Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Journal of Sound and Vibration 281 (2005) 261–273
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Localization of elastic waves in periodic rib-stiffened rectangular plates under axial compressive load

F.M. Li[a,*], Y.S. Wang[a], C. Hu[b], W.H. Huang[b]

[a] *Institute of Engineering Mechanics, School of Civil Engineering, Beijing Jiaotong University, Beijing 100044, People's Republic of China*

[b] *Department of Aerospace Engineering and Mechanics, Harbin Institute of Technology, Harbin 150001, People's Republic of China*

Received 12 June 2003; accepted 15 January 2004
Available online 18 September 2004

Abstract

Based on the theory of elastic dynamics, the problem of wave localization in disordered periodic multi-span rib-stiffened plates is investigated. The transfer matrix method is employed to obtain the transfer matrix of the system, and the method for calculating the Lyapunov exponents in continuous dynamical systems presented by Wolf is used to determine the localization factors in discrete dynamical systems. As examples, the numerical results of the localization factors are given for a disordered periodic multi-span rib-stiffened plate under axial compressive load. The effects of the degree of disorder of span length and the structural parameters on the elastic wave localization are analyzed. The larger the degree of disorder, the larger the degree of localization. The larger the dimensionless torsional and flexural rigidities of the rib, the larger the degree of localization.

1. Introduction

Plates are commonly used in many engineering applications such as airplanes, buildings, bridges and ships. In order to enhance the ability of resisting axial instability and transverse rigidity of plates, ribs are often added in them. If the ribs are added to them in period form, the

*Corresponding author.
E-mail address: llffcc@163.net (F.M. Li).

doi:10.1016/j.jsv.2004.01.055

plates became periodic rib-stiffened ones. In complete periodic rib-stiffened plates, waves can propagate throughout all the structures. But disorder can lead to the appearance of localization of elastic waves in mistuned periodic structures. Localization leads to a spatial decay of wave amplitude, and the associated exponential decay constant is known as localization factor. So localization factor characterizes the average exponential rates of decay of wave amplitudes in disordered periodic structures. Up to now, many people have studied this problem [1–3].

Some people have studied the problem of static buckling mode localization in disordered periodic rib-stiffened plates and shells under axial compressive load [4,5]. When rib-stiffened plates and shells are subjected to transverse dynamical load, their motion equations are relevant to the time and it is more difficult to solve the problem. Therefore, fewer people have investigated localization of flexural waves in periodic rib-stiffened plates and shells.

Elishakoff et al. [6] studied the buckling mode localization in disordered two-span and three-span rib-stiffened plates under axial compressive load using the method of model analysis. They discovered that small span-length disorder of the plates could lead to the appearance of buckling mode localization in the structures. Subsequently, they investigated the buckling mode localization in multi-span rib-stiffened plates applying the method of finite difference [7]. They considered the effects of span-length disorder and torsional stiffness of ribs on the localization and discovered that the torsional stiffness of ribs remarkably influences the buckling mode localization.

Xie [8] studied the static buckling mode localization in disordered periodic rib-stiffened plate and presented the transfer matrix of structure and the formulation of localization factor. Using a modified finite element method, Sridharan and Zeggane [9] analyzed the buckling mode localization of rib-stiffened plates and shells and presented numerical results.

Based on the theory of elastic dynamics, localization of elastic waves in disordered periodic rib-stiffened plates is studied. The method of transfer matrix is applied to obtain the transfer matrix of the system. The method for calculating Lyapunov exponents in continuous dynamical systems by Wolf [10] is used to determine them in discrete ones. The expression for computing the localization factor of the system is further presented. As examples, the numerical results of localization factors are given for disordered periodic multi-span rib-stiffened plates. The effects of the disorder of span length and structural parameters on localization of elastic waves are analyzed.

This paper is organized as follows. In Section 2, the wave motion equation and transfer matrix of multi-span rib-stiffened plates are given. In Section 3, the formulation for calculating localization factors of multi-coupled systems is presented. As examples, the numerical results of localization factors for disordered periodic rib-stiffened plates are calculated and analyzed in Section 4. The conclusions from this study are drawn in Section 5.

2. Wave motion equation and transfer matrix

The localization of elastic waves in a homogeneous periodic multi-span rib-stiffened plate as shown in Fig. 1 is studied. There are n ribs and $n+1$ spans in the rib-stiffened plate. The local coordinate of each span is depicted in Fig. 1. The rectangular plate is compressed in its middle plane by forces uniformly distributed along the sides $x_1 = 0$ and $x_{n+1} = a_{n+1}$. The magnitude of

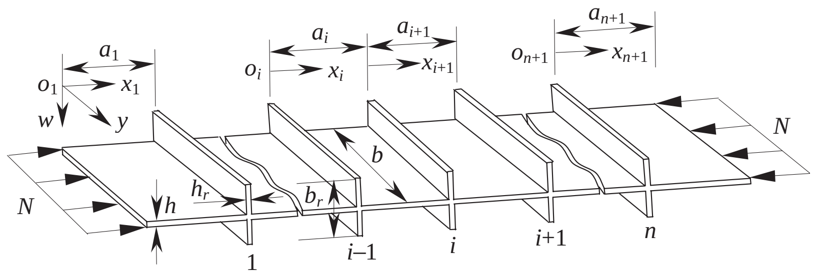

Fig. 1. Simply supported periodic rib-stiffened rectangular plate under axial compressive load.

the compressive force per unit length of the edge is N. The boundary conditions of the plate are assumed to be simply supported.

The equation of motion of flexural waves in plate uniformly compressed in x-direction can be written as the following form [11]:

$$D\nabla^2\nabla^2 w + N\frac{\partial^2 w}{\partial x^2} + \rho h\frac{\partial^2 w}{\partial t^2} = q, \tag{1}$$

where $w(x, y, t)$ is the transverse displacement, $D = Eh^3/12(1 - v^2)$ the bending stiffness of plate, E the Young's modulus, v the Poisson ratio, ρ and h the density and the thickness of plate, N the magnitude of compressive force per unit length of the edge in x-direction of plate, $\nabla^2 = \partial^2/\partial x^2 + \partial^2/\partial y^2$ the Laplacian operator, t the time and q the transverse load. Considering free bending motion, q is fixed at zero.

Steady solution of elastic wave motion in plate is studied. According to the boundary conditions of simple support on the edges $y=0$ and b, the transverse deflection of the plate can be expressed as

$$w = W(x)\sin\frac{\pi y}{b}\exp(-\mathrm{i}\omega t), \tag{2}$$

where ω is the circular frequency of elastic wave and b is the width of the plate in direction y. Substituting Eq. (2) and $q=0$ into Eq. (1) leads to the following fourth-order linear ordinary differential equation for $W(x)$

$$\frac{\mathrm{d}^4 W}{\mathrm{d}x^4} + \left(\frac{N}{D} - \frac{2\pi^2}{b^2}\right)\frac{\mathrm{d}^2 W}{\mathrm{d}x^2} + \left(\frac{\pi^4}{b^4} - k^4\right)W = 0, \tag{3}$$

where $k = (\rho h\omega^2/D)^{1/4} = 2\pi/\lambda$ is the wavenumber, and λ is the wavelength.

The eigenvalues of Eq. (3) are given by

$$r^2 = \left(\frac{\pi^2}{b^2} - \frac{N}{2D}\right) \pm \sqrt{\frac{N}{2D}\left(\frac{N}{2D} - \frac{2\pi^2}{b^2}\right) + k^4}. \tag{4}$$

It is known that the critical buckling load N_{cr} for a simply supported compressed rectangular plate is [11,12]

$$N_{\mathrm{cr}} = 4\pi^2 D/b^2. \tag{5}$$

In the present study, the case that the rectangular plate does not buckle is considered. So the axial compressive load N satisfies the following condition:

$$0 \leqslant N \leqslant N_{\mathrm{cr}}. \tag{6}$$

In order to investigate the localization of elastic waves, at least one pair of opposite propagating waves are taken into account. When there are one pair of propagating waves and one pair of attenuating waves in the plate, the solutions of Eq. (4) can be expressed as

$$r_{1,3} = \pm\alpha_1, \quad r_{2,4} = \alpha_2 \pm \mathrm{i}\beta_2. \tag{7a}$$

For the case of two pairs of propagating waves in the plate, the solutions of Eq. (4) can be written as

$$r_{1,3} = \alpha_1 \pm \mathrm{i}\beta_1, \quad r_{2,4} = \alpha_2 \pm \mathrm{i}\beta_2, \tag{7b}$$

where $\mathrm{i} = \sqrt{-1}$. The real and imaginary parts of Eqs. (7a) and (7b) satisfy the following conditions:

$$\alpha_i \leqslant 0, \ \ \beta_i \geqslant 0, \quad (i = 1, 2). \tag{8}$$

So, the general solution of Eq. (3) is given by

$$W(x) = A \exp(r_1 x) + B \exp(r_2 x) + C \exp(r_3 x) + D \exp(r_4 x), \tag{9}$$

where A, B, C and D are unknown coefficients to be determined by the continuity and the boundary conditions.

For a typical span i, $i = 1,2,\ldots,n+1$, the transverse deflection can be written as

$$w_i(x_i, y) = [A_i \exp(r_1 x) + B_i \exp(r_2 x) + C_i \exp(r_3 x) + D_i \exp(r_4 x)] \sin\frac{\pi y}{b} \exp(-\mathrm{i}\omega t), \tag{10}$$

where A_i, B_i, C_i and D_i are coefficients to be determined by the continuity and the boundary conditions.

The continuity conditions between the two typical neighboring spans i and $i+1$ are given by

$$w_i|_{x_i=a_i} = w_{i+1}\big|_{x_{i+1}=0}, \tag{11a}$$

$$\left.\frac{\partial w_i}{\partial x_i}\right|_{x_i=a_i} = \left.\frac{\partial w_{i+1}}{\partial x_{i+1}}\right|_{x_{i+1}=0}, \tag{11b}$$

$$M_x^{(i+1)}\big|_{x_{i+1}=0} - M_x^{(i)}\big|_{x_i=a_i} = G_r J_r \left.\frac{\partial^3 w_{i+1}}{\partial x_{i+1}\partial y^2}\right|_{x_{i+1}=0} + \rho_r J_r \frac{\partial^2}{\partial t^2}\left.\left(\frac{\partial w_{i+1}}{\partial x_{i+1}}\right)\right|_{x_{i+1}=0}, \tag{11c}$$

$$V_x^{(i+1)}\big|_{x_{i+1}=0} - V_x^{(i)}\big|_{x_i=a_i} = E_r I_r \left.\frac{\partial^4 w_{i+1}}{\partial y^4}\right|_{x_{i+1}=0} + \rho_r A_r \left.\frac{\partial^2 w_{i+1}}{\partial t^2}\right|_{x_{i+1}=0}, \tag{11d}$$

where $G_r J_r$ and $E_r I_r$ are the torsional and flexural rigidities of the ribs, respectively, $\rho_r J_r$ the moment of inertia per unit length of the ribs, ρ_r the density of the ribs and A_r the cross-sectional area of the ribs. $M_x^{(i)}$ and $V_x^{(i)}$ are the bending moment and shear force in span i, and they can be expressed as

$$M_x^{(i)} = -D\left(\frac{\partial^2 w_i}{\partial x_i^2} + v\frac{\partial^2 w_i}{\partial y^2}\right), \tag{12a}$$

$$V_x^{(i)} = -D\left[\frac{\partial^3 w_i}{\partial x_i^3} + (2-v)\frac{\partial^3 w_i}{\partial x_i\,\partial y^2}\right]. \tag{12b}$$

Substituting Eqs. (12a) and (12b) into Eqs. (11c) and (11d) leads to the following expressions:

$$\begin{aligned}&-\left(\frac{\partial^2 w_{i+1}}{\partial x_{i+1}^2} + v\frac{\partial^2 w_{i+1}}{\partial y^2}\right)\Bigg|_{x_{i+1}=0} + \left(\frac{\partial^2 w_i}{\partial x_i^2} + v\frac{\partial^2 w_i}{\partial y^2}\right)\Bigg|_{x_i=a_i}\\ &= \frac{G_r J_r}{D}\frac{\partial^3 w_{i+1}}{\partial x_{i+1}\,\partial y^2}\bigg|_{x_{i+1}=0} + \frac{\rho_r J_r}{D}\frac{\partial^2}{\partial t^2}\left(\frac{\partial w_{i+1}}{\partial x_{i+1}}\right)\bigg|_{x_{i+1}=0},\end{aligned} \tag{13a}$$

$$\begin{aligned}&-\left[\frac{\partial^3 w_{i+1}}{\partial x_{i+1}^3} + (2-v)\frac{\partial^3 w_{i+1}}{\partial x_{i+1}\,\partial y^2}\right]\Bigg|_{x_{i+1}=0} + \left[\frac{\partial^3 w_i}{\partial x_i^3} + (2-v)\frac{\partial^3 w_i}{\partial x_i\,\partial y^2}\right]\Bigg|_{x_i=a_i}\\ &= \frac{E_r I_r}{D}\frac{\partial^4 w_{i+1}}{\partial y^4}\bigg|_{x_{i+1}=0} + \frac{\rho_r A_r}{D}\frac{\partial^2 w_{i+1}}{\partial t^2}\bigg|_{x_{i+1}=0}.\end{aligned} \tag{13b}$$

Substituting Eq. (10) into Eqs. (11a), (11b), (13a) and (13b), one can get the following formulations:

$$A_i\exp(r_1a_i) + B_i\exp(r_2a_i) + C_i\exp(r_3a_i) + D_i\exp(r_4a_i) - A_{i+1} - B_{i+1} - C_{i+1} - D_{i+1} = 0, \tag{14a}$$

$$\begin{aligned}&A_ir_1\exp(r_1a_i) + B_ir_2\exp(r_2a_i) + C_ir_3\,exp(r_3a_i) + D_ir_4\exp(r_4a_i)\\ &\quad - A_{i+1}r_1 - B_{i+1}r_2 - C_{i+1}r_3 - D_{i+1}r_4 = 0,\end{aligned} \tag{14b}$$

$$\begin{aligned}&A_ip_1\exp(r_1a_i) + B_ip_2\exp(r_2a_i) + C_ip_3\exp(r_3a_i) + D_ip_4\exp(r_4a_i)\\ &\quad + A_{i+1}\xi_1 + B_{i+1}\xi_2 + C_{i+1}\xi_3 + D_{i+1}\xi_4 = 0,\end{aligned} \tag{14c}$$

$$\begin{aligned}&A_ir_1q_1\exp(r_1a_i) + B_ir_2q_2\exp(r_2a_i) + C_ir_3q_3\exp(r_3a_i) + D_ir_4q_4\exp(r_4a_i)\\ &\quad + A_{i+1}\zeta_1 + B_{i+1}\zeta_2 + C_{i+1}\zeta_3 + D_{i+1}\zeta_4 = 0,\end{aligned} \tag{14d}$$

where

$$\begin{aligned}&p_i = r_i^2 - v(\pi/b)^2,\;\; q_i = r_i^2 - (2-v)(\pi/b)^2,\\ &\xi_i = (G_rJ_rr_i/D)(\pi/b)^2 - r_i^2 + v(\pi/b)^2 + \rho_rJ_r\omega^2r_i/D,\\ &\zeta_i = (-E_rI_r/D)(\pi/b)^4 - r_i^3 + (2-v)r_i(\pi/b)^2 + \rho_rA_r\omega^2/D.\end{aligned}$$

The following dimensionless quantities are introduced:

$$\hat{a}_i = \pi a_i/b, \quad \hat{k} = bk/\pi, \quad \hat{h} = h/b, \quad \hat{h}_r = h_r/b, \quad \hat{b}_r = b_r/b, \tag{15}$$

where $\hat{a}_i$, $\hat{k}$, $\hat{h}$, $\hat{h}_r$ and $\hat{b}_r$ are the dimensionless span length, wavenumber, thickness of plate, thickness of rib and width of rib. Then the following non-dimensional quantities are employed:

$$\hat{N} = \frac{Nb^2}{\pi^2 D} = \frac{12(1-v^2)}{\pi^2 \hat{h}^2}\left(\frac{\sigma}{E}\right), \quad \hat{r}^2 = \left(\frac{b}{\pi}\right)^2 r^2 = \left(1 - \frac{\hat{N}}{2}\right) \pm \sqrt{\frac{\hat{N}}{2}\left(\frac{\hat{N}}{2} - 2\right) + \hat{k}^4},$$

$$\hat{E}_r = \frac{\pi E_r I_r}{bD} = \pi(1-v^2)\hat{h}_r \left(\frac{\hat{b}_r}{\hat{h}}\right)^3 \left(\frac{E_r}{E}\right), \quad \hat{G}_r = \frac{\pi G_r J_r}{bD} = \frac{\pi(1-v^2)}{2(1+v_r)}\hat{b}_r \frac{\hat{h}_r(\hat{h}_r^2 + \hat{b}_r^2)}{\hat{h}^3}\left(\frac{E_r}{E}\right),$$

$$\hat{\omega}_{r1} = \frac{b\rho_r J_r \omega^2}{\pi D} = \pi^3 \frac{\hat{h}_r \hat{b}_r (\hat{h}_r^2 + \hat{b}_r^2)}{12\hat{h}}\left(\frac{\rho_r}{\rho}\right)\hat{k}^4, \quad \hat{\omega}_{r2} = \frac{b^3 \rho_r A_r \omega^2}{\pi^3 D} = \pi \frac{\hat{h}_r \hat{b}_r}{\hat{h}}\left(\frac{\rho_r}{\rho}\right)\hat{k}^4, \tag{16}$$

where σ is the axial compressive stress of the plate, v_r the Poisson ratio of the ribs, $\hat{E}_r$ and $\hat{G}_r$ the dimensionless flexural and torsional rigidities of the ribs, respectively.

Substituting Eqs. (15) and (16) into Eqs. (14a)–(14d) leads to the following dimensionless expressions:

$$A_i \exp(\hat{r}_1\hat{a}_i) + B_i \exp(\hat{r}_2\hat{a}_i) + C_i \exp(\hat{r}_3\hat{a}_i) + D_i \exp(\hat{r}_4\hat{a}_i) - A_{i+1} - B_{i+1} - C_{i+1} - D_{i+1} = 0, \tag{17a}$$

$$\begin{aligned} &A_i\hat{r}_1 \exp(\hat{r}_1\hat{a}_i) + B_i\hat{r}_2 \exp(\hat{r}_2\hat{a}_i) + C_i\hat{r}_3 \exp(\hat{r}_3\hat{a}_i) + D_i\hat{r}_4 \exp(\hat{r}_4\hat{a}_i) \\ &\quad - A_{i+1}\hat{r}_1 - B_{i+1}\hat{r}_2 - C_{i+1}\hat{r}_3 - D_{i+1}\hat{r}_4 = 0, \end{aligned} \tag{17b}$$

$$\begin{aligned} &A_i\hat{p}_1 \exp(\hat{r}_1\hat{a}_i) + B_i\hat{p}_2 \exp(\hat{r}_2\hat{a}_i) + C_i\hat{p}_3 \exp(\hat{r}_3\hat{a}_i) + D_i\hat{p}_4 \exp(\hat{r}_4\hat{a}_i) \\ &\quad + A_{i+1}\hat{\xi}_1 + B_{i+1}\hat{\xi}_2 + C_{i+1}\hat{\xi}_3 + D_{i+1}\hat{\xi}_4 = 0, \end{aligned} \tag{17c}$$

$$\begin{aligned} &A_i\hat{r}_1\hat{q}_1 \exp(\hat{r}_1\hat{a}_i) + B_i\hat{r}_2\hat{q}_2 \exp(\hat{r}_2\hat{a}_i) + C_i\hat{r}_3\hat{q}_3 \exp(\hat{r}_3\hat{a}_i) + D_i\hat{r}_4\hat{q}_4 \exp(\hat{r}_4\hat{a}_i) \\ &\quad + A_{i+1}\hat{\zeta}_1 + B_{i+1}\hat{\zeta}_2 + C_{i+1}\hat{\zeta}_3 + D_{i+1}\hat{\zeta}_4 = 0, \end{aligned} \tag{17d}$$

where $\hat{p}_i = \hat{r}_i^2 - v$, $\hat{q}_i = \hat{r}_i^2 - (2-v)$, $\hat{\xi}_i = \hat{G}_r\hat{r}_i - \hat{r}_i^2 + v + \hat{\omega}_{r1}\hat{r}_i$, $\hat{\zeta}_i = -\hat{E}_r - \hat{r}_i^3 + (2-v)\hat{r}_i + \hat{\omega}_{r2}$.

Solving Eqs. (17a)–(17d) for A_{i+1}, B_{i+1}, C_{i+1} and D_{i+1} in terms of A_i, B_i, C_i and D_i results in matrix equation

$$\mathbf{v}_{i+1} = \mathbf{T}_i \mathbf{v}_i, \tag{18}$$

where $\mathbf{v}_i = \{A_i, B_i, C_i, D_i\}^{\mathrm{T}}$ is the state vector of the ith span and $\mathbf{T}_i$ is the 4×4 transfer matrix, the elements of which are given by

$$T_{11} = \frac{[\hat{E}_r - \hat{r}_1^3 - \hat{r}_3\hat{r}_4(\hat{r}_1 - \hat{r}_2) - \hat{r}_1\hat{r}_2(\hat{r}_3 + \hat{r}_4) + \hat{r}_1(\hat{r}_1 + \hat{G}_r + \hat{\omega}_{r1})(\hat{r}_2 + \hat{r}_3 + \hat{r}_4) - \hat{\omega}_{r2}]\exp(\hat{r}_1\hat{a}_i)}{(\hat{r}_2 - \hat{r}_1)(\hat{r}_1 - \hat{r}_3)(\hat{r}_1 - \hat{r}_4)},$$

$$T_{12} = \frac{[\hat{E}_r + (\hat{G}_r + \hat{\omega}_{r1})\hat{r}_2(\hat{r}_2 + \hat{r}_3 + \hat{r}_4) - \hat{\omega}_{r2}]\exp(\hat{r}_2\hat{a}_i)}{(\hat{r}_2 - \hat{r}_1)(\hat{r}_1 - \hat{r}_3)(\hat{r}_1 - \hat{r}_4)},$$

$$T_{13}=\frac{[\hat{E}_r+(\hat{G}_r+\hat{\omega}_{r1})\hat{r}_3(\hat{r}_2+\hat{r}_3+\hat{r}_4)-\hat{\omega}_{r2}]\exp(\hat{r}_3\hat{a}_i)}{(\hat{r}_2-\hat{r}_1)(\hat{r}_1-\hat{r}_3)(\hat{r}_1-\hat{r}_4)},$$

$$T_{14}=\frac{[\hat{E}_r+(\hat{G}_r+\hat{\omega}_{r1})\hat{r}_4(\hat{r}_2+\hat{r}_3+\hat{r}_4)-\hat{\omega}_{r2}]\exp(\hat{r}_4\hat{a}_i)}{(\hat{r}_2-\hat{r}_1)(\hat{r}_1-\hat{r}_3)(\hat{r}_1-\hat{r}_4)},$$

$$T_{21}=\frac{[\hat{E}_r+(\hat{G}_r+\hat{\omega}_{r1})\hat{r}_1(\hat{r}_1+\hat{r}_3+\hat{r}_4)-\hat{\omega}_{r2}]\exp(\hat{r}_1\hat{a}_i)}{(\hat{r}_1-\hat{r}_2)(\hat{r}_2-\hat{r}_3)(\hat{r}_2-\hat{r}_4)},$$

$$T_{22}=\frac{[\hat{E}_r-\hat{r}_2^3-\hat{r}_3\hat{r}_4(\hat{r}_2-\hat{r}_1)-\hat{r}_1\hat{r}_2(\hat{r}_3+\hat{r}_4)+\hat{r}_2(\hat{r}_1+\hat{r}_3+\hat{r}_4)(\hat{r}_2+\hat{G}_r+\hat{\omega}_{r1})-\hat{\omega}_{r2}]\exp(\hat{r}_2\hat{a}_i)}{(\hat{r}_1-\hat{r}_2)(\hat{r}_2-\hat{r}_3)(\hat{r}_2-\hat{r}_4)},$$

$$T_{23}=\frac{[\hat{E}_r+(\hat{G}_r+\hat{\omega}_{r1})\hat{r}_3(\hat{r}_1+\hat{r}_3+\hat{r}_4)-\hat{\omega}_{r2}]\exp(\hat{r}_3\hat{a}_i)}{(\hat{r}_1-\hat{r}_2)(\hat{r}_2-\hat{r}_3)(\hat{r}_2-\hat{r}_4)},$$

$$T_{24}=\frac{[\hat{E}_r+(\hat{G}_r+\hat{\omega}_{r1})\hat{r}_4(\hat{r}_1+\hat{r}_3+\hat{r}_4)-\hat{\omega}_{r2}]\exp(\hat{r}_4\hat{a}_i)}{(\hat{r}_1-\hat{r}_2)(\hat{r}_2-\hat{r}_3)(\hat{r}_2-\hat{r}_4)},$$

$$T_{31}=\frac{[\hat{E}_r+(\hat{G}_r+\hat{\omega}_{r1})\hat{r}_1(\hat{r}_1+\hat{r}_2+\hat{r}_4)-\hat{\omega}_{r2}]\exp(\hat{r}_1\hat{a}_i)}{(\hat{r}_1-\hat{r}_3)(\hat{r}_3-\hat{r}_2)(\hat{r}_3-\hat{r}_4)},$$

$$T_{32}=\frac{[\hat{E}_r+(\hat{G}_r+\hat{\omega}_{r1})\hat{r}_2(\hat{r}_1+\hat{r}_2+\hat{r}_4)-\hat{\omega}_{r2}]\exp(\hat{r}_2\hat{a}_i)}{(\hat{r}_1-\hat{r}_3)(\hat{r}_3-\hat{r}_2)(\hat{r}_3-\hat{r}_4)},$$

$$T_{33}=\frac{[\hat{E}_r-\hat{r}_3^3-\hat{r}_3\hat{r}_4(\hat{r}_1+\hat{r}_2)-\hat{r}_1\hat{r}_2(\hat{r}_3-\hat{r}_4)+\hat{r}_3(\hat{r}_1+\hat{r}_2+\hat{r}_4)(\hat{r}_3+\hat{G}_r+\hat{\omega}_{r1})-\hat{\omega}_{r2}]\exp(\hat{r}_3\hat{a}_i)}{(\hat{r}_1-\hat{r}_3)(\hat{r}_3-\hat{r}_2)(\hat{r}_3-\hat{r}_4)},$$

$$T_{34}=\frac{[\hat{E}_r+(\hat{G}_r+\hat{\omega}_{r1})\hat{r}_4(\hat{r}_1+\hat{r}_2+\hat{r}_4)-\hat{\omega}_{r2}]\exp(\hat{r}_4\hat{a}_i)}{(\hat{r}_1-\hat{r}_3)(\hat{r}_3-\hat{r}_2)(\hat{r}_3-\hat{r}_4)},$$

$$T_{41}=\frac{[\hat{E}_r+(\hat{G}_r+\hat{\omega}_{r1})\hat{r}_1(\hat{r}_1+\hat{r}_2+\hat{r}_3)-\hat{\omega}_{r2}]\exp(\hat{r}_1\hat{a}_i)}{(\hat{r}_1-\hat{r}_4)(\hat{r}_4-\hat{r}_2)(\hat{r}_4-\hat{r}_3)},$$

$$T_{42}=\frac{[\hat{E}_r+(\hat{G}_r+\hat{\omega}_{r1})\hat{r}_2(\hat{r}_1+\hat{r}_2+\hat{r}_3)-\hat{\omega}_{r2}]\exp(\hat{r}_2\hat{a}_i)}{(\hat{r}_1-\hat{r}_4)(\hat{r}_4-\hat{r}_2)(\hat{r}_4-\hat{r}_3)},$$

$$T_{43}=\frac{[\hat{E}_r+(\hat{G}_r+\hat{\omega}_{r1})\hat{r}_3(\hat{r}_1+\hat{r}_2+\hat{r}_3)-\hat{\omega}_{r2}]\exp(\hat{r}_3\hat{a}_i)}{(\hat{r}_1-\hat{r}_4)(\hat{r}_4-\hat{r}_2)(\hat{r}_4-\hat{r}_3)},$$

$$T_{44}=\frac{[\hat{E}_r-\hat{r}_4^3-\hat{r}_3\hat{r}_4(\hat{r}_1+\hat{r}_2)+\hat{r}_1\hat{r}_2(\hat{r}_3-\hat{r}_4)+\hat{r}_4(\hat{r}_1+\hat{r}_2+\hat{r}_3)(\hat{r}_4+\hat{G}_r+\hat{\omega}_{r1})-\hat{\omega}_{r2}]\exp(\hat{r}_4\hat{a}_i)}{(\hat{r}_1-\hat{r}_4)(\hat{r}_4-\hat{r}_2)(\hat{r}_4-\hat{r}_3)}.$$

In Eq. (18), the state vector $\mathbf{v}_{i+1}$, which is a measure of the transverse deflection of the $(i+1)$th span, is related to that at the ith span through the transfer matrix $\mathbf{T}_i$. Beginning from the first span and iteratively employing Eq. (18), it can be seen that the state vector of the $(i+1)$th span is related to that of the first span by a product of transfer matrices

$$v_{n+1}=\mathbf{\Gamma}_n v_1, \tag{19}$$

where $\mathbf{\Gamma}_n=\mathbf{T}_n\mathbf{T}_{n-1}\cdots\mathbf{T}_1$ is the total transfer matrix.

3. Localization of elastic waves

Lyapunov exponent measures the average exponential rate of convergence or divergence between two neighboring phase orbits in phase space and qualitatively and quantitatively describes the dynamical characters of chaos systems. When studying elastic wave localization in periodic structures, by employing the concept of Lyapunov exponent, one can get a measurable index about the rate of decay of wave amplitudes. According to the symmetry of periodic structures, it can be proved that Lyapunov exponents always occur in pairs, i.e. if λ_i is a Lyapunov exponent then $-\lambda_i$ is also a Lyapunov exponent [1,2]. Therefore, for $2m \times 2m$ transfer matrices, the m pairs of Lyapunov exponents have the following property, $\lambda_1 \geqslant \lambda_2 \geqslant \cdots \geqslant \lambda_m \geqslant \lambda_{m+1}(= -\lambda_m) \geqslant \lambda_{m+2}(= -\lambda_{m-1}) \geqslant \cdots \geqslant \lambda_{2m}(= -\lambda_1)$.

Localization factor is used to characterize the average exponential rate of decay of wave amplitudes. It is defined by the smallest positive Lyapunov λ_m. Since λ_m represents the wave which has the least amount of decay and transmits energy farther along the structure than any other waves. So, it characterizes the main decay behavior of elastic waves.

In this paper, the algorithm for calculating Lyapunov exponents for continuous dynamical system due to Wolf [10] is applied to calculate the Lyapunov exponents for the discrete dynamical system, Eq. (18). Assuming the dimension of the transfer matrices is $2m \times 2m$. In order to calculate the kth Lyapunov exponent, $1 \leqslant k \leqslant 2m$, k orthogonal unit vectors $\mathbf{u}_1^{(1)}, \mathbf{u}_1^{(2)}, \ldots, \mathbf{u}_1^{(k)}$ whose dimension is $2m$ are chosen as the initial state vectors. Eq. (18) is used to compute the state vectors iteratively. At the lth iteration, $v_{l+1}^{(j)} = T_l u_l^{(j)}$ $(l = 1,2,\ldots;\ j = 1,2,\ldots,k)$. The Gram–Schmidt orthonormalization procedure is now applied

$$\hat{\mathbf{v}}_{l+1}^{(1)} = \mathbf{v}_{l+1}^{(1)}, \quad \mathbf{u}_{l+1}^{(1)} = \frac{\hat{\mathbf{v}}_{l+1}^{(1)}}{\left\|\hat{\mathbf{v}}_{l+1}^{(1)}\right\|},$$

$$\hat{\mathbf{v}}_{l+1}^{(2)} = \mathbf{v}_{l+1}^{(2)} - (\mathbf{v}_{l+1}^{(2)}, \mathbf{u}_{l+1}^{(1)})\mathbf{u}_{l+1}^{(1)}, \quad \mathbf{u}_{l+1}^{(2)} = \frac{\hat{v}_{l+1}^{(2)}}{\left\|\hat{v}_{l+1}^{(2)}\right\|},$$

$$\vdots$$

$$\hat{v}_{l+1}^{(k)} = v_{l+1}^{(k)} - (v_{l+1}^{(k)}, u_{l+1}^{(k-1)})u_{l+1}^{(k-1)} - \cdots - (v_{l+1}^{(k)}, u_{l+1}^{(1)})u_{l+1}^{(1)}, \quad u_{l+1}^{(k)} = \frac{\hat{v}_{l+1}^{(k)}}{\left\|\hat{v}_{l+1}^{(k)}\right\|}.$$

After the k orthonormal unit vectors, $u_l^{(1)}, u_l^{(2)}, \ldots, u_l^{(k)}$, operated by transfer matrix $\mathbf{T}_l$ and orthonormalized by Gram–Schmidt procedure, the volume of a k-dimensional hypersphere is $||\hat{\mathbf{v}}_{l+1}^{(1)}|| \cdot ||\hat{\mathbf{v}}_{l+1}^{(2)}|| \cdots ||\hat{\mathbf{v}}_{l+1}^{(k)}|| = \prod_{j=1}^{k} ||\hat{\mathbf{v}}_{l+1}^{(j)}||$. Hence, after the initial vectors, $u_1^{(1)}, u_1^{(2)}, \ldots, u_1^{(k)}$, operated by a product of transfer matrices, $T_n T_{n-1} \cdots T_1$, the volume of a k-dimensional hypersphere becomes

$$V = \prod_{l=1}^{n} \left(\prod_{j=1}^{k} \left\| \hat{\mathbf{v}}_{l+1}^{(j)} \right\| \right). \tag{20}$$

For an n-dimensional dynamical system in phase space, a k-dimensional volume defined by the k principal axes evolves on the average as $\exp[(\lambda_1 + \lambda_2 + \cdots + \lambda_k)n]$, where $\lambda_1, \lambda_2, \ldots, \lambda_k$ are the k Lyapunov exponents. Hence, combined with Eq. (20), the expression for determining the kth

Lyapunov exponent is derived as

$$\lambda_k = \lim_{n\to\infty} \frac{1}{n} \sum_{l=1}^{n} \ln \left\| \hat{v}_{l+1}^{(k)} \right\|. \tag{21}$$

By means of the above expression, each of the m pairs of contrary Lyapunov exponents can be calculated. The mth Lyapunov exponent λ_m is the localization factor. Then it is implied that the wave amplitudes decay at the magnitude $exp(-\lambda_m)$ when they propagate through each span of the multi-span rib-stiffened plate. For this study, the dimensions of the transfer matrix $\mathbf{T}_i$ for the multi-span plate are 4×4. So, the second Lyapunov exponent λ_2 is the localization factor.

4. Example and discussions

As example, the localization of elastic wave in a periodic rib-stiffened multi-span plate is studied. The material property is considered to be same, i.e. $E = E_r$ and $\rho = \rho_r$. The span length a_i ($i = 1, 2, \ldots, n+1$) is assumed to be a uniformly distributed random variable with mean a and coefficient of variation δ, namely, a_i is a uniformly distributed random number between $[a(1-\sqrt{3}\delta),\ a(1+\sqrt{3}\delta)]$. Hence, if z_i is a standard uniformly distributed random variable, i.e. $z_i \in (0, 1)$, then a_i can be written as

$$a_i = a[1 + \sqrt{3}\delta(2z_i - 1)]. \tag{22}$$

In the present research, four values of the coefficient of variation of the random span length are considered, i.e. $\delta = 0$, 0.02, 0.05 and 0.1. The case $\delta = 0$ corresponds to the ordered periodic plate, namely, there is no misplacement in the ribs.

For the case of $\hat{h}_r = \hat{h} = 1/20$, $\hat{b}_r = 1/4$, Figs. 2(a)–(d) display the variation of localization factors versus dimensionless wavenumber $\hat{k}$ for $\hat{N} = 1.0, 2.0, 3.0$ and 4.0. For $\hat{N} = 2.0$, $\hat{h}_r = 1/40$, $\hat{b}_r = 1/4$, the localization factors versus dimensionless wavenumber are plotted in Fig. 3 for the case of $\hat{h} = 1/20$ and 1/40. For $\hat{N} = 0.2$, $\hat{h} = 1/20$, $\hat{b}_r = 1/4$, Fig. 4 displays the variation of localization factors versus non-dimensional wavenumber $\hat{k}$ for $\hat{h}_r = h/2$ and $\hat{h}_r = \hat{h}$. As $\hat{N} = 0.2$, $\hat{h} = 1/20$ and $\hat{h}_r = \hat{h}$, the curves of localization factors versus non-dimensional wavenumber $\hat{k}$ at different $\hat{b}_r$ are shown in Fig. 5. For $\hat{h} = 1/40$, $\hat{h}_r = \hat{h}$, $\hat{b}_r = 1/4$, the localization factors versus dimensionless axial compressive force are shown in Fig. 6 for the case of $\hat{k} = 2.0$, 3.0, 4.0 and 5.0. The discussions are as follows:

1. From Figs. 2–5, it can be seen that tuned periodic multi-span rib-stiffened plates have the properties of frequency passband and stopband and localization phenomenon can occur in mistuned periodic multi-span plates. For example, in Fig. 2(a) the values of curve 1 are zero at interval $\hat{k} \in (4.7, 5.4)$. This interval is known as passband. And at this interval, the values in curves 2, 3 and 4 are bigger than zero and the incident elastic waves are localized. The values of curve 1 are bigger than zero at interval $\hat{k} \in (5.4, 6.9)$. And this interval is called as stopband.
2. Due to the influence of axial compressive force N, one can observe from Figs. 2(a)–(d) that the intervals of dimensionless wavenumber $\hat{k}$ for propagating waves that satisfy Eqs. (7a) and (7b) are different for different non-dimensional axial force $\hat{N}$. When $0 \leqslant \hat{N} \leqslant 2.0$, the interval of

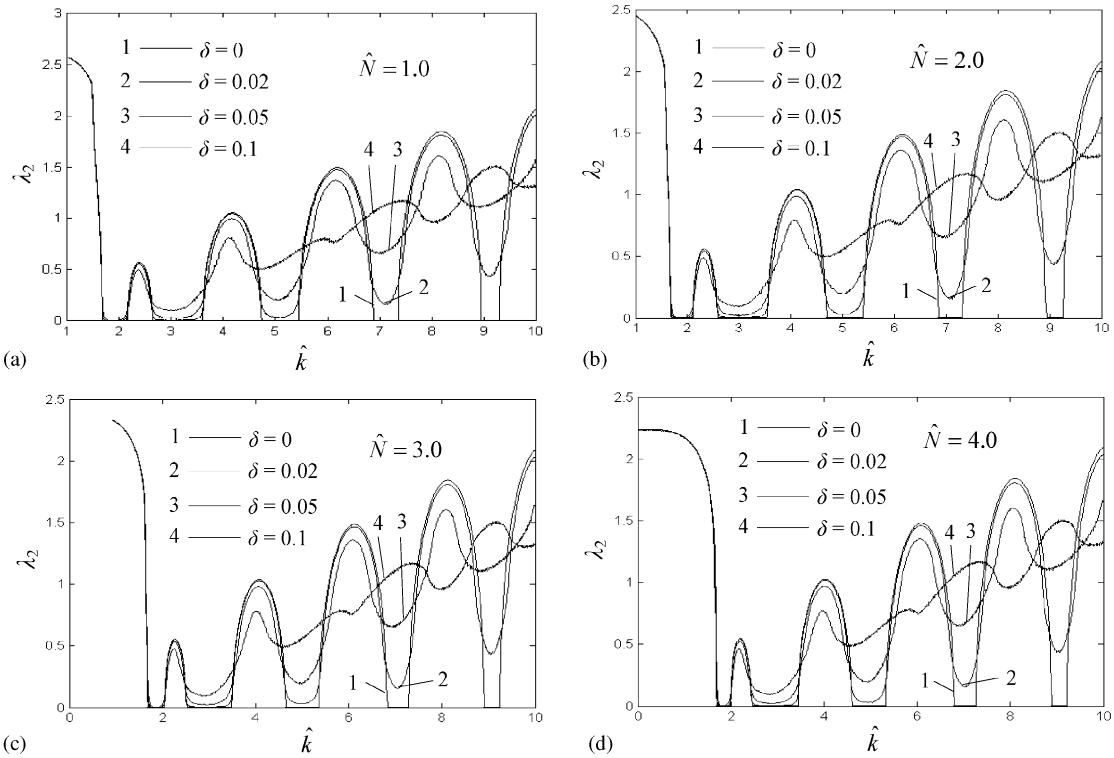

Fig. 2. Localization factors in rib-stiffened plates versus non-dimensional wavenumber ($\hat{h}=1/20$, $\hat{h}_r=\hat{h}$, $\hat{b}_r=1/4$).

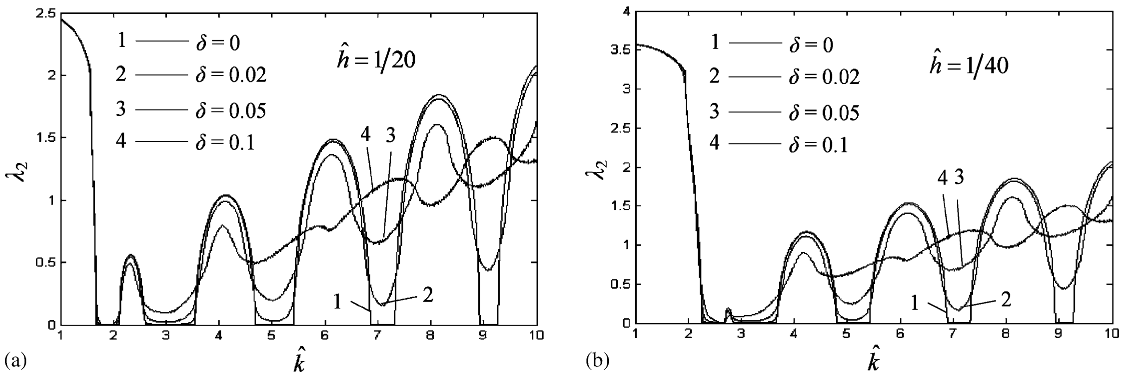

Fig. 3. Localization factors in rib-stiffened plates versus non-dimensional wavenumber ($\hat{N}=2.0$, $\hat{h}_r=1/40$, $\hat{b}_r=1/4$).

dimensionless wavenumber is $\hat{k}\geqslant 1.0$. When $2.0<\hat{N}\leqslant 4.0$, $\hat{k}$ can get values at interval (0, 1) and the range of $\hat{k}$ at this interval increases with the increase of $\hat{N}$.

3. With the decrease of the dimensionless thickness of the plate, it can be observed from Fig. 3 that the localization is stronger in lower frequency regions and there are less changes in

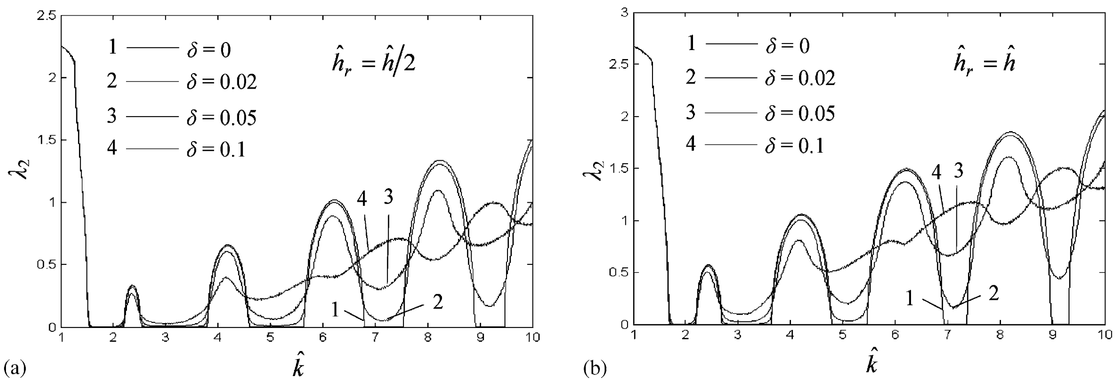

Fig. 4. (a, b) Localization factors in rib-stiffened plates versus non-dimensional wavenumber ($\hat{N} = 0.2$, $\hat{h} = 1/20$, $\hat{b}_r = 1/4$).

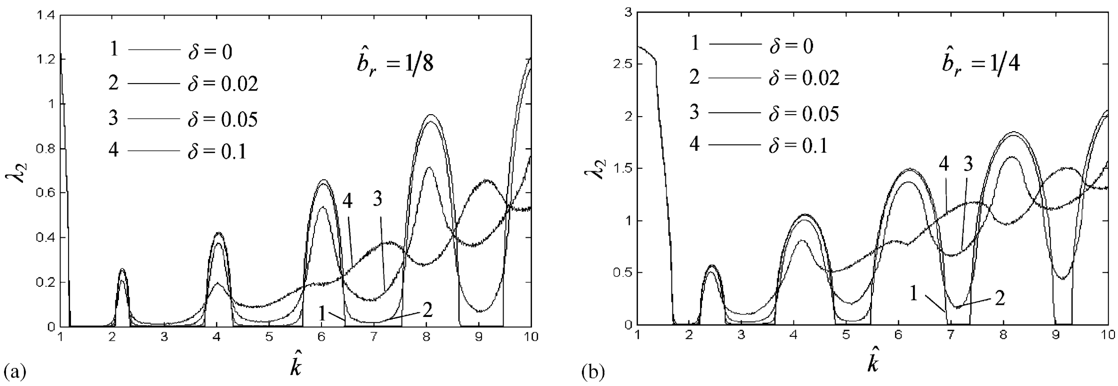

Fig. 5. (a, b) Localization factors in rib-stiffened plates versus non-dimensional wavenumber ($\hat{N} = 0.2$, $\hat{h} = 1/20$, $\hat{h}_r = \hat{h}$).

higher-frequency regions. For example, in Fig. 3(a) the interval of $\hat{k} \in (1.0, 1.6)$ is stopband for $\hat{h} = 1/20$. But this interval will increase to $\hat{k} \in (1.0, 2.2)$ when $\hat{h}$ decreases to $\hat{h} = 1/40$ in Fig. 3(b) and the degree of localization is strengthened.

4. With the increase of the dimensionless thickness of the rib, Fig. 4 shows that localization factors will be increased for a certain non-dimensional wavenumber and the passband and the stopband will become narrower and wider, respectively. Hence, the degree of localization will be increased. For example, the frequency passband $\hat{k} \in (6.8, 7.5)$ will be decreased to $\hat{k} \in (6.95, 7.30)$ and the stopband $\hat{k} \in (5.7, 6.8)$ will be increased to $\hat{k} \in (5.4, 6.95)$ when the dimensionless thickness of the rib is increased from $\hat{h}/2$ to $\hat{h}$. The reason is that with the increase of the dimensionless thickness $\hat{h}_r$ of the rib the dimensionless torsional and flexural rigidities will also be increased, respectively.
5. With the increase of the dimensionless width of the rib, Fig. 5 shows that localization factors will be increased for a certain non-dimensional wavenumber and the passband and the stopband will become narrower and wider, respectively. Hence, the degree of localization will

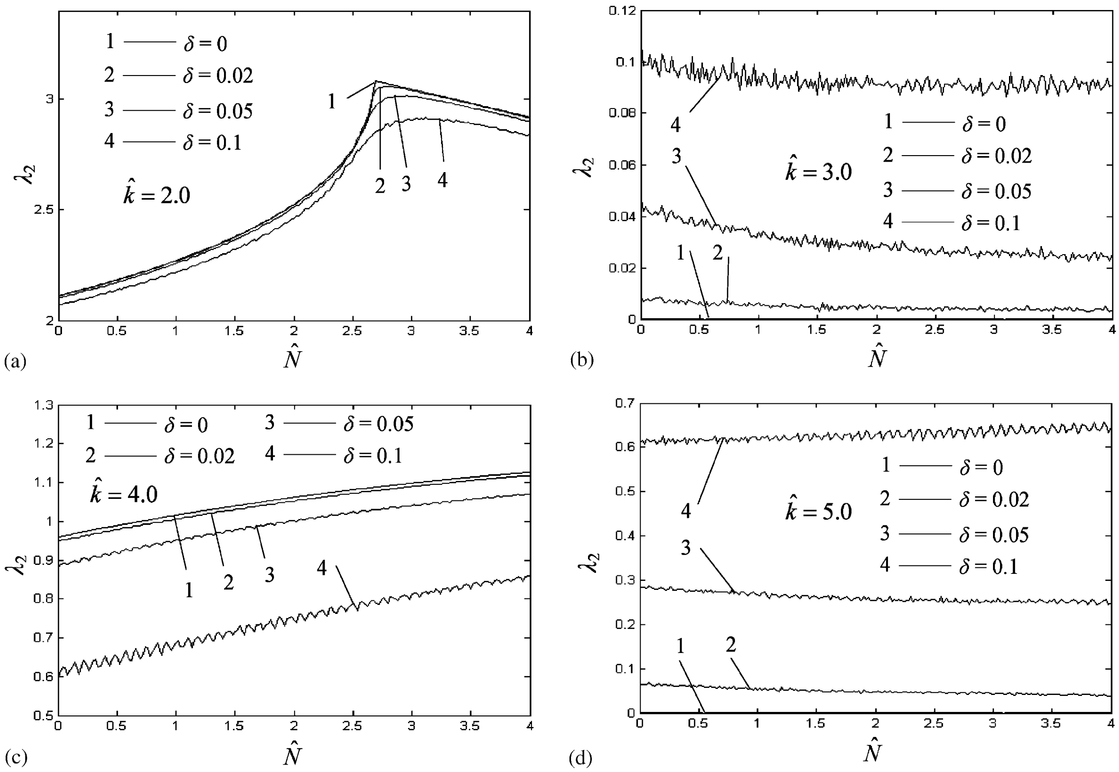

Fig. 6. (a)–(d) Localization factors in rib-stiffened plates versus non-dimensional wavenumber ($\hat{h} = 1/40$, $\hat{h}_r = \hat{h}$, $\hat{b}_r = 1/4$).

be increased. For example, the frequency passband $\hat{k} \in (2.3, 3.8)$ will be decreased to $\hat{k} \in (2.7, 3.7)$ and the stopband $\hat{k} \in (3.8, 4.3)$ will be increased to $\hat{k} \in (3.7, 4.8)$ when the dimensionless width of the rib is increased from $\frac{1}{8}$ to $\frac{1}{4}$. The reason is that with the increase of the dimensionless width $\hat{b}_r$ of the rib the dimensionless torsional and flexural rigidities will also be increased, respectively.

6. For different dimensionless wavenumber, it can be seen in Fig. 6 that the variation of localization factors versus dimensionless axial compressive force is very different. For example, for $\hat{k} = 2.0$ the interval $\hat{N} \in (0, 4)$ for dimensionless axial compressive force is stopband. But for $\hat{k} = 3.0$, this interval will become passband. So, when designing the dynamical intensity of a periodic structure, the dynamical analysis should be performed according to actual structural dynamical status, but the standards of static intensity design should not be completely applied.

5. Conclusions

In this study, the localization of elastic waves in disordered periodic multi-span rib-stiffened plates is studied using the method of transfer matrix. The method for calculating the Lyapunov exponents in continuous dynamical systems presented by Wolf is employed to determine the localization factors in discrete dynamical systems. The main findings of this work are as follows:

1. Tuned periodic multi-span rib-stiffened plates have the properties of frequency passband and stopband. Localization phenomenon can occur in mistuned periodic multi-span plates, and the larger the degree of disorder, the larger the degree of localization.
2. With the increase of the dimensionless torsional and flexural rigidities of the rib, the passband and the stopband will become narrower and wider, respectively. So, the degree of localization will be increased.
3. For different dimensionless wavenumber, the variation of the localization factor versus the dimensionless axial compressive force is very complicated, and it is necessary to study the problem of wave localization according to the actual structural dynamical status.
4. Applying these properties of periodic structures, disordered periodic structures can be designed according to different purposes to localize the amplitudes of elastic waves and vibration, to reduce the vibration of important substructures and to realize the structural vibration control.

Acknowledgements

The authors wish to express gratitude for the supports provided by the National Science Fund for Distinguished Young Scholars under Grant No. 10025211, the China National Natural Science Foundation under Grant No. 19972018, the NJTU Paper Foundation of China under grant No. PD248 and the China Postdoctoral Science Foundation for this research work.

References

[1] M.P. Castanier, C. Pierre, Lyapunov exponents and localization phenomena in multi-coupled nearly periodic systems, *Journal of Sound and Vibration* 183 (1995) 493–515.
[2] G.J. Kissel, Localization factor for multichannel disordered systems, *Physical Review A* 44 (1991) 1008–1014.
[3] F.M. Li, C. Hu, W.H. Huang, One-dimensional localization of elastic waves in rib-stiffened plates, *Chinese Journal of Aeronautics* 15 (2002) 208–212.
[4] W.C. Xie, A. Ibrahim, Buckling mode localization in rib-stiffened plates with misplaced stiffeners—a finite strip approach, *Chaos, Solitons and Fractals* 11 (2000) 1543–1558.
[5] W.C. Xie, I. Elishakoff, Buckling mode localization in rib-stiffened plates with misplaced stiffeners—Kantorovich approach, *Chaos, Solitons and Fractals* 11 (2000) 1559–1574.
[6] I. Elishakoff, Y.W. Li, J.H. Starnes Jr., Buckling mode localization in elastic plates due to misplacement in the stiffener location, *Chaos, Solitons and Fractals* 5 (1995) 1517–1531.
[7] I. Elishakoff, Y.W. Li, J.H. Starnes Jr., Passive control of buckling deformation via Anderson localization phenomena, *Chaos, Solitons and Fractals* 8 (1997) 59–75.
[8] W.C. Xie, Buckling mode localization in rib-stiffened plates with randomly misplaced stiffeners, *Computers and Structures* 67 (1998) 175–189.
[9] S. Sridharan, M. Zeggane, Stiffened plates and cylindrical shells under interactive buckling, *Finite Elements in Analysis and Design* 38 (2001) 155–178.
[10] A. Wolf, J.B. Swift, H.L. Swinney, J.A. Vastano, Determining Lyapunov exponents from a time series, *Physica D* 16 (1985) 285–317.
[11] C.L. Dym, I.H. Shames, *Solid Mechanics: A Variational Approach*, McGraw-Hill, New York, 1973.
[12] S.P. Timoshenko, J.M. Gere, *Theory of Elastic Stability*, McGraw-Hill, New York, 1961.