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# Studies in dynamic design using updated models ☆

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## Abstract

Model updating techniques are used to update a finite element model of a structure so that an updated model predicts more accurately the dynamics of a structure. The application of such an updated model in dynamic design demands that it also predict the effects of structural modifications with a reasonable accuracy. This paper presents studies that deal with updating of a finite element model of a structure and its subsequent use for predicting the effects of structural modifications. Updated models have been obtained by a recently proposed method of model updating based on constrained optimization and by an iterative method of model updating based on the modal data. The suitability of updated models for predicting the effect of structural modifications is evaluated by some computer and laboratory experiments. First, a study is performed using a simulated fixed–fixed beam. Cases of complete, incomplete and noisy data are considered. The simulated study is followed by a study involving actual measured data for the case of an F-shape test structure. Structural modifications in terms of mass and beam modifications are then introduced to evaluate the updated models obtained in the simulated and the experimental studies for their usefulness in dynamic design. The prediction results obtained on the basis of an updated model based on the proposed method for the simulated and the experimental studies are found to be comparatively better than those obtained on the basis of an updated model based on the iterative method.


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## 1. Introduction

Availability of an accurate dynamic finite element model of a structure is very important to design engineers as it allows them to improve the dynamic design of the structure at the computer level. Such a model provides a basis for arriving at an optimized design apart from resulting savings in terms of money and time. But there are some inaccuracies or uncertainties associated with a finite element model. The discretization error, arising due to approximation of a continuous structure by a finite number of individual elements, is inherent in the finite element technique. Additionally, other inaccuracies there are due to the assumptions and simplifications made by the analyst with regards to the choice of elements, modelling of boundary conditions, joints, etc. This gets reflected in the difference between the finite element model predictions and the dynamic test data. Given the availability of an accurate data acquisition and measuring equipment, the measured test data are generally considered to be more accurate than analytical model predictions. But at times the difference between the test and the analysis results may be due to the inaccuracies in both the experimental data as well as the analytical model. The modal testing and modal extraction methods [1,2] are also well developed for obtaining a reliable estimate of the modal data. This has formed the basis for adjustment or correction of a finite element model in the light of measured test data, which is referred to as model updating.

A number of model updating methods have been proposed in recent years [3–5]. A significant number of methods, like in Refs. [6,7], which were first to emerge, used modal data, but were one-step procedures and have come to be called direct methods. The resulting updated matrices, though they reproduce measured modal data exactly, do not generally maintain structural connectivity and the suggested corrections are not physically meaningful. The method proposed in Ref. [8] is another method in the direct category but is based on the formulation of the stiffness and mass error matrices.

Analytical model updating using modal data in an iterative framework was first proposed in Ref. [9]. The updating equations were based on the first order approximation for the eigenvalues and the eigenvectors in terms of the updating parameters. A method based on matrix perturbation has been proposed [10] for recalculation of eigensolution and evaluation of eigendata sensitivities. A strategy for parameterization of a welded joint and a clamped end is proposed in Ref. [11] and the parameters updated using modal data. In Ref. [12] a penalty function based on the modal assurance criterion is used while in Ref. [13] the difficulties that are likely to be encountered in simultaneously using frequency, mode shape and modal assurance criterion in a penalty function are assessed. Numerical ill-conditioning due to large sensitivity discrepancies is pointed out as a major difficulty.

Updating on the basis of only natural frequency data using the iterative method is found to be a more widely used approach. The use of mode shape information in the process of updating has posed problems because the mode shape data is less sensitive to the updating parameters than the sensitivity of the natural frequency data. Secondly, since, in general, the mode shapes are measurable with lesser accuracy than the natural frequencies, the noise in the mode shape data may affect the progress of the iterations in an undesired way. A model updating method was recently proposed by the authors [14] in which these difficulties have been attempted to be addressed. The success of the method has been demonstrated using simulated data for the case of a beam structure. This paper presents studies that deal with updating of a finite element model of a structure and its subsequent use for predicting the effects of structural modifications. Updated models have been obtained by a recently proposed method of model updating based on constrained optimization and by an iterative method of model updating based on the modal data. The suitability of updated models for predicting the effect of structural modifications is evaluated by some computer and laboratory experiments.

## 2. Theory

Two methods of model updating, one of which is a recently proposed method based on constrained optimization [14], and the other is an iterative method based on modal data, proposed in Ref. [9] and referred to as inverse eigensensitivity method, have been employed in this work for obtaining updated models. The basic theory of these methods is briefly presented here. In both methods the physical parameters of the model are proposed to be used as updating variables.

### *2.1. Model updating using constrained optimization (MUCO)*

In this method [14] the problem of updating of a finite element model in the light of measured data is framed as a constrained nonlinear optimization problem which when solved yields corrections to the selected updating parameters. An objective function representing the error between the measured and the analytical versions of the natural frequencies and the mode shapes is minimized subjected to constraints using nonlinear optimization. The development of the objective function and the constraints both of which are based on the modal data is explained below.

Let $\{\lambda_x\}$ and $\{\lambda_A\}$ be the vectors of the measured and the analytical eigenvalues, respectively, to be used in updating. These vectors can be established by first identifying Correlated Mode Pairs (CMPs), which is essentially a list indicating correspondence between measured and analytical modes, using a correlation tool like Modal Assurance Criterion (MAC) [15]. The Euclidean norm-based normalized percentage error in eigenvalues is written as

$$F_1 = \frac{\left\|(\{\lambda_X\} - \{\lambda_A\})/\{\lambda_X\}\right\|}{\left\|\{\lambda_X\}/\{\lambda_X\}\right\|} \times 100. \tag{1}$$

The individual eigenvalue errors in Eq. (1) have been weighted, as shown, by the corresponding measured eigenvalues so that the relative weighting of the individual eigenvalue errors is balanced. This was done by weighting each occurrence of measured eigenvalue in the numerator as well as denominator of the equation for $F_1$ by the corresponding eigenvalue. Eq. (1) then represents the error in the eigenvalues in an average sense. The total percentage error in m number of eigenvalues will be

$$\varepsilon_1 = m \times F_1, \tag{2}$$

where $\varepsilon_1$ represents % eigenvalue error. The normalized percentage error in m number of mode shapes is written as

$$F_2 = 100 \times \sum_{i=1}^{m} \frac{\left\| \{\phi_X\}^i - \{\phi_A\}^i \right\|}{\left\| \{\phi_X\}^i \right\|}, \tag{3}$$

where $\{\phi_X\}^i$ and $\{\phi_A\}^i$ are the $i$th simulated experimental and analytical mode shape vectors. The term inside the summation sign in Eq. (3) represents an average error in one measured coordinate of the $i$th mode shape vector. Multiplication of this term by $n$, the number of measured coordinates, gives a measure of error in an entire $i$th mode-shape-vector. In this way, the total percentage error in m number of mode shapes can be written as

$$\varepsilon_2 = n \times F_2. \tag{4}$$

The objective function to be minimized, based on error in natural frequencies and mode shapes given by Eqs. (2) and (4) respectively, is constructed as

$$\varepsilon(u) = W_1 \times \varepsilon_1 + W_2 \times \varepsilon_2. \tag{5}$$

$W_1$ and $W_2$ in Eq. (5) are the weights to be given to the natural frequencies-based and the mode shapes-based errors. Provision for $W_1$ and $W_2$ allows to account for any known uncertainties associated with the measured eigenvalues and the mode shapes by taking a relatively higher weight for that data that is supposed to have been measured with a lesser level of uncertainty.

Two sets of constraints are imposed on the objective function to be minimized. The first set of constraints are the lower and upper bounds on the vector of correction factors $\{u\}$, whose elements represent the unknown fractional corrections to be made to the chosen updating variables, and is written as

$$\{u\}_{\text{LB}} \leqslant \{u\} \leqslant \{u\}_{\text{UB}}, \tag{6}$$

where the subscripts LB and UB represent lower and upper bound, respectively. The second set of constraints seeks to put a lower bound on the level of correlation between the experimental and the analytical modes. MAC is a correlation tool that has been widely used to determine the degree of correlation between mode shapes. However, when the vectors are incompletely described, MAC can indicate correlation between vectors that are linearly independent. This can also occur if the measurement points are not sufficient on modally sensitive or active regions, which may be inaccessible during modal testing. Cross orthogonalty check is another tool that is often used to establish correlation. It is based on the property of orthogonalization of modal vectors with respect to the mass matrix and is expected to give zero off-diagonal terms when the modes are not correlated. Pseudo-orthogonality check, proposed in Ref. [16], is an attempt to overcome the difficulties, associated with coordinate incompatibility between measured modal vectors and the mass matrix, encountered while applying a cross orthogonalty check. In the present work constraints relating mode shape correlation have been framed on the basis of MAC as its calculation does not require any mass matrix. This set of constraints are imposed to ensure that the level of correlation between measured and analytical modes shapes, represented by the MAC-value, existing before updating is at least retained after updating. The constraint for $i$th CMP is given by the inequality

$$(\text{MAC}(\{\phi_A\}^i, \{\phi_X\}^i))^j \geqslant (\text{MAC}(\{\phi_A\}^i, \{\phi_X\}^i))^0, \tag{7}$$

where the right and left side of the inequality represent MAC-value corresponding to the *i*th CMP before updating (denoted by superscript '0') and at the *j*th iteration, respectively. If the unconstrained version of the method is used, where the MAC-value constraints are not imposed, then CMPs will have to be established at each iteration. The MAC-value is calculated as

$$\mathrm{MAC}(\{\phi_A\}^i, \{\phi_X\}^i) = \frac{(|\{\phi_A\}^{iH}, \{\phi_X\}^i|)^2}{(|\{\phi_A\}^{iH}\{\phi_X\}^i|)^2(|\{\phi_X\}^{iH}\{\phi_X\}^i|)}, \tag{8}$$

where the superscript '*H*' represents the conjugate transpose operation. The objective function given by Eq. (5) is minimized subject to inequality constraints given by Eqs. (6) and (7). The analytical natural frequencies and mode shapes appearing in the objective f unction and the constraints are related to the analytical stiffness matrix $[K_A]$ and analytical mass matrix $[M_A]$ by an eigenvalue problem, which for the *i*th mode is written as

$$[K_A]\{\phi_A\}^i = \lambda_A^i[M_A]\{\phi_A\}^i, \tag{9}$$

where $\lambda_A = (2\pi f_A)^2$. The $[K_A]$ and $[M_A]$ are in turn the functions of the selected updating parameters. The optimization problem is solved using a routine for constrained minimization of nonlinear functions available in MATLAB [17]. The routine is based on sequential quadratic programming in which at every iteration a quadratic programming sub-problem is formulated and solved. This requires the first- and second- order derivatives of the objective function and the constraints. The first- order derivatives were derived and supplied to the routine while the Hessian matrix, the matrix of second- order derivatives, is constructed by the routine itself. The method of MUCO does not require any model reduction to be performed.

### 2.2. Inverse eigensensitivity method (IESM)

This method uses modal data, namely the natural frequencies, the mode shapes and the damping ratios, which are obtained in practice by modal analysis of measured FRFs. The updating parameters corresponding to an analytical model are corrected to bring the analytical modal data closer to that of the experimentally derived. Most often updating equations are based on a linear approximation of the modal data that is generally a nonlinear function of updating parameters. For the *r*th eigenvalue, $\lambda^r$ (square of the *r*th natural frequency in rad/sec), and the *r*th eigenvector, $\{\Phi\}^r$ (the mode shape), linearization gives

$$\lambda_X^r = \lambda_A^r + \sum_{i=1}^{nu}\left(\frac{\partial \lambda_A^r}{\partial p_i}\Delta p_i\right), \tag{10}$$

$$\{\phi\}_X^r = \{\phi\}_A^r + \sum_{i=1}^{nu}\left(\frac{\partial \{\phi\}_A^r}{\partial p_i}\Delta p_i\right), \tag{11}$$

where the eigenvalue and the eigenvector sensitivities, represented by the derivatives, can be calculated from the relationships [18]

$$\frac{\partial \lambda_A^r}{\partial p_i} = \{\phi\}_A^{rT} \left[ \frac{\partial [K]}{\partial p_i} - \lambda_A^r \frac{\partial [M]}{\partial p_i} \right] \{\phi\}_A^r, \tag{12}$$

$$\frac{\partial \{\phi\}_A^r}{\partial p_i} = \sum_{j=1,\neq r}^{N} \left( \frac{\{\phi\}_A^{jT} \frac{\partial([K]-\lambda_A^r[M])}{\partial p_i} \{\phi\}_A^j}{(\lambda_r - \lambda_j)} \right) \{\phi\}_A^j - \frac{1}{2} \{\phi\}_A^{rT} \frac{\partial [M]}{\partial p_i} \{\phi\}_A^r \{\phi\}_A^r. \tag{13}$$

Using Eqs. (12) and (13), Eqs. (10) and (11) can be written for the chosen $m$ number of modes. These equations together, after dividing and multiplying by $p_i$ and then writing $u_i$ in place of $\Delta p_i/p_i$, can be written in the following matrix form,

$$\begin{bmatrix} p_1 \cdot \frac{\partial \lambda_A^1}{\partial p_1} / \lambda_A^1 & p_2 \cdot \frac{\partial \lambda_A^1}{\partial p_2} / \lambda_A^1 & \cdots & \cdots & \cdots & p_{nu} \cdot \frac{\partial \lambda_A^1}{\partial p_{nu}} / \lambda_A^1 \\ p_1 \cdot \frac{\partial \{\phi\}_A^1}{\partial p_1} & p_2 \cdot \frac{\partial \{\phi\}_A^1}{\partial p_2} & \cdots & \cdots & \cdots & p_{nu} \cdot \frac{\partial \{\phi\}_A^1}{\partial p_{nu}} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ p_1 \cdot \frac{\partial \lambda_A^m}{\partial p_1} / \lambda_A^m & p_2 \cdot \frac{\partial \lambda_A^m}{\partial p_2} / \lambda_A^m & \cdots & \cdots & \cdots & p_{nu} \cdot \frac{\partial \lambda_A^m}{\partial p_{nu}} / \lambda_A^m \\ p_1 \cdot \frac{\partial \{\phi\}_A^m}{\partial p_1} & p_2 \cdot \frac{\partial \{\phi\}_A^m}{\partial p_2} & \cdots & \cdots & \cdots & p_{nu} \cdot \frac{\partial \{\phi\}_A^m}{\partial p_{nu}} \end{bmatrix} \cdot \begin{Bmatrix} u_1 \\ u_2 \\ \vdots \\ \vdots \\ \vdots \\ u_{nu} \end{Bmatrix} = \begin{Bmatrix} (\lambda_X^1 - \lambda_A^1)/\lambda_A^1 \\ \{\phi\}_X^1 - \{\phi\}_A^1 \\ \vdots \\ \vdots \\ \vdots \\ (\lambda_X^m - \lambda_A^m)/\lambda_A^m \\ \{\phi\}_X^m - \{\phi\}_A^m \end{Bmatrix} \tag{14a}$$

Note that in the above equation the eigenvalue errors have been balanced by the corresponding eigenvalue. The above equation can be represented in a compact form as

$$[S]_{(n+1)m \times nu} \{u\}_{nu \times 1} = \{\Delta e\}_{(n+1)m \times 1}, \tag{14b}$$

where $n$ represents the number of degrees of freedom at which measurements are available and $\{\Delta e\}$ is a vector whose elements are equal to the difference between experimental and analytical eigenvalues and eigenvectors. The above matrix equation is solved for $\{u\}$ using a routine for finding pseudo-inverse of a matrix, $[S]$ in the present case, available in MATLAB [17]. The pseudo-inverse, calculated by the routine using singular value decomposition of a matrix, is related to the least-squares problem, as the value of $\{u\}$ that minimizes $||[S]\{u\} - \{\Delta e\}||^2$ can be given by $\{u\} = [S]^+\{\Delta e\}$ [2]. The superscript '+' denotes pseudo-inverse. The $\{u\}$ so found is used to update the vector of physical variables $\{p\}$ and then the updated version of an analytical finite element model is built using this new set of physical variables. This process is repeated in an iterative way until convergence is obtained.

## 3. Structural modification using an updated model

The dynamic design using an updated model requires that the model predict the changes in the dynamic characteristics due to potential modifications with a reasonable accuracy. With this purpose the structural modifications in the form of mass and beam modifications have been considered. An updated undamped finite element model for a structure is available in terms of a stiffness matrix and a mass matrix denoted by $[K_A]$ and $[M_A]$, respectively. If $[\Delta K]$ and $[\Delta M]$ represent the modification matrices due to a modification then the modified structure's stiffness and mass matrix denoted by $[K_m]$ and $[M_m]$, respectively can be written as

$$[K_m] = [K_A] + [\Delta K], \tag{15}$$

$$[M_m] = [M_A] + [\Delta M] \tag{16}$$

Consider the case of mass modification by assuming that a mass $m_0$ kg is added at $i$th node. The $[\Delta M]$ matrix is obtained by making the diagonal entries corresponding to the translational degrees of freedom for the $i$th node equal to '$+m_0$' [19]. The rotary inertia of the modification can also be accounted for by making the diagonal entries corresponding to the rotational degrees of freedom for the $i$th node equal to the rotary inertia about the corresponding axes. For the case of a 2D FE model with frame elements (two translational and one rotational degree of freedom) the mass modification matrix is given as

$$[\Delta M] = \begin{bmatrix} 0 & \cdots & \cdots & \cdots & \cdots & 0 \\ \vdots & \ddots & & & & \vdots \\ \vdots & & +m_0 & & & \vdots \\ \vdots & & & +m_0 & & \vdots \\ \vdots & & & & +I_m & \vdots \\ 0 & \cdots & \cdots & \cdots & \cdots & 0 \end{bmatrix}, \tag{17}$$

where $I_m$ is the rotary inertia of the added mass. The modified stiffness matrix remains the same as the $[K_A]$ for the case of mass modification.

For the case of beam modification the $[K_m]$ and $[M_m]$ are essentially obtained by assembling the FE-model for the added beam member with that of the FE-model of the unmodified structure. Predictions on the basis of the updated model can be made by assembling the FE-model for the added beam member with that of the updated FE-model of the unmodified structure. Thus, in general, the number of finite elements, the number of nodes and consequently the size of the modified model will be higher than that for the unmodified model. In terms of Eqs. (15) and (16) the $[K_A]$ and $[M_A]$ represent the structural matrices, expanded to the size of the modified model, corresponding to either an updated or baseline FE-model depending upon which model is made the basis for making predictions. The modification matrices $[\Delta K]$ and $[\Delta M]$ represent a FE-model for the added beam-member expanded to the size of the modified model. It can be noted that for the case of beam modification both the mass and stiffness matrices are affected.

Once for a given modification the $[K_m]$ and $[M_m]$ are established via equations (15) and (16) the eigenvalues $[\lambda_m]$ and the eigenvectors $[\phi_m]$ of the modified structure predicted by a model can be obtained by resolving the eigenvalue problem

$$[K_m][\phi_m] = [M_m][\phi_m][\lambda_m]. \tag{18}$$

## 4. Dynamic design of a beam structure using simulated data-based updated models

A simulated study on a fixed–fixed beam is conducted for evaluating the suitability of updated models for dynamic design. The dimensions of the beam are $9100 \times 50 \times 5$ mm. The modulus of elasticity and density are taken as $2.0(10^{11})\,\mathrm{N/m^2}$ and $7800\,\mathrm{kg/m^3}$, respectively. The beam is modelled using 30 beam elements with nodes at ends fixed giving a total of 29 nodes with three degrees of freedom (two displacements and one rotation) each. The simulated modal data, which is treated as experimental data, is obtained by generating a finite element model by introducing certain known discrepancies in the thickness of all the finite elements with respect to the analytical model, the details of which are given in Table 1. The frequency range from 0 to 1 kHz covering seven modes is taken as the measured frequency range.

In the present study using simulated data the individual thickness of all the finite elements are taken as the updating parameters and the eigendata corresponding to the first six modes of the structure has been utilized as the target data.

First, the case of a complete data set is considered where it is assumed that all the degrees of freedom of a finite element model are measured. Thus, in this case all the eigenvalues and the corresponding eigenvectors falling in the measurement frequency range will be known. Fig. 1 gives a comparison of the fractional correction factors to the updating parameters obtained using the methods of MUCO (model updating using constrained optimization) and IESM (inverse eigensensitivity method). The identified correction factors are found to be exactly identical with the introduced discrepancies.

In practice, it is not realistic that all the coordinates specified in the analytical FE model have been measured either due to physical inaccessibility or due to difficulties faced in the measurement like that for rotational degrees of freedom. The second case considered therefore is that of an incomplete measured data set. It is assumed that only lateral degree of freedom have been measured at 15 alternate nodes leaving 82.7% degrees of freedom unmeasured. Fig. 2 gives a comparison of the fractional correction factors to the updating parameters obtained using the methods of MUCO and IESM. The correction factors identified by the MUCO method are found to be almost identical with the introduced discrepancies while those obtained by the IESM method are found to be little away from the exact values.

Table 1
Initial discrepancies between the finite element and the simulated experimental model

| Element number | 3 | 5 | 11 | 16 | 25 | 29 | All other elements |
|---|---|---|---|---|---|---|---|
| % Deviation in thickness | +20 | +40 | +25 | +40 | +30 | +30 | +10 |

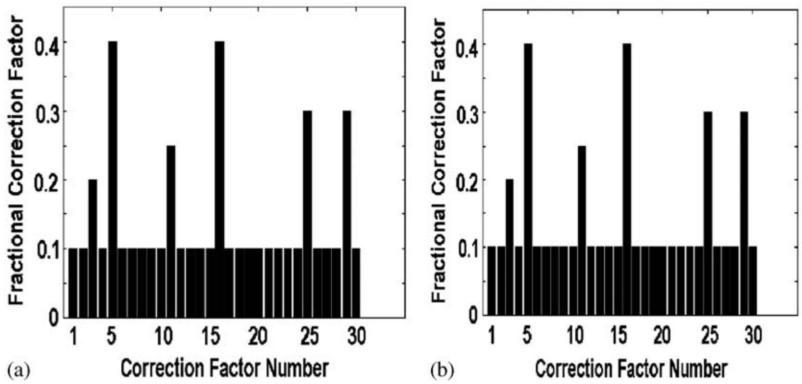


Fig. 1. Comparison of final values of fractional correction factors for the case of complete data: (a) MUCO, (b) IESM.

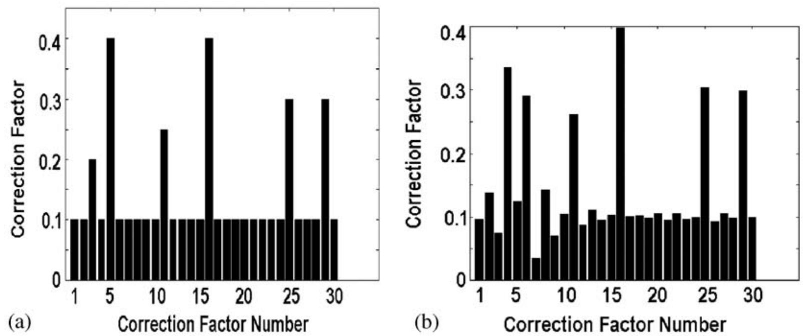


Fig. 2. Comparison of final values of fractional correction factors for the case of incomplete data: (a) MUCO, (b) IESM.

In practical measurement, the measured FRFs will be contaminated by measurement noise and consequently the extracted modal parameters will also be affected. In the third case, the natural frequencies and the mode shapes corresponding to simulated experimental data are polluted with random errors. While the mode shapes are polluted with 2% random noise, the natural frequencies are polluted with 0.2% noise. Fig. 3 gives the fractional correction factors to the updating parameters obtained by the two methods. After comparing Fig. 3 with Fig. 1 it is seen that the correction factors have been identified reasonably though not very accurately by the two methods.

Two cases of structural modification are now considered to evaluate the suitability of updated models for dynamic design. The first case is that of a mass modification in terms of an addition of a lumped mass of 0.5 kg at the node number 16 which is around the middle of the beam as shown in Fig. 4(a). The modified mass is accounted for in the analytical model as described in the previous section. Results for the case of incomplete and noisy data only are given here as the results for this case are found to be very close to the results for the cases of complete and incomplete data. A comparison of the natural frequency and the mode shape correlation as predicted by the updated models based on the two methods is shown in Table 2. The double line in

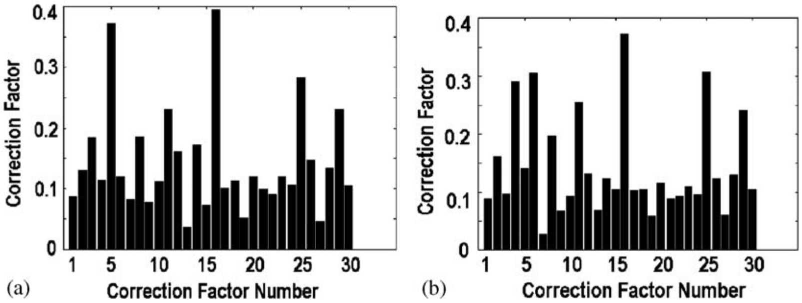


Fig. 3. Comparison of final values of fractional correction factors for the case of incomplete and noisy data: (a) MUCO, (b) IESM.

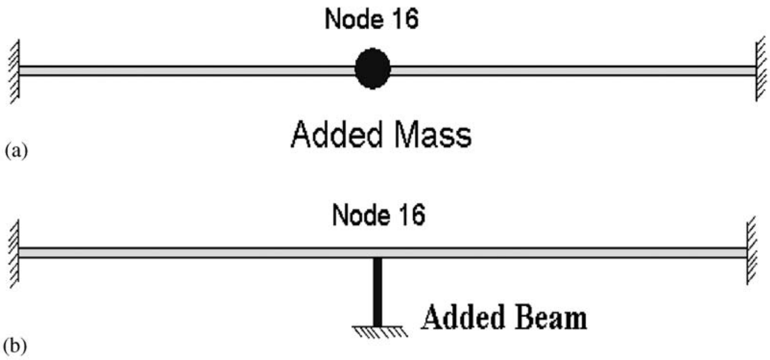


Fig. 4. Beam-structure with (a) mass modification, (b) beam modification.

Table 2
Comparison of the natural frequency and the mode shape correlation for *the mass-modified simulated structure* as predicted by the updated models based on the two methods

| Mode no. | Actual frequency (Hz) | MUCO | | | IESM | | |
|---|---|---|---|---|---|---|---|
| | | Frequency (Hz) | % Error | MAC-value | Frequency (Hz) | % Error | MAC-value |
| 1 | 27.88 | 27.87 | −0.03 | 0.9999 | 27.87 | −.03 | 0.9999 |
| 2 | 96.30 | 96.40 | 0.10 | 0.9999 | 96.30 | −.06 | 0.9999 |
| 3 | 167.83 | 167.77 | −0.04 | 0.9999 | 167.72 | −.06 | 0.9999 |
| 4 | 310.56 | 310.71 | 0.04 | 0.9998 | 310.21 | −.11 | 0.9999 |
| 5 | 431.81 | 431.86 | 0.01 | 0.9998 | 431.63 | −.04 | 0.9999 |
| 6 | 639.82 | 640.30 | 0.07 | 0.9998 | 639.28 | −.08 | 0.9999 |
| 7 | 819.67 | 818.35 | −0.16 | 0.9999 | 819.50 | −.02 | 0.9996 |

Table 3
Comparison of the natural frequency and the mode shape correlation for the *beam-modified simulated structure* as predicted by the updated models based on the two methods

| Mode no. | Actual frequency (Hz) | MUCO | | | IESM | | |
|---|---|---|---|---|---|---|---|
| | | Frequency (Hz) | % Error | MAC-value | Frequency (Hz) | % Error | MAC-value |
| 1 | 123.85 | 123.54 | −0.25 | 0.9999 | 123.45 | −0.32 | 0.9999 |
| 2 | 144.15 | 143.67 | −0.33 | 0.9999 | 144.39 | 0.16 | 0.9999 |
| 3 | 344.20 | 343.29 | −0.26 | 0.9999 | 342.70 | −0.43 | 0.9999 |
| 4 | 406.10 | 403.83 | −0.55 | 0.9999 | 406.80 | 0.17 | 0.9999 |
| 5 | 673.20 | 673.17 | 0.0 | 0.9999 | 670.77 | −0.36 | 0.9999 |
| 6 | 791.13 | 793.69 | 0.32 | 0.9998 | 793.73 | 0.32 | 0.9999 |
| 7 | 1138.68 | 1134.77 | −0.34 | 0.9997 | 1138.63 | 0.01 | 0.9996 |

the table indicates that the six of modes were used for obtaining an updated model while the predictions were made for the next mode not used in updating to serve as a measure of quality of the updated model. It is seen that the prediction of natural frequencies and the mode shapes on the basis of the two updated models are very close to the actual changes.

The second case, a little more complex, is that of a beam modification. A beam member of length 0.06 m, of the same cross-section as that of the unmodified beam, is added at node number 16 as shown in Fig. 4(b). The added beam member is modelled by two finite elements and is accommodated in the analytical model as described in the previous section. Again the results for the case of incomplete and noisy data only are given here as the results for this case are found to be very close to the results for the cases of complete and incomplete data. A comparison of the natural frequency and the mode shape correlation as predicted by the updated models based on the two methods is shown in Table 3. It is again seen that the predictions on the basis of the two updated model are very close to the actual changes. In terms of comparison the predictions on the basis of two updated models are quite similar and do not differ much even though the identified correction factors on the basis of the MUCO during updating were closer to the exact values than those identified using IESM. A comparison of overlays of one of the FRFs before and after updating has also been given in Fig. 5.

## 5. Dynamic design of an F-shape structure using experimental data-based updated models

The suitability of updated models for dynamic design is now evaluated for the case of an F-shape structure, shown in Fig. 6(a), using experimental data. The F-shape structure has been constructed by bolting horizontally the two beam members to a vertical beam member, which in turn has been welded to a base plate at the bottom. All the beam members have a square cross-section with 37.7 mm as one of its sides. A finite element model of the structure is built using 48 2D-frame elements with three degrees of freedom (two displacements and one rotation) at each of the nodes. The values for the modulus of elasticity and the density corresponding to basic steel are taken as $2.0(10^{11})\,\mathrm{N/m^2}$ and $7800\,\mathrm{kg/m^3}$, respectively. An undamped eigenvalue problem is set and

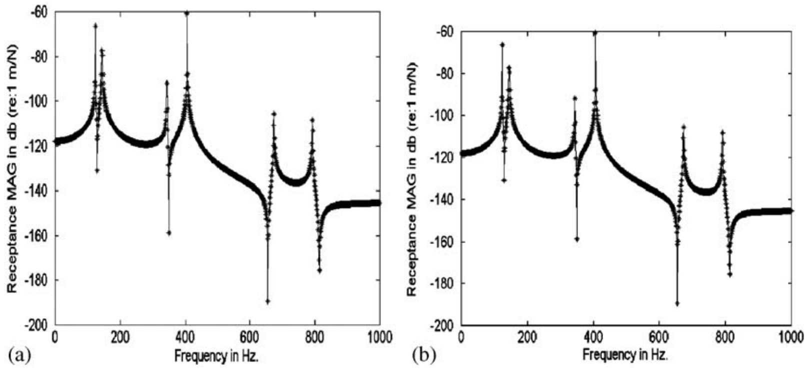


Fig. 5. Comparison of the overlay of the FRF of the *beam-modified structure* as predicted by the updated models (exact modified FRF (___) overlaid on the modified FRF (***) predicted by an updated model). (a) MUCO predictions, (b) IESM predictions.

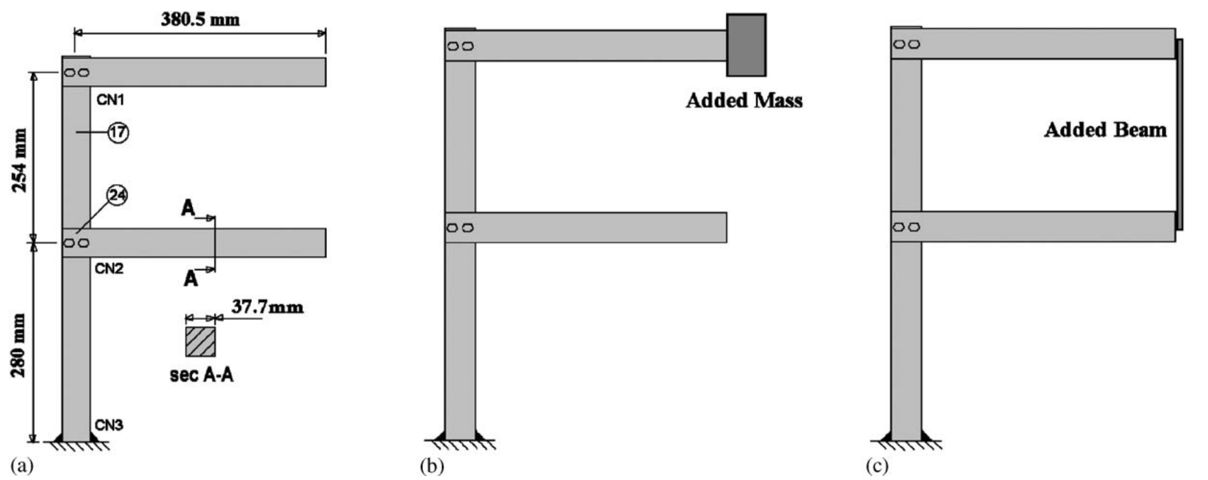


Fig. 6. (a) F-shape-structure (b) with mass modification (c) with beam modification.

solved in order to obtain an analytical estimate of undamped natural frequencies and mode shapes. The modal test is performed by exciting the structure with an impact hammer at 16-locations and measuring the response by an accelerometer kept fixed at one of the locations. The frequency response functions so acquired are analyzed using a global curve-fitting method available in ICATS [20] to obtain an experimental set of modes in the range of 0–1600 Hz.

The correlation between the analytical and the experimental set of modal data is now performed using the modal assurance criterion (MAC) [15]. On the basis of the MAC-matrix the correlated mode pairs are established and then the existing level of differences between the corresponding natural frequencies is ascertained. The results of such an exercise carried out for the above case are shown in Table 4 indicating corresponding experimental and analytical natural

Table 4
Correlation of measured and FE-model based modal data

| Mode no. | Measured frequency in Hz | FE-model predictions | | MAC-value |
|---|---|---|---|---|
| | | Frequency in Hz | % Error | |
| 1 | 34.95 | 43.05 | 23.17 | 0.9901 |
| 2 | 104.02 | 123.67 | 18.89 | 0.9470 |
| 3 | 133.96 | 185.21 | 38.26 | 0.9265 |
| 4 | 317.52 | 385.17 | 21.30 | 0.9054 |
| 5 | 980.16 | 1020.06 | 4.07 | 0.7299 |
| 6 | 1057.8 | 1084.79 | 2.55 | 0.8040 |
| 7 | 1531.45 | 1925.76 | 25.74 | 0.8798 |

frequencies, percentage difference between them and the corresponding MAC-value for the seven modes. The double line in the table indicates that five modes were used for obtaining an updated model while the predictions were made for the next two mode not used in updating to serve as a measure of quality of the updated model. An overlay of the measured FRF($24 \times 17$x) (i.e.. response at location 24 in the direction X and force at location 17 in the direction X, refer to Fig. 6) the corresponding FE model FRF is also shown in Fig. 7. It is observed that though the mode shape correlation is reasonably good the error in the analytical natural frequencies is significantly high. To improve the correlation of the analytical modal data with the experimental data the FE-model is now updated.

One of the most important issues in the FE-model updating is the selection of the updating parameters. The selection of updating parameters will decide to a great extent whether the process of updating results only in the improvement of correlation of the FE-model in terms of modal data or also in terms of its ability to correctly represent the stiffness and mass distribution of the structure. Choice of updating parameters on the basis of engineering judgment about the possible locations of modelling error in a structure is a widely used strategy to ensure that only meaningful corrections are made. In the present case due to the presence of three joints the modelling of stiffness at these places is expected to be a dominant source of inaccuracy in the FE-model assuming that the values of the material and the geometric parameters are correctly known. The three joints are modelled by taking coincident nodes at each of them. A horizontal, a vertical and a torsion spring couples the two nodes at each of such coincident pair of nodes. The stiffness of these springs, Kx, Ky and Kt, respectively, are the potential updating parameters allowing to account for the deviation in the stiffness of the regions covered by the joints. Table 5 shows the sensitivities of the first five eigenvalues with respect to the stiffness of these springs. In this table CN1, CN2 and CN3, as depicted in Fig. 6, represent the coincident node pair at the joints between the upper-horizontal and the vertical beam member, between the lower horizontal and the vertical beam member and at the welded joint, respectively. It is noticed that the eigenvalues are much more sensitive to the torsional stiffness at three joints than to other spring–stiffness parameters. In the light of this observation the three torsional stiffness parameters are chosen as updating variables. The other stiffness values are taken to be very large to represent rigid coupling of those degrees of freedom.

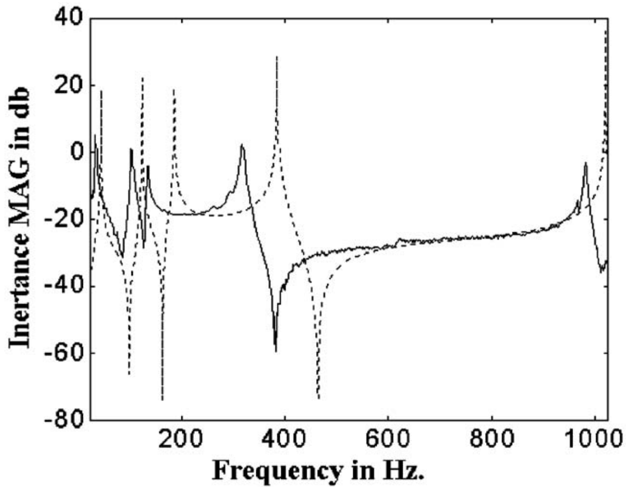


Fig. 7. Overlay of the measured FRF $24 \times 17 \times$ (response at location 24 and in the X-direction and force at location 17 and in the X-direction) (___) and the corresponding FE model FRF (- - - - -) before updating.

Table 5
Sensitivity of eigenvalues to the spring stiffness parameters

| Eigen value No. | Kx | | | Ky | | | Kt | | |
|---|---|---|---|---|---|---|---|---|---|
| | CN1 | CN2 | CN3 | CN1 | CN2 | CN3 | CN1 | CN2 | CN3 |
| 1 | 4.7E−5 | 5.3E−6 | 1.6E−4 | 1.7E−5 | 9.4E−6 | 5.2E−5 | 1.3E−3 | 7.4E−4 | 6.9E−2 |
| 2 | 1.7E−5 | 5.1E−5 | 3.3E−4 | 3.9E−4 | 9.9E−5 | 1.0E−4 | 3.2E−2 | 8.2E−3 | 1.9E−2 |
| 3 | 2.5E−4 | 1.5E−4 | 1.7E−3 | 6.4E−5 | 1.0E−3 | 1.6E−3 | 5.5E−3 | 9.4E−2 | 3.7E−2 |
| 4 | 1.7E−3 | 3.7E−3 | 2.4E−3 | 1.6E−4 | 7.2E−6 | 2.5E−4 | 2.4E−2 | 1.0E−3 | 4.8E−2 |
| 5 | 7.9E−6 | 4.8E−4 | 1.6E−4 | 7.6E−3 | 1.7E−5 | 1.5E−2 | 1.0E−2 | 4.0E−4 | 1.2E−3 |

First updating is carried out for matching only the first five natural frequencies and updating the three stiffness parameters as described above. Table 6 shows that the final values of the updating parameters obtained by the two methods are identical. A comparison of the correlation between the measured and the updated model natural frequencies and the mode shapes is given in Table 7. It is seen that the correlation of the natural frequencies has significantly improved inside the updating range though the MAC-value of one of the modes has been reduced. For both the methods there is also a significant reduction in the average natural frequency error for the modes beyond the updating range. An overlay comparison of the measured FRF($24 \times 17$x) and the corresponding updated model FRFs based on the two methods is also shown in Fig. 8. It is seen that there is very good match between the updated model FRF and the measured FRF.

Next, updating is carried out by also including mode shapes corresponding to the first five natural frequencies in the updating process. For the case of IESM, equations corresponding to the eigenvalues are balanced by dividing them by the corresponding eigenvalues. For the case of

Table 6
Corrected values of the updating variables when updating is carried out using only natural frequencies

| Updating variable | Initial value | Final value | |
|---|---|---|---|
| | | MUCO | IESM |
| Kt1 | 3.28E+06 | 3.45E+05 | 3.45E+05 |
| Kt2 | 3.28E+06 | 2.81E+05 | 2.81E+05 |
| Kt3 | 3.28E+06 | 3.30E+05 | 3.30E+05 |

Table 7
Comparison of the correlation between the measured and the updated models natural frequencies and the mode shapes when updating is carried out using only natural frequencies

| Mode no. | Measured frequency (Hz) | Dynamic characteristics of the MUCO-based updated-model | | | Dynamic characteristics of the IESM-based updated-model | | |
|---|---|---|---|---|---|---|---|
| | | Frequency (Hz) | % Error | MAC-value | Frequency (Hz) | % Error | MAC-value |
| 1 | 34.95 | 34.86 | −0.25 | 0.9954 | 34.86 | −0.25 | 0.9954 |
| 2 | 104.02 | 103.36 | −0.63 | 0.9638 | 103.36 | −0.63 | 0.9638 |
| 3 | 133.96 | 134.03 | 0.05 | 0.9410 | 134.03 | 0.05 | 0.9410 |
| 4 | 317.52 | 320.63 | 0.98 | 0.9370 | 320.63 | 0.98 | 0.9370 |
| 5 | 980.16 | 980.21 | 0.00 | 0.4231 | 980.21 | 0.00 | 0.4231 |
| 6 | 1057.8 | 1003.27 | −5.15 | 0.5517 | 1003.27 | −5.15 | 0.5517 |
| 7 | 1531.45 | 1509.78 | −1.41 | 0.9629 | 1509.78 | −1.41 | 0.9629 |

updating using MUCO the mode shapes are included in the form of MAC-based constraints. Table 8 gives the initial values and the final values of the updating parameters as obtained by the two methods. A comparison of the correlation between the measured and the updated model natural frequencies and the mode shapes is given in Table 9. It is seen that in the case of IESM though there is a very good improvement in the mode shape correlation but the natural frequency correlation has not improved to the same extent as for the case when only the eigenvalues were the targets. It therefore seems that the progress of the iterations is more influenced by the mode shapes as compared to the natural frequencies. In case of MUCO, the average error in the prediction of natural frequencies based on the updated model is 3.81%, which is a very significant reduction as compared to the finite element model. In comparison with the first case, when MAC-constraints were not included, though the natural frequency error is on the higher side, the mode shape correlation is better. This appears to be because of the fact that as constraints on the MAC-value of mode pairs are included, the algorithm seeks to correct the parameters in such a way that the correlation of the natural frequencies is improved but without any loss in the correlation of the mode shapes. Therefore, inclusion of mode shape information in the updating process, which has been achieved indirectly in the present method by imposing constraints on their MAC-value, has

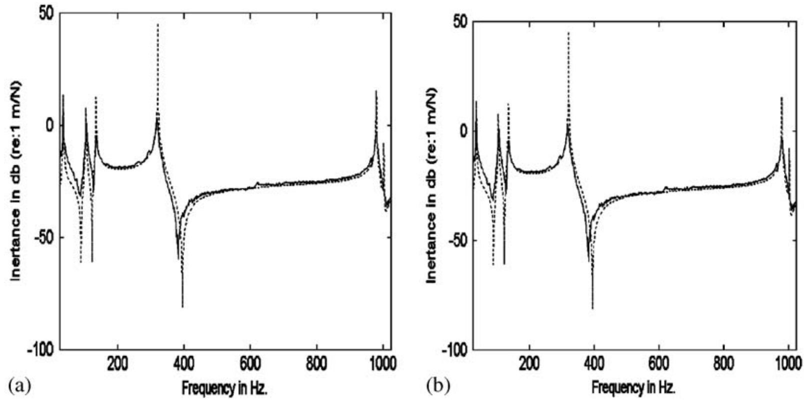


Fig. 8. Comparison of the overlays of the experimental FRF(___) and the Updated model FRF(-----) after updating. (a) MUCO based, (b) IESM based.

Table 8
Corrected values of the updating variables when updating is carried out using natural frequencies as well as mode shapes

| Updating variable | Initial value | Final Value | |
|---|---|---|---|
| | | MUCO | IESM |
| Kt1 | 3.28E+06 | 1.03E+06 | 3.75E+06 |
| Kt2 | 3.28E+06 | 3.31E+05 | 3.68E05 |
| Kt3 | 3.28E+06 | 2.66E+05 | 1.71E+05 |

Table 9
Comparison of the correlation between the measured and the updated models natural frequencies and the mode shapes when updating is carried out using natural frequencies as well as mode shapes

| Mode no. | Measured frequency (Hz) | Dynamic characteristics of the MUCO-based updated-model | | | Dynamic characteristics of the IESM-based updated-model | | |
|---|---|---|---|---|---|---|---|
| | | Frequency (Hz) | % Error | MAC-value | Frequency (Hz) | % Error | MAC-value |
| 1 | 34.95 | 33.58 | −3.92 | 0.9943 | 30.39 | −13.0 | 0.9877 |
| 2 | 104.02 | 109.99 | 5.73 | 0.9587 | 111.56 | 7.24 | 0.9693 |
| 3 | 133.96 | 137.75 | 2.82 | 0.9344 | 138.13 | 3.1 | 0.9669 |
| 4 | 317.52 | 332.18 | 4.61 | 0.9122 | 333.13 | 4.91 | 0.9118 |
| 5 | 980.16 | 999.72 | 1.99 | 0.7275 | 1004.81 | 2.51 | 0.9925 |
| 6 | 1057.8 | 1008.81 | −4.63 | 0.8048 | 1015.69 | −3.98 | 0.9222 |
| 7 | 1531.45 | 1521.97 | −0.61 | 0.9571 | 1501.78 | −1.93 | 0.9471 |

caused some sacrifice in the improvement of the correlation of the natural frequencies. One of the possible reasons for this outcome could be the inconsistency between the experimental natural frequency and mode shape data due to noise and other measurement and processing errors. The other possibility, which probably is much more likely to occur, could be that it is possible to improve further the choice of the parameters. This means that the updating parameters selected at present are not sufficient to parameterize all sources of modelling inaccuracies present in the finite element model.

The updated models obtained above by the two methods by matching natural frequencies and then both the natural frequencies and the mode shapes, are now used for predicting the effects of potential design modifications made to the structure. The predictions are made first for a mass modification and then for a beam modification.

A mass modification is introduced by attaching a mass of 1.8 kg at the tip of the upper horizontal beam member as shown in Fig. 6(b). The FRFs for the mass-modified structure are then acquired. The mass modification is also introduced analytically in the updated models obtained by the two methods. The mass and stiffness matrix for the modified structure, and subsequently its modal data and the FRFs, corresponding to the updated models are obtained as explained in Section 3. A comparison of the modified modal data as predicted by the updated models based on the two methods is shown in Table 10 while a comparison of the predicted FRFs for the modified structure is given in Fig. 9. It is observed that the predicted dynamic characteristic on the basis of the updated models based on the two methods are reasonably closer to the measured characteristics for the modified structure. For example, the percentage average error in the predicted natural frequencies inside the updating range is 3.45% for both the methods.

Next, the predictions are made using the updated model obtained by matching both the natural frequencies and the mode shapes. A comparison of the modified modal data as predicted by the updated models based on the two methods is shown in Table 11. It is seen that the MUCO

Table 10
Comparison of the natural frequency and the mode shape correlation for the *mass-modified structure* as predicted by the updated models obtained when updating is carried out using only natural frequencies

| Mode no. | Measured frequency for the modified structure (Hz) | Predictions for the modified structure on the basis of the MUCO-based updated-model | | | Predictions for the modified structure on the basis of the IESM-based updated-model | | |
|---|---|---|---|---|---|---|---|
| | | Frequency (Hz) | % Error | MAC-value | Frequency (Hz) | % Error | MAC-value |
| 1 | 27.32 | 28.95 | 5.95 | 0.9853 | 28.95 | 5.95 | 0.9853 |
| 2 | 74.53 | 74.94 | 0.54 | 0.9924 | 74.94 | 0.54 | 0.9924 |
| 3 | 133.38 | 130.73 | −1.9 | 0.9919 | 130.73 | −1.9 | 0.9919 |
| 4 | 280.11 | 299.14 | 6.79 | 0.7685 | 299.14 | 6.79 | 0.7685 |
| 5 | 745.12 | 760.60 | 2.07 | 0.6547 | 760.60 | 2.07 | 0.6547 |
| 6 | 1050.47 | 1002.04 | −4.6 | 0.9842 | 1002.04 | −4.6 | 0.9842 |
| 7 | 1522.66 | 1491.49 | −2.0 | 0.9635 | 1491.49 | −2.0 | 0.9635 |

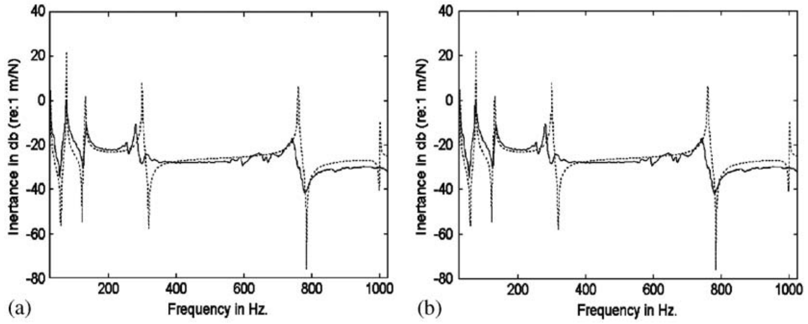


Fig. 9. A comparison of the overlays of the measured modified FRF(___) and those predicted by the updated models (-----), obtained when updating is carried out using only natural frequencies, for the case of *mass modification*. (a) MUCO based, (b) IESM based.

Table 11
Comparison of the natural frequency and the mode shape correlation for the *mass-modified structure* as predicted by the updated models obtained when updating is carried out using natural frequencies as well as mode shapes

| Mode no. | Measured frequency for the modified structure (Hz) | Predictions for the modified structure on the basis of the MUCO-based updated-model | | | Predictions for the modified structure on the basis of the IESM-based updated-model | | |
|---|---|---|---|---|---|---|---|
| | | Frequency(Hz) | % Error | MAC-value | Frequency (Hz) | % Error | MAC-value |
| 1 | 27.32 | 28.07 | 2.76 | 0.9837 | 25.66 | −6.09 | 0.9811 |
| 2 | 74.53 | 79.59 | 6.8 | 0.9929 | 72.55 | −2.64 | 0.9922 |
| 3 | 133.38 | 134.56 | 0.88 | 0.9917 | 117.27 | −12.08 | 0.9929 |
| 4 | 280.11 | 305.37 | 9.01 | 0.8035 | 292.14 | 4.29 | 0.7768 |
| 5 | 745.12 | 782.94 | 5.07 | 0.6822 | 771.44 | 3.53 | 0.6620 |
| 6 | 1050.47 | 1007.80 | −4.06 | 0.9864 | 997.62 | −5.03 | 0.9847 |
| 7 | 1522.66 | 1501.29 | −1.40 | 0.9614 | 1423.74 | −6.49 | 0.9590 |

updated model-based predictions are closer to the measured modified characteristics than the predictions based on the IESM updated model. Thus, the MUCO updated model has given reasonable predictions of both the natural frequencies and the mode shapes for the case of mass modification.

It is also noticed that these predictions are inferior in terms of natural frequency prediction than the predictions based on the updated model obtained by matching only the natural frequencies. This indicates that the correlation of natural frequencies of an updated model with the test data has a much greater influence on the accuracy of the updated model-based predictions than the correlation of the mode shapes.

Next, a beam modification is introduced in the form of a stiffener of width 38.2 mm and thickness 5 mm. The stiffener is attached between the tips of the lower and the upper horizontal

Table 12
Comparison of the MAC-values for the mass-modified and the MAC-values for the beam-modified structure (MAC calculated using the measured and the realized mode shapes for the unmodified and the modified)

| MAC-value for mass-modified structure( MAC-value for beam-modified structure) | | | | | | |
|---|---|---|---|---|---|---|
| *.97*(.97) | .16(.04) | .12(.18) | .17(.08) | .02(.00) | .03(.02) | .00(.00) |
| .00(.36) | *.93*(.33) | .07(.50) | .08(.05) | .08(.00) | .02(.06) | .01(.00) |
| .21(.02) | .03(.00) | *.99*(.13) | .00(.70) | .05(.00) | .01(.00) | .02(.05) |
| .04(.05) | .04(.60) | .05(.23) | *.40*(.01) | .00(.00) | .13(.03) | .04(.00) |
| .18(.00) | .21(.36) | .02(.23) | .06(.05) | *.40*(.15) | .04(.02) | .00(.00) |
| .00(.00) | .02(.00) | .10(.00) | .02(.00) | .42(.01) | *.68*(.82) | .04(.04) |
| .05(.00) | .00(.08) | .01(.01) | .05(.00) | .00(.19) | .11(.00) | *.93*(.59) |

beam members as shown in Fig. 6(c). The modal data for the modified structure is again obtained by performing a modal test on the modified structure. The extent to which the modified-structure mode shapes differ in comparison to the unmodified-structure mode shapes can be gauged by calculating the MAC-matrix between the mode shapes of the modified and the unmodified structure. Table 12 gives a comparison of the MAC-matrix calculated in this way for the case of beam modification (values given inside the bracket) and for the case of mass modification at the tip of the upper horizontal member (values given outside the bracket). The comparison shows that in the case of the mass modification, the diagonal nature of the matrix is still retained (values shown in bold and italic) indicating that there is a clear one-to-one correspondence between the modes of the modified and the unmodified structure. A relatively high MAC-value also indicates that the mode shapes have not altered much. On the contrary, in the case of beam modification one-to-one correspondence does not seem to be easily and reliably identifiable and the presence of low MAC-values also indicate that relatively the mode shapes have altered drastically. In the light of this comparison it therefore appears that the beam modification considered here is a much more complex modification than the mass modification.

Table 13 gives a comparison of the predicted natural frequencies and the mode shape correlation based on the updated models that were obtained by using only the natural frequencies as the targets. The table indicates that both the updated models have been able to predict with reasonable accuracy and the percentage average error in the predicted natural frequencies inside the updating range is 3.18% for both the methods. The predictions of FRFs on the basis of the two updated models are also compared in Fig. 10. The prediction of the FRF also seem to be reasonably good as the predicted curve seem to be following the measured curve closely though in the higher frequency range, the difference seems to be higher. From this figure and also from Fig. 9 earlier it appears that there is an additional experimental mode close to the third mode. However, no meaningful mode shape could be extracted during curve fitting as observed using the animation facility in the software. Secondly, overlay of FRFs after updating indicates a very good match between experimental and analytical FRF that otherwise could not have been obtained, had there been a missing mode (refer Fig. 8). It is also seen from the Figs. 8–10 that the pattern of resonances and antiresonances for analytical model FRF matches with the measured FRF. In view of the above observations, the appearance of some additional peaks close to mode 3 in

Table 13
Comparison of the natural frequency and the mode shape correlation for the *beam-modified structure* as predicted by the updated models obtained when updating is carried out using only natural frequencies

| Mode no. | Measured frequency for the modified structure (Hz) | Predictions for the modified structure on the basis of the MUCO-based updated-model | | | Predictions for the modified structure on the basis of the IESM-based updated-model | | |
|---|---|---|---|---|---|---|---|
| | | Frequency (Hz) | % Error | MAC-value | Frequency (Hz) | % Error | MAC-value |
| 1 | 33.95 | 34.24 | 0.85 | 0.9882 | 34.24 | 0.85 | 0.9882 |
| 2 | 117.30 | 122.28 | 4.25 | 0.9927 | 122.28 | 4.25 | 0.9927 |
| 3 | 309.98 | 313.57 | 1.15 | 0.8653 | 313.57 | 1.15 | 0.8653 |
| 4 | 376.89 | 405.25 | 7.52 | 0.7253 | 405.25 | 7.52 | 0.7253 |
| 5 | 648.34 | 662.28 | 2.15 | 0.9823 | 662.28 | 2.15 | 0.9823 |
| 6 | 1001.21 | 922.18 | −7.89 | 0.9551 | 922.18 | −7.89 | 0.9551 |
| 7 | 1489.98 | 1487.0 | −2.33 | 0.9576 | 1487.0 | −2.33 | 0.9576 |

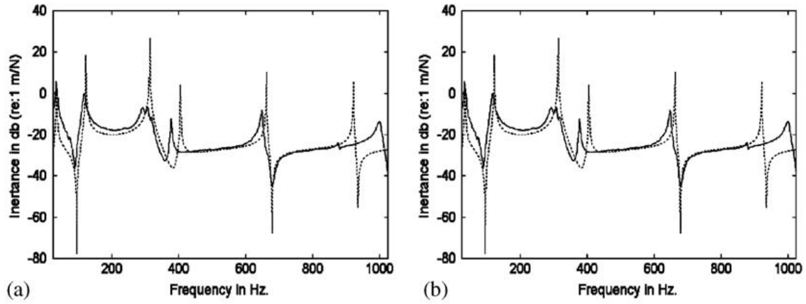


Fig. 10. A comparison of the overlays of the measured modified FRF(___) and those predicted by the updated models (-----), obtained when updating is carried out using only natural frequencies, for the case of *beam modification*. (a) MUCO based, (b) IESM based.

Figs. 9 and 10 may be due to the excitation of an out-of-plane mode for some of the measurement points.

Next, the predictions are made using the updated model obtained by matching both the natural frequencies and the mode shapes. A comparison of the modified modal data as predicted by the updated models based on the two methods is shown in Table 14. For this case it is observed that the MUCO updated model-based predictions are closer to the measured modified characteristics than the predictions based on the IESM updated model. Thus, the MUCO updated model is giving reasonable predictions of both the natural frequencies and the mode shapes for the case of mass as well as beam modifications when updating was performed by including natural frequencies as well as mode shapes as the targets. It also appears that the updating parameters chosen in the present case were reasonable enough to yield an updated model that gives predictions of acceptable accuracy.

Table 14
Comparison of the natural frequency and the mode shape correlation for the *beam-modified structure* as predicted by the updated models obtained when updating is carried out using natural frequencies as well as mode shapes

| Mode No. | Measured frequency for the modified structure (Hz) | Predictions for the modified structure on the basis of the MUCO-based updated-model | | | Predictions for the modified structure on the basis of the IESM-based updated-model | | |
|---|---|---|---|---|---|---|---|
| | | Frequency (Hz) | % Error | MAC-value | Frequency (Hz) | % Error | MAC-value |
| 1 | 33.95 | 32.94 | −2.94 | 0.9907 | 29.79 | −12.24 | 0.9936 |
| 2 | 117.30 | 126.39 | 7.74 | 0.9927 | 125.49 | 6.98 | 0.9924 |
| 3 | 309.98 | 322.16 | 3.92 | 0.8585 | 321.73 | 3.78 | 0.8470 |
| 4 | 376.89 | 405.30 | 7.53 | 0.7103 | 405.32 | 7.54 | 0.6963 |
| 5 | 648.34 | 674.02 | 3.96 | 0.9757 | 679.29 | 4.77 | 0.9710 |
| 6 | 1001.21 | 939.12 | −6.20 | 0.9622 | 948.79 | −5.23 | 0.9651 |
| 7 | 1489.98 | 1491.71 | 0.11 | 0.8484 | 1467.82 | −1.48 | 0.9354 |

## 6. Conclusion

This paper presents analytical and experimental studies about the application of updated FE-models for performing dynamic design. The dynamic design capability of updated models is evaluated by undertaking a simulated study on a beam structure and experimental an study on an F-shape structure. The finite element model for these structures is updated by using a method of model updating based on constrained optimization and an iterative method of model updating based on modal data. The dynamic design at the computer level has been demonstrated via mass and beam/stiffener modifications using these updated models. The simulated and the experimental studies demonstrate that the modified dynamic behavior due to potential structural modifications can be predicted with reasonable accuracy on the basis of updated models. Selection of updating parameters during updating is very important for making reliable predictions using updated models.

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