Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Journal of Sound and Vibration 281 (2005) 311–321
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Dynamic response analysis of linear stochastic truss structures under stationary random excitation

Wei Gao*, Jianjun Chen, Mingtao Cui, Yi Cheng

School of Electronic Mechanical Engineering, Xidian University, Xi'an, 710071, P.R. China

Received 13 December 2002; accepted 21 January 2004
Available online 27 September 2004

Abstract

This paper presents a new method for the dynamic response analysis of linear stochastic truss structures under stationary random excitation. Considering the randomness of the structural physical parameters and geometric dimensions, the computational expressions of the mean value, variance and variation coefficient of the mean square value of the structural displacement and stress response under the stationary random excitation are developed by means of the random variable's functional moment method and the algebra synthesis method from the expressions of structural stationary random response of the frequency domain. The influences of the randomness of the structural physical parameters and geometric dimensions on the randomness of the mean square value of the structural displacement and stress response are inspected by the engineering examples.

1. Introduction

The theoretical framework of classical random vibration has been established and is already available. The analysis of structures with deterministic characteristic to random excitations has been reported extensively in the literature. Nigam and Narayanan [1] considered different types of loading in this group of problems. As a matter of fact, the randomness of structure and applied loads existed and must be considered. Therefore, studying the problem of stationary random response of random structure is of much realistic engineering background and important theoretical significance.

*Corresponding author. Present address: School of Mechanical and Manufacturing Engineering, University of New South Wales, Sydney, NSW 2052, Australia.

E-mail addresses: wigao@sohu.com (W. Gao), jjchen@xidian.edu.cn (J. Chen).

0022-460X/$ - see front matter
doi:10.1016/j.jsv.2004.01.014

Because the random dynamic response analysis of linear stochastic structure is very complicated and difficult, it is only in the recent years that stochastic finite element method based on perturbation technique has begun to be used for solving the dynamic response of a structure with random parameters under random excitation. Wall et al. [2] studied the dynamic effects of uncertainty in structural properties when the excitation is random by use of the perturbation stochastic finite element method (PSFEM). Liu et al. [3] discussed the secular terms resulted from PSFEM in transient analysis of such a random dynamic system. Jensen and Iwan [4] studied the response of systems with uncertain parameters to random excitation by extending the orthogonal expansion method. Zhao and Chen [5] studied the vibration for structures with stochastic parameters to random excitation by using the dynamic Neumann stochastic finite element method, in which the random equation of motion for structure is transformed into a quasi-static equilibrium equation for the solution of displacement in time domain. Lin et al. [6] studied the stationary random response of a structure with stochastic parameters, in which the random excitations are first transformed into sinusoidal ones in terms of the pseudo-excitation method (PEM), that turns the joint-random problem into a single-random problem for which only structural parameters remain random. Li et al. [7] expanded the orthogonal expansion method with the pseudo-excitation method for analyzing the dynamic response of structures with uncertain parameters under external random excitation.

In this paper, the problems of the stationary random dynamic responses of random structures are studied, a new method (random factor method) is proposed, in which the influence of each parameter on the structural dynamic response can be reflected. The truss structures are taken as analyzing objects, in which the randomness of physical parameters (elastic module and mass density) of structural materials and structural geometric dimensions (length and cross-sectional area) are considered simultaneously. The expressions of numerical characteristics of the mean square value of the structural displacement and stress response are developed by use of the random variable's functional moment method and the algebra synthesis method.

2. Structural stationary random dynamic response analysis

Suppose that there are *ne* elements in the analyzed truss structure. The stiff matrix $[K]$ and mass matrix $[M]$ of truss structure in global coordinate can be, respectively, expressed as

$$[M] = \sum_{e=1}^{ne} \left[M^{(e)}\right] = \sum_{e=1}^{ne} \left\{\tfrac{1}{2}\rho^{(e)} A^{(e)} l^{(e)} [I]\right\}, \tag{1}$$

$$[K] = \sum_{e=1}^{ne} \left[K^{(e)}\right] = \sum_{e=1}^{ne} \left\{\frac{E^{(e)} A^{(e)}}{l^{(e)}} [G]\right\}, \tag{2}$$

where $\left[K^{(e)}\right]$ is the eth element's stiffness matrix, $\left[M^{(e)}\right]$ is the eth element's mass matrix. $E^{(e)}$, $A^{(e)}$, $l^{(e)}$ and $\rho^{(e)}$ are the eth element's elastic module, cross-sectional area, length and mass density, respectively. $[I]$ and $[G]$ do not include structural parameters, they are the determinate parts of $\left[M^{(e)}\right]$ and $\left[K^{(e)}\right]$, and have the same order as $[M]$ and $[K]$, respectively.

The dynamic equation of the structure under stationary excitation can be given by

$$[M]\{\ddot{u}(t)\} + [C]\{\dot{u}(t)\} + [K]\{u(t)\} = \{P(t)\}, \quad (3)$$

where $[M]$, $[C]$ and $[K]$ are the mass, damping and stiffness matrices, respectively. $\{u(t)\}$, $\{\dot{u}(t)\}$, and $\{\ddot{u}(t)\}$ are the structural displacement vector, velocity vector and acceleration vector, respectively. $\{P(t)\}$ is the stationary random load force vector. In the following, the Wilson's damping hypothesis will be adopted.

Eq. (3) is a set of differential equations coupled to each other. Its formal solution can be obtained in terms of the decoupling transformation and Duhamel integral, that is

$$\{u(t)\} = \int_0^t [\phi][h(\tau)][\phi]^{\mathrm{T}}\{P(t-\tau)\}\ \mathrm{d}\tau, \quad (4)$$

where $[\phi]$ is the normal modal matrix of the structure, $[h(t)]$ is the impulse response function matrix of the structure and can be expressed as

$$[h(t)] = \mathrm{diag}(h_j(t)), \quad (5)$$

where

$$h_j(t) = \begin{cases} \dfrac{1}{m_j\bar{\omega}_j}\exp(-\xi_j\omega_j t)\ \sin\ \bar{\omega}_j t, & t \geqslant 0, \\ & \qquad (j = 1, 2, \ldots, s)' \\ 0, & t < 0, \end{cases} \quad (6)$$

where ω_j and ξ_j are the jth order inherence frequency and mode damping of structure, respectively. $\bar{\omega}_j = \omega_j(1-\xi_j^2)^{1/2}$.

From Eq. (4), the correlation function matrix of the displacement response of the structure $[R_u(\varepsilon)]$ can be obtained

$$\begin{aligned}[R_u(\varepsilon)] &= E(\{u(t)\}\{u(t+\varepsilon)\}^{\mathrm{T}}) \\ &= \int_0^t\int_0^t [\phi][h(\tau)][\phi]^{\mathrm{T}}[R_P(\tau-\tau_1+\varepsilon)][\phi][h(\tau_1)]^{\mathrm{T}}[\phi]^{\mathrm{T}}\ d\tau\ d\tau_1, \end{aligned} \quad (7)$$

where $[R_P(\tau-\tau_1+\varepsilon)]$ is the correlation function matrix of $\{P(t)\}$.

By performing a $[R_u(\varepsilon)]$ Fourier transformation, the power spectral density matrix of the displacement response $[S_u(\omega)]$ is

$$[S_u(\omega)] = [\phi][H(\omega)][\phi]^{\mathrm{T}}[S_P(\omega)][\phi][H^*(\omega)][\phi]^{\mathrm{T}}, \quad (8)$$

where $[S_P(\omega)]$ is the power spectral density matrix of $\{P(t)\}$, $[H^*(\omega)]$ is the conjugate matrix of $[H(\omega)]$, $[H(\omega)]$ is the frequency response function matrix of the structure and can be expressed as

$$[H(\omega)] = \mathrm{diag}\,[H_j(\omega)], \quad (9)$$

where

$$H_j(\omega) = \frac{1}{\omega_j^2 - \omega^2 + \mathrm{i}\,2\xi_j\omega_j\omega} \quad (\mathrm{i} = \sqrt{-1}) \quad (j = 1, 2, \ldots, s), \quad (10)$$

Integrating $[S_u(\omega)]$ within the frequency domain, the mean square value matrix of the structural displacement response, that is, $[\psi_u^2]$ can be obtained:

$$[\psi_u^2] = \int_0^\infty [S_u(\omega)]\ \mathrm{d}\omega = \int_0^\infty [\phi][H(\omega)][\phi]^\mathrm{T}[S_P(\omega)][\phi][H^*(\omega)][\phi]^\mathrm{T}\ \mathrm{d}\omega. \quad (11)$$

Then the mean square value of the kth degree of freedom of the structural dynamic displacement response can be expressed as

$$\psi_{uk}^2 = \vec{\phi}_k \cdot \int_0^\infty [H(\omega)][\phi]^\mathrm{T}[S_P(\omega)][\phi][H^*(\omega)]\ \mathrm{d}\omega \cdot \vec{\phi}_k^\mathrm{T} \qquad (k = 1, 2, \ldots, n), \quad (12)$$

where $\vec{\phi}_k$ is the kth line vector of the matrix $[\phi]$.

According to the relationship between node displacement and element stress, the stress response of the eth element in the truss structure can be expressed as

$$\{\sigma(t)^{(e)}\} = E^{(e)} \cdot [B] \cdot \{u(t)^{(e)}\} \qquad (e = 1, 2, \ldots, n_e), \quad (13)$$

where $\{u(t)^{(e)}\}$ is the displacement response of the nodal point of the eth element, $\{\sigma(t)^{(e)}\}$ is the stress response of the eth element, $[B]$ is the geometric matrix of the eth element. $E^{(e)}$ is the elastic module of the eth element.

From Eq. (13) the correlation function matrix of the eth element stress response $[R_\sigma^{(e)}(\tau)]$ can be obtained

$$[R_\sigma^{(e)}(\tau)] = E(\{\sigma(t)^{(e)}\}\{\sigma(t+\tau)^{(e)}\}^\mathrm{T}) = E^{(e)}[B][R_u^{(e)}(\tau)][B]^\mathrm{T}E^{(e)} \quad (14)$$

Furthermore, the power spectral density matrix of the stress response of the eth element $[S_\sigma^{(e)}(\omega)]$ can be obtained:

$$[S_\sigma^{(e)}(\omega)] = E^{(e)}[B][S_u^{(e)}(\omega)][B]^\mathrm{T}E^{(e)}. \quad (15)$$

Then, the mean square value matrix of the eth element stress response $[\psi_{\sigma^{(e)}}^2]$ can be expressed as

$$[\psi_{\sigma^{(e)}}^2] = E^{(e)}[B][\psi_{u^{(e)}}^2][B]^\mathrm{T}E^{(e)}. \quad (16)$$

3. Numerical characteristics of the stationary random response of the random structure

3.1. Numerical characteristics of natural frequency random variable

Considering the randomness of the structural physical parameters ($\rho^{(e)}$, $E^{(e)}$) and geometric dimensions ($A^{(e)}$, $l^{(e)}$) simultaneously, that is, they are all random variables. From Eqs. (1) and (2), it can be obtained that the matrices $[M]$ and $[K]$are random variables too. The randomness of physical parameters and geometric dimension will lead the inherence frequency ω as well as randomness. The structural dynamic characteristic based on probability will be analyzed.

Suppose the material of each element is equal, E and ρ are elastic module and mass density, respectively. Then the elastic module and mass density can be written, respectively, as $E = \alpha\tilde{E}$; $\rho = \beta\tilde{\rho}$, where α and β are the determinate quantities of E and ρ, respectively. $\tilde{E}$ and $\tilde{\rho}$ are the random variable factors of E and ρ, respectively. The mean values of $\tilde{E}$ and $\tilde{\rho}$ are all 1.0. The variation coefficients of $\tilde{E}$ and $\tilde{\rho}$ are v_E and v_ρ, respectively.

The length and cross-sectional area of bars are two kinds of random variables. Then the length and cross-sectional area of the eth element can be written, respectively, as: $l^{(e)} = \lambda^{(e)}\tilde{l}$ $A^{(e)} = \eta^{(e)}\tilde{A}$ $(e = 1, \ldots, ne)$, where $\lambda^{(e)}$ is the determinate quantity that denotes the nominal length of eth bar, $\eta^{(e)}$ is the determinate quantity that denotes the nominal cross-sectional area of the eth bar; $\tilde{l}$ is the random variable factor of all bars' length, its mean value is 1.0 and variation coefficient is v_l. $\tilde{A}$ is the random variable factor of all bars' cross-sectional area, its mean value is 1.0 and variation coefficient is v_A.

From Eq. (1), it can be seen easily that, when $\rho^{(e)}$, $A^{(e)}$ and $l^{(e)}$ are all random variables, $[M^{(e)}]$ and $[M]$ are all random variables too.

Let

$$[M^{(e)}] = \tilde{\rho}\tilde{A}\tilde{l}[M^{(e)}]^{\#}, \tag{17}$$

where $[M^{(e)}]^{\#}$ is the determinate part in mass matrices $[M^{(e)}]$. The expression shows that the mass matrix $[M^{(e)}]$ can be divided into the product of two parts, i.e., the random variables $\tilde{\rho}$, $\tilde{A}$, $\tilde{l}$ and the constant matrix $[M^{(e)}]^{\#}$ and that the randomness of $[M^{(e)}]$ is dependent on the randomness of $\rho^{(e)}$, $A^{(e)}$, and $l^{(e)}$. Therefore, constructing the matrix $[M^{(e)}]^{\#}$ is the same as constructing the mass matrix of the element before, just taking the parameters as: $l^{(e)} = \lambda^{(e)}$, $A^{(e)} = \eta^{(e)}$, $\rho^{(e)} = \beta[M]$ can be written as

$$[M] = \sum_{e=1}^{ne}[M^{(e)}] = \sum_{e=1}^{ne}(\tilde{\rho}\tilde{A}\tilde{l}[M^{(e)}]^{\#}) = \tilde{\rho}\tilde{A}\tilde{l}[M]^{\#}. \tag{18}$$

From Eq. (2) the conclusion can be obtained that when $E^{(e)}$, $A^{(e)}$and $l^{(e)}$ are all random variables, $[K^{(e)}]$ and $[K]$ are all random variables too.

Let

$$[K^{(e)}] = \frac{\tilde{E}\tilde{A}}{\tilde{l}}[K^{(e)}]^{\#}, \tag{19}$$

where $[K^{(e)}]^{\#}$ is the determinate part in stiff matrices $[K^{(e)}]$. The expression shows that the stiff matrix $[K^{(e)}]$ can be divided into the product of two parts, i. e., the random variables $\tilde{E}$, $\tilde{A}$, $\tilde{l}$ and the constant matrix $[K^{(e)}]^{\#}$ and that the randomness of $[K^{(e)}]$ is dependent on the randomness of $E^{(e)}$, $A^{(e)}$, and $l^{(e)}$. Therefore, constructing the matrix $[K^{(e)}]^{\#}$ is the same as constructing the mass matrix of the element before, just taking the parameters as: $l_m^{(e)} = \lambda^{(e)}$, $A^{(e)} = \eta^{(e)}$ and $E = \alpha$. Suppose the material of each element is equal, $[K]$ can be written as

$$[K] = \sum_{e=1}^{ne}[K^{(e)}] = \sum_{e=1}^{ne}\left(\frac{\tilde{E}\tilde{A}}{\tilde{l}}[K^{(e)}]^{\#}\right) = \frac{\tilde{E}\tilde{A}}{\tilde{l}}[K]^{\#}. \tag{20}$$

Suppose that jth order inherence frequency and mode shape of structure are ω_j and $\{\phi\}_j$, respectively, by using the Rayliegh's quotient expression, the random variable of jth inherence frequency can be expressed as

$$\omega_j^2 = \frac{\{\phi\}_j^{\mathrm{T}}[K]\{\phi\}_j}{\{\phi\}_j^{\mathrm{T}}[M]\{\phi\}_j}. \tag{21}$$

Substituting the stiff and mass matrices of structure into the above formula,

$$\omega_j^2 = \frac{\tilde{E}\tilde{A}}{\tilde{\rho}\tilde{A}\tilde{l}^2}\ \frac{K_j^{\#}}{M_j^{\#}} = \frac{\tilde{E}\tilde{A}}{\tilde{\rho}\tilde{A}\tilde{l}^2}(\omega_j^{\#})^2, \tag{22}$$

where $K_j^{\#}, M_j^{\#}, \omega_j^{\#}$ are all determinate quantities, they are the jth order main stiffness, main mass and inherence frequency of the structure when the parameters $A^{(e)} = \eta^{(e)},\ l^{(e)} = \lambda^{(e)},\ E = \alpha$ and $\rho = \beta$.

According to previous formulae and paying attention to the structural physical parameters and geometric dimension of the two kinds of independent variables, the mean value μ_{ω_j} and mean variance σ_{ω_j} of jth order inherence frequency ω_j can be deduced by means of the algebra synthesis method [8].

$$\begin{aligned}\mu_{\omega_j} = \omega_j^{\#}\left(\frac{\mu_E}{\mu_\rho\mu_Z}\right)^{1/2} & \left\{\left[1 + v_A^2 + v_Z^2 + v_\rho^2 + v_A^2v_Z^2 + v_A^2v_\rho^2 + v_Z^2v_\rho^2 + v_A^2v_Z^2v_\rho^2\right.\right. \\ & - c_{E\rho}\left(v_A^2 + v_E^2 + v_A^2v_E^2\right)^{1/2}\left.\left(v_A^2 + v_Z^2 + v_\rho^2 + v_A^2v_Z^2 + v_A^2v_\rho^2 + v_Z^2v_\rho^2 + v_A^2v_Z^2v_\rho^2\right)^{1/2}\right]^2 \\ & - \tfrac{1}{2}\left[2v_A^2 + v_E^2 + v_Z^2 + v_\rho^2 + v_A^2v_Z^2 + v_A^2v_E^2 + v_A^2v_\rho^2 + v_Z^2v_\rho^2 + v_A^2v_Z^2v_\rho^2\right. \\ & \left.\left. - 2c_{E\rho}(v_A^2 + v_E^2 + v_A^2v_E^2)^{1/2}(v_A^2 + v_Z^2 + v_\rho^2 + v_A^2v_Z^2 + v_A^2v_\rho^2 + v_Z^2v_\rho^2 + v_A^2v_Z^2v_\rho^2)^{1/2}\right]\right\}^{1/4}\end{aligned}$$

$$\begin{aligned}\sigma_{\omega_j} = \omega_j^{\#}\left(\frac{\mu_E}{\mu_\rho\mu_Z}\right)^{1/2} & \left\{\left[1 + v_A^2 + v_Z^2 + v_\rho^2 + v_A^2v_Z^2 + v_A^2v_\rho^2 + v_Z^2v_\rho^2 + v_A^2v_Z^2v_\rho^2\right.\right. \\ & \left. -c_{E\rho}\left(v_A^2 + v_E^2 + v_A^2v_E^2\right)^{1/2}\left(v_A^2 + v_Z^2 + v_\rho^2 + v_A^2v_Z^2 + v_A^2v_\rho^2 + v_Z^2v_\rho^2 + v_A^2v_Z^2v_\rho^2\right)^{1/2}\right] \\ & - \left\{\left[1 + v_A^2 + v_Z^2 + v_\rho^2 + v_A^2v_Z^2 + v_A^2v_\rho^2 + v_Z^2v_\rho^2 + v_A^2v_Z^2v_\rho^2\right.\right. \\ & \left. -c_{E\rho}\left(v_A^2 + v_E^2 + v_A^2v_E^2\right)^{1/2}\left(v_A^2 + v_Z^2 + v_\rho^2 + v_A^2v_Z^2 + v_A^2v_\rho^2 + v_Z^2v_\rho^2 + v_A^2v_Z^2v_\rho^2\right)^{1/2}\right]^2 \\ & - \tfrac{1}{2}\left[2v_A^2 + v_E^2 + v_Z^2 + v_\rho^2 + v_A^2v_Z^2 + v_A^2v_E^2 + v_A^2v_\rho^2 + v_Z^2v_\rho^2 + v_A^2v_Z^2v_\rho^2\right. \\ & \left.\left.\left. -2c_{E\rho}\left(v_A^2 + v_E^2 + v_A^2v_E^2\right)^{1/2}\left(v_A^2 + v_Z^2 + v_\rho^2 + v_A^2v_Z^2 + v_A^2v_\rho^2 + v_Z^2v_\rho^2 + v_A^2v_Z^2v_\rho^2\right)^{1/2}\right]\right\}^{1/2}\right\}^{1/2}\end{aligned} \tag{23}$$

$$v_z = v_{l^2} = \frac{\sqrt{4v_l^2 + v_l^4}}{1 + v_l^2}, \tag{24}$$

where the symbol v denotes variation coefficient, $c_{E\rho}$ is the correlation coefficient of variables E and ρ.

If variables E and ρ are independent, then $c_{E\rho} = 0$. If variable E is completely correlative with ρ, then $c_{E\rho} = 1$. They are two kinds of extreme situations. According to the observation on the property of common material, it can be found that the elastic module E is usually positively correlated with mass density ρ, and that the degree of correlation is rather higher. So in practical computation, it is suggested that the correlative coefficient $c_{E\rho} = 0.5-0.9$.

From the above formula, it is easily seen that the values of variation coefficient of every inherence frequency v_{ω_j} are equal to each other. They are only dependent on the physical parameters E, ρ and the random variable factor of geometric dimension l, A, as wall as the correlative coefficient $c_{E\rho}$, but they are independent of the order number of structural vibration model. According to the determinate stiffness and mass matrices $[K]^{\#}$ and $[M]^{\#}$, the determinate values of every order inherence frequency $\omega_j^{\#}$ can be obtained by means of the conventional dynamic analysis method.

The randomness of the structural dynamic characteristics and the stochastic excitation will lead the structural dynamic response (dynamic displacement and dynamic stress) as well as randomness. The statistical description of random variables is represented by using its numerical characteristic. In the following, the expressions of numerical characteristics of the structural stationary response random variables will be derived.

3.2. Numerical characteristics of the stationary random response of the random structure

From Eq. (12), the mean value $\mu_{\psi^2_{uk}}$ and mean variance $\sigma_{\psi^2_{uk}}$ of the mean square value of the kth degree of freedom of the structural dynamic displacement response can be deduced by means of the random variable's functional moment method [9]:

$$\mu_{\psi^2_{uk}} = \mu_{\vec{\phi}_k} \int_0^{\omega_c} \mu_{[H(\omega)]}\mu_{[\phi]^{\mathrm{T}}}\mu_{[S_P(\omega)]}\mu_{[\phi]}\mu_{[H^*(\omega)]}\ \mathrm{d}\omega\ \mu_{\vec{\phi}_k^{\mathrm{T}}}, \tag{25}$$

$$\sigma_{\psi^2_{uk}} = \mu_{\vec{\phi}_k}\left\{\int_0^{\omega_c}\left\{\mu^2_{[H(\omega)]}(\mu_{[\phi]^{\mathrm{T}}}\mu_{[S_P(\omega)]}\mu_{[\phi]})^2\sigma^2_{[H^*(\omega)]} + \sigma^2_{[H(\omega)]}(\mu_{[\phi]^{\mathrm{T}}}\mu_{[S_P(\omega)]}\mu_{[\phi]})^2\mu^2_{[H^*(\omega)]} + \sigma_{[H(\omega)]}\mu_{[\phi]^{\mathrm{T}}}\mu_{[S_P(\omega)]}\mu_{[\phi]}\sigma_{[H^*(\omega)]}\right\}\mathrm{d}\omega\right\}^{1/2}\mu_{\vec{\phi}_k^{\mathrm{T}}} \quad (j = 1, 2 \ldots, s), \tag{26}$$

where

$$\sigma_{[H(\omega)]} = \mathrm{diag}\left[\frac{-2\mu_{\omega_j} - \mathrm{i}\,2\xi_j\omega}{(\mu^2_{\omega_j} - \omega^2 + \mathrm{i}\,2\xi_j\mu_{\omega_j}\omega)^2}\sigma_{\omega_j}\right] \qquad (j = 1, 2, \ldots, s), \tag{27}$$

From Eqs. (25) and (26), the variation coefficient $v_{\psi^2_{uk}}$ of the random variable ψ^2_{uk} can be obtained:

$$v_{\psi^2_{uk}} = \sigma_{\psi^2_{uk}}/\mu_{\psi^2_{uk}}. \tag{28}$$

From Eq. (16), the mean value, mean variance and variation coefficient of the mean square value of the eth element stress response can be deduced by means of the algebra synthesis method .

$$\mu_{\left[\psi^2_{\sigma(e)}\right]} = (\mu_{E^2} + \sigma_{E^2})[B]\mu_{\left[\psi^2_{u(e)}\right]}[B]^{\mathrm{T}} \qquad (e = 1, 2, \ldots, n_e), \tag{29}$$

$$\begin{aligned}\sigma_{\left[\psi^2_{\sigma(e)}\right]} = \{&(\mu_{E^2} + \sigma_{E^2})^2([B]\sigma_{\left[\psi^2_{u(e)}\right]}[B]^{\mathrm{T}})^2 + (4\mu_{E^2}\sigma_{E^2} + 2\sigma_{E^4})([B]\mu_{\left[\psi^2_{u(e)}\right]}[B]^{\mathrm{T}})^2 \\ &+ (4\mu_{E^2}\sigma_{E^2} + 2\sigma_{E^4})([B]\sigma_{\left[\psi^2_{u(e)}\right]}[B]^{\mathrm{T}})^2\}^{1/2} \quad (e = 1, 2, \ldots, n_e),\end{aligned} \tag{30}$$

$$v_{\left[\psi^2_{\sigma(e)}\right]} = \sigma_{\left[\psi^2_{\sigma(e)}\right]} \Big/ \mu_{\left[\psi^2_{\sigma(e)}\right]} \qquad (e = 1, 2, \ldots, n_e). \tag{31}$$

4. Examples

According to the preceding computing formula and the solving method, the corresponding computational program is designed. A 20-bar planar intelligent truss structure is used as example (Fig. 1). The material of this structure is steel. The elastic module E, mass density ρ and cross-sectional area A are all random variables: $\mu_E = 2.058 \times 10^5$ (MPa), $\mu_p = 76.5$ (kN/m^3), $\mu_A = 4 \times 10^{-4}$ (m^2). Length l is a random variable too, and let $\xi_j = \xi = 0.01$. A ground level acceleration $P(t)$ act on the structure, $P(t)$ is a Gauss stochastic process and its mean value is zero. Its self-power spectral density can be expressed as [6]

$$S_{PP}(\omega) = \frac{1 + 4(\xi_g\omega/\omega_g)^2}{(1 - \omega^2/\omega_g^2)^2 + 4(\xi_g\omega/\omega_g)^2} S_0,$$

where $\omega_g = 16.5$, $\xi_g = 0.7$, $S_0 = 15.6$ cm^2/s^3.

In order to investigate the effect of the dispersal degree of random variables E, ρ, l and A on the structural dynamic response, different models are selected and the values of variation coefficients of parameters E, ρ, l and A are, respectively, taken as a different group. The computational results of the mean value $\mu_{\psi^2_{X9}}$, mean variance $\sigma_{\psi^2_{X9}}$ and variation coefficient $v_{\psi^2_{X9}}$ of the mean square value of displacement response ψ^2_{X9} of the nodal 9 of the X-axis are listed in Table 1. In addition, in order to verify the effectiveness of the random factor method, the computational results that obtained by the Monte–Carlo simulation method are given in Table 1, too. The computational results of the mean value $\mu_{\psi^2_{\sigma1}}$, mean variance $\sigma_{\psi^2_{\sigma1}}$ and variation coefficient $v_{\psi^2_{\sigma1}}$ of the mean square value of stress response $\psi^2_{\sigma1}$ of the first element are listed in Table 2.

5. Conclusions

1. The analytical results of the method proposed in this paper are in accordance with that of the Monte–Carlo simulation method, by which the effectiveness of our method is verified.
2. The effects of the randomness of E, ρ, l and A on the randomness of the mean square value of the structural displacement and stress response are different; the randomness of bar's length l produces the greatest effect on the randomness of the mean square value of the structural

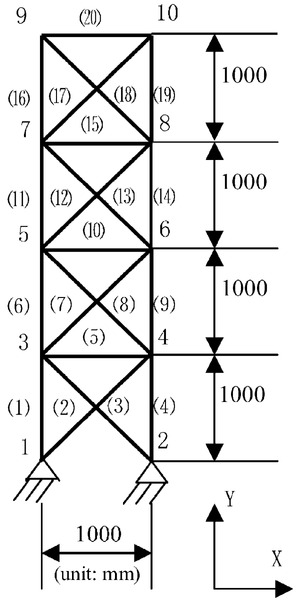

Fig. 1. 20-bar planar truss structure.

Table 1
The mean value $\mu_{\psi^2_{X9}}$, mean variance $\sigma_{\psi^2_{X9}}$ and variation coefficient $v_{\psi^2_{X9}}$ of the mean square value of displacement response ψ^2_{X9} of the nodal 9 of the X-axis (*Monte–Carlo simulation method)

Model	$\mu_{\psi^2_{X9}}$ (mm^2)	$\sigma_{\psi^2_{X9}}$ (mm^2)	$v_{\psi^2_{X9}}$
$v_E = 0.1,\ v_\rho = v_A = v_l = 0$	3.8127	0.1586	0.04159
$v_\rho = 0.1,\ v_E = v_A = v_l = 0$	3.8127	0.1570	0.04118
$v_E = v_\rho = 0.1,\ v_A = v_l = 0$	3.8127	0.1994	0.05229
$v_A = 0.1,\ v_E = v_\rho = v_l = 0$	3.8127	0.1796	0.04711
$v_l = 0.1,\ v_E = v_\rho = v_A = 0$	3.8127	0.6253	0.1640
$v_A = v_l = 0.1,\ v_E = v_\rho = 0$	3.8127	0.6302	0.1653
$v_E = v_\rho = v_A = v_l = 0.1$	3.8127	0.6493	0.1703
$v_E = 0.2,\ v_\rho = v_A = v_l = 0$	3.8127	0.3216	0.08435
$v_\rho = 0.2,\ v_E = v_A = v_l = 0$	3.8127	0.3086	0.08094
$v_E = v_\rho = 0.2,\ v_A = v_l = 0$	3.8127	0.3977	0.1043
$v_A = 0.2,\ v_E = v_\rho = v_l = 0$	3.8127	0.3582	0.09395
	3.8135*	0.3585*	0.09402*
$v_l = 0.2,\ v_E = v_\rho = v_A = 0$	3.8127	0.7783	0.2041
	3.8135*	0.7821*	0.2051*
$v_A = v_l = 0.2,\ v_E = v_\rho = 0$	3.8127	0.8079	0.2119
	3.8135*	0.8111*	0.2127*
$v_E = v_\rho = v_A = v_l = 0.2$	3.8127	0.8550	0.2243
	3.8135*	0.8576*	0.2249*

Table 2
The mean value $\mu_{\psi^2_{\sigma1}}$, mean variance $\sigma_{\psi^2_{\sigma1}}$ and variation coefficient $v_{\psi^2_{\sigma1}}$ of the mean square value of stress response $\psi^2_{\sigma1}$ of the first element

Model	$\mu_{\psi^2_{\sigma1}}$ (Mpa^2)	$\sigma_{\psi^2_{\sigma1}}$ (Mpa)	$v_{\psi^2_{\sigma1}}$
$v_E = 0.1,\ v_\rho = v_A = v_l = 0$	2761.5	548.2	0.1985
$v_\rho = 0.1,\ v_E = v_A = v_l = 0$	2734.2	35.07	0.01283
$v_A = 0.1,\ v_E = v_\rho = v_l = 0$	2734.2	40.11	0.01467
$v_l = 0.1,\ v_E = v_\rho = v_A = 0$	2734.2	139.44	0.05049
$v_E = v_\rho = 0.1,\ v_A = v_l = 0$	2761.5	549.2	0.1989
$v_E = v_\rho = 0,\ v_A = v_l = 0.1$	2734.2	140.7	0.05146
$v_E = v_\rho = v_A = v_l = 0.1$	2761.5	551.4	0.1997
$v_E = v_\rho = v_A = v_l = 0.2$	2843.6	1116.3	0.3926

displacement response, however, the randomness of the elastic module E produces the greatest effect on the randomness of the mean square value of the structural stress response.

3. When the variation coefficients of physical parameters are equal to those of the geometric dimensions, the randomness of physical parameters will produce greater effect on the randomness of the mean square value of structural stress response, however, the randomness of geometric dimensions will produce a greater effect on the randomness of the mean square value of structural displacement response.
4. Along with the increase in the variation coefficients of E, ρ, l and A, the dispersal degree of the structure's dynamic response will increase.

The computational expressions of the mean value, variance and variation coefficient of the mean square value of the structural displacement and stress response under the stationary random excitation are developed in this paper, the dynamic response analysis results of the random structure to the stochastic excitation can be obtained expediently. The examples show that the model and method of the stationary random dynamic response analysis of stochastic structure presented in this paper are rational and feasible.

Reference

[1] N.C. Nigam, S. Narayanan, *Applications of Random Vibrations*, Narosa, New Delh, 1994.
[2] F.J. Wall, C.G. Bucher, Sensitivity of expected rate of SDOP system response to statistical uncertainties of loading and system parameters, *Probabilistic Engineering Mechanics* 2 (1987) 138–146.
[3] W.K. Liu, G. Besterfield, T. Belytschko, Transient probabilistic systems, *Computational Methods and Applications in Mechanical Engineering* 67 (1988) 27–54.
[4] H. Jensen, W.D. Iwan, Response of system with uncertain parameters to stochastic excitation, *Journal of Engineering Mechanics* 118 (1992) 1012–1025.
[5] Lei Zhao, Chen Qiu, Neumann dynamic stochastic finite element method of vibration for structures with stochastic parameters to random excitation, *Computers & Structures* 77 (2000) 651–657.

[6] Jiahao Lin, Ping Yi, Stationry random response of structures with stochastic parameters, *Chinese Journal of Computational Mechanics* 18 (2001) 402–408.
[7] Jie Li, Songtao Liao, Dynamic response of linear stochastic structures under random excitation, *Acta Mechanica Sinica* 34 (2002) 416–424.
[8] W. Geo, J.J. Chen, H.B. Ma, X.S. Ma, M.T. Cui, Dynamic response analysis of closed loop control system for intelligent truss structures based on probability, *Structural Engineering and Mechanics* 15 (2003) 239–248.
[9] W. Gao, J.J. Chen, H.B. Ma, X.S. Ma, Optimal placement of active bars in active vibration control for piezoelectric intelligent truss structures with random parameters, *Computers & Structures* 81 (2003) 53–60.