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Discussion

On the use of the cyclic power spectrum in rolling element bearings diagnostics

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In a recent paper [1], a method based on the cyclic power spectrum (CPS) was presented to detect mechanical signals with periodic amplitude modulation, such as those characterising faulty states in rolling element bearings. The method was claimed "to obtain more information than other conventional methods, such as the frequency domain and the envelope detection". Actually, this conclusion is not new and it is only partly true. In order to explain why this assertion should be revised, we first prove that the theoretical argument by which the authors of Ref. [1] arrive at advocating the use of the CPS is wrong; we then give one acceptable reason why the CPS is a relevant tool for analysing rolling element bearing signals; we finally refute the authors' claim that the CPS is a more effective tool than envelope analysis and we prove why.

1. Terminology

The aim of this first section is to introduce the notation used in the following discussion and which is consistent with that of Ref. [1]. Let us refer to *first-order cyclostationary* (CS1) signals as to those signals whose ensemble average $m_x(t) = E\{\mathbf{x}(t)\}$ has a Fourier series expansion $m_x(t) = \sum_{\alpha_i \in A} m_x^{\alpha_i} e^{j2\pi\alpha_i t}$, where A is a countable set of cyclic frequencies. Similarly, let us refer to *second-order*

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cyclostationary (CS2) signals as to those signals whose instantaneous autocorrelation function $R_x(t,\tau) = E\{\mathbf{x}(t)\mathbf{x}(t-\tau)^*\}$ has a Fourier series expansion $R_x(t,\tau) = \sum_{\alpha_i \in A} R_x^{\alpha_i}(\tau)e^{j2\pi\alpha_i t}$, where the Fourier coefficients $R_x^{\alpha_i}(\tau)$ are known as the *cyclic autocorrelation functions*. The Fourier transform of the cyclic autocorrelation function at a given cyclic frequency α_i then defines the *cyclic power spectrum* (CPS): $S_x^{\alpha_i}(f) = F\{R_x^{\alpha_i}(\tau)\}$. The family of CPS's indexed by all $\alpha_i \in A$ is related to the *spectral correlation* (SC)—i.e. the double Fourier transform of $R_x(t,\tau)$ with respect to t and τ—in the following way:

$$S_x(\alpha,f) = \iint_{R^2} R_x(t,\tau)e^{-j2\pi f\tau}e^{-j2\pi\alpha t}dtd\tau = \sum_{\alpha_i \in A} S_x^{\alpha_i}(f)\delta(\alpha-\alpha_i).$$

Note that the SC is a surface-wise density (power/Hz^2) whereas the CPS is a line-wise density (power/Hz). In other words the 2-dimensional SC $S_x(\alpha,f)$ of a CS2 signal is made of a series of slices parallel to the f-axis, each of which indexed by a given cyclic frequency α_i and having its shape specified by the 1-dimensional CPS $S_x^{\alpha_i}(f)$. Because the SC and the CPS of a cyclostationary signal are so closely related and actually display the same spectral information, we shall indifferently refer to one or the other in the following discussion.

2. Quasi-periodic signals should not be analysed with the CPS

The authors of Ref. [1] proposed to model the rolling element bearing signal by a periodically modulated sine wave of the type

$$x(t) = \left(\beta_0 + \sum_{i=1}^{N} \beta_i \cos(2\pi f_{m_i} t + \phi_i)\right)\cos(2\pi f_0 t). \tag{1}$$

It must be recognised that the so-defined $x(t)$ is a perfectly *deterministic* and *quasi-periodic* signal, whose SC is simply obtained from

$$S_x(\alpha,f) = \sum_{\alpha} S_x^{\alpha_i}(f)\delta(\alpha-\alpha_i) = X(f+\alpha)X^*(f) \tag{2}$$

where $X(f)$ is the Fourier transform of $x(t)$. This is illustrated in Fig. 1. Eq. (2) and Fig. 1 clearly evidence that, in the case of a quasi-periodic signal, the SC is constructed from interference terms between the peaks of $X(f)$. In other words, for a quasi-periodic signal, the SC and a fortiori the CPS contain no more information than the Fourier transform $X(f)$. Indeed, computing a long Fourier transform on the data would perform just as well—if not better—than the CPS for detecting a quasi-periodic signal buried in additive noise. In brief, the reason why the CPS is inappropriate here is because the signal defined by Eq. (1) is simply CS1 and therefore does not require second-order tools.

3. The reason why the CPS is justified for analysing rolling element bearing signals

As a matter of fact, the CPS was originally introduced for analysing *purely random* signals with a cyclostationary behaviour, i.e. signals for which the existence of a Fourier series expansion of

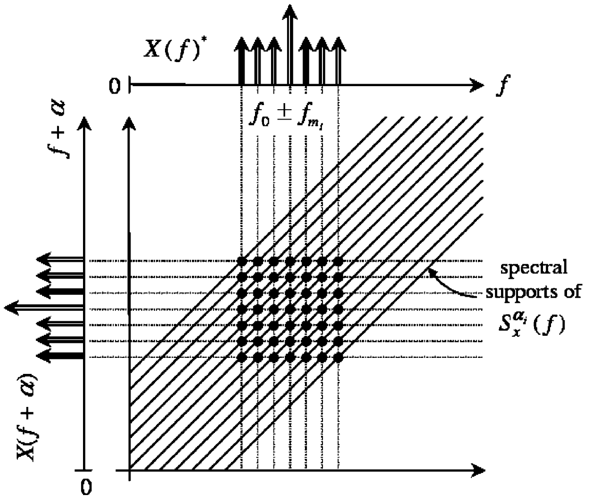

Fig. 1. SC magnitude of the signal x(t) defined by Eq. (1). It consists solely of interferences between $X(f)^*$ and $X(f-\alpha)$, whenever $\alpha = \alpha_i \in A$. This yields a spectral structure both discrete in the f and in the α frequency variables, materialised by the black dots in the above picture. The parallel diagonal lines show the directions spanned by the family of CPS's $S_x^{\alpha_i}(f), \alpha_i \in A$.

their autocorrelation function is not solely due to interferences from deterministic quasi-periodic components. Such signals are said to be *purely* second-order cyclostationary signals and are radically different from quasi-periodic signals which are CS1. Differentiating between CS1 and CS2 signals is of fundamental importance, since the processing tools dedicated to the former are not the same as those dedicated to the latter. Whereas the Fourier transform or Fourier series works perfectly for detecting CS1 characteristics, they are unable to recognise CS2 signals whose spectra are continuous functions of frequency f. Second-order tools such as the CPS are necessary to detect the hidden periodicities of purely CS2 signals.

Concerning rolling element bearing signals, it actually turns out that they are well modelled by CS2 signals [2–5], but this requires replacing the quasi-periodic model of Eq. (1) by a more realistic stochastic model:

$$\mathbf{x}(t) = \sum_{i=-\infty}^{\infty} \mathbf{A}_i s(t - \mathbf{T}_i), \tag{3}$$

where $s(t\text{-}\mathbf{T}_i)$ is the waveform generated by the i-th impact at the time $\mathbf{T}_i$ and $\mathbf{A}_\mathrm{i}$ its amplitude which accounts for possible periodic modulations. Due to the presence of various sources of randomness (in particular random slips of the cage) in the bearings, the variables $\mathbf{T}_i$ and $\mathbf{A}_i$ are random variables in general. It has been shown in Refs. [3–5] that the signal $\mathbf{x}(t)$ of Eq. (3) contains a negligible quasi-periodic part at high frequencies, so that its high-pass-filtered version (or typically a band-pass-filtered version around a high-frequency resonance) is purely CS2. This property then justifies the analysis by means of the SC or CPS, and theoretically yields a SC which is continuous in the f-frequency variable, and discrete in the α-frequency variable. A typical example is provided in Fig. 2.

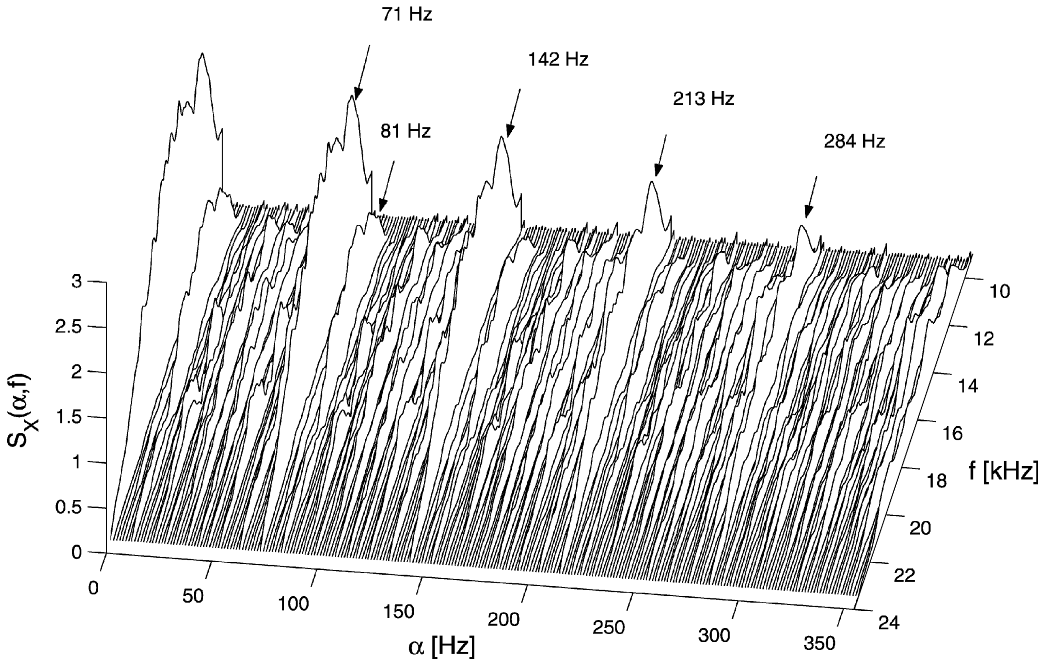

Fig. 2. The SC magnitude of a rolling element bearing signal with an inner race fault. The signature of the fault comes out in the frequency range [10;20] kHz, at the discrete frequencies $\alpha_i = i \cdot 71$ Hz, $i = 1, \ldots, 4$ (ball-pass frequencies on the inner race). Note the 10 Hz modulation due to the shaft rotation, giving discrete frequencies at $\alpha_i \pm 10$ Hz. Note also that, as expected from a purely CS2 signal, the SC is a continuous function in the f-frequency variable whereas discrete in the α-frequency variable. The arrows point at the CPS's indexed by the cyclic frequencies of the fault.

Note that Refs. [2–4], at least, were published well before Ref. [1].

Note also that our original model in Ref. [3] claimed that bearing signals were CS2, whereas the more refined models in Refs. [4–5] demonstrate that they are not strictly CS2, but can usefully be treated as such, since the low order "harmonics" in the α (cyclic frequency) direction are very narrow band even if not strictly discrete.

4. The relationship between the CPS and the envelope spectrum

The authors of Ref. [1] claimed that the CPS performs better than envelope analysis. This may be true for the wavelet envelope as tested in their paper, but not for the squared-envelope obtained from squaring the modulus of the (analytic) band-pass-filtered signal. Indeed, a close relationship exists between the CPS and the squared-envelope spectrum, which has been proved in Refs. [2,3]. Let us denote by $\tilde{x}(t)$ the filtered version of signal $x(t)$ in the frequency band $[f_1;f_2]$—for instance around a high-frequency resonance where the signal-to-noise is maximised. Then the following holds true

$$\int_{f_1}^{f_2} S_x^{\alpha_i}(f)\,\mathrm{d}f = \lim_{T\to\infty} \frac{1}{T} \int_{-T/2}^{T/2} |\tilde{x}(t)|^2 \mathrm{e}^{-\mathrm{j}2\pi\alpha_i t}\,\mathrm{d}t \tag{4}$$

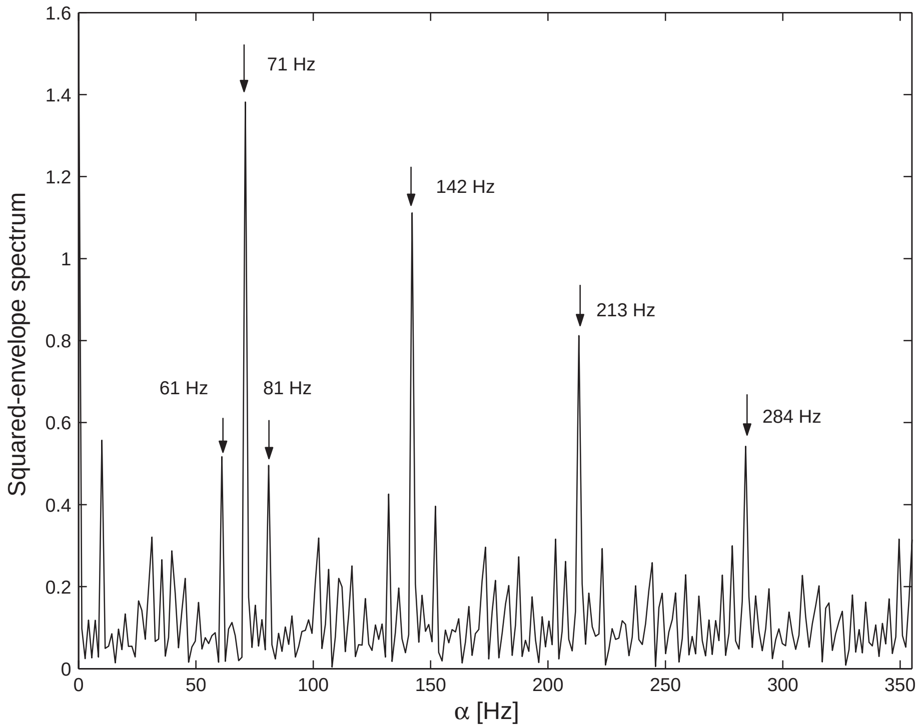

Fig. 3. The squared-envelope spectrum as obtained from projecting the 2D density of Fig. 2 on the α-frequency axis. In this particular case, the information is even more readable on the squared-envelope spectrum than on the SC.

provided that $\alpha_i \ll f_2 - f_1$ (this condition is always satisfied when dealing with rolling element bearings; for illustration, in the above example the largest scrutinised cyclic frequency $\alpha_i = 355$ Hz is about two orders of magnitude smaller than $f_2 - f_1 \approx 10$ kHz). Eq. (4) clearly proves that the squared-envelope spectrum contains the same information as the CPS does. It is seen that the squared-envelope spectrum can be interpreted as the projection of the CPS on the α-frequency axis. This projection usually summarises the CPS in a very efficient manner, as illustrated in Fig. 3.

In Ref. [6] it was shown that in the majority of cases, the squared-envelope spectrum is superior to the normal envelope spectrum.

5. Conclusion

We have shown that the use of the CPS for analysing rolling element bearing signals cannot be justified from a quasi-periodic model of the vibration signals, as done in Ref. [1]. Indeed, the correct justification of using the CPS is based on first demonstrating that rolling element bearing signals can be approximated as *purely second-order cyclostationary* [2–5]. Differentiating between quasi-periodic signals and purely second-order random cyclostationary signals is essential, since the two types of signals require different processing tools. These aspects are discussed in depth in Ref. [7].

We have also pointed out that the CPS is closely related to the squared-envelope spectrum, which means that there is no rational reason why the latter should not be as effective as the former. The effectiveness of the envelope spectrum (frequently referred to as the High Frequency Resonance Technique) has actually been reported in many instances, at least for detecting incipient and localised faults. From our experience, there are many examples where the information displayed by the squared-envelope spectrum is more readable than the information displayed by the CPS. A special situation where the CPS was found more advantageous than the envelope spectrum has been discussed in Refs. [2,3], for differentiating the spectral signatures of distributed bearing faults from the spectral signatures of gear faults.

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