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Experimental verification of a method of detection of multiple cracks in beams based on frequency measurements

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Abstract

A method for prediction of location and size of multiple cracks based on measurement of natural frequencies has been verified experimentally for slender cantilever beams with two and three normal edge cracks. The analysis is based on energy method and representation of a crack by a rotational spring. For theoretical prediction the beam is divided into a number of segments and each segment is considered to be associated with a damage index. The damage index is an indicator of the extent of strain energy stored in the rotational spring. The crack size is computed using a standard relation between stiffness and crack size. Number of measured frequencies equal to twice the number of cracks is adequate for the prediction of location and size of all the cracks. The accuracy of prediction of crack details is encouraging. The maximum error in predicting location of cracks decreases with an increase in the number of cracks. It is less than 10% and 20% for two and three cracks respectively. The maximum error in predicting the crack size is less than 12% and 30% respectively for the two cases. A strategy to overcome failure in the prediction for cases with one of the cracks located near an anti-node has been suggested.

1. Introduction

To help in a continuous safety assessment of a machine or structure it is very necessary to constantly assess the health of its critical components. This calls for a continuous assessment of

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changes in their static and/or dynamic behaviour. The changes have very often their origin in local reduction of structural stiffness caused by cracks or crack-like defects. The development of a crack does not necessarily make a component instantly useless, but it is a signal that its behaviour has to be monitored more carefully. Such monitoring can play a significant role in assuring an uninterrupted operation in service by the component. This has made the vibration-based monitoring of components consisting of cracks or crack-like defects in service very important and the study of vibration of components with crack very wide. Very good reviews on these issues have been presented by Dimarogonas [1], Wauer [2] and Salawu [3].

Most studies concerning the forward (i.e. determination of natural frequencies from crack details) and inverse (i.e. determination of crack details from the measurement of vibration parameters) problems in beams deal with a single crack. The case of multiple cracks has not received the same degree of attention. Ostachowicz and Krawczuk [4], Liang et al. [5], Hu and Liang [6], Choy et al. [7], Ruotolo and Surace [8], Tsai and Wang [9], Shrifin and Ruotolo [10], Sekhar [11], Kisa and Brandon [12], Zheng and Fan [13], Khiem and Lien [14], Li [15], Sinha [16], etc., have studied such problems.

While papers dealing with multiple cracks mostly address the forward problem, Liang et al. [5], Hu and Liang [6], Choy et al. [7], Ruotolo and Surace [8], and Sinha et al. [16], address the inverse analysis. In solving the inverse problem involving slender beams, some of the investigators consider that a crack reduces the section modulus (EI) of a small segment around itself [7,16]. Others consider that a crack causes a local discontinuity in the slope of the deflection curve or reduces the energy content of the beam. They also consider that the overall mode shapes of the cracked and the corresponding uncracked beams differ a little except near the crack location, where there is a jump in slope. In a physical representation of the beam these are embedded by invoking at the crack location a rotational spring, which adds the additional rotational flexibility or acts as an energy sink. Experimental results have been presented by a few researchers [8,16–17], that too, involving at the most two cracks. Ruotolo and Surace [8] give the results for cracks with two fixed locations but varying sizes in cantilever beams and the solution is obtained using the genetic algorithm. Sinha et al. [16] present similar results for free–free beams and the method of detection is based on sensitivity matrix update technique. Cawley and Adams [17] consider two-dimensional plates and prediction is based on the error sensitivity analysis. All the above approaches use finite element method as a tool for analysis and they are iterative and require an initial guess. As a result, the error in the solution is markedly influenced by the initial guess. Experimental results involving more than two cracks are not available.

Liang et al. [5] and Patil [18] have approached the problem of multiple cracks by representing a crack by a rotational spring too and breaking the beam into a number of segments, each of which can have only one crack. Though the size of crack in a segment decides the extent of the local effects, they have assumed the effect to be spread uniformly over the whole segment. They present a linear relationship between the changes in natural frequency with the damage parameters through energy considerations. Liang et al. [5] have obtained the relationship through symbolic computation, which is basically employed to obtain the mode shapes of the corresponding uncracked beam. Patil [18] have obtained the exact relationship directly without resorting to the symbolic computation. This is facilitated by the transfer matrix method. Measurement of a change in a number of natural frequencies permits determination of the damage parameters and detection of crack details. Unlike Liang et al. [5], Patil [18] show that the crack details can be obtained from the damage parameters directly. This method does not require any initial guess for prediction of crack details.

In this paper experimental studies, which help to verify the accuracy and effectiveness of the method [18], are presented. Cantilever beams with two and three cracks are examined. The beams are considered slender so as to be able to neglect the shear deformation and rotational inertia. Further, the overall mode shapes of the cracked and the corresponding uncracked beams are considered to differ a little except near the crack location, where there is a local slope discontinuity.

2. Theoretical formulation

For completeness of presentation the formulation is briefly given here. The crack is represented by a rotational spring (Fig. 1). The rotational spring stiffness (K) for a through-the-thickness crack is explicitly given by

$$K = \frac{EBh^2}{72\pi f(\eta)}, \tag{1}$$

where $\eta = a/h$, a is crack size, B is beam width or thickness, h is its depth, E is modulus of elasticity and

$$f(\eta) = 0.6384(\eta)^2 - 1.035(\eta)^3 + 3.7201(\eta)^4 - 5.1773(\eta)^5 + 7.553(\eta)^6 - 7.3324(\eta)^7 + 2.4909(\eta)^8. \tag{2}$$

This relationship is very accurate up to $\eta \leqslant 0.6$ [19].

The strain energy (U) of a beam containing a crack reduces because the beam can deform easily to the same extent as the uncracked beam. This reduction is equal to the extent of energy stored in the fictitious rotational spring, which represents the crack. That is,

$$U = U_n - \frac{M^2}{2K}, \tag{3}$$

where U_n is the energy stored in the corresponding uncracked beam in mode n, M is the bending moment at the location of the crack in the uncracked beam and K is the rotational spring stiffness. This idea can be extended to multiple cracks. If there are m cracks the energy levels of the cracked

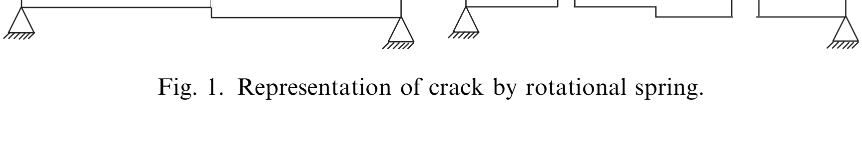

Fig. 1. Representation of crack by rotational spring.

and uncracked beams are related by

$$U = U_n - \sum_{i=1}^{m} \frac{M_i^2}{2K_i}, \tag{4}$$

where M_i and K_i are the bending moment at the crack location $x = l_i$ and stiffness associated with crack i, respectively.

The natural frequencies (ω_{nc} and ω_c) of a beam without and with a crack are given respectively by

$$\omega_{\mathrm{nc}}^2 = \frac{U_n}{V_n}, \tag{5}$$

$$\omega_c^2 = \frac{U}{V} = \left\{ U_n - \frac{M_1^2}{2K_1} - \frac{M_2^2}{2K_2} - \cdots - \frac{M_m^2}{2K_m} \right\} \Big/ V, \tag{6}$$

where $U_n = 1/2 \int_0^L EI(\mathrm{d}^2 Z/\mathrm{d}x^2)^2 \, \mathrm{d}x$, $V_n = 1/2 \int_0^L \rho A Z^2 \, \mathrm{d}x$, A and I are the cross-sectional area and second moment of inertia, respectively, ρ is the density and L is the total length of the beam. Z represents mode shape of the beam without a crack. V is ratio of kinetic energy of the beam with a crack and ω_c^2. Generally, for a beam with an edge normal crack, there is no loss of material and, hence, change in the mass. With the assumption that there is a negligible difference between the transverse mode shapes of a virgin beam and the corresponding beam with a crack, $V = V_n$ [20]. Therefore,

$$\omega_{\mathrm{nc}}^2 - \omega_c^2 = \frac{U_n}{V_n} - \frac{U_n - (M_1^2/2K_1 + \cdots + M_i^2/2K_i + \cdots + M_m^2/2K_m)}{V_n}, \tag{7}$$

$$\frac{\omega_{\mathrm{nc}}^2 - \omega_c^2}{\omega_{\mathrm{nc}}^2} = \frac{M_1^2/2K_1 + \cdots + M_i^2/2K_i + \cdots + M_m^2/2K_m}{U_n}, \tag{8}$$

$$\frac{\Delta\omega_n(\omega_{\mathrm{nc}} + \omega_c)}{\omega_{\mathrm{nc}}^2} = \frac{(S_1 \Psi_{n1} + \cdots + S_i \Psi_{ni} + \cdots + S_m \Psi_{nm})}{U_n}, \tag{9}$$

where $\Psi_{ni} = \int_{l_i} EI/2(\mathrm{d}^2 Z/dx^2)^2 \, \mathrm{d}x$ and $S_i = (M_i^2/2K_i)/\Psi_{ni}$. S_i is called damage index; it is an indicator of extent of strain energy stored in the rotational spring. If the beam is divided arbitrarily into r ($\geqslant m$) segments, assuming $S_i = 0$ for no crack, $S_i = 1$ for complete separation, and $\omega_{\mathrm{nc}} + \omega_c \cong 2\omega_{\mathrm{nc}}$, above equation can be rewritten as

$$\frac{2\Delta\omega_n}{\omega_{\mathrm{nc}}} = \frac{(S_1 \Psi_{n1} + \cdots + S_i \Psi_{ni} + \cdots + S_r \Psi_{nr})}{U_n}, \tag{10}$$

$$\frac{\Delta\omega_n}{\omega_{\mathrm{nc}}} = \frac{1}{2} \frac{\sum_{j=1}^{r} S_j \Psi_{nj}}{U_n}. \tag{11}$$

The coefficients ($\Psi_{nj}/2U_n$) of damage parameter S_j depends only on the mode shape Z of the uncracked beam. Thus for a beam divided into r segments and with q number of known frequencies, Eq. (11) gives rise to a set of simultaneous equations

$$\left\{\frac{\Delta\omega_n}{\omega_{\mathrm{nc}}}\right\}_{q\times 1} = [H]_{q\times r}\{S\}_{r\times 1}, \tag{12}$$

where a typical element of $[H]$ matrix, $h_{ij} = \Psi_{ij}/2U_i$, $i=1,2,\ldots,q$, and $j=1,2,\ldots,r$; i refers to the mode and j refers to the segment number. For example, for a cantilever beam, the ith mode shape is given by

$$Z(x) = \cos(px) - \cosh(px) - \sigma[\sin(px) - \sinh(px)], \tag{13}$$

where $\sigma = (\cos(pL) + \cosh(pL))/(\sin(pL) + \sinh(pL))$ and $p^4 = \rho A\omega_{\mathrm{nc}}^2/EI$. Therefore,

$$\frac{\mathrm{d}Z}{\mathrm{d}x} = p\{-\sin(px) - \sinh(px) - \sigma[\cos(px) - \cosh(px)]\}, \tag{14}$$

$$\frac{\mathrm{d}^2Z}{\mathrm{d}x^2} = p^2\{-\cos(px) - \cosh(px) - \sigma[-\sin(px) - \sinh(px)]\}. \tag{15}$$

Explicitly,

$$h_{ij} = \frac{G(x_j) - G(x_{j-1})}{2G(L)}, \tag{16}$$

where

$$\begin{aligned} G(x) = \frac{EIp^3}{8}\{&(\sigma^2 + 1)[\sinh(2px) + 4\cosh(px)\,\sin(px)] \\ &- (\sigma^2 - 1)[\sin(2px) + 4\,\cos(px)\,\sinh(px)] \\ &+ 4px - 4\sigma[\sin(px) + \sinh(px)]^2\}. \end{aligned} \tag{17}$$

For solving the inverse problem Eq. (12) is the basis. The numerical values of the damage parameters S_i obtained after solving Eq. (12) furnish information about the extent of damage in the segments. If the number of measured frequencies q is less than r, unknown q damage parameters can be obtained through pseudo-inverse technique [5].

In order to locate the crack in segment i with $S_i>0$, the value of $M_i^2/2K_i$ is determined from

$$\frac{M_i^2}{2K_i} = S_i\int_{l_i}\frac{EI}{2}\left(\frac{\mathrm{d}^2Z}{\mathrm{d}x^2}\right)^2\mathrm{d}x. \tag{18}$$

Assuming $\lambda_i = M_i^2/2 = 1/2[EI(\mathrm{d}^2Z/\mathrm{d}x^2)]_{x=\beta L}^2$ and $C_i=1/K_i$, for the segment, $\lambda_i C_i$=constant. Taking a position β for the crack in the span, λ_i can be calculated, and then using $\lambda_i C_i$=constant, C_i can be determined. Thus a variation of K_i vs. crack position (β) can be obtained for a particular mode. This can be repeated for more than two modes. The intersection of the curves will give directly the crack location in the segment and the corresponding spring stiffness. Using relation (1), the crack size can then be evaluated.

3. Experimental studies

Specimens were made out of an aluminium alloy. The material properties are: modulus of elasticity $E = 70.06$ GPa and mass density $\rho = 2645.19\,\text{kg/m}^3$. The beam dimensions are included in the tables later.

Different crack positions and sizes (a/h in the range 0.1–0.65) have been examined. The cracks are made by wire cut machining. The width of wire cut is 0.15 mm. For clamping a specimen a readymade fixture [21] has been used. This consists of a heavy base plate with a top cover plate. The cover plate is secured in position by six bolts.

The experimental setup is shown in Fig. 2. An accelerometer (Bruel and Kjaer, Type 4344) with a mass of 2 gm is fixed on the top edge of the beam using wax at a distance of about 15 mm from the fixed end. The output of the accelerometer is supplied to a charge amplifier (Bruel and Kjaer, Type 5974). The charge amplifier output is analyzed by an FFT analyzer (Tektronix TDS 220). The analyzer has a digital readout for the peaks.

During testing a specimen is lightly tapped by a finger or sound hammer in the transverse direction. The beam response is analyzed by the FFT analyzer. From the responses, the first few natural frequencies are noted. More experimental details are available in Ref. [21].

3.1. Results

Table 1 presents a typical set of the first five measured natural frequencies for cantilever beams with two cracks. For the prediction of crack details using these data as input a program has been written in MATLAB based on the theory given earlier. In each case the beam is divided into 10 segments, i.e. $r = 10$, unless otherwise specified. From the five frequencies for a beam (Table 1) with and without cracks, a set of simultaneous equations involving the damage parameters are obtained using Eq. (12). The uncracked beam frequencies are required for zero setting [21]. Since the number of measured frequencies (five) is less than number of segments (10), the pseudo-inverse technique of the MATLAB package is employed to solve for the damage parameters iteratively.

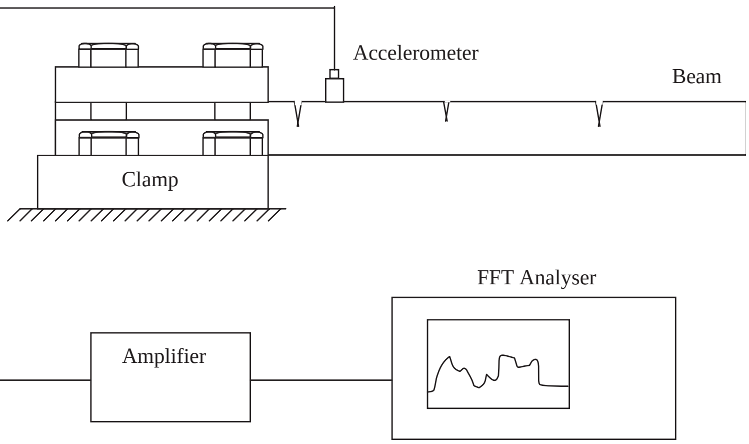

Fig. 2. Experimental set-up.

Table 1
Measured natural frequencies for cantilever beam with two cracks

Case no.	Crack location and size				Natural frequencies (Hz)				
	β_1	a_1/h	β_2	a_2/h	ω_1	ω_2	ω_3	ω_4	ω_5
	Uncracked beam				237.50	1460.0	3880.0	7210.0	9740.0
1	0.2	0.2	0.498	0.43	220.0	1260.0	3790.0	6275.0	9320.0
2	0.25	0.455	0.50	0.25	200.0	1355.0	3475.0	6185.0	9305.0
3	0.10	0.450	0.496	0.450	185.0	1220.0	3700.0	6300.0	9120.0
4	0.25	0.46	0.50	0.15	205.0	1405.0	3490.0	6285.0	9160.0
5	0.15	0.30	0.496	0.455	210.0	1250.0	3800.0	6225.0	8725.0
6	0.25	0.434	0.50	0.35	205.0	1345.0	3570.0	6245.0	8925.0
7	0.25	0.42	0.5	0.4	207.5	1310.0	3585.0	6190.0	9065.0
8	0.25	0.15	0.5	0.45	220.0	1235.0	3720.0	6100.0	9495.0

$E = 70.06$ GPa, $\rho = 2645.19\,\text{kg/m}^3$, $h = 0.0191$ m, $B = 0.0064$ m and $L = 0.24$ m.

The iteration continued till all the damage parameters are found to be positive. If a damage parameter was negative after iteration, it was set equal to zero and further iteration was carried out [6].

To predict the location of crack in a segment Eq. (18), which can be re-written as follows, is employed:

$$\frac{M^2_{x=\beta L}}{2K} = S_i \int_{l_i} \frac{EI}{2}\left(\frac{\mathrm{d}^2 Z}{\mathrm{d}x^2}\right)^2 \mathrm{d}x. \tag{19}$$

For a particular mode, varying β (where $\beta = x/L$) in the above relation a plot of K vs. β is obtained. Similar plots for two or more modes are considered. These are shown for a few cases in Fig. 3. The intersection of these curves corresponding to the three modes gives the crack location and the associated rotational spring stiffness. If the curves do not intersect at a single point, the centre of gravity of the three pair of intersection points is taken as the crack location [21]. The crack size is then obtained using Eq. (1). The accuracy of predictions (Table 2) is good. The maximum absolute error is 9.2% in crack location and 12% in crack size.

The same cases were again examined considering only the first four frequencies. The predicted results are compared in Table 3. The results show similar accuracy in the prediction of both crack location and size. This therefore indicates that a minimum of $2n$ frequencies are required to predict location and size of n cracks, i.e. $2n$ unknowns.

Cantilever beams with three cracks were also examined and the experimental results and theoretical predictions are presented in Tables 4 and 5, respectively. In this case the first six frequencies were measured and are used for the predictions. The maximum absolute error is 19.5% in crack location and 15.1% in crack size. In cases 5 and 8 (Table 5) one of the cracks, the second and third respectively, is located near an anti-node. They could not be predicted employing all the six measured frequencies. These two cases were again tried out by considering only the four measured frequencies ω_3–ω_6. The results obtained thereby are shown in Table 5. They too show good accuracy for the prediction of location. The error in crack size prediction however increases to 30%.

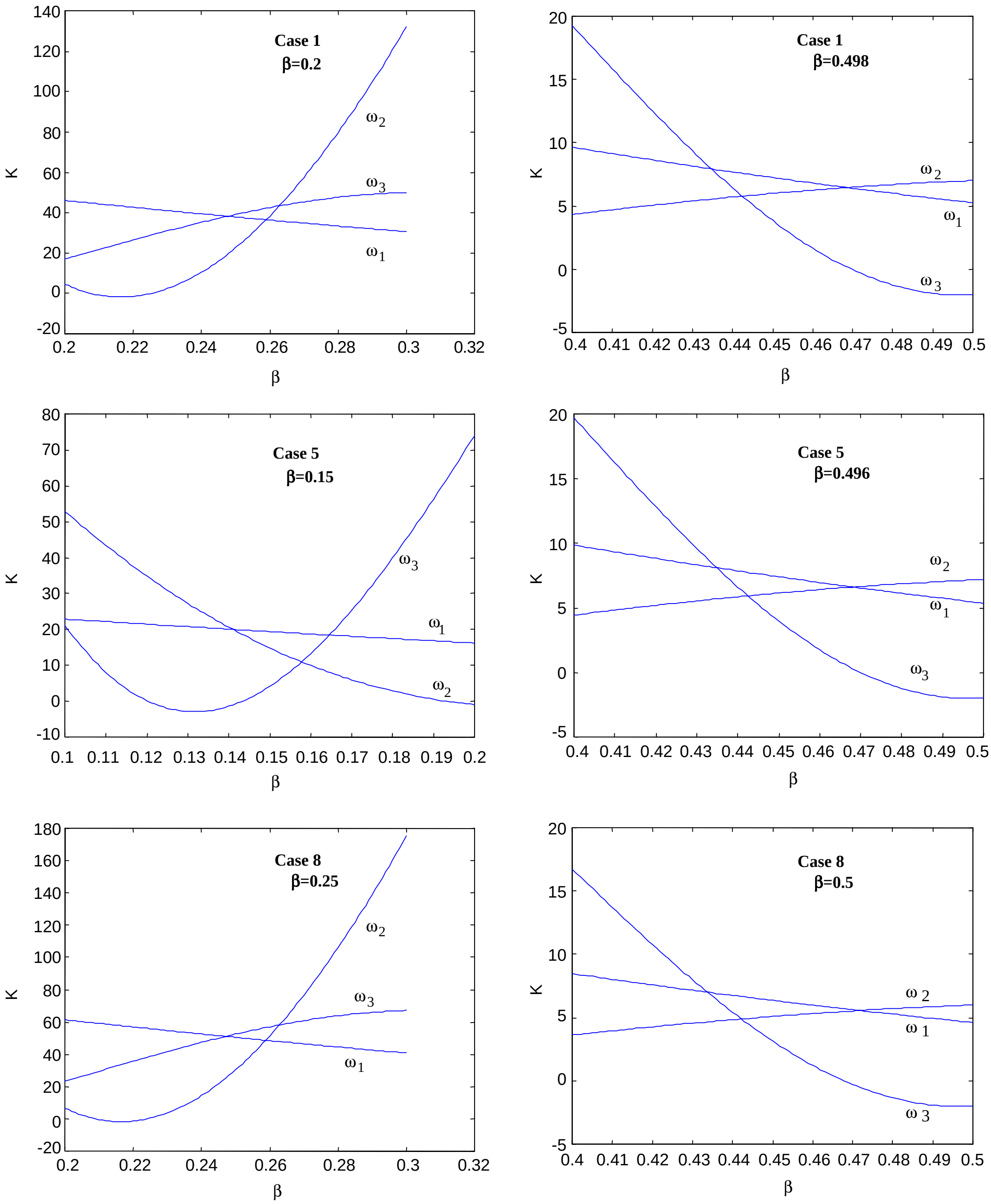

Fig. 3. Sample plots of K vs. β for cantilever beam with two cracks (Table 2).

Table 2
Comparison of actual and predicted crack location and size for cantilever beams of Table 1

Case no.	Actual data		Predicted location and crack size				
	β	a/h	β	%error	K	a/h	%error
1	0.2	0.2	0.259	5.9	38.0	0.1787	−2.13
	0.498	0.43	0.442	5.6	6.5	0.4064	−3.56
2	0.25	0.455	0.272	2.2	7.0	0.394	−6.1
	0.5	0.25	0.410	−9.0	14.5	0.2856	3.56
3	0.1	0.45	0.157	5.7	4.0	0.494	4.4
	0.496	0.45	0.560	6.4	7.8	0.376	−7.4
4	0.25	0.46	0.272	2.2	7.0	0.394	−6.6
	0.50	0.15	0.411	−8.9	37.0	0.181	3.1
5	0.15	0.30	0.157	0.7	16.0	0.273	−2.7
	0.496	0.455	0.442	−5.4	6.8	0.398	−5.7
6	0.25	0.434	0.272	2.2	9.0	0.354	−8.0
	0.50	0.35	0.408	−9.2	12.0	0.311	−3.9
7	0.25	0.42	0.272	2.2	13.0	0.300	−12.0
	0.5	0.40	0.408	−9.2	8.3	0.366	−4.2
8	0.25	0.15	0.259	0.9	52	0.152	0.2
	0.5	0.45	0.442	−5.8	5.5	0.435	−1.5

3.2. Ruotolo and Surace's [8] experimental results on cantilever beams

Ruotolo and Surace's [8] measured natural frequencies are shown in Table 6 for steel cantilever beams. The beam span (L) is 0.8 m and depth (h) is 0.02 m. They considered two cracks at locations 0.254 m ($\beta = 0.317$) and 0.545 m ($\beta = 0.681$) from the fixed end with sizes 20% and 30%, respectively. For prediction of crack locations the beam is divided into 11 segments, the same as considered in Ref. [8]. The predictions (Table 7) were made using the measured frequencies (Table 6). The maximum absolute error in prediction of crack location is 0.3% and that in size is 6.3%. Ruotolo and Surace predicted correctly only the elements in which cracks are located in the cases C1 and C3 (Tables 6 and 7).

4. Discussions

The present experimental results involving cantilever beams with two and three cracks and Ruotolo and Surace's [8] experimental data have helped to examine the accuracy of the method [18] of crack prediction. Results on three cracks are reported for the first time. On the whole the method accuracy is good for the prediction of crack location. The accuracy is lower for the prediction of crack size. The maximum absolute error is 19.5% for the prediction of location and

Table 3
Comparison of actual and predicted crack location and size for cantilever beams of Table 1 considering first four natural frequencies

Case no.	Actual data		Predicted location and crack size			
	β	a/h	β	%error	a/h	%error
1	0.2	0.2	0.157	−4.3	0.169	−3.1
	0.498	0.43	0.442	5.6	0.398	−3.2
2	0.25	0.455	0.156	−9.4	0.420	−3.5
	0.5	0.25	0.555	−5.5	0.2046	4.54
3	0.1	0.45	0.157	5.7	0.494	4.4
	0.496	0.45	0.560	6.4	0.376	−7.4
4	0.25	0.46	0.156	−9.4	0.394	−6.6
	0.50	0.15	0.444	−5.6	0.089	−6.1
5	0.15	0.30	0.157	0.7	0.273	−2.7
	0.496	0.455	0.442	−5.4	0.398	−5.7
6	0.25	0.434	0.158	−9.2	0.394	−4.0
	0.50	0.35	0.556	5.6	0.23	−12.0
7	0.25	0.42	0.157	−9.3	0.39	−3.0
	0.5	0.40	0.557	5.7	0.29	−11.0
8	0.25	0.15	0.157	−9.3	0.14	−1.0
	0.5	0.45	0.442	−5.8	0.435	−1.5

Table 4
Measured natural frequencies for cantilever beam with three cracks

Case no.	Crack location and size						Natural frequencies (Hz)					
	β_1	a_1/h	β_2	a_2/h	β_3	a_3/h	ω_1	ω_2	ω_3	ω_4	ω_5	ω_6
			Uncracked				125	770	2106	4020	6520	9355
1	0.353	0.398	0.55	0.25	0.75	0.25	120	712	1945	3935	6000	9200
2	0.10	0.15	0.40	0.20	0.73	0.53	120	700	1775	3705	6380	8720
3	0.10	0.30	0.39	0.32	0.65	0.50	112	655	1825	4000	5850	8255
4	0.1	0.15	0.4	0.15	0.62	0.52	120	665	1905	3990	5815	8810
5	0.28	0.52	0.5	0.2	0.75	0.2	110	740	1865	3715	6470	8555
6	0.10	0.50	0.40	0.30	0.62	0.31	98	670	1995	3780	5815	8350
7	0.10	0.40	0.45	0.40	0.73	0.42	105	660	1700	3400	6140	8450
8	0.23	0.65	0.45	0.15	0.70	0.15	95	765	1855	3875	6400	9135

$E = 70.06$ GPa, $\rho = 2645.19$ kg/m^3, $h = 0.0191$ m, $B = 0.0064$ m and $L = 0.34$ m.

it is 30% for the prediction of crack size. The accuracy reduces as the number of cracks increases. Number of measured frequencies equal to twice the number of cracks is adequate for prediction of their locations and sizes.

Table 5
Comparison of actual and predicted crack location and size for beams of Table 4

Case no.	Predicted crack location and size											
	β_1	%error	a_1/h	%error	β_2	%error	a_2/h	%error	β_3	%error	a_3/h	%error
1	0.358	0.5	0.316	−8.19	0.531	−1.9	0.169	−8.1	0.641	−10.9	0.24	−1.0
2	0.046	−5.4	0.132	−1.8	0.388	−1.2	0.245	4.5	0.734	0.46	0.43	−9.9
3	0.106	0.6	0.289	−1.1	0.294	−9.72	0.174	−15.1	0.645	−0.5	0.48	−2.0
4	0.17	7.0	0.141	−0.9	0.452	5.2	0.231	8.1	0.640	1.94	0.43	−9.4
5	0.195	−19.5	0.455	−6.86	—		—		0.716	−3.4	0.34	14.0
					0.453[a]	−4.7	0.231[a]	3.1	0.70[a]	−5.0	0.45[a]	25.0
6	0.104	0.4	0.477	−2.3	0.452	5.2	0.241	−5.9	0.630	0.65	0.28	−2.9
7	0.195	9.5	0.483	8.3	0.452	0.2	0.370	−3.0	0.725	−0.59	0.44	2.12
8	0.195	−3.44	0.678	−7.8	0.294	−15.6	0.09	−6.0	—		—	
									0.695[a]	−0.5	0.45[a]	30.0

[a]Obtained considering frequencies ω_3–ω_6.

Table 6
Measured natural frequencies of Ruotolo and Surace [8] for cantilever beams

Case no.	Crack location and size				Natural frequencies (Hz)			
	β_1	a_1/h	β_2	a_2/h	ω_1	ω_2	ω_3	ω_4
C1		Uncracked beam			24.248	152.026	424.457	823.160
	0.317	0.2	0.681	0.2	24.066	150.612	416.579	820.399
C2		Uncracked beam			24.175	152.103	424.455	824.209
	0.317	0.2	0.681	0.3	24.044	149.268	409.287	818.150
C3		Uncracked beam			24.145	151.873	424.328	823.749
	0.317	0.3	0.681	0.2	23.892	150.260	411.265	819.811

$E = 2.06\,\text{GPa}$, $\rho = 7850\,\text{Kg/m}^3$, $h = B = 0.02\,\text{m}$ and $L = 0.8\,\text{m}$.

Table 7
Comparison of predicted crack location and size by present method with actual data Table 6

Case no.	Actual data		Predicted location and crack size				
	β	a/h	β	%error	K	a/h	%error
C1	0.317	0.2	0.319	0.2	115	0.183	−1.7
	0.681	0.2	0.682	0.1	105.5	0.190	−1.0
C2	0.317	0.2	0.32	0.3	168	0.15	−5.0
	0.681	0.3	0.683	0.2	38.8	0.31	1.0
C3	0.317	0.3	0.32	0.3	68.5	0.237	−6.3
	0.681	0.2	0.682	0.1	110	0.187	−1.3

Cases where one of the cracks is located near an anti-node require a careful handling. A subset of the measured frequencies has been found useful to overcome the failures in the prediction. However, such a strategy increases the error in the prediction of crack size. It must be noted here that failures to predict crack details even for the case of a single crack located near an anti-node of the beam has been reported in the literature [21].

Very good accuracy of the method associated with the prediction of location can be exploited at least for quickly locating a crack in practice for long beam-like components. Further refinements in the prediction of location are possible by employing any standard conventional NDT method. Thereby time and cost involved in the inspection can be reduced.

References

[1] A.D. Dimarogonas, Vibration of cracked structures: state of the art review, *Engineering Fracture Mechanics* 55 (1996) 831–857.

[2] J. Wauer, On the dynamics of cracked rotors: a literature survey, *Applied Mechanics Reviews* 43 (1) (1990) 13–17.

[3] O.S. Salawu, Detection of structural damage through changes in frequency: a review, *Engineering Structures* 19 (9) (1997) 718–723.

[4] W.M. Ostachowicz, M. Krawczuk, Analysis of the effect of cracks on the natural frequencies of a cantilever beam, *Journal of Sound and Vibration* 150 (1991) 191–201.

[5] R.Y. Liang, J. Hu, F.K. Choy, Quantitative NDE technique for assessing damage in beam structures, *Journal of Engineering Mechanics* 118 (7) (1992) 1468–1487.

[6] J. Hu, R.Y. Liang, An integrated approach to detection of cracks using vibration characteristics, *Journal of Franklin Institute* 330 (5) (1993) 841–853.

[7] F.K. Choy, R. Liang, P. Xu, Fault identification in beams on elastic foundation, *Computers and Geotechnics* 17 (2) (1995) 157–176.

[8] R. Ruotolo, C. Surace, Damage assessment of multiple cracked beams: numerical results and experimental validation, *Journal of Sound and Vibration* 206 (4) (1997) 567–588.

[9] T.C. Tsai, Y.Z. Wang, The vibration of a multi-crack rotor, *International Journal of Mechanical Science* 39 (9) (1997) 1037–1053.

[10] E.I. Shifrin, R. Ruotolo, Natural frequencies of a beam with an arbitrary number of cracks, *Journal of Sound and Vibration* 222 (3) (1999) 409–423.

[11] A.S. Sekhar, Vibration characteristics of a cracked rotor with two open cracks, *Journal of Sound and Vibration* 223 (4) (1999) 497–512.

[12] M. Kisa, J.A. Brandon, Free vibration analysis of multiple open-edge cracked beams by component mode synthesis, *Structural Engineering Mechanics* 10 (1) (2000) 81–92.

[13] D.Y. Zheng, S.C. Fan, Natural frequencies of a non-uniform beam with multiple cracks via modified Fourier series, *Journal of Sound and Vibration* 242 (4) (2001) 701–717.

[14] N.T. Khiem, T.V. Lien, A simplified method for natural frequency analysis of a multiple cracked beam, *Journal of Sound and Vibration* 245 (4) (2001) 737–751.

[15] Q.S. Li, Vibration characteristics of multi-step beams with an arbitrary number of cracks and concentrated masses, *Applied Acoustics* 62 (2001) 691–706.

[16] J.K. Sinha, M.I. Friswell, S. Edwards, Simplified models for the location of cracks in beam structures using measured vibration data, *Journal of Sound and Vibration* 251 (1) (2002) 13–38.

[17] P. Cawley, R.D. Adams, The location of defects in structures from measurements of natural frequencies, *Journal of Strain Analysis* 14 (2) (1979) 49–57.

[18] D.P. Patil, Modelling to Facilitate Detection of Single and Multiple Cracks in Beams of Various Geometries Based on Transverse Vibration Frequencies, PhD Dissertation, Mechanical Engineering Department, Indian Institute of Technology, Bombay, 2003.

[19] H. Tada, P.C. Paris, G.R. Irwin, *The Stress Analysis of Cracks Handbook*, third ed, ASME Press, Professional Engineering Publishing, New York, 2000.
[20] P. Gudmundson, Eigenfrequency changes of structures due to cracks, notches and other geometric changes, *Journal of the Mechanics and Physics of Solids* 30 (1982) 339–353.
[21] B.P. Nandwana, On Foundation for Detection of Crack Based on Measurement of Natural Frequencies, PhD Dissertation, Mechanical Engineering Department, Indian of Institute of Technology, Bombay, 1997.