


Fig. 4. Displacement PDF's of the closed-loop system. (a) PDF for $p = 1$ and various $\alpha$'s; (b) PDF for $\alpha = 10^{-0.5}$ and various p's.

The resulting joint PDF of the closed-loop system displacement for various $p$'s and $\alpha$'s are plotted in Fig. 4. Only one excitation with $\mathbf{S_w} = 0.5\pi\,\mathrm{cm}^2/\mathrm{s}^3$ is considered and other parameters are chosen as follows; $\xi_0 = 0.02$, $\omega_0 = \pi$ rad/s, $\mathbf{b_1} = -1$, $b_2 = 0.01\,\mathrm{kg}^{-1}$ and $u_{\max} = 1$ N. In Fig. 4(a), it is shown that the PDF concentrates more to the center as $\alpha$ becomes smaller implying higher reduction of the response. On the other hand, Fig. 4(b) shows that $p$ has insignificant influence on the responses. However, since $p$ influences the peak control gain, which is closely related to the stability of closed-loop system, as shown in Fig. 3, its value should be selected carefully.

## 4. Design and evaluation of the controller

To design a structural controller for given random excitations, the peak response of the structure needs to be defined stochastically. To define the peak response for the design of the proposed stochastic control strategy, the first-passage failure probability is employed. This probability is difficult to obtain analytically and thereby the solution is often obtained numerically [20,21]. In this study, to make the design process simple, the following failure probability proposed by Vanmarke for the first-passage problem of the Gaussian narrow band process [22] is employed:

$$P_F(x_b, t_s) = 1 - \exp\left(-\nu_{|X|}(x_b)t_s\,\frac{1 - \exp\left(-q^{1.2}\sqrt{\pi/2}\,x_b/\sigma_X\right)}{1 - \exp\left(-x_b^2/(2\sigma_X^2)\right)}\right), \tag{16}$$

where $t_s$ is the duration time, $v_{|X|}(x_b)$ is the double-barrier crossing rate of the process, $X$, over the barrier level, $x_b$, $\sigma_X$ is the standard deviation, and $q$ is a parameter associated with the frequency band size. $v_{|X|}(x_b)$ can be calculated exactly by using the joint PDF of the closed-loop system given by Eq. (13), while $q$ can be calculated only approximately by using the formula for a linear sdof oscillator as [23]

$$q = \sqrt{1 - \frac{1}{1-\xi_{\mathrm{eq}}^2}\left(1 - \frac{2}{\pi}\tan^{-1}\frac{\xi_{\mathrm{eq}}}{\sqrt{1-\xi_{\mathrm{eq}}^2}}\right)^2}, \tag{17}$$

where $\xi_{\mathrm{eq}}$ is an equivalent damping ratio derived from the well-known formula on the crossing rate of a linear sdof oscillator subjected to the Gaussian white noise excitation, and is given as

$$\xi_{\mathrm{eq}} = -\frac{\pi \mathbf{b}_1 \mathbf{S}_{\mathbf{w}} \mathbf{b}_1^{\mathrm{T}}}{x_b^2 \omega_0^3} \ln\left(\frac{\pi v_{|x|}(x_b)}{\omega_0}\right), \tag{18}$$

It should be noted that the failure probability presented in Eq. (16) is an approximation for the closed-loop system with the proposed controller and Gaussian random excitation, since the response of such a nonlinear system to the Gaussian random excitation is non-Gaussian.

The design of the proposed controller involves determinations of three parameters associated with the nonlinear control gain in Eq. (8), i.e., $u_{\max}$, $\alpha$ and $p$, that meet the required first-passage failure probability of the controlled structure. At the initial design stage, $u_{\max}$, meeting the target failure probability of the structure for prescribed $\alpha$ and $p$, is searched using an appropriate method. As an example, a process to determine $u_{\max}$ based on the bisection algorithm is illustrated in Fig. 5, where $u_{\max,u}$ and $u_{\max,l}$ are the upper and lower bounds of $u_{\max}$, respectively, and $P_0$ is the target failure probability.

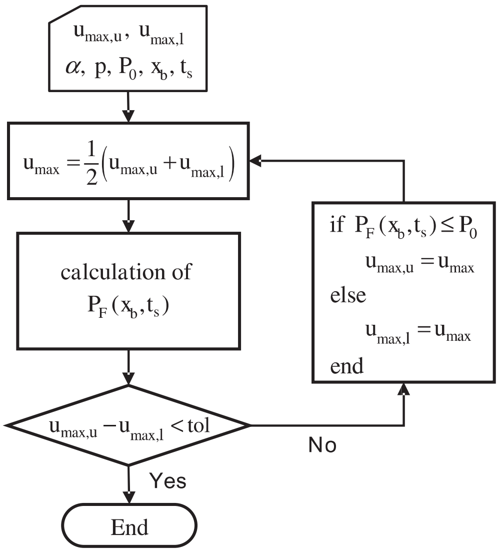


Fig. 5. Determination of $u_{\max}$ by the bisection algorithm.

## 5. Numerical analysis

To investigate the influence of design parameters, $\alpha$ and $p$, on the transient response property, a time history analysis of an sdof structure is performed. Structural parameters are chosen as follows: $\xi_0 = 0.02$, $\omega_0 = \pi\,\mathrm{rad/s}$, $\mathbf{b_1} = -1$, $b_2 = 0.01\,\mathrm{kg}^{-1}$. A sample function of the ground acceleration is generated by passing a white noise excitation through the Kanai–Tajimi filter rather than using the original white noise directly, for more realistic simulation. This can be justified by the fact that the response of the sdof structure has little difference for the white noise and the filtered white noise, if the natural frequency of the Kainai–Tajimi filter is sufficiently larger than that of the sdof structure. The natural frequency and damping ratio of the filter are set to be 15.6 rad/s and 0.6, respectively, as suggested by Kanai representing the firm soil condition [24]. The PSD of the filter input white noise is assumed to be $0.5\pi\,\mathrm{cm^2/s^3}$. For comparison, a linear velocity feedback controller that provides the closed-loop system with 8% damping ratio is designed as well. The control force bound, $u_{\max}$, of the nonlinear controller is set to be 0.9212 N, which is the maximum control force of the linear controller for the aforementioned ground acceleration.

First, simulations are carried out for varying $\alpha$ with fixed $p$. The results are presented in Fig. 6. In case of $\alpha = 1.25$, the result is not as good as the linear controller. This is because the control force is very suppressed by $V(H)$ over a region of relatively small energy, $H$. In case of $\alpha \leqslant 0.25$, the proposed controller is better than the linear controller, and the controlled responses are almost the same regardless of $\alpha$. However, in case of $a = 0.05$, the abrupt direction change of the control force occurs frequently, which is necessary to be prevented.

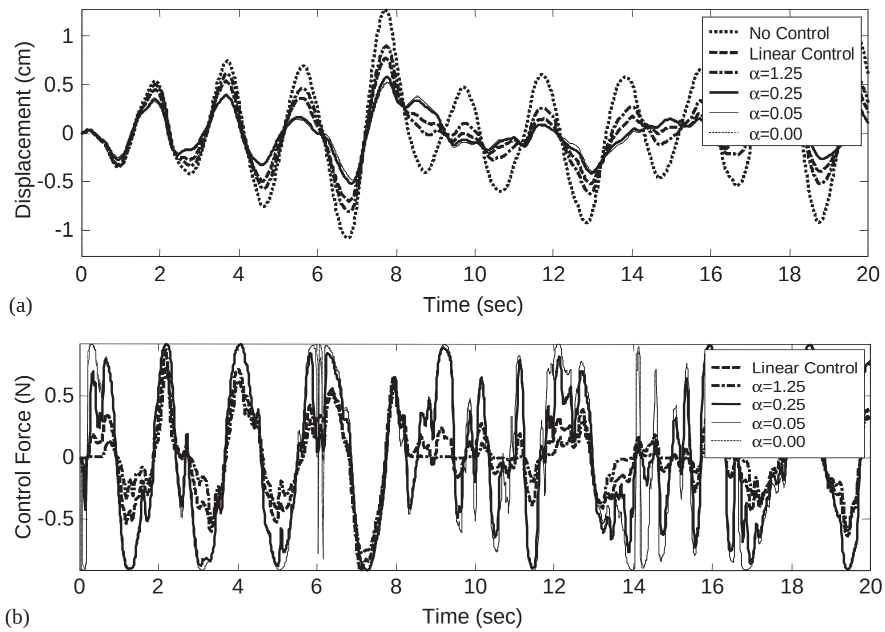


Fig. 6. Conrol results for various $\alpha$'s ($p = 2$). (a) Relative displacement; (b) control force.

Second, the effect of $p$ on control performance is investigated for varying $p$ with fixed $\alpha$. The result is shown in Fig. 7. Results indicate that the responses change little for the variation of $p$ while the required peak control forces are smaller as $p$ becomes smaller.

Next, the effect of the excitation magnitude on control performance is investigated. Simulation results for the scaled ground acceleration by the factor of 0.2 are presented in Fig. 8. This investigation is important because the excitation with the magnitude smaller than the design level happens more frequently from a probabilistic point of view. It is shown that as $\alpha$ decreases, more response reduction is achieved. However, for $\alpha = 0.05$, the control force exhibits a lot of high-frequency contents, which may cause a spillover effect on the higher structural modes when applied to the control of mdof structure. Therefore, $\alpha$ should be selected carefully, considering the magnitude of the excitation on which the controller operates. Additionally, simulation results for the scaled ground acceleration by a factor of 2.0 are presented in Fig. 9. For comparison, the control force of the linear controller is cut down at $u_{\max}$. Little difference between the response of the linear control and that of the nonlinear control is observed due to the saturation effect of the former.

Finally, statistical verification of the approximate failure probability calculation procedure is performed for the sdof structure using the same Kanai–Tagimi filter for the preceding analyses. The following envelop function is multiplied to the white noise input for the filter in order to emulate the non-stationary characteristics of earthquake:

$$f_{\text{env}}(t) = \begin{cases} (t/t_1)^2 & (t < t_1), \\ 1 & (t_1 \leqslant t \leqslant t_2), \\ \mathrm{e}^{-c(t-t_2)} & (t_2 < t), \end{cases} \tag{19}$$

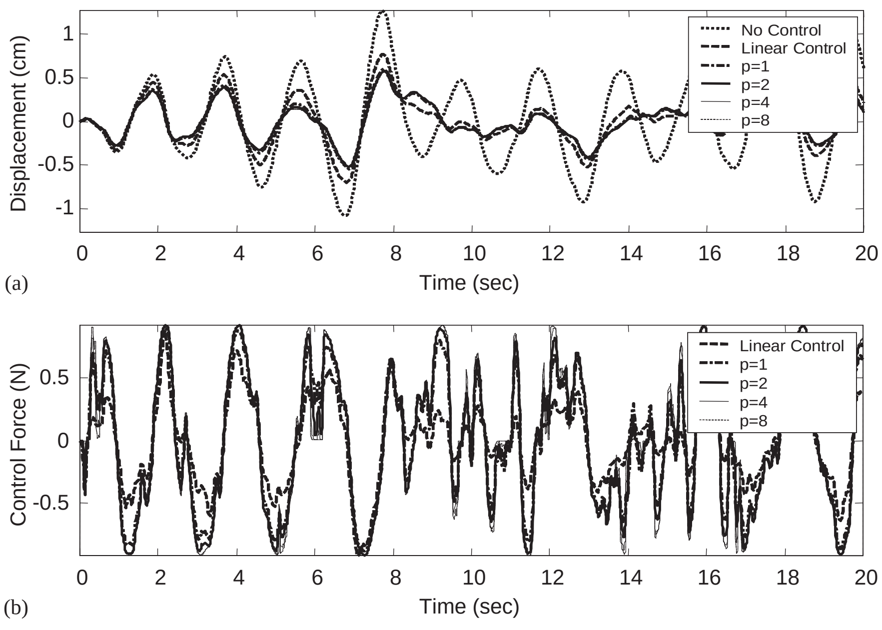


Fig. 7. Control results for various $p$'s ($\alpha = 0.25$). (a) Relative displacement; (b) control force.

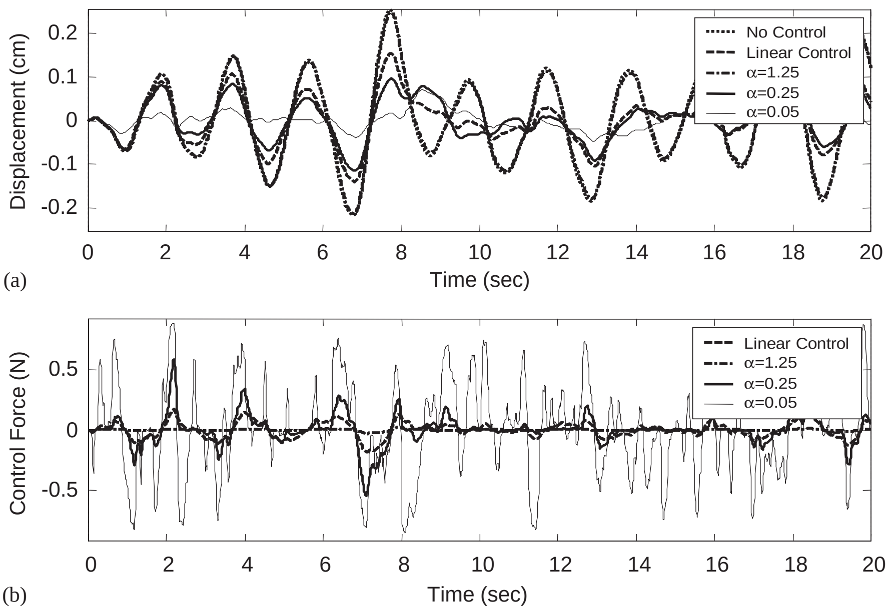


Fig. 8. Control result for the excitation scaled down by 0.2 ($p = 1$). (a) Relative displacement; (b) control force.

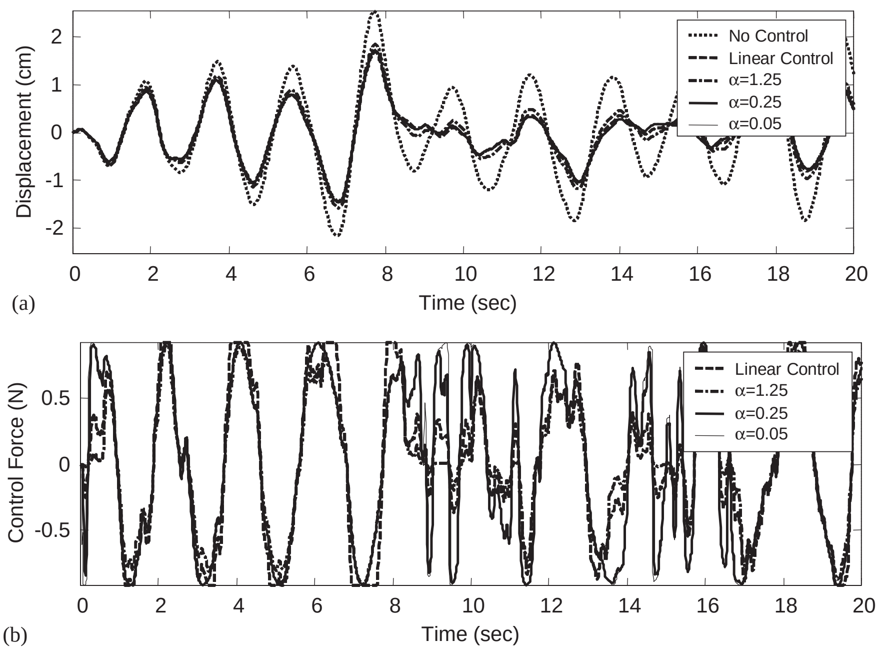


Fig. 9. Control result for the excitation scaled up by 2.0 ($p = 1$). (a) Relative displacement; (b) control force.

Table 1
Maximum control force and first-passage failure probability

| Target displacement reduction ratio | Maximum control force (N) | | | First-passage failure probability (%) | |
|---|---|---|---|---|---|
| | Linear control (A) | Nonlinear control (B) | Ratio (B/A) | Linear control | Nonlinear control |
| 0.8 | 0.4278 | 0.2961 | 0.69 | 4.6 | 4.0 |
| 0.6 | 1.0242 | 0.7072 | 0.69 | 5.5 | 4.1 |
| 0.4 | 2.0750 | 1.4096 | 0.68 | 5.5 | 4.6 |

where $t_1$, $t_2$ and $c$ are set as 20s, 40s, and 0.15, respectively. This envelope function, proposed by Jennings [25], has a stationary region in the middle where the stationary assumption of the excitation used in the derivation of the joint PDF of the nonlinear closed-loop system is valid. Further, if the frequency band of the excitation is sufficiently large compared to the natural frequency of the structure, the excitation can be approximated by the white noise.

The uncontrolled maximum displacement is found to be 2.701 cm. Linear and nonlinear velocity feedback controllers are designed to reduce the maximum displacement by 80, 60 and 40% with 5% first-passage failure probability. Parameters $a$ and $p$ of the nonlinear controller are selected to be 0.01 and 1, respectively. The maximum control forces of the linear and the nonlinear controller are compared in Table 1. The maximum control force for a linear controller is a probabilistically estimated one by the sum of mean and standard deviation of the maximum control force using Davenport's formula [26]. Results show that the proposed nonlinear controllers achieve same target displacement reduction ratios using only about 70% of the maximum control force of linear controllers. Also, the first-passage failure probabilities calculated from time history analyses for 1000 sample functions of ground acceleration is provided in Table 1. Results indicate that the failure probabilities are close to the design failure probability, 5%, and the proposed approximation method is considerably accurate.

## 6. Conclusions

A new nonlinear control algorithm, *energy damping control*, is proposed for the sdof structure subjected to random excitations. For the design and evaluation of the proposed controller, the joint PDF of the closed-loop system is derived and the approximate first-passage failure probability calculation procedure is proposed. By proper selection of the parameter $\alpha$ and $p$ of the nonlinear control gain, the abrupt change or waste of control force, which are important issues in actual implementation, can be prevented.

Numerical analyses results indicate that the required maximum control forces expended by the proposed nonlinear controllers are only about 70% of that of the linear controller in order to achieve the same response reduction ratio. Further, simulation results show that the proposed approximation of the failure probability for the nonlinear closed-loop system subjected to a filtered Gaussian white noise excitation is considerably accurate.

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