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Vibration in screw jack mechanisms: experimental results

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Abstract

It is well known that vibrations can occur in screw jack mechanisms under certain conditions, especially during downward motion. Several models have been proposed in the literature in order to explain this vibratory phenomenon due to system instability. Nevertheless, to the best of our knowledge, complete and accurate experimental results have never been carried out before. In this paper, the mechanical system made up of a screw and a nut is analyzed. Then a 2 dof model is introduced. In particular, this model shows that the system is unstable when the moment of inertia J_1 of a mass clamped to the free end of the screw is in a range between two boundary values J_1 min and J_1 max. These values depend on the mechanical characteristics of the system. The existence of this range is experimentally observed. Moreover, it is shown that theoretical values J_1 min and J_1 max are in good agreement with the experimental ones.

From a design point of view, the second main contribution of this work consists in providing a simple but effective way to avoid instability in screw jack mechanisms: in order to prevent the mechanism from vibrating (instability), it is sufficient to clamp (when allowed) an inertia mass to the free end of the screw.

1. Introduction

Although screw jack mechanisms have been widely employed in industry, their dynamic behavior is still poorly understood. In particular, from a design point of view, it is essential to know the conditions for avoiding vibrations. In fact, one can find many practical instances where

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vibrations may occur. Not only vibrations produce loud noise, they can also damage the whole mechanism.

Olofsson [1] built an instrumented prototype representing a simple screw jack mechanism in order to gain an insight into the problem of vibrations. He experimentally found some conditions that could lead to vibrations. However, he did not provide a theoretical explanation of the phenomenon.

It has been proven that the phenomenon of vibrations in a screw jack mechanism, under certain conditions, is due to the so-called *friction-induced instability*. Among others, Dupont [2,3] by means of a simple 1 dof model, gave an elegant explanation of the phenomenon. He found that, in a non-backdrivable screw jack mechanism, the stability condition is given by a relationship involving the masses of the nut and the screw, the friction coefficient and the screw helix angle.

This theory has been improved by Gallina et al. [4] by introducing a 2 dof model and taking into account both axial and torsional stiffnesses of the screw. In such a model, the two generalized coordinates are the vertical displacement of the screw and the torsional displacement of the screw. The results they found are in good agreement with the conclusion given by Dupont. Moreover, the new model led to the following important result from a design point of view: stability condition requires that the ratio between the axial and torsional natural frequencies of the screw jack do not exceed a given limit. By means of this model, it has been possible to eliminate vibrations that occurred during the operation of a big platform (140 ton in weight) of a stage in a theater. Practically, this was obtained by placing a layer of cellular rubber between the surface of the concrete beam and the plate, where the top of the screw was suspended to. In this way, the axial natural frequency of the screw as well as the ratio between the axial and the torsional natural frequencies of the screw jack was reduced.

In this work, exploiting a similar model, it is shown that vibrations can be avoided by simply increasing the moment of inertia of a mass fixed to the free end of the screw. This solution has two advantages with respect to the one proposed in Ref. [4]: first of all, modifying the moment of inertia of a body is easier than providing a given axial stiffness; secondly, this solution does not require to disassemble the whole system.

In Section 2, the 2 dof model is described. Stability analysis is carried out in Section 3. Eventually, in Section 4, the model is verified by means of experimental results.

2. Two degrees of freedom model

The 2 dof model here presented is schematized in Fig. 1.

The mechanism could be analyzed by introducing a continuum model of the screw, as far as torsional vibrations are concerned. Such a model would complicate the mathematical treatment since infinite degrees of freedom would be introduced. Therefore, in order to gain an insight into the problem, here the screw jack is schematized by an equivalent rigid screw coupled to a nut of mass m. r denotes the mean radius of the thread. The screw is placed in a vertical position; its length is l. One end of the screw is clamped to the frame, while the other one is free. The nut engages at the free end of the screw. ϕ is the screw angular position while u is the screw axial displacement along the screw axis. The screw torsional elasticity is schematized by a torsional spring of stiffness k_ϕ. In the same way, the screw axial elasticity is schematized by a spring of

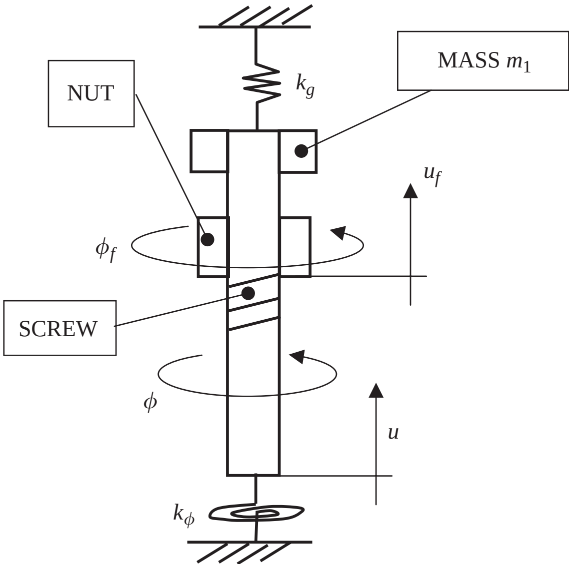

Fig. 1. Schematic diagram of the system.

stiffness k_g placed between the screw and the frame. According to the theory of the continuum model of a circular section shaft, if the frame is assumed to be perfectly rigid, the torsional and the axial stiffnesses assume the values

$$k_g = GI_p/l,$$
$$k_\phi = EA_r/l, \tag{1}$$

where G is the modulus of rigidity, $I_p = (\pi/2)r^4$ is the polar moment of inertia, E is the Young's modulus and $A_r = \pi r^2$ is the screw section area.

In order to simplify the mathematical model, both nut and screw are assumed to be rigid bodies.

A mass m_1 is clamped at the free end of the screw. Its moment of inertia is J_1. It will be shown that this mass has the function of stabilizing the system.

The nut is assumed always to be in contact with the screw. Let ϕ_f be the nut angular position and u_f the nut vertical position. Since the model is made up of two rigid bodies interacting with each other, two dynamic equations can be carried out. The first one is related to the undamped rotational equation of motion of the screw

$$(J_\phi + J_1)\ddot{\phi} + k_\phi \phi = T, \tag{2}$$

where T is the torque the nut exerts on the screw. The torque T is assumed to be positive when counter-clockwise oriented (top view). The nut is assumed to move upwards when it rotates counter-clockwise (top view).

J_ϕ represents the moment of inertia of the equivalent screw.

The undamped translational equation of motion of the screw along its axis is

$$(m_1 + m_v)\ddot{u} + k_g u = -P - (m_1 + m_v)g, \tag{3}$$

where m_v is the equivalent screw mass and g is the gravity acceleration. P is the vertical load the nut exerts on the screw. The nut vertical displacement u_f, its angular position ϕ_f and the screw angular position ϕ must satisfy the following kinematic relationship:

$$u_f = \phi_f r \tan\delta - \phi r \tan\delta + u, \tag{4}$$

where $\delta = \arctan(p/2\pi r)$ is the screw helix angle, p is the screw pitch and r is the screw mean radius. The last equation to complete the 2 dof model comes from the expression of the torque T.

In this regard, it must be highlighted that two kinds of screws are employed in common screw jack mechanisms: square-threaded screws and V-thread screws. In both cases, the nut exerts a vertical load P on the screw.

Case 1: *Square-threaded screw*

By applying the analogy between a screw thread and the inclined plane, one obtains the torque the nut exerts on the screw [4].

$$T = rP(\tan\delta + f\,\mathrm{sgn}\,\omega_{\mathrm{rel}})/(1 - \tan(\delta) f\,\mathrm{sgn}\,\omega_{\mathrm{rel}}), \tag{5}$$

where f is the friction coefficient. $\omega_{\mathrm{rel}} = \dot{\phi}_f - \dot{\phi}$ represents the relative angular velocity between the screw and the nut. Eq. (4) is carried out by observing that the screw jack mechanism can be thought of as a wedge, where the rotation movement of the nut is replaced by an equivalent horizontal one.

By exploiting the relationship $f = \tan\eta_s$, where η_s is the angle of friction, Eq. (5) becomes

$$T = rP\tan(\delta + \eta_s\,\mathrm{sgn}\,\omega_{\mathrm{rel}}), \tag{6}$$

where η_s is a function of the sliding velocity between the nut and the thread. Therefore, it can be written as

$$\eta_s = \eta_s(v_s) = \eta_s(r|\omega_{\mathrm{rel}}|), \tag{7}$$

where v_s represents the absolute value of the sliding speed. For the sake of simplicity, it is assumed that the angle of friction depends only on the sign of the sliding speed, but not on its value (coulomb friction model), therefore relationship (7) can be simplified in the following:

$$\eta_s = \eta_s(v_s) = \eta, \tag{8}$$

where η is constant.

Therefore, when the nut rotates counter-clockwise ($\dot{\phi}_f > 0$) and the term $\dot{\phi}$ is reasonably assumed to be negligible when compared to the nut's speed, the sliding speed is positive and the torque the nut exerts on the screw is $T = rP\tan(\delta + \eta)$. On the contrary, when the nut rotates clockwise (downward motion of the nut), the torque is $T = rP\tan(\delta - \eta)$.

Case 2: *Triangular-threaded screw* (*or V-thread screw*)

This case is more complicated, since it is necessary to take into account the angle on a normal section perpendicular to the helix ψ_n [5]. Referring to Fig. 2, the plane is a portion of the thread on the surface at the mean radius r. For the sake of clarity an xyz local frame is given. The plane is defined by means of the unit vectors **i** and **j** which are inclined at an angle of ψ_n and δ, respectively.

Vectors represented by thick lines are the forces the screw exerts on the nut. In particular, $F_n\mathbf{s}$ is the contact force between the screw and the nut normal to the contact surface: F_n is the force

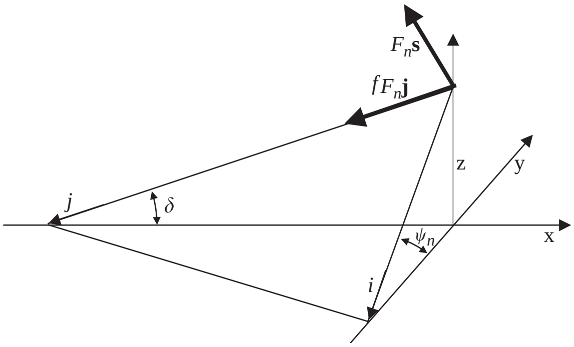

Fig. 2. Forces in a triangular-threaded screw.

value while the unit vector $\mathbf{s}$ is normal to the thread surface. $f\,\mathrm{sgn}\,\omega_{\mathrm{rel}}F_n j$ is the friction force, $\mathbf{j}$ being the nut motion direction. Since $\mathbf{j} = \{-\cos\,\delta, 0, -\sin\,\delta\}^{\mathrm{T}}$ and $\mathbf{i} = \left\{0, -\cos\,\psi_n, -\sin\,\psi_n\right\}^{\mathrm{T}}$, it yields

$$\mathbf{s} = \{s_x, s_y, s_z\} = (\mathbf{j}\times\mathbf{i})/||\mathbf{j}\times\mathbf{i}||. \tag{9}$$

The whole force the screw exerts on the nut is

$$F_w = F_n\mathbf{s} + f\,\mathrm{sgn}\,\omega_{\mathrm{rel}}F_n\mathbf{j} = F_n\left(\begin{Bmatrix} s_x \\ s_y \\ s_z \end{Bmatrix} + f\,\mathrm{sgn}\,\omega_{\mathrm{rel}}\begin{Bmatrix} -\cos\,\delta \\ 0 \\ -\sin\,\delta \end{Bmatrix}\right). \tag{10}$$

The first component of F_w represents the force that produces the torque

$$T = rF_n(s_x - f\,\mathrm{sgn}\,\omega_{\mathrm{rel}}\,\cos\,\delta) \tag{11}$$

which acts on the nut.

The third component of F_w represents the vertical load the screw exerts on the nut; therefore, the vertical load the nut exerts on the screw is

$$P = -F_n(s_z - f\,\mathrm{sgn}\,\omega_{\mathrm{rel}}\,\sin\,\delta). \tag{12}$$

By exploiting Eqs. (11) and (12), it can be inferred that the relationship between the torque T the nut exerts on the screw and the vertical load P is

$$T = Pr\Lambda \tag{13}$$

where $\Lambda = -(s_x - f\,\mathrm{sgn}\,\omega_{\mathrm{rel}}\,\cos\,\delta)/(s_z - f\,\mathrm{sgn}\,\omega_{\mathrm{rel}}\,\sin\,\delta)$. f is a function of the sliding velocity between the nut and the thread

$$f = f(v_s) = f(r|\omega_{\mathrm{rel}}|). \tag{14}$$

It is assumed that the friction coefficient depends only on the sign of the sliding speed, but not on its value; therefore, it can be written as

$$f = f(v_s) = f_r, \tag{15}$$

where f_r is constant.

Therefore, during upward motion Λ assumes the value $\Lambda_u = -(s_x - f_r \cos\delta)/(s_z - f_r \sin\delta)$, while during downward motion, it becomes

$$\Lambda_d = -(s_x + f_r \cos\delta)/s_z + f_r \sin\delta. \tag{16}$$

In conclusion, the torque the nut exerts on the screw in the two cases is summarized in Table 1.

In the literature [2–4], it has been shown that instability could occur in a screw jack mechanism only during downward motion. Therefore, in the following, only downward motion ($\dot{\phi}_f < 0$) is considered. Moreover, the screw thread is assumed to be triangular in order to analyze a more general situation.

The vertical equation of motion of the nut leads to

$$P = m(g + \ddot{u}_f) = m(g - \ddot{\phi} r \tan\delta + \ddot{u}), \tag{17}$$

where m is the nut mass and P is the vertical load the screw exerts on the nut. The second part of the equation is obtained by deriving Eq. (4) and assuming that the nut velocity is constant. Then, Eq. (13) can be replaced by Eq. (2). Eqs. (2) and (3) form the following system:

$$\begin{aligned}&(J_\phi + J_1)\ddot{\phi} + k_\phi \phi = Pr\Lambda_d,\\&(m_1 + m_v)\ddot{u} + k_g u = -P - (m_1 + m_v)g.\end{aligned} \tag{18}$$

By making the value of the vertical load explicit by means of Eq. (17), system (18) becomes

$$\begin{aligned}&((J_\phi + J_1) + \Lambda_d m r^2 \tan\delta)\ddot{\phi} - \Lambda_d m r \ddot{u} + k_\phi \phi = g\Lambda_d m r,\\&- mr\tan\delta\ddot{\phi} + (m + m_1 + m_v)\ddot{u} + k_g u = -(m + m_1 + m_v)g.\end{aligned} \tag{19}$$

Using a matrix notation, the second-order linear differential equation system (19) becomes

$$M\begin{Bmatrix}\ddot{\phi}\\ \ddot{u}\end{Bmatrix} + K\begin{Bmatrix}\phi\\ u\end{Bmatrix} = \begin{Bmatrix}g\Lambda_d m r\\ -(m_1 + m + m_v)g\end{Bmatrix}, \tag{20}$$

where

$$\begin{aligned}M &= \begin{bmatrix}J_1 + J_\phi + \Lambda_d m r^2 \tan\delta & -\Lambda_d m r\\ -mr\tan\delta & m + m_v + m_1\end{bmatrix},\\ K &= \begin{bmatrix}k_\phi & 0\\ 0 & k_g\end{bmatrix}.\end{aligned} \tag{21}$$

Here it can be recalled that this mass matrix M represents the case in which the screw jack mechanism is moving downward ($\dot{\phi}_f < 0$). Instead, during upward motion the mass

Table 1
The torque the nut exerts on the screw in case of square thread or V-thread during upward or downward motion

	Upwards	Downwards
Square thread	$T = Pr\tan(\delta + \eta)$	$T = Pr\tan(\delta - \eta)$
V-thread	$T = Pr\Lambda_u$	$T = Pr\Lambda_d$

matrix would be

$$M_u = \begin{bmatrix} J_1 + J_\phi + \Lambda_u mr^2 \tan\delta & -\Lambda_u mr \\ -mr \tan\delta & m + m_v + m_1 \end{bmatrix}.$$

Notice that the mass matrix M_u during upward motion and mass matrix M during downward motion are not the same. Moreover, if the friction coefficient as null, the mass matrix would be symmetric and unique. As a consequence, the system would be stable in the sense of weak stability. We conclude that friction is responsible for instability.

It is reminded that for a conservative system both stiffness and inertia matrices have to be symmetric.

System (20) is a *non-conservative system* since the inertia matrix is not symmetric because of a non-null friction coefficient. This is understandable since a coulomb friction force is not a conservative force.

System (20) can be given a better form by premultiplying by the matrix

$$M_a = \begin{bmatrix} 1/(J_1 + J_\phi) & 0 \\ 0 & 1/(m + m_v + m_1) \end{bmatrix}. \tag{22}$$

Therefore, system (20) becomes

$$\tilde{M}\begin{Bmatrix} \ddot{\phi} \\ \ddot{u} \end{Bmatrix} + \tilde{K}\begin{Bmatrix} \phi \\ u \end{Bmatrix} = \begin{Bmatrix} g\Lambda_d mr/(J_1 + J_\phi) \\ -g \end{Bmatrix}, \tag{23}$$

where

$$\tilde{M} = \begin{bmatrix} 1 + (\Lambda_d mr^2 \tan\delta)/(J_1 + J_\phi) & -(\Lambda_d mr)/(J_1 + J_\phi) \\ -(mr \tan\delta)/(m + m_v + m_1) & 1 \end{bmatrix},$$

$$\tilde{K} = \begin{bmatrix} \omega_\phi^2 & 0 \\ 0 & \omega_g^2 \end{bmatrix}. \tag{24}$$

The components of the stiffness matrix represent, respectively, the natural torsional frequency of the screw included the stabilizing mass m_1, without the nut, while the second is the natural axial frequency of the screw included the mass of the nut

$$\omega_\phi = \sqrt{k_\phi/(J_1 + J_\phi)},$$

$$\omega_g = \sqrt{k_g/(m + m_v + m_1)}. \tag{25}$$

Notation (23) is preferable to notation (20) because, from an experimental point of view, the natural frequencies ω_ϕ and ω_g are easier to measure than the stiffnesses k_ϕ and k_g.

3. Stability analysis

System (23) is a non-conservative undamped linear system [6] since the inertia matrix is non-symmetric and the damping matrix is absent. In order to perform the stability analysis, the method proposed in Ref. [6] will be followed. Such a method gives sufficient and necessary conditions for a linear non-conservative undamped system to be stable, in a BIBO sense (weak stability). The main feature of this method is that it does not require eigenvectors calculation. The method and its application to system (23) will be explained in the Appendix.

In particular, it will be shown that the system is stable when

$$\begin{aligned} &A \geqslant 0, \\ &B \geqslant 0, \\ &B^2 - 4AC \geqslant 0, \end{aligned} \tag{26}$$

where the coefficient A, B and C are

$$\begin{aligned} A &= 1 + \frac{\Lambda_d m r^2 \tan \delta}{J_1 + J_\phi}\left(1 - \frac{m}{m + m_v + m_1}\right), \\ B &= \omega_\phi^2 + \omega_g^2\left(1 + \frac{\Lambda_d m r^2 \tan \delta}{J_1 + J_\phi}\right), \\ C &= \omega_\phi^2 \omega_g^2. \end{aligned} \tag{27}$$

In conclusion, when conditions (26) are not simultaneously satisfied, the system that describes the dynamic behavior of the screw jack turns out to be unstable, since there exists at least one eigenvector of system (23) which has a positive real part. As a consequence, both axial and torsional vibrations occur. This phenomenon can be recorded by a set of accelerometers. In fact, it produces a loud noise.

4. Experimental results

In order to validate the 2 dof model, a simple experimental apparatus has been built (see Fig. 3 on the left). It consists of a screw fixed to the frame, a cylindrical nut (of mass m) and a mechanism clamped to the free end of the nut of mass m_1. This latter (see Fig. 3 on the right) is made up of a parallelepiped with two screws located in symmetric positions. By screwing the two screws, it is possible to change the moment of inertia J_1 of mass m_1. The nut is manually operated: downward motion is obtained by a counter-clockwise rotation.

In order to measure axial and torsional vibrations, two accelerometers (type PCB 352C22) are employed. They have a voltage sensitivity of 10 mV/g, and a weight of 0.0005 kg. Accelerometer 1 is fixed to the free end of the screw in order to measure axial vibrations. Accelerometer 2 is fixed to the mass m_1 at a distance of 0.015 m far away from the screw axis so as to measure torsional vibrations.

According to the theory of the previous section, in order to predict instability, the three values A, B and $B^2 - 4AC$ have to be calculated. In particular, since the main goal of this work consists

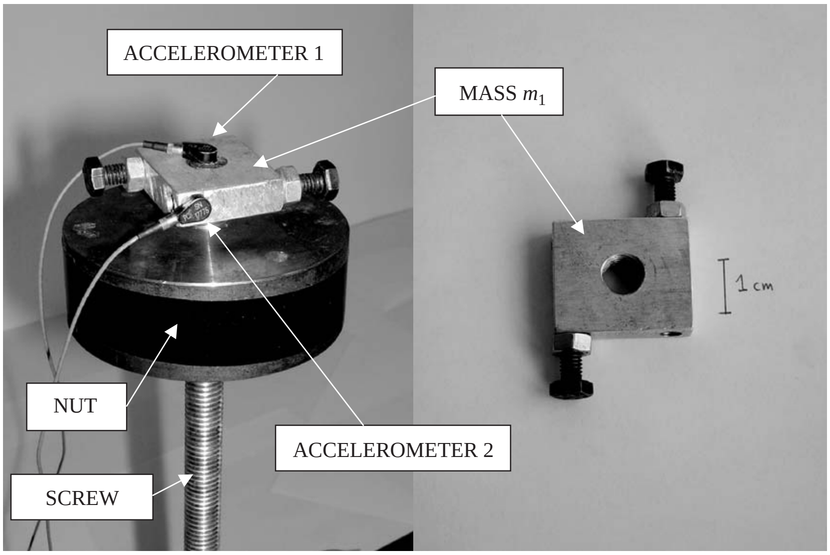

Fig. 3. Screw jack prototype. Nut, screw and the mass m_1 with the two accelerometers on the left; the mass m_1 on the right.

Table 2
Parameters related to the screw jack prototype

m (kg)	m_1 (kg)	r (m)	δ (rad)	f_r	ψ_n (rad)
1	0.02	0.005	0.054	0.19	0.524

in analyzing the effect of the moment of inertia J_1 as far as instability is concerned, the coefficients given by Eq. (26) will be calculated and plotted versus the parameter J_1.

Since the coefficients A, B and $B^2 - 4AC$ depend on many screw jack mechanical parameters, it is necessary to estimate these parameters for obtaining the aforesaid plot.

The screw jack prototype is characterized by data given in Table 2. While m, m_1, r, δ and ψ_n are directly measured, the friction coefficient f_r is estimated by exploiting Eqs. (13) and (15). In fact, given a known vertical load P and measuring the minimum torque T that can move the nut, it is possible to get the value f_r.

Once f_r is estimated, it is possible to calculate Λ_d by means of Eq. (16).

The natural frequencies involved in Eq. (27) are estimated in the following way. First of all, according to Saint-Venant theory, the axial and torsional stiffnesses are

$$
\begin{aligned}
k_g &= GI_p/l,\\
k_\phi &= EA_r/l,
\end{aligned}
\tag{28}
$$

where G is the modulus of rigidity, $I_p = (\pi/2)r^4$ is the polar moment of inertia, E is the Young's modulus and $A_r = \pi r^2$ is the section area. Screw material characteristics are given in Table 3. m_v

Table 3
Screw material characteristics

ρ (kg/m^3)	E (N/m^2)	G (N/m^2)
7800	2.1×10^{11}	8×10^{10}

and J_ϕ are the most difficult parameters to estimate. In fact, as explained in Section 2, for the sake of simplicity, the real screw is modeled as an equivalent rigid screw. Such a screw has mass m_v and moment of inertia J_ϕ which are lower than the total mass of the screw $m_{\text{tot}} = \rho\pi r^2 l$ and its moment of inertia $J_{\text{tot}} = \frac{1}{2} m_{\text{tot}} r^2$. In order to find the value m_v, it is assumed that the first axial natural frequency of the real screw has to equal the axial frequency of the equivalent one. From the theory of vibration of continuum systems [7], it is known that the first natural frequency of the axial oscillation of the screw is

$$\omega_l = \frac{\sqrt{E/\rho}\pi}{2l}. \tag{29}$$

The same frequency has to be obtained by considering a mass m_v of the equivalent screw connected to the frame by means of the spring of stiffness k_g

$$\omega_l = \sqrt{\frac{k_g}{m_v}}. \tag{30}$$

By comparing Eqs. (29) and (30), the mass m_v results as follows:

$$m_v = k_g \bigg/ \left(\frac{\sqrt{E/\rho}\pi}{2l} \right)^2. \tag{31}$$

As far as the torsional vibrations are concerned, following the same line of reasoning, the first natural frequency of the torsional oscillation of the screw is

$$\omega_t = \frac{\sqrt{G/\rho}\pi}{2l}. \tag{32}$$

The same frequency has to be obtained by considering a moment of inertia J_ϕ of the equivalent screw connected to the frame by means of the torsional spring of stiffness k_ϕ:

$$\omega_t = \sqrt{\frac{k_\phi}{J_\phi}} \;\Rightarrow\; J_\phi = k_\phi \bigg/ \left(\frac{\sqrt{G/\rho}\pi}{2l} \right)^2. \tag{33}$$

The mass m_1 can be easily measured. It results in $m_1 = 0.02$ kg. Now that all the parameters are involved in the expression of the coefficients A, B and $B^2 - 4AC$ have been obtained, it is possible to study the stability of the system.

In the experiment, three different screw jack configurations have been considered: $l = l_1 = 0.43$ m, $l = l_2 = 0.53$ m, $l = l_3 = 0.63$ m. For each configuration, the values A, B and $B^2 - 4AC$

are calculated for different values of J_1. Figs. 4 and 5 represent the theoretical values of A and B versus J_1 for the three configurations. Plots of A and B are always greater than zero. Therefore, the first two conditions of (26) are always satisfied for every J_1.

The plot of Fig. 6 is more interesting . It represents the value of $B^2 - 4AC$ versus J_1 for the three configurations. Let us recall that the system is stable when $B^2 - 4AC > 0$. J_1 min and J_1 max represent the intersections of the curve of $B^2 - 4AC$ with the abscissa axis. When $J_1 < J_1$ min, the system is stable since the value $B^2 - 4AC$ is positive. When J_1 min $< J_1 < J_1$ max, the system is unstable. Eventually, when J_1 exceeds the value of J_1 max, the system is stable again. As one can see, the values of J_1 min and J_1 max are poorly affected by the value of l. Such theoretical values of J_1 min and J_1 max are summarized in the third column of Table 4.

As it is shown in the following, the existence of these two theoretical values of J_1 min and J_1 max is proved by experimental evidence. The experimental protocol is made up of the following steps:

1. The length of the screw is set to $l = l_1 = 0.43$ m.
2. By completely screwing the two screws of the mechanism of Fig. 3 (right figure), J_1 is set to its lowest value. With this set-up, the nut is manually rotated both in clockwise and counter-clockwise directions. According to the theory, during upward motion (corresponding to the counter-clockwise direction), vibrations do not occur. The same effect is obtained during downward motion.
3. The value of J_1 is increased by operating the two screws of the mechanism of mass m_1. The nut is then rotated in both directions. If no vibration occurs, J_1 is increased again. This procedure

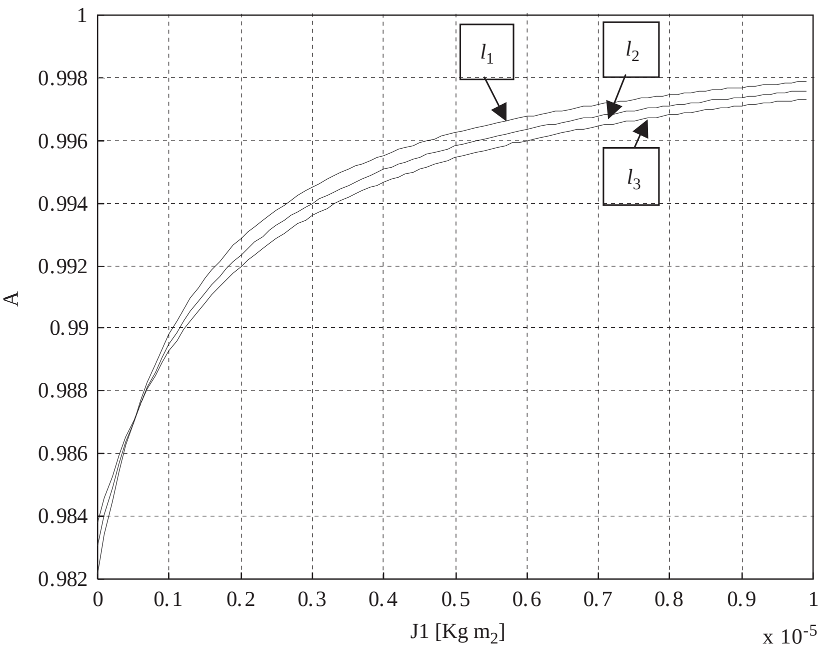

Fig. 4. Theoretical value of coefficient A versus J_1.

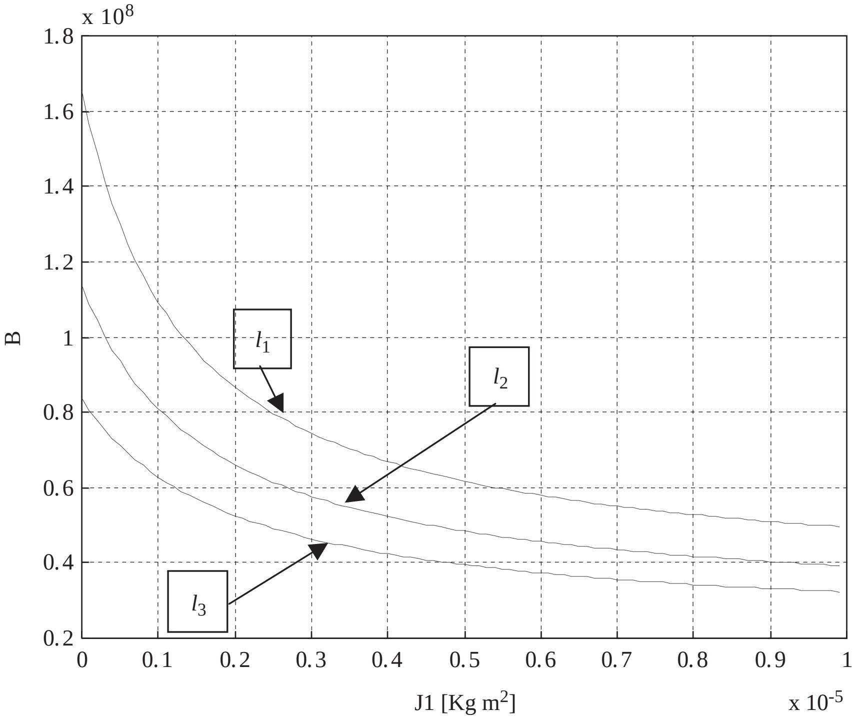

Fig. 5. Theoretical value of coefficient B versus J_1.

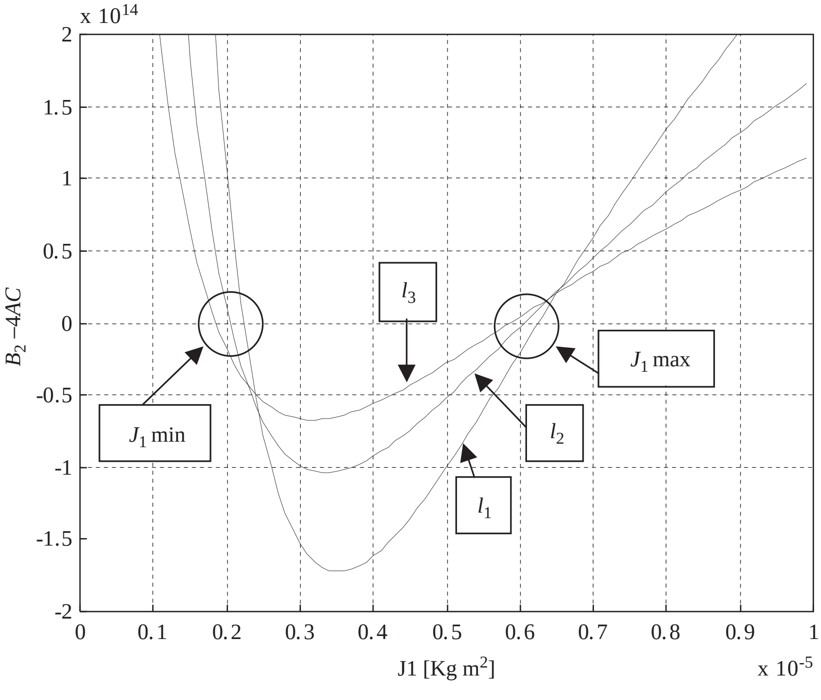

Fig. 6. Theoretical value of coefficient $B^2 - 4AC$ versus J_1.

Table 4
Comparison between theoretical and experimental data

		Theoretical data (kg/m^2)	Experimental data (kg/m^3)	Error (%)
$l = l_1 = 0.43\,\text{m}$	J_1 min	2.24×10^{-6}	2.03×10^{-6}	9.38
	J_1 max	6.25×10^{-6}	8.34×10^{-6}	33.44
$l = l_2 = 0.53\,\text{m}$	J_1 min	2.05×10^{-6}	1.78×10^{-6}	13.17
	J_1 max	6.06×10^{-6}	8.04×10^{-6}	32.67
$l = l_3 = 0.63\,\text{m}$	J_1 min	1.85×10^{-6}	1.35×10^{-6}	27.03
	J_1 max	5.87×10^{-6}	7.91×10^{-6}	34.75

is repeated, increasing the value of J_1 each time, until a vibration occurs (during downward motion). In fact, according to the theory, a value of J_1 was found so that, during downward motion, the typical vibration due to the instability has been recorded. This value corresponds to the experimental value of $J_1\,\text{min}$.

4. Then the value of J_1 is slightly increased. As in step 3, the nut is rotated in both directions. If the vibration is still present, J_1 is increased again. This procedure is repeated, increasing the value of J_1 each time, until the vibration disappears (during downward motion). In fact, according to the theory, a value of J_1 was found so that, during downward motion, no vibration due to the instability was recorded. This value corresponds to the experimental value of $J_1\,\text{max}$.

The same procedure, from step (1) to step (4), is repeated for the other two screw heights l_2 and l_3. The experimental values of J_1 min and J_1 max are summarized in the fourth column of Table 4.

Table 4 represents the most important result achieved by this work. In fact, Table 4 compares experimental and theoretical results. In particular, by the 2 dof theoretical model, it has been inferred that there exists an instability range given by $J_1\,\text{min} < J_1 < J_1\,\text{max}$. What is relevant is that this range has been also experimentally observed. Not only does this range exist, but its theoretical lower and upper boundaries J_1 min and J_1 max are also in good agreement with the experimental values. In this regard it is sufficient to compare the third and the fourth columns of Table 4.

In the fifth column, errors between theoretical and experimental results are given. We believe that such errors are so high because of the high variability of some parameter values such as the friction coefficient and screw geometry. Nevertheless, the existence of the boundaries J_1 min and J_1 max has clearly been proven. This conclusion leads to the following important result from a design point of view: in order to eliminate the vibration in a screw jack mechanism it is sufficient to fix a mass (with a proper moment of inertia) to the screw.

In order to give an example of the instability behavior, both axial and torsional vibrations have been recorded. For the sake of brevity, just the configuration with $l = l_3$ is taken into account here. The moment of inertia of mass m_1 has been set to the value $J_1 = 5.58 \times 10^{-6}\,\text{kg}\,\text{m}^2$. As it was

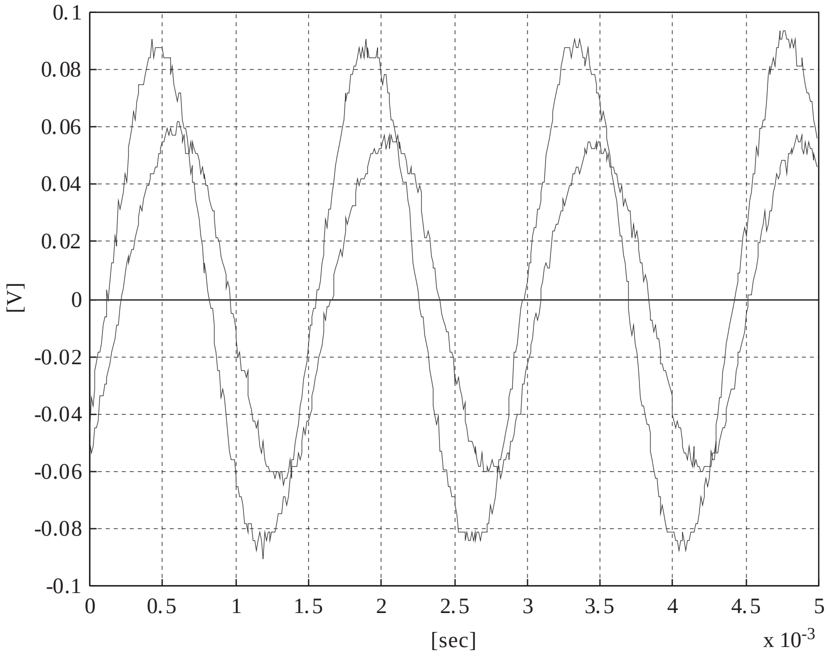

Fig. 7. Vibrations recorded by the two accelerometers.

predicted by the plot of Fig. 6, system (23) turns out to be unstable with this value of J_1. Such an instability is experimentally shown in Fig. 7. The solid line represents the signal coming from accelerometer 2, while the dotted line represents the signal coming from accelerometer 1. Since the axial vibration signal was very low, it was amplified 100 times before being plotted. As one can see, there is an axial vibration coupled with a torsional one. The experimental vibration frequency is $f_{\rm ev} = 692\,\text{Hz}$.

On the other hand, by calculating the eigenvalues of system (23), it is easy to obtain a theoretical vibration frequency $f_{\rm tv} = 683\,\text{Hz}$. Therefore, once again, the theoretical result is in good agreement with the experimental one.

5. Conclusions

Although vibration in screw jack mechanisms under certain conditions, especially during downward motion is a known and remarkable problem, still the phenomenon has not been fully analyzed. To the best of our knowledge, in literature there is no comparison between the experimental and teh simulated results, as far as vibrations in screw jack mechanisms are concerned. In this study a 2 dof model is proposed. The model takes into account both the square- and triangular-threaded screws. Such a model has been validated by means of experimental results.

Both theoretical and experimental results lead to the following important result from a design point of view: in order to eliminate the vibration in a screw jack mechanism, it is sufficient to fix a mass (with a proper moment of inertia) to the screw.

Appendix A

The results of the theory presented in Ref. [6] are briefly summarized here.

A non-conservative undamped linear system is expressed in the form

$$\mathbf{M}\ddot{\mathbf{x}} + \mathbf{K}\mathbf{x} = 0, \tag{A.1}$$

where $\mathbf{M}$ and $\mathbf{K}$ are arbitrary real square matrices of order n and $\mathbf{x}$ is the state vector. The characteristic polynomial associated with system (A.1) has the structure

$$\left|\mathbf{M}\lambda^2 + \mathbf{K}\right| = a_0\lambda^{2n} + a_1\lambda^{2(n-1)} + \cdots + a_{n-1}\lambda^2 + a_n, \tag{A.2}$$

where $a_0, a_1, \ldots, a_n$ are real coefficients.

It is possible to rewrite polynomial (A.2) by replacing $\mu = \lambda^2$. The characteristic equation becomes

$$f_e(\mu) = a_0\mu^n + a_1\mu^{n-1} + \cdots + a_{n-1}\mu + a_n = 0. \tag{A.3}$$

Polynomial (A.3) is referred to as *reduced polynomial* in the variable μ.

Some useful definitions are given:

Definition 1. Given a polynomial $f_e(\mu) = a_0\mu^n + a_1\mu^{n-1} + \cdots + a_{n-1}\mu + a_n$, the following $2n \times 2n$ matrix is called *discrimination matrix*:

$$\Delta(f_e) = \begin{bmatrix} a_0 & a_1 & a_2 & \cdots & a_n & 0 & & & \\ 0 & na_0 & (n-1)a_1 & \cdots & a_{n-1} & 0 & & & \\ & a_0 & a_1 & \cdots & a_{n-1} & a_n & 0 & & \\ & & na_0 & \cdots & 2a_{n-1} & a_{n-1} & & & \\ & & & \cdots & \cdots & & & & \\ & & & \cdots & \cdots & & & & \\ & & & 0 & a_0 & a_1 & \cdots & a_n & \\ & & & & 0 & na_0 & \cdots & a_{n-1} & \end{bmatrix}. \tag{A.4}$$

The discrimination matrix, in turn, can be thought of as associated with the undamped system or with the characteristic polynomial of the undamped system.

Definition 2. The sequence

$$\{D_1, \ldots, D_n\}, \tag{A.5}$$

where D_i is the determinant of the sub-matrix of the *discrimination matrix* formed by the first $2i$ rows and $2i$ columns, is called the *discriminant sequence* of the polynomial $f(\mu) = a_0\mu^n + a_1\mu^{n-1} + \cdots + a_{n-1}\mu + a_n$. Sometimes, D_i's were called *sub-discriminants* or *principal sub-resultants*.

Theorem. *Consider a linear undamped system. Let its polynomial characteristic be* $a_0\lambda^{2n} + a_1\lambda^{2(n-1)} + \cdots + a_{n-1}\lambda^2 + a_n$ *and its reduced polynomial be* $f_e(\mu) = a_0\mu^n + a_1\mu^{n-1} + \cdots + a_{n-1}\mu + a_n$. *A necessary and sufficient condition for the system to be weakly stable is that all the elements of the discriminant sequence of the reduced polynomial are non-negative and all the coefficients of the polynomial are all non-positive or all non-negative. If such conditions are not satisfied, the system is unstable, or, in other words, there exists an eigenvalue with a positive real part. It is reminded that a system is weakly stable when all its eigenvalues are pure complex.*

The theorem is now applied to the homogeneous system (23). The characteristic polynomial of the undamped system is

$$\left|\tilde{M}\lambda^2 + \tilde{K}\right| = A\lambda^4 + B\lambda^2 + C = 0, \tag{A.6}$$

where

$$\begin{aligned} A &= 1 + \frac{\Lambda_d m r^2 \tan\delta}{J_1 + J_\phi}\left(1 - \frac{m}{m + m_v + m_1}\right), \\ B &= \omega_\phi^2 + \omega_g^2\left(1 + \frac{\Lambda_d m r^2 \tan\delta}{J_1 + J_\phi}\right), \\ C &= \omega_\phi^2\omega_g^2. \end{aligned} \tag{A.7}$$

The theorem can be applied to the following reduced polynomial:

$$f_e(\mu) = A\mu^2 + B\mu + C = 0. \tag{A.8}$$

Its discrimination matrix is

$$\Delta(g) = \begin{bmatrix} A & B & C & 0 \\ 0 & 2A & B & 0 \\ 0 & A & B & C \\ 0 & 0 & 2A & B \end{bmatrix}. \tag{A.9}$$

Computing the discriminant sequence, it yields

$$\left\{ D_1 = \det\begin{bmatrix} A & B \\ 0 & 2A \end{bmatrix}, \quad D_2 = \det\begin{bmatrix} A & B & C & 0 \\ 0 & 2A & B & 0 \\ 0 & A & B & C \\ 0 & 0 & 2A & B \end{bmatrix} \right\}. \tag{A.10}$$

According to the theorem, in order for the system to be weakly stable, all the elements of the discriminant sequence of the reduced polynomial must be non-negative; therefore,

$$\begin{aligned} D_1 \geqslant 0 \quad &\Rightarrow \quad A \geqslant 0, \\ D_2 \geqslant 0 \quad &\Rightarrow \quad B^2 - 4AC \geqslant 0. \end{aligned} \tag{A.11}$$

Moreover, the coefficients of the polynomial must be all non-positive or all non-negative. Since $C = \omega_\phi^2\omega_g^2 > 0$ it has to be

$$A \geqslant 0 \quad \text{and} \quad B \geqslant 0. \tag{A.12}$$

In conclusion, the system is stable (weakly stable) when

$$\begin{aligned} &A \geqslant 0, \\ &B \geqslant 0, \\ &B^2 - 4AC \geqslant 0. \end{aligned} \tag{A.13}$$

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