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Control of kinetic energy of a one-dimensional structure using multiple vibration neutralizers

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Abstract

There are two cases presented in this paper: the effects of multiple vibration neutralizers on the kinetic energy of a continuous structure, all tuned to a particular natural frequency, and when the tuning ratio of each neutralizer is optimally adjusted at each excitation frequency using quadratic minimization technique. In the first case, theoretical formulations are developed to calculate the width of the separation between the new resonances and the kinetic energy reduction. The width between the new resonances is found to be a function of the total mass of the neutralizers and the modal amplitudes at the neutralizer's location, whereas the kinetic energy reduction is determined by their total damping ratios, total mass and also the modal amplitudes at their respective locations. In the second case, it is found that the reduction of the kinetic energy in the frequency range of interest increases with the number of optimally detuned neutralizers, and the reduction is comparable to that of the feedforward active control method. Simulations are presented to facilitate better understanding on the control effects using multiple vibration neutralizers.

1. Introduction

The vibration neutralizer has been used in many applications since invented. In the early years, the device was used to control the response of a lumped-mass structure or the displacement of a

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single-degree-of-freedom system. The neutralizer was designed in such a way that its natural frequency either (1) coincides with the problematic resonance frequency of the host structure or (2) coincides with the frequency of the primary force [1]. The role of the vibration neutralizer on the frequency response of such a problematic structure has been well understood and has appeared in many text books, for example by Den Hartog [2], Hunt [3] and Beards [4], just to name a few.

The first known attempt to use a neutralizer as a control device for the vibration of a continuous structure was conducted by Young in 1952 [5–7]. He conducted an investigation on the displacement of a cantilever at the neutralizer attachment point with the neutralizer tuned to the first natural frequency of the cantilever. It was then followed by other researchers, for example by Neubert [8] on longitudinal vibration of a bar, Jones et al. [9] on a clamped–clamped beam and Ozguven and Candir [7] on a cantilever using two neutralizers.

The application of vibration neutralizer has been broadened when Sun and his co-workers conducted an experiment on the global control of vibration and sound power transmission through an aircraft panel. They concluded that the vibration neutralizer is effective in suppressing the vibration transmission from the aircraft propellers [10]. In 1996 and 1997, Charette and his co-workers published two papers concerned with experimental work on the control of sound radiation from a plate using two globally detuned tunable vibration neutralizers [11,12]. Later, Huang and Fuller [13,14] carried out their investigation on a cylindrical shell to reduce the interior sound field with a reduction of more than 20 dB in the acoustical potential energy at a certain frequency.

In respect to global vibration control, Dayou and Brennan [15] proposed an optimization method of the tuning ratio of a single vibration neutralizer. The method, which is by using the quadratic minimization technique, provides a way to determine the optimal tuning ratio that minimizes the kinetic energy of a continuous structure. The optimization was then verified on a simply supported beam [16]. They found that an optimal neutralizer can be as effective as an active control method for a single-frequency excitation problem if it is properly optimized.

The investigation in this paper is an extension to the previous work reported in Refs. [15,16] on the global vibration control of a structure, but by using multiple vibration neutralizers. Theoretical formulations on the effects of changing the location, mass and damping ratios of multiple vibration neutralizers, tuned to a particular natural frequency of the host structure, are presented. The effects of multiple optimally detuned vibration neutralizers and comparison with active control, particularly the feedforward technique, are also discussed. Throughout the paper, a simply supported beam in an infinite baffle is used as a host structure with the dimensions of $1 \times 0.0381 \times 0.00635\,\mathrm{m}^3$. The density, Young's modulus and modal damping of the beam is $7870\,\mathrm{kg/m}^3$, 207E9 and 0.005, respectively, and the beam is excited by an acoustic plane wave of single frequency with unity pressure amplitude and incident at 45°.

2. Kinetic energy of a structure

In general, the dynamic response of a general type of structure (Fig. 1) at any point in terms of its displacement can be expressed as [17]

$$w(x) = \mathbf{\Phi}^{\mathrm{T}}\mathbf{q}, \tag{1}$$

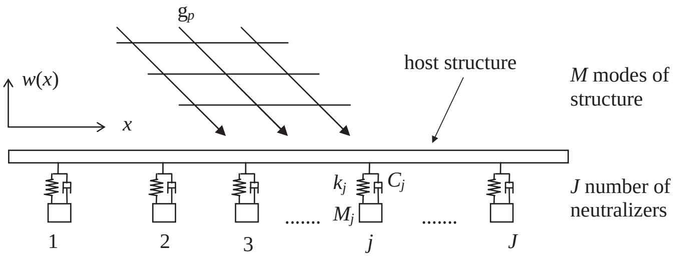

Fig. 1. A general structure with multiple neutralizers attached.

where $\mathbf{\Phi}$ and $\mathbf{q}$ are the $M \times 1$ vectors of the normalized mode shape of the structure evaluated at point x and the $M \times 1$ vector of the modal displacement amplitudes, respectively. The superscript T indicates the transpose of the vector and $e^{j\omega t}$ time dependency in Eq. (1) is suppressed for clarity.

The modal displacement amplitude of the structure is

$$\mathbf{q} = \mathbf{A}\mathbf{g}_p, \tag{2}$$

where $\mathbf{A}$ and $\mathbf{g}_p$ are the $M \times M$ diagonal matrix of the complex modal amplitudes of the structure and $M \times 1$ vector of the generalized primary force acting on the structure, respectively. The mth component of the complex modal amplitudes is $A_m = 1/M_b(\omega_m^2 - \omega^2 + i2\zeta_m\omega\omega_m)$, where M_b, ζ_m, ω_m, and ω are the total mass of the structure, modal damping ratio of the structure, its circular natural frequency, and circular frequency of the excitation force, respectively, and i is the imaginary number, $\sqrt{-1}$.

In order to minimize the vibration amplitude of the structure, secondary forces may be applied. In this case, the modal displacement amplitudes in Eq. (2) is written as

$$\mathbf{q} = \mathbf{A}(\mathbf{g}_p + \mathbf{g}_c), \tag{3}$$

where $\mathbf{g}_c$ is the secondary control force term in its generalized form which is

$$\mathbf{g}_c = \mathbf{\Psi}\mathbf{f}_c, \tag{4}$$

with $\mathbf{\Psi}$ being the $M \times J$ matrix of normalized mode shape of the structure, where the entry of ϕ_{mj} is the modal amplitude of the structure at the jth secondary force location, and $\mathbf{f}_c$ is the j-length vector of the secondary forces amplitude acting on the structure. If $\mathbf{f}_c$ is the vector of the secondary forces generated by J-number of neutralizers (refer to Fig. 1), then it can be written in terms of the neutralizer's dynamic stiffness matrix, which is $\mathbf{f}_c = -\mathbf{K}\mathbf{w}(x_j)$. $\mathbf{K}$ is a diagonal $J \times J$ dynamic stiffness matrix of the neutralizers and $\mathbf{w}(x_j)$ is the J-length displacement vector of the host structure evaluated at the location of the jth neutralizer. The dynamic stiffness of the jth neutralizer is

$$K_j = -M_j\omega^2\left(\frac{1 + i2\zeta_j\mathbf{\Omega}_j}{1 - \mathbf{\Omega}_j^2 + i2\zeta_j\mathbf{\Omega}_j}\right), \tag{5}$$

where ζ_j is the damping ratio and $\mathbf{\Omega}_j = \omega/\omega_j$ is the tuning ratio of the jth neutralizer, ω_j being the natural frequency of the neutralizer given by $(k_j/M_j)^{1/2}$, and k_j and M_j are the neutralizer stiffness constant and mass, respectively. Using Eq. (1), the vector of the generalized forces of the neutralizers can be written as

$$\mathbf{f}_c = -\mathbf{K}\boldsymbol{\Psi}^{\mathrm{T}}\mathbf{q}. \tag{6}$$

By combining Eqs. (3) and (6), the vector of the modal displacement amplitudes of the coupled system—host structure and neutralizers—can be written as

$$\mathbf{q} = [\mathbf{I} + \mathbf{A}\boldsymbol{\Psi}\mathbf{K}\boldsymbol{\Psi}^{\mathrm{T}}]^{-1}\mathbf{A}\mathbf{g}_p, \tag{7}$$

where $\mathbf{I}$ is the identity matrix. For a single neutralizer, Eq. (7) can be simplified, which enables one to carry out theoretical investigation rather than numerical simulations only [17].

The above derivations can be used for all type structures with all boundary conditions. However, a simply supported beam is assumed in this paper for the simplicity of discussions. In this case, the time-averaged kinetic energy of the beam can be expressed as

$$\mathrm{KE} = \frac{M_b\omega^2}{4}\,\mathbf{q}^{\mathrm{H}}\mathbf{q}, \tag{8}$$

where M_b is the total mass of the beam and the superscript H denotes the Hermitian transpose. The kinetic energy of the beam with neutralizers attached can be determined by substituting the modal displacement amplitude's vector in Eq. (7) into Eq. (8), while kinetic energy of uncontrolled beam can be obtained by substituting Eq. (2) into Eq. (8), or simply by setting the $\mathbf{K}$ in Eq. (7) zero.

3. Control of kinetic energy using multiple vibration neutralizers tuned to a natural frequency of a structure

In classical applications, a neutralizer is used for local control of a point response, collocated or non-collocated. For that case, the neutralizer is either tuned to a problematic resonance frequency or to a troublesome excitation frequency away from the resonance region. It is therefore interesting to investigate how the neutralizer affects the global response of a continuous structure such as kinetic energy in comparison with the classical applications. Some of these control effects have been discussed by the author elsewhere for a point force excitation with a single neutralizer attached on the structure.

In global vibration control, neutralizers can be used in three distinct ways: (1) The resonance frequency of the neutralizers is fixed to a problematic frequency; (2) The resonance frequency are always tuned to the excitation frequency (tuned neutralizers); and (3) The resonance frequency of the neutralizers is optimally tuned to a value that minimizes the global vibration amplitude (optimally detuned). The aim of this section is to investigate the global effects of multiple neutralizers tuned to a particular frequency of the structure (case no. 1), which is the natural frequency.

3.1. Frequency separation of the new resonances with multiple tuned neutralizers attached

To illustrate the effects of the tuned neutralizer on the kinetic energy of a structure, the application of a single neutralizer is first investigated numerically. It is applied to a simply supported beam at its mid-point. The beam is excited by acoustic plane wave where the derivation of its generalized force is given in the appendix. The acoustic plane wave is used to provide distributed force rather than a point force as in the previous investigations. The neutralizer is then applied at few other locations for comparisons. For the same neutralizer location, simulation is carried out to visualize the effects of changing the neutralizer mass and damping ratio. In the investigation, the resonance frequency of the neutralizer is tuned to the first natural frequency of the beam.

Fig. 2(a) shows the kinetic energy of the beam, in the frequency range close to its first natural frequency, for three neutralizer locations. The neutralizer mass is fixed at 5% of the beam ($\mu = 0.05$) and damping ratio is $\zeta = 0.001$. Similarly, Fig. 2(b) shows the kinetic energy when the neutralizer location is fixed at $x = 0.5L$ with the same value of damping ratio ($\zeta = 0.001$) but for different mass. A numerical simulation is also carried out for different values of neutralizer damping ratio but the location and mass are fixed at $x = 0.5L$ and $\mu = 0.05$, respectively, and this is shown in Fig. 2(c).

The neutralizer effects on the kinetic energy shown in Fig. 2 are similar to that of a two-degree-of-freedom system where application of an auxiliary system splits the host resonance frequency into two. It is well known that for the two-degree-of-freedom system, the width between the new resonances is determined by the neutralizer mass alone. In the case of the kinetic energy of a beam, the width of the new resonance is also a function of the neutralizer location on the host structure and is given by [17]

$$\Delta\omega_m = \omega_m\sqrt{\mu_a\phi_m^2(x_a)}. \tag{9}$$

Eq. (9) is the width of the frequency separation for a single neutralizer.

Using the procedure described by Dayou and Brennan [18], the separation between the two resonances when multiple vibration neutralizers are attached can be derived as follows. For a single neutralizer, the modal displacement amplitudes in Eq. (7) can be simplified to

$$\mathbf{q}^{(1)} = \mathbf{A}^{(1)}\mathbf{g}_p, \tag{10}$$

where

$$\mathbf{A}^{(1)} = \left[\mathbf{I} - \frac{K_1}{D}\,\mathbf{A}\boldsymbol{\Phi}(x_1)\boldsymbol{\Phi}^{\mathrm{T}}(x_1)\right]\mathbf{A} \tag{11}$$

and

$$D = 1 + K_1\boldsymbol{\Phi}^{\mathrm{T}}(x_1)\mathbf{A}\boldsymbol{\Phi}(x_1). \tag{12}$$

Superscript (1) shows that only one device is attached and K_1 is the dynamic stiffness of the neutralizer. Consequently, for J number of neutralizers, the modal displacement amplitudes can be written as

$$\mathbf{q}^{(J)} = \mathbf{A}^{(J)}\mathbf{g}_p, \tag{13}$$

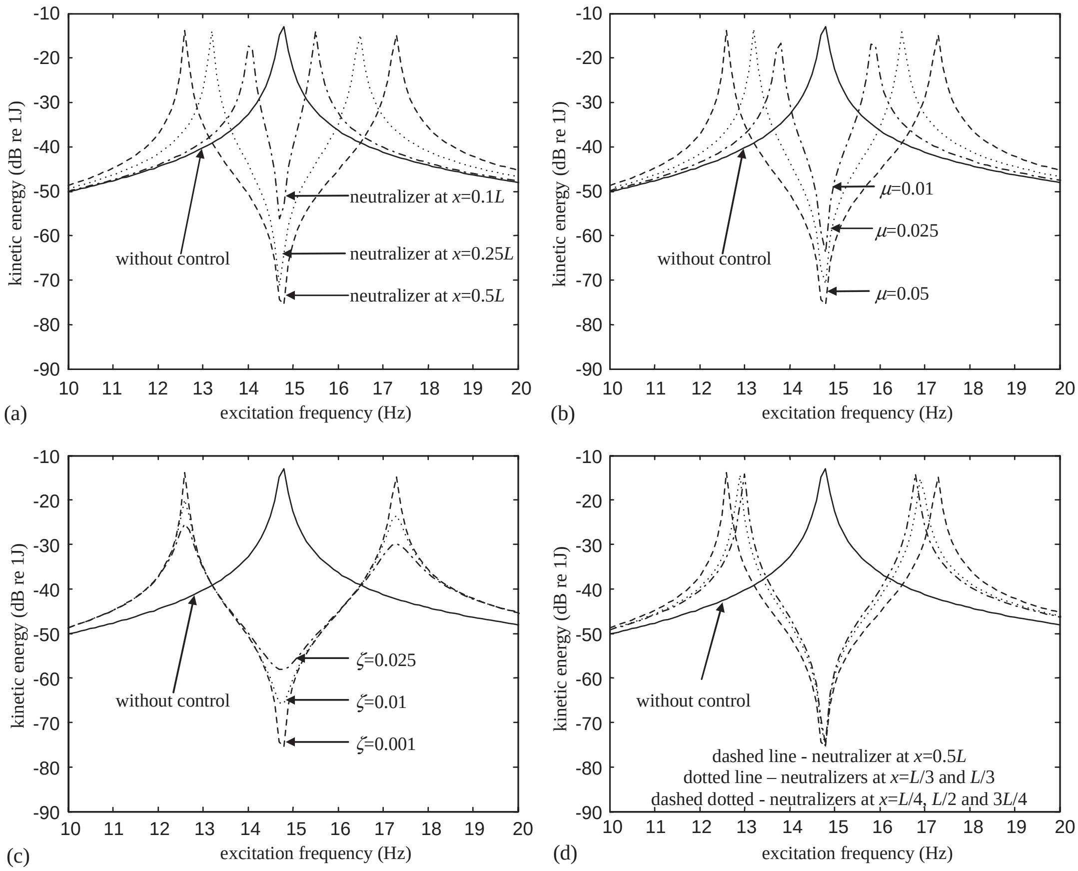

Fig. 2. Kinetic energy of the simply supported beam in the vicinity of its first natural frequency, without and with neutralizer/s attached, where the neutralizer/s is/are tuned to this natural frequency. The first natural frequency of the beam is 15 Hz. In (d), the mass for each distributed neutralizer is equal and their total mass is kept the same to the single neutralizer. (a) Neutralizer with different location ($\mu = 0.05, \zeta = 0.001$); (b) neutralizer with different mass ($x = 0.5L, \zeta = 0.001$); (c) neutralizer with different damping ratio ($x = 0.5, \mu = 0.05$); (d) single and distributed neutralizers with equal mass.

where

$$\mathbf{A}^{(J)} = \left[\mathbf{I} - \frac{K_J}{D^{(J-1)}}\,\mathbf{A}^{(J-1)}\boldsymbol{\Phi}(x_J)\boldsymbol{\Phi}^{\mathrm{T}}(x_J)\right]\mathbf{A}^{(J-1)} \tag{14}$$

and

$$D^{(J-1)} = 1 + K_J\boldsymbol{\Phi}^{\mathrm{T}}(x_J)\mathbf{A}^{(J-1)}\boldsymbol{\Phi}(x_J) \tag{15}$$

with K_J as the dynamic stiffness of the Jth neutralizer. Around and at the mth resonance frequency, the modal displacement amplitudes with one neutralizer attached can be well approximated by [17]

$$q_m = \left[\frac{g_{pm}}{A_m^{-1} + K_1\phi_m^2(x_1)}\right]. \tag{16}$$

Following the procedures described in Eqs. (10)–(15), the modal displacement amplitudes around the resonance frequency can also be approximated as, after some mathematical manipulations

$$q_m = \left[\frac{g_{pm}}{A_m^{-1} + K_1\phi_m^2(x_1) + K_2\phi_m^2(x_2) + \cdots + K_J\phi_m^2(x_J)}\right] \tag{17}$$

or

$$q_m = \frac{g_{pm}}{M_b\omega_m^2 - M_b\omega^2 + \mathrm{i}2M_b\zeta_m\omega\omega_m - \left\{\sum_{j=1}^{J}\frac{M_j\omega^2\phi_m^2(x_j)(1+\mathrm{i}2\zeta_j\mathbf{\Omega}_j)}{1-\mathbf{\Omega}_j^2+\mathrm{i}2\zeta_j\mathbf{\Omega}_j}\right\}}. \tag{18}$$

Setting the damping in each neutralizers and the beam to zero would result in an infinite modal amplitude at the new maxima. This means that the denominator of Eq. (18) is zero. Since all neutralizers are tuned to the same frequency, it can be written that $\Omega_1 = \Omega_2 = \cdots = \Omega_J = \Omega$, and the denominator of Eq. (18) is simplified to

$$M_b\omega_m^2 - M_b\omega^2 - M_b\omega_m^2\Omega^2 + M_b\omega^2\Omega^2 - \sum_{j=1}^{J} M_j\omega^2\phi_m^2(x_j) = 0. \tag{19}$$

Substituting $\Omega = \omega/\omega_m$ and $\mu_j = M_j/M_b$ we get

$$\frac{\omega^4}{\omega_m^4} - \left(2 + \sum_{j=1}^{J}\mu_j\phi_m^2(x_j)\right)\frac{\omega^2}{\omega_m^2} + 1 = 0 \tag{20}$$

which has the solution of

$$\frac{\omega_{m1,m2}}{\omega_m} = \sqrt{\left(1 + \frac{\sum_{j=1}^{J}\mu_j\phi_m^2(x_j)}{2}\right) \pm \sqrt{\frac{\left(\sum_{j=1}^{J}\mu_j\phi_m^2(x_j)\right)^2}{4} + \sum_{j=1}^{J}\mu_j\phi_m^2(x_j)}}, \tag{21}$$

where ω_{m1} and ω_{m2} are the frequencies where the lower and upper maxima occur. Defining the frequency separation between the two peaks as $\Delta\omega_m = \omega_{m2} - \omega_{m1}$, we can write

$$\Delta\omega_m = \omega_m\sqrt{\sum_{j=1}^{J}\mu_j\phi_m^2(x_j)}. \tag{22}$$

Examination of Eq. (22) shows that the frequency separation of the new resonances is a function of each neutralizer's mass ratio and their location on the host structure. However, the damping in the neutralizer has no contribution to the frequency separation, and the numerical simulation in Fig. 2(c) for a single neutralizer verified this. Eq. (22) has the same form as for the two-degree-of-freedom system but with additional terms, which is the neutralizer's location on the structure. Eq. (22) is also similar to Eq. (9) except for the summation sign.

The effect of using distributed neutralizers is shown in Fig. 2(d) in comparison with a single neutralizer but with the same total mass. The damping ratio of each neutralizer is fixed at the same value, which is 0.001. It can be seen that the modal amplitude terms (ϕ_m) play an important role in determining the width of the frequency separation.

3.2. Kinetic energy reduction at the problematic natural frequency

From Fig. 2(c), it can be seen that reducing the neutralizer damping ratio results in a lower kinetic energy at the tuned frequency. From Fig. 2(a) and (b), it can also be seen that the reduction in kinetic energy of the beam is also a function of the neutralizer mass and location. In this section, the effect of these parameters on the kinetic energy of the structure at the tuned frequency is investigated for a multiple neutralizer's case.

The mth modal amplitude of the beam with the multiple neutralizers tuned to the same natural frequency of interest (the mth natural frequency) in Eq. (18) can be simplified as (because $\Omega = 1$)

$$q_m = \frac{g_{pm}}{\mathrm{i}2M_b\zeta_m\omega_m^2 + \sum_{j=1}^{J}\frac{\mathrm{i}M_j\omega_m^2\phi_m^2(x_j)}{2\zeta_j}} \tag{23}$$

and the kinetic energy is

$$\mathrm{KE}_{m(\mathrm{tuned})} = \frac{M_b\omega_m^2}{4}\left(\frac{g_{pm}}{2M_b\zeta_m\omega_m^2 + \sum_{j=1}^{J}\frac{M_j\omega_m^2\phi_m^2(x_j)}{2\zeta_j}}\right)^2. \tag{24}$$

$\mathrm{KE}_{m(\mathrm{tuned})}$ denotes the kinetic energy of the host structure at the mth natural frequency with neutralizers tuned to this frequency. The kinetic energy when no control device is attached is given by

$$\mathrm{KE}_m = \frac{M_b\omega_m^2}{4}\left(\frac{g_{pm}}{2M_b\zeta_m\omega_m^2}\right)^2. \tag{25}$$

By defining the kinetic energy reduction as the ratio of the kinetic energy of the structure, with tuned neutralizers attached, to the kinetic energy of the structure alone, it can be written that

$$\mathrm{KE}_{m(\mathrm{red})} = \mathrm{KE}_{m(\mathrm{tuned})}/\mathrm{KE}_m = \left(1 + \sum_{j=1}^{J}\frac{\mu_j\phi_m^2(x_j)}{4\zeta_j\zeta_m}\right)^{-2}, \tag{26}$$

where $\mathrm{KE}_{m(\mathrm{red})}$ is the kinetic energy reduction of the beam at its mth natural frequency. If the damping ratio in the beam and neutralizers are small enough so that $4\zeta_j\zeta_m \ll \mu_j\phi_m^2(x_j)$, then Eq. (26) can be reduced to

$$\mathrm{KE}_{m(\mathrm{red})} = \mathrm{KE}_{m(\mathrm{tuned})}/\mathrm{KE}_m = \left(\sum_{j=1}^{J}\frac{4\zeta_j\zeta_m}{\mu_j\phi_m^2(x_j)}\right)^2. \tag{27}$$

It is clear that the beam kinetic energy is further reduced with a higher mass and smaller damping ratios of neutralizers. For the same value of neutralizer mass and damping ratios, higher reduction in kinetic energy can be achieved if neutralizers are fitted at the anti-node of the mode of interest, and neutralizers located at the nodal point have no effect to the kinetic energy. Again, as shown in Fig. 2(d), distributing the neutralizers on the structure has no effect on the total kinetic energy reduction at the natural frequency if their total mass and damping ratios are kept the same as the mass and damping ratio of the single neutralizer, respectively.

4. Global control of vibration using multiple optimally detuned vibration neutralizers

4.1. Theoretical development

Suppose active control actuators are used to control the kinetic energy of a structure, and the structure is excited by single-frequency external forces. In this condition, the vector of the modal amplitudes in Eq. (3) can be written as

$$\mathbf{q} = \mathbf{d} + \mathbf{G}\mathbf{f}_c, \tag{28}$$

where

$$\mathbf{d} = \mathbf{A}\mathbf{g}_p \quad \text{and} \quad \mathbf{G} = \mathbf{A}\boldsymbol{\Psi}. \tag{29, 30}$$

Substituting for $\mathbf{q}$ in Eq. (8) and expanding gives the kinetic energy in the standard Hermitian quadratic form as

$$\mathrm{KE} = \frac{M_b\omega^2}{4}\{\mathbf{f}_c^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{G}\mathbf{f}_c + \mathbf{f}_c^{\mathrm{H}}\mathbf{G}^{\mathrm{H}}\mathbf{d} + \mathbf{d}^{\mathrm{H}}\mathbf{G}\mathbf{f}_c + \mathbf{d}^{\mathrm{H}}\mathbf{d}\}. \tag{31}$$

The kinetic energy in Eq. (31) has a minimum value when the vector of the secondary forces is [19]

$$\mathbf{f}_{\mathrm{co}} = -[\mathbf{G}^{\mathrm{H}}\mathbf{G}]^{-1}\mathbf{G}^{\mathrm{H}}\mathbf{d}. \tag{32}$$

The corresponding optimum vector of modal amplitudes can be written as [16]

$$\mathbf{q}_o = \{\mathbf{I} - \mathbf{G}[\mathbf{G}^{\mathrm{H}}\mathbf{G}]^{-1}\mathbf{G}^{\mathrm{H}}\}\mathbf{d}. \tag{33}$$

As has been discussed before, $\mathbf{K}$ in Eq. (6) is the dynamic stiffness of the neutralizers. However, $\mathbf{K}$ can also be considered as the equivalent dynamic stiffness of the control forces, $\mathbf{f}_c$. If $\mathbf{f}_c$ is optimized, it is therefore possible to determine the optimum dynamic stiffness of any control device that minimizes the kinetic energy. The optimum dynamic stiffness of the neutralizers can be determined by the following.

Equating the optimum vector of secondary forces in Eq. (32) to the feedback forces from the neutralizers given in Eq. (6) results in

$$\mathbf{f}_{\mathrm{co}} = -\mathbf{K}_o\boldsymbol{\Psi}^{\mathrm{T}}\mathbf{q}_o. \tag{34}$$

In this equation, $\mathbf{K}_o$ is considered as the dynamic stiffness of the optimal secondary forces, $\mathbf{f}_{\mathrm{co}}$. If this optimal condition is imposed on neutralizers, then $\mathbf{K}_o$ is the required dynamic stiffness that has to be generated by the devices so that the kinetic energy can be minimized. Therefore, in this case Eq. (34) is better to be written as

$$\mathbf{f}_{co} = -\mathbf{K}_r \mathbf{\Psi}^{\mathrm{T}} \mathbf{q}_o, \tag{35}$$

where $\mathbf{K}_r$ is the required dynamic stiffness that has to be generated by the neutralizers.

Eq. (35) has no direct solution for the required dynamic stiffness. In order to get the required dynamic stiffness for each neutralizer, individual calculation has to be carried out as in the following. The jth diagonal term in $\mathbf{K}_r$ is the required dynamic stiffness for the jth neutralizer (ijth components are zero because $\mathbf{K}_r$ is diagonal) and can be written as

$$K_{rj} = -f_{coj}[\mathbf{\Phi}^{\mathrm{T}}(x_j)\mathbf{q}_o]^{-1} \tag{36}$$

where f_{coj} is the optimum amplitude of the jth secondary force. Eq. (36) can be used in the determination of the optimum tuning ratio of the jth neutralizer which can be written as [15]

$$\mathbf{\Omega}_{oj} = \sqrt{1 - \frac{M_j \omega^2}{\mathrm{Re}\{K_{rj}\}}}, \tag{37}$$

where Re{} denotes the real part of the terms inside the bracket. Some detailed discussions regarding the characteristics of the optimal vibration neutralizers can be found in Ref. [15]. Eq. (37) can now be substituted into Eq. (5) to get the optimal dynamic stiffness of the jth neutralizer. Combining with Eqs. (7) and (8) gives the kinetic energy of the structure with optimal neutralizers.

4.2. Simulations

In active vibration control, it is customary to apply the control device at location which does not coincide to the nodal point of any mode of interest. Therefore, in the following investigation $x = 9L/20$ was chosen as the location to apply the control device on the structure, both for the active device and also for the neutralizer. It has to be mentioned again that the performance by the active control device is used as a reference to evaluate the performance of the optimal neutralizer.

In the investigation, it is assumed that only the neutralizer stiffness is adjustable but not the mass and the damping ratio. Therefore, in the real situation, the optimum mass has to be predetermined before fabricating the neutralizer and the way to do this can be found in Ref. [17]. In the investigation in this paper, μ_a/ζ_a is found to be 40 with neutralizer damping ratio as $\zeta_a = 0.001$, and therefore the neutralizer mass was fixed at 4% of that beam. The same beam properties and excitation conditions are used as in the previous section.

Using an active control device fixed at $9L/20$, a substantial reduction in kinetic energy can be achieved and this is shown as a dashed line in Fig. 3. At the first natural frequency, the reduction is as high as 60 dB, and between 10 and 30 dB at other natural frequencies. Using Eq. (36), the required dynamic stiffness that minimizes the kinetic energy can be determined and is shown in Fig. 4(a) and (b). The required dynamic stiffness has both real and imaginary parts, and can be positive or negative. When the real part is positive, the control device is required to produce a stiffness-like behavior but when it is negative, mass-like behavior is required. On the other hand, when the imaginary part has a positive value, the control device is required to absorb energy from the host structure but it is required to supply energy to the host structure when it is negative. Similar behaviors were observed by Dayou and Brennan [15] for point force excitation.

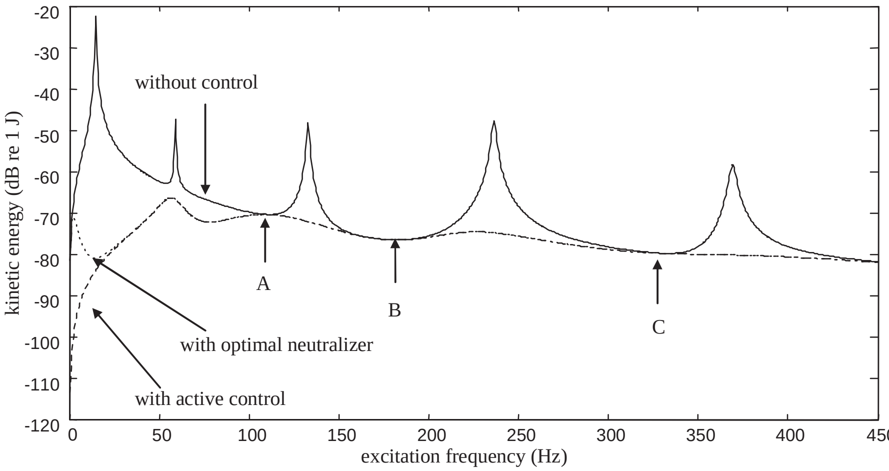

Fig. 3. Comparison of the kinetic energy of the beam with the feedforward active control method (dashed line) and the optimal neutralizer (dotted line), both located at $9L/20$. μ_a/ζ_a for the neutralizer is 40 with $\zeta_a = 0.001$.

It is well known that a neutralizer can change its behavior either to act mass- or stiffness-like depending on the value of its tuning ratio. Therefore, one can expect that if a neutralizer is fixed at the same location as the active device ($x = 9L/20$), a similar behavior can be produced. Comparison with Fig. 4(c) and (a) shows that the optimal neutralizer behaves mass-like or stiffness-like accordingly, similar to that of the real part of the required dynamic stiffness of the control device. It can also be seen that except at a very low frequency (below the first natural frequency), the optimal neutralizer produces a similar amplitude to that of the real part of the required dynamic stiffness. This is because the mass of the neutralizer is optimized only at this frequency and above.

Examination on Fig. 4(d) shows that the imaginary part of the optimal dynamic stiffness of the neutralizer has only positive value. This means the neutralizer can only absorb energy from the host structure but cannot supply energy into it. Although the neutralizer has such a limitation, in general the reduction in kinetic energy by the application of the device is comparable to that of the active control device. This interesting finding has been verified in Ref. [16] for point force excitation.

There are frequency ranges where the imaginary part of the optimal dynamic stiffness of the neutralizer has no value. These ranges are coinciding with the frequency ranges where no reduction in kinetic energy can be achieved even with an active control device. In the example discussed in Fig. 3, these frequency ranges are 110–112, 160–190 and 326–334 Hz, which are labeled as A, B and C, respectively. However, the control device is still required to produce a very large dynamic stiffness. This can be seen in Fig. 4(a) and (b) for the real and imaginary parts of the required dynamic stiffness, respectively, where the corresponding frequency ranges are also labeled as A, B and C.

Further examination of Fig. 4(e) shows that at these frequency ranges, the tuning ratio of the neutralizer has zero value (also labeled as A, B and C). The possible reason for this is that the selected neutralizer mass is too large, and the optimum tuning ratio defined by Eq. (37) is purely

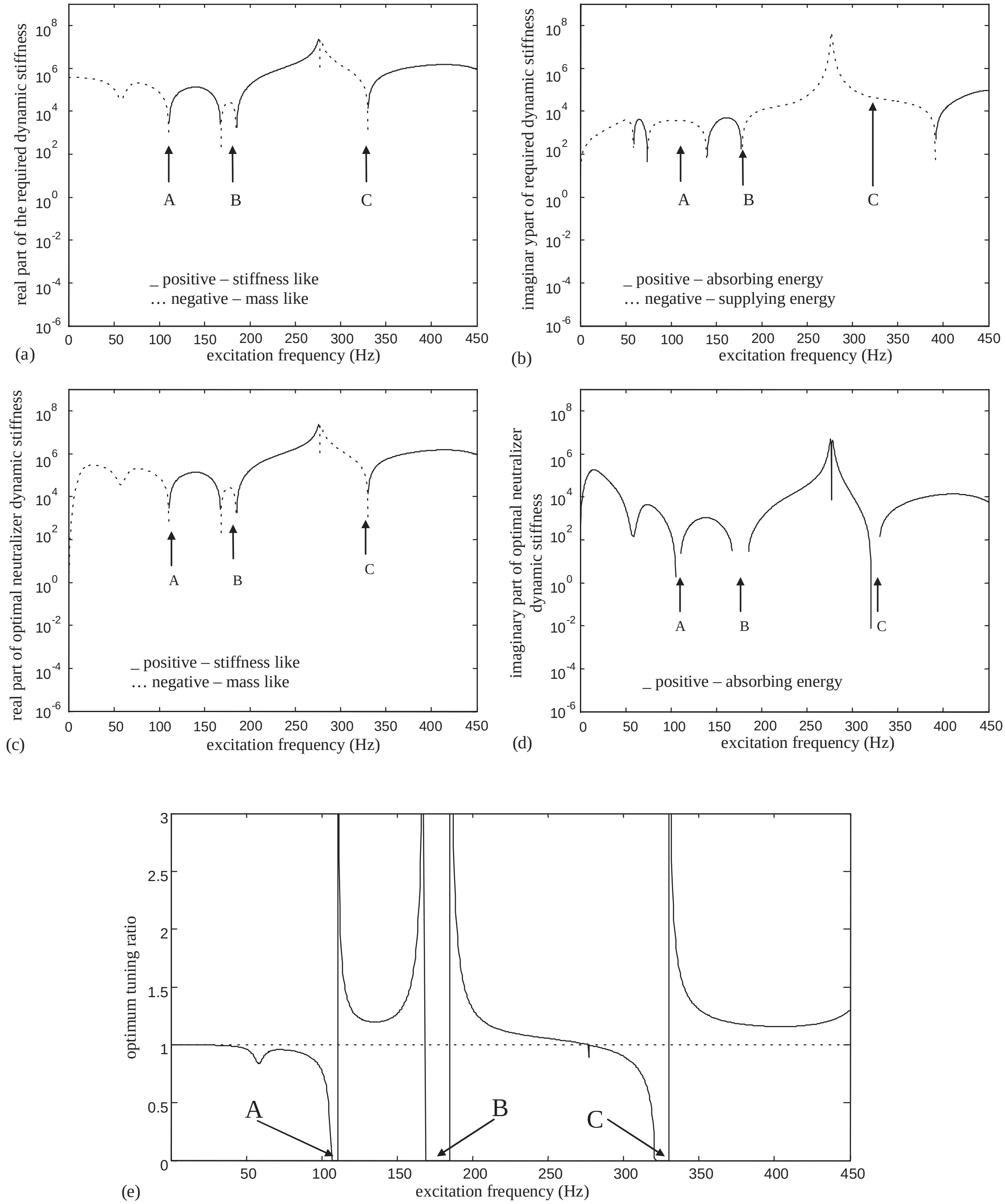

Fig. 4. Characteristics of the optimal control device that minimizes the kinetic energy of the simply supported beam. (a) Real part of the required dynamic stiffness; (b) imaginary part of the required dynamic stiffness; (c) real part of the optimal neutralizer dynamic stiffness; (d) imaginary part of the optimal neutralizer dynamic stiffness; (e) optimum turning ratio of the neutralizer.

imaginary. In this situation, it is therefore possible to simply remove the device or find another location where some reductions in kinetic energy can be achieved.

It is well known that in the active control method, the reduction in kinetic energy increases with the number of active control devices. This is demonstrated in Fig. 5 when two active control devices fitted at $x = 9L/20$ and $L/6$, and the optimal characteristics of the first and second control device are shown in Figs. 6(a) and (b), and 7(a) and (b), respectively. In comparison with the single active device in Fig. 3, additional reduction of 5 dB in average can be achieved in the whole frequency range. At certain frequencies, additional reduction of more than 10 dB can be achieved which is between 120 and 180 Hz.

It is interesting to note that the application of a second neutralizer also result in further reduction in the kinetic energy, and this is also shown in Fig. 5. It can be seen that the overall reduction by the neutralizer is comparable with that of the active control device, except again at a low frequency. With the second neutralizer fitted, the optimal characteristics of the first neutralizer in Fig. 6(c)–(e) are significantly changed compared to when the neutralizer is acting alone on the structure (Fig. 4(c)–(e)). There are frequency ranges where the first neutralizer has no contribution to the reduction in kinetic energy (labeled D and E in Fig. 6(d)) and these ranges are also different from Fig. 4(d).

The behavior of the second control device is shown in Fig. 7. Fig. 7(a) and (b) are the real and the imaginary parts of the required dynamic stiffness, respectively, whereas Fig. 7(c) and (d) are the real and imaginary parts of the optimal dynamic stiffness of the neutralizer and its optimum tuning is shown in Fig. 7(e). Overall, the neutralizer has to be almost tuned, and always contributes to the reduction in the kinetic energy.

Higher reduction in kinetic energy of the host structure can be achieved when more control devices are attached. This is demonstrated in Fig. 8 for three control devices fitted at $9L/20$, $L/6$ and $11L/20$ and four control devices at $9L/20$, $L/6$, $11L/20$ and $5L/6$, both for active control devices and neutralizers. Again, it is interesting to note that the reduction caused by the

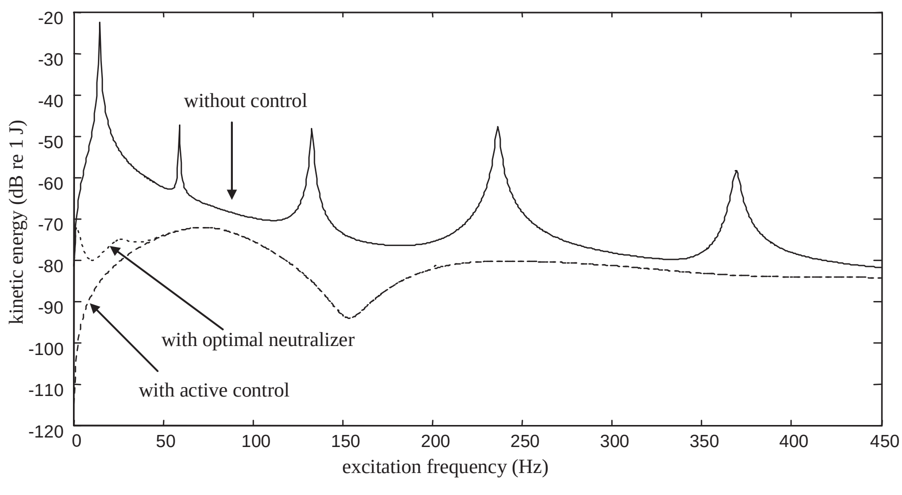

Fig. 5. Kinetic energy of the beam without control (solid line), with two active devices (dashed line) and with two neutralizers (dotted line) respectively, fitted at $x = 9L/20$ and $L/6$.

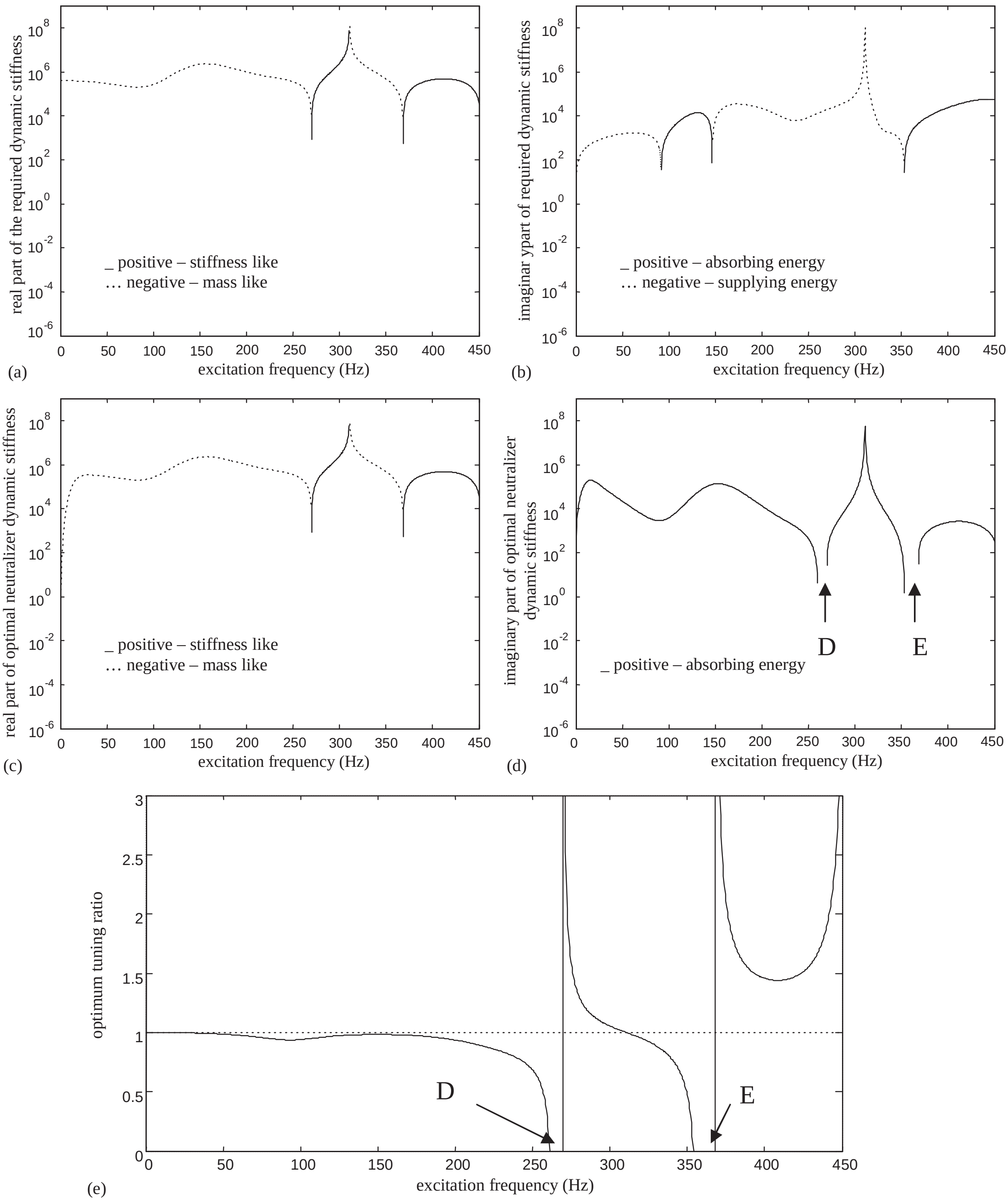

Fig. 6. Optimum characteristics of the first control device attached at $x = 9L/20$. (a) Real part of the required dynamic stiffness; (b) imaginary part of the required dynamic stiffness; (c) real part of the optimal neutralizer dynamic stiffness; (d) imaginary part of the optimal neutralizer dynamic stiffness; (e) optimum turning ratio of the neutralizer.

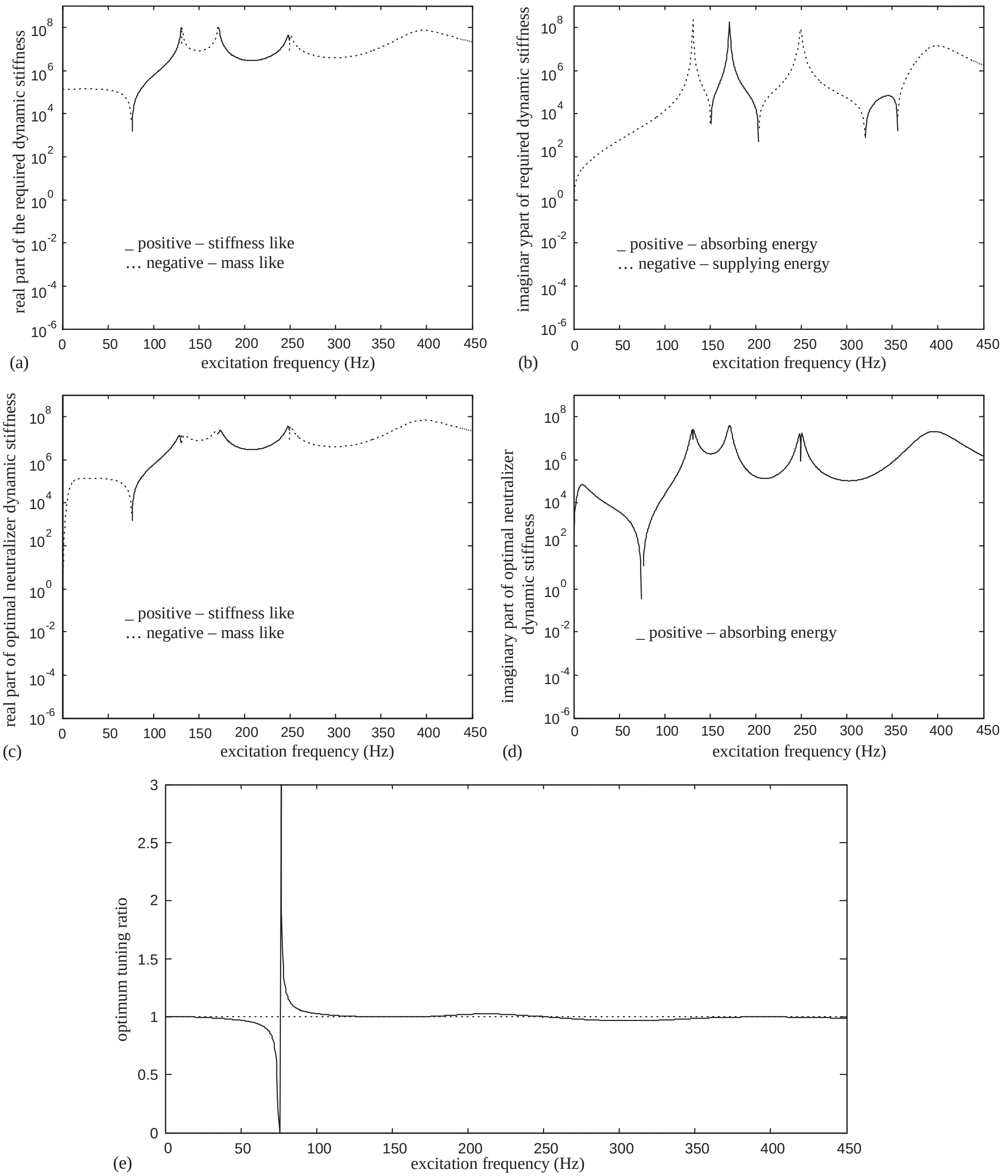

Fig. 7. Optimum characteristics of the second control device attached at $x = L/6$. (a) Real part of the required dynamic stiffness; (b) imaginary part of the required dynamic stiffness; (c) real part of the optimal neutralizer dynamic stiffness; (d) imaginary part of the optimal neutralizer dynamic stiffness; (e) optimum tuning ratio of the neutralizer.

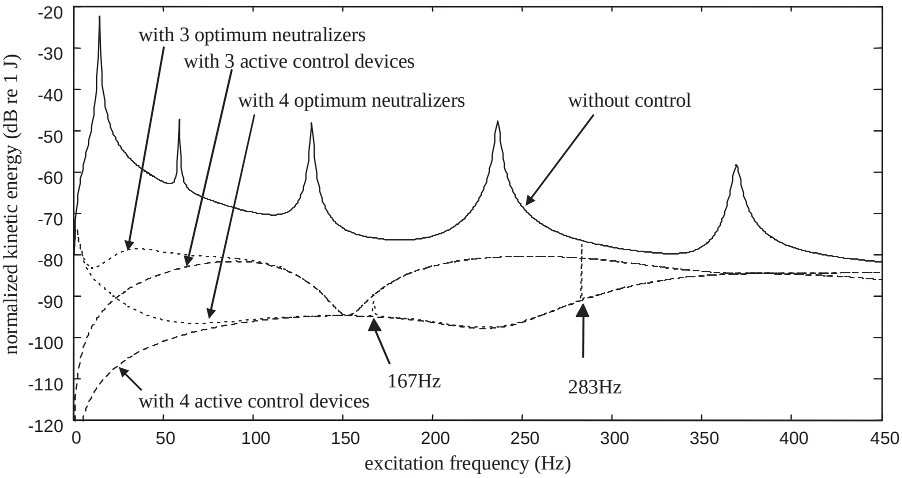

Fig. 8. Comparison of the kinetic energy of the simply supported beam with three active devices and three optimal tunable vibration neutralizers located at $9L/20, L/6$ and $11L/20$, and also with four active devices and four optimal tunable vibration neutralizers located at $9L/20$, $L/6$, $11L/20$ and $5L/6$.

neutralizers is comparable to that of the active control devices except in the low-frequency region. However, the frequency range where the discrepancy occurs between these two methods increases with the number of control devices.

There are certain frequencies where although some reductions still can be achieved by using neutralizers, the reductions are very small compared to that of the active control method. In Fig. 8, these frequencies are 167 and 283 Hz with four neutralizers attached at $9L/20$, $L/6$, $11L/20$ and $5L/6$. This occurrence may be explained as follows. In the optimization procedure suggested in this paper, it is assumed that only the tuning ratio is adjustable. Therefore, only single-parameter optimization is possible. At the frequency mentioned above, two-parameter optimization may be required by optimizing both the tuning and the damping ratios of the neutralizers simultaneously. However, the frequency range where multi-dimensional optimization is required is very small and therefore one-dimensional optimization is still very useful.

5. Summary and conclusion

In this paper, the role of the mass, the damping ratio and the positioning of multiple vibration neutralizers tuned to a particular natural frequency of a continuous structure namely a simply supported beam, have been discussed. In addition, a method to minimize the kinetic energy of the structure using multiple vibration neutralizers has also been proposed. For neutralizers tuned to a particular natural frequency of the host structure, the total of all neutralizers mass multiplied by their respective locations determines the width of the separation between the two new resonance frequencies. On the other hand, the neutralizer's mass, damping ratios and their locations on the structure influence the reduction that can be achieved in the kinetic energy. These are interesting findings because of the similarity effects with the two-degree-of-freedom system and with additional term which is the modal amplitude.

As the number of optimal neutralizer's increases, the overall reduction in the kinetic energy of the structure also increases. Besides that, the reduction in the structural kinetic energy is also comparable to the reduction that can be achieved by using the feedforward active control technique. This shows that the optimization procedure discussed in this paper can be used to provide a good alternative to globally control the vibration of a continuous structure.

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Appendix A

In this appendix, the generalized forcing pressure acting on a simply supported beam in an infinite baffle is derived. The incident pressure is assumed to be an obliquely incident traveling plane wave, and is derived from the paper by Roussos [20]. For a one-dimensional structure such as a beam, one of the incident angles is zero leaving the incident pressure as, for stationary wave

$$p_i = P_i \mathrm{e}^{-\mathrm{i}k\,\sin(\theta_i)x}, \tag{A.1}$$

where P_i, k and θ_i are the amplitude of the incident acoustic pressure, acoustic wavenumber in air ($k = \omega/c, c$ is sound speed in air) and the incident angle of the plane wave, respectively. Therefore, the generalized forcing pressure per unit length can be written as

$$g_{pm} = \int_0^L 2P_\mathrm{i} \mathrm{e}^{-\mathrm{i}k\,\sin(\theta_i)x} \phi_m(x)\,\mathrm{d}x. \tag{A.2}$$

L is the length of the beam and ϕ_m is the beam normalized mode shape function. For a simply supported beam, the mode shape function is $\phi_m = \sqrt{2}\sin(m\pi x/L)$.

The integration in Eq. (A.2) can be done in closed form to obtain the generalized forcing pressure for each mode

$$g_{pm} = 2\sqrt{2} p_i I_m, \tag{A.3}$$

where

$$I_m = \begin{cases} -\dfrac{\mathrm{j}}{2}\,\mathrm{sgn}[\sin\theta_i] & \text{for } [m\pi]^2 = [\sin\theta_i(kL)]^2, \\ \dfrac{m\pi[1-(-1)^m \mathrm{e}^{-\mathrm{j}\,\sin\theta_i(kL)}]}{[m\pi]^2 - [\sin\theta_i(kL)]^2} & \text{for } [m\pi]^2 \neq [\sin\theta_i(kL)]^2. \end{cases} \tag{A.4}$$

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