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An explicit finite element method for dynamic analysis in fluid saturated porous medium-elastic single-phase medium-ideal fluid medium coupled systems and its application

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Abstract

In this paper, an explicit finite element method to analyze the dynamic responses of three-media coupled systems with any terrain is developed on the basis of the numerical simulation of the continuous conditions on the boundaries among fluid saturated porous medium, elastic single-phase medium and ideal fluid medium. This method is a very effective method with the characteristic of high calculating speed and small memory needed because the formulae for this explicit finite element method have the characteristic of decoupling and does not need to solve the system of linear equations. The method is applied to analyze the seismic responses in a reservoir with the consideration of the dynamic interactions among water, dam, sediment and foundation rock. The vertical displacement at the top point of dams is calculated and some conclusions are given.

1. Introduction

In the process of analyzing the dynamic responses of complex construction sites and large systems with the consideration of soil–structure–fluid interactions, there are always many kinds of media, such as fluid saturated porous media, elastic single-phase media and ideal fluid media, which, in most cases, are coupled with each other. The fluid saturated porous medium–elastic

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single-phase medium–ideal fluid medium coupled system is namely advanced with the deepening of the research on dynamic soil–structure interactions. At present, although many researchers made some research on the dynamic response of the above-mentioned three-media coupled systems [1–4], as the complexity of this problem, how to deal with it efficiently is still the aim of researchers who have research interests in this problem.

On the basis of the explicit finite element expressions for the dynamic analysis in fluid saturated porous media [5], ideal fluid media [6] and elastic single-phase media [7], the explicit finite element expressions for the dynamic analysis of the nodes on the joint line among fluid saturated porous media, elastic single-phase media, and ideal fluid media are proposed in this paper. An explicit finite element method to analyze the dynamic responses of a structure in three-media coupled systems with any terrain is developed. This method is a very effective method with the characteristic of high calculating speed and small memory needed because the formulae for this explicit finite element method have the characteristic of decoupling and does not need to assemble the global stiffness matrix and damping matrix and solve the system of linear equations. The method is applied to analyze the seismic responses of the dams, considering the dynamic interactions among water, dam, sediment and foundation rock. The calculation results have shown that for a dynamic problem with 10,000 time steps and 2000 nodes in a three-media coupled system, only 168 s CPU time is needed on a 700 MHz PC-Pentium ¢.

2. Explicit finite element expression of nodes in each medium

2.1. Explicit finite element expressions of nodes in elastic single-phase media

Liao [7] proposed the explicit finite element expressions of internal nodes in elastic single-phase media:

$$\{u_{s1}^{p+1}\} = \{u_{s1}^{p}\} + \Delta t\{\dot{u}_{s1}^{p}\} - \tfrac{1}{2}\Delta t^2 M_{s1}^{-1}\{k_{1i1}\{u_{si}^{p}\} - F_{si1}^{p}\}, \tag{1}$$

$$\{\dot{u}_{s1}^{p+1}\} = \{\dot{u}_{s1}^{p}\} - \frac{\Delta t}{2} M_{s1}^{-1}\{k_{1i1}\{\{u_{si}^{p}\} + \{u_{si}^{p+1}\}\} - \{F_{si1}^{p} + F_{si1}^{p+1}\}\}. \tag{2}$$

2.2. Explicit finite element expressions of nodes in ideal fluid media

Wang [6] proposed the explicit finite element expressions of internal nodes in ideal fluid media:

$$\{u_{f1}^{p+1}\} = \{u_{f1}^{p}\} + \Delta t\{\dot{u}_{f1}^{p}\} - \tfrac{1}{2}\Delta t^2 M_{f1}^{-1}\{k_{3i1}\{u_{fi}^{p}\} - F_{fi1}^{p}\}, \tag{3}$$

$$\{\dot{u}_{f1}^{p+1}\} = \{\dot{u}_{f1}^{p}\} - \frac{\Delta t}{2} M_{f1}^{-1}\{k_{3i1}\{\{u_{fi}^{p}\} + \{u_{fi}^{p+1}\}\} - \{F_{fi1}^{p} + F_{fi1}^{p+1}\}\}. \tag{4}$$

2.3. Explicit finite element expressions of nodes in fluid saturated porous media

Zhao [5] proposed the explicit finite element expressions of internal nodes in fluid saturated porous media

$$\{u_1^{p+1}\} = \{u_1^{p}\} + \Delta t\{\dot{u}_1^{p}\} - \frac{\Delta t^2}{2} M_{u1}^{-1}\{C_{i1}(\{\dot{u}_i^{p}\} - \{\dot{U}_i^{p}\}) + k_{11i1}\{u_i^{p}\} + k_{12i1}\{U_i^{p}\} - F_{ui1}^{p}\}, \tag{5}$$

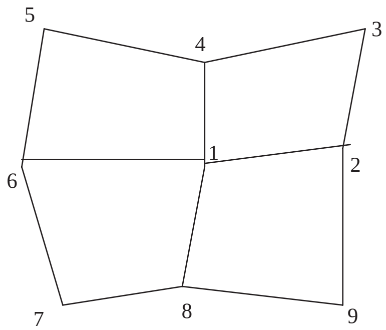

Fig. 1. Local element and node system.

$$\{U_1^{p+1}\} = \{U_1^p\} + \Delta t\{\dot{U}_1^p\} - \frac{\Delta t^2}{2} M_{U1}^{-1}\{C_{i1}(\{\dot{U}_i^p\} - \{\dot{u}_i^p\}) + k_{12i1}\{u_i^p\} + k_{13i1}\{U_i^p\} - F_{Ui1}^p\}, \quad (6)$$

$$\{\dot{u}_1^{p+1}\} = \{\dot{u}_1^p\} - M_{u1}^{-1}\{C_{i1}\{(\{u_i^{p+1}\} - \{u_i^p\}) - (\{U_i^{p+1}\} - \{U_i^p\})\} + \frac{\Delta t}{2}\{k_{11i1}(\{u_i^{p+1}\} + \{u_i^p\}) + k_{12i1}(\{U_i^{p+1}\} + \{U_i^p\}) - (F_{ui1}^p + F_{ui1}^{p+1})\}\}, \quad (7)$$

$$\{\dot{U}_1^{p+1}\} = \{\dot{U}_1^p\} - M_{U1}^{-1}\{C_{i1}\{(\{U_i^{p+1}\} - \{U_i^p\}) - (\{u_i^{p+1}\} - \{u_i^p\})\} + \frac{\Delta t}{2}\{k_{12i1}(\{u_i^{p+1}\} + \{u_i^p\}) + k_{13i1}(\{U_i^{p+1}\} + \{U_i^p\}) - (F_{Ui1}^p + F_{Ui1}^{p+1})\}\}. \quad (8)$$

Fig. 1 shows a local element system of the node 1 and other nodes adjacent to it. Only these adjacent nodes have contribution to it. In Eqs. (1)–(8), $\{u_{s1}\}, \{\dot{u}_{s1}\}, \{u_{f1}\}, \{\dot{u}_{f1}\}, \{u_1\}, \{\dot{u}_1\}, \{U_1\}, \{\dot{U}_1\}$ represent the displacement and velocity of the internal node (node 1) in elastic single-phase media, ideal fluid media, and the solid phase and liquid phase of fluid saturated porous media, respectively; $M_{s1}, M_{f1}, M_{u1}, M_{U1}; k_{1i1}, k_{3i1}, k_{11i1}, k_{12i1}, k_{13i1}$ represent the local combination mass matrix and the stiffness matrix of the internal node (node 1) in the three kinds of media, respectively; $\{F_{si1}\}, \{F_{fi1}\}, \{F_{ui1}\}, \{F_{Ui1}\}$ represent the external force of the internal node (node 1) in the three kinds of media, respectively; $\mathbf{C}_{i1}$ represent the damp matrix of the internal node (node 1) in fluid saturated porous media; the subscript i represents 1 to 9, the superscript p and $p+1$ represent time steps; f/t represents the length of every fixed time step. (The meanings of these symbols are shown in Refs. [5,6].)

3. Explicit finite element expressions of nodes on interfaces among different media

Based on the explicit finite element expressions for the dynamic analysis of fluid saturated porous media, ideal fluid media and elastic single-phase media, the explicit finite element expressions for the dynamic analysis of the nodes on boundary joint line among different kinds of medium will be deduced in this section. The illustration of interface between any two kinds of media is shown in Fig. 2.

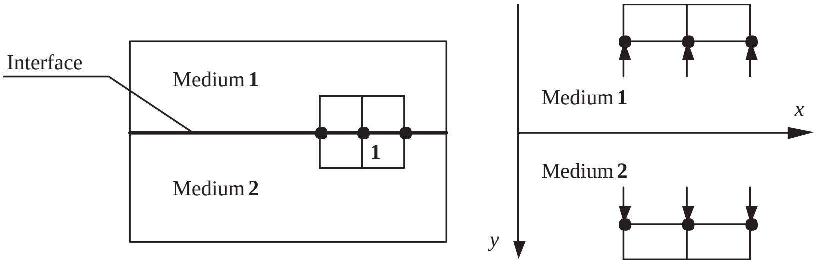

Fig. 2. Illustration of node 1 on the joint line.

3.1. Explicit finite element expressions of nodes on boundary joint nodes between fluid saturated porous medium and ideal fluid medium

3.1.1. Continuous conditions on joint nodes

Robert [8] gave the continuous conditions on the joint line between fluid saturated porous medium and ideal fluid medium.

(1) The movements are continuous in the normal direction of the interface

$$\{u_{fx}\} = (1-n)\{u_x\} + n\{U_x\}, \tag{9}$$

$$\{\dot{u}_{fx}\} = (1-n)\{\dot{u}_x\} + n\{\dot{U}_x\}. \tag{10}$$

(2) The global stress is continuous in the normal direction of the interface

$$\sigma_x - np = -p_w. \tag{11}$$

(3) The pressure intensity of liquid is continuous

$$p = p_w. \tag{12}$$

(4) The stress in the tangential direction of the interface is zero

$$\tau_{xy} = 0. \tag{13}$$

In Eqs. (9)–(13), σ_x and τ_{xy} are the normal and tangential stress of the solid skeleton of fluid saturated porous media separately, p is the pressure of the liquid phase of fluid saturated porous media, and p_w is the pressure of ideal fluid.

3.1.2. Displacement explicit finite element expressions of nodes on the joint line

Based on the above explicit finite element expressions for the dynamic analysis of the fluid saturated porous medium and ideal fluid medium (Eqs. (1)–(8)), and the continuous conditions on the interface (Eqs. (9)–(13)), the displacement explicit finite element expressions for the dynamic analysis of the nodes on the joint line can be developed as shown below:

(1) The normal displacements of node 1 on the interface between the fluid saturated porous medium and ideal fluid medium:

$$u_{1x}^{p+1} = (DE - BF)/(AD - BC), \tag{14}$$

$$U_{1x}^{p+1} = (AF - CE)/(AD - BC), \tag{15}$$

$$\{u_{f1x}^{p+1}\} = (1-n)\{u_{1x}^{p+1}\} + n\{U_{1x}^{p+1}\}, \tag{16}$$

where

$$A = (1-n)^2 M_{f1} + M_{u1}, \quad B = n(1-n)M_{f1},$$

$$C = n(1-n)M_{f1}, \quad D = M_{U1} + n^2 M_{f1},$$

$$\begin{aligned} E = {} & A(u_{1x}^p + \Delta t \dot{u}_{1x}^p) + C(U_{1x}^p + \Delta t \dot{U}_{1x}^p) \\ & - \frac{\Delta t^2}{2} m_x \{C_{i1}(\{\dot{u}_i^p\} - \{\dot{U}_i^p\}) + k_{11i1}\{u_i^p\} + k_{12i1}\{U_i^p\} + (1-n)k_{3i1}\{u_{fi}^p\}\}, \\ F = {} & B(u_{1x}^p + \Delta t \dot{u}_{1x}^p) + D(U_{1x}^p + \Delta t \dot{U}_{1x}^p) \\ & - \frac{\Delta t^2}{2} m_x \{-C_{i1}(\{\dot{u}_i^p\} - \{\dot{U}_i^p\}) + k_{11i1}\{u_i^p\} + k_{12i1}\{U_i^p\} + (1-n)k_{3i1}\{u_{fi}^p\}\}, \end{aligned}$$

and

$$m_x = \begin{bmatrix} 1 \\ 0 \end{bmatrix}.$$

(2) The tangential displacements of node 1 on the interface between the fluid saturated porous medium and ideal fluid medium:

$$\{u_{1y}^{p+1}\} = \{u_{1y}^p\} + \Delta t\{\dot{u}_{1y}^p\} - \frac{\Delta t^2}{2} m_y M_{u1}^{-1}\{C_{i1}(\{\dot{u}_i^p\} - \{\dot{U}_i^p\}) + k_{11i1}\{u_i^p\} + k_{12i1}\{U_i^p\}\}, \tag{17}$$

$$\{U_{1y}^{p+1}\} = \{U_{1y}^p\} + \Delta t\{\dot{U}_{1y}^p\} - \frac{\Delta t^2}{2} m_y M_{U1}^{-1}\{C_{i1}(\{\dot{U}_i^p\} - \{\dot{u}_i^p\}) + k_{12i1}\{u_i^p\} + k_{13i1}\{U_i^p\}\}, \tag{18}$$

$$\{u_{f1y}^{p+1}\} = \{u_{f1y}^p\} + \Delta t\{\dot{u}_{f1y}^p\} - \tfrac{1}{2}\Delta t^2 m_y M_{f1}^{-1}\{k_{3i1}\{u_{fi}^p\}\}. \tag{19}$$

(3) The normal velocities of node 1 on the interface between the fluid saturated porous medium and ideal fluid medium:

$$\dot{u}_{1x}^{p+1} = (DE' - BF')/(AD - BC), \tag{20}$$

$$\dot{U}_{1x}^{p+1} = (AF' - CE')/(AD - BC), \tag{21}$$

$$\{\dot{u}_{f1x}^{p+1}\} = (1-n)\{\dot{u}_{1x}^{p+1}\} + n\{\dot{U}_{1x}^{p+1}\}, \tag{22}$$

where

$$E' = A\{\dot{u}_{1x}^{p}\} + C\{\dot{U}_{1x}^{p}\} - \frac{\Delta t}{2} m_x \left\{ \frac{2}{\Delta t} C_{i1}[(\{u_i^{p+1}\} - \{u_i^p\}) - (\{U_i^{p+1}\} - \{U_i^p\})] + k_{11i1}(\{u_i^{p+1}\} + \{u_i^p\}) + k_{12i1}(\{U_i^{p+1}\} + \{U_i^p\}) + (1-n)k_{3i1}(\{u_{fi}^{p+1}\} + \{u_{fi}^p\}) \right\},$$

$$F' = B\{\dot{u}_{1x}^{p}\} + D\{\dot{U}_{1x}^{p}\} - \frac{\Delta t}{2} m_x \left\{ \frac{2}{\Delta t} C_{i1}[-(\{u_i^{p+1}\} - \{u_i^p\}) + (\{U_i^{p+1}\} - \{U_i^p\})] + k_{12i1}(\{u_i^{p+1}\} + \{u_i^p\}) + k_{13i1}(\{U_i^{p+1}\} + \{U_i^p\}) + nk_{3i1}(\{u_{fi}^{p+1}\} + \{u_{fi}^p\}) \right\}.$$

(4) The tangential displacements of node 1 on the interface between the fluid saturated porous medium and ideal fluid medium:

$$\{\dot{u}_{1y}^{p+1}\} = \{\dot{u}_{1y}^{p}\} - m_y M_{u1}^{-1} \left\{ C_{i1}\{(\{u_i^{p+1}\} - \{u_i^p\}) - (\{U_i^{p+1}\} - \{U_i^p\})\} + \frac{\Delta t}{2}\{k_{11i1}(\{u_i^{p+1}\} + \{u_i^p\}) + k_{12i1}(\{U_i^{p+1}\} + \{U_i^p\})\} \right\}, \tag{23}$$

$$\{\dot{U}_{1y}^{p+1}\} = \{\dot{U}_{1y}^{p}\} - m_y M_{U1}^{-1} \left\{ C_{i1}\{(\{U_i^{p+1}\} - \{U_i^p\}) - (\{u_i^{p+1}\} - \{u_i^p\})\} + \frac{\Delta t}{2}\{k_{12i1}(\{u_i^{p+1}\} + \{u_i^p\}) + k_{13i1}(\{U_i^{p+1}\} + \{U_i^p\})\} \right\}, \tag{24}$$

$$\{\dot{u}_{f1y}^{p+1}\} = \{\dot{u}_{f1y}^{p}\} - \frac{\Delta t}{2} m_y M_{f1}^{-1} k_{3i1}(\{u_{fi}^p\} + \{u_{fi}^{p+1}\}). \tag{25}$$

3.2. Explicit finite element expressions of nodes on boundary joint nodes between fluid saturated porous medium and elastic single-phase medium

3.2.1. Continuous conditions on joint nodes

Zhao [9] gave the continuous conditions on the joint line between fluid saturated porous medium and elastic single-phase media.

(1) The movements of elastic single-phase media and those of the solid phase and liquid phase of fluid saturated porous media are continuous in the normal direction of the inclined interface

$$\{u_{sx}\} = \{U_x\} = \{u_x\}. \tag{26}$$

(2) The movements of the solid phase of fluid saturated porous media and elastic single-phase media are continuous in the tangential direction of the inclined interface

$$\{u_{sy}\} = \{u_y\}. \tag{27}$$

(3) The global stress is continuous in the normal direction of the interface

$$\sigma_x = \sigma_{sx}. \tag{28}$$

(4) The stresses of the solid phase of fluid saturated porous media and elastic single-phase media are continuous in the tangential direction of the inclined interface

$$\tau_{xy} = \tau_{sxy}. \tag{29}$$

In Eqs. (28) and (29), σ_{sx} and τ_{sxy} are the normal and tangential stress of the elastic single-phase medium separately.

3.2.2. Displacement explicit finite element expressions of nodes on the joint line

Based on the explicit finite element expressions for the dynamic analysis of the fluid saturated porous medium and elastic single-phase medium, and the continuous conditions on the interface, the displacement explicit finite element expressions for the dynamic analysis of the nodes on the joint line can be developed.

(1) The normal displacements of node 1 on the interface between the fluid saturated porous medium and elastic single-phase medium:

$$\{u_{1x'}^{p+1}\} = \{u_{1x}^{p}\} + \Delta t\{\dot{u}_{1x}^{p}\} - \frac{\Delta t^2}{2} m_x \frac{1}{M_{S1} + M_{u1} + M_{U1}} \times (k_{1i1}\{u_{si}^{p}\} + k_{11i1}\{u_i^{p}\} + k_{12i1}\{U_i^{p}\} + k_{12i1}\{u_i^{p}\} + k_{13i1}\{U_i^{p}\}), \tag{30}$$

$$U_{1x}^{p+1} = u_{1x}^{p+1} = u_{s1x}^{p+1}. \tag{31}$$

(2) The tangential displacements of node 1 on the interface between the fluid saturated porous medium and elastic single-phase medium:

$$\{u_{1y}^{p+1}\} = \{u_{1y}^{p}\} + \Delta t\{\dot{u}_{1y}^{p}\} - \frac{\Delta t^2}{2} m_y \frac{1}{M_{s1} + M_{u1}} \{C_{i1}(\{u_i^{p}\} - \{U_i^{p}\}) + k_{11i1}\{u_i^{p}\} + k_{12i1}\{U_i^{p}\} + k_{1i1}\{u_{si}^{p}\}\}, \tag{32}$$

$$\{U_{1y}^{p+1}\} = \{U_{1y}^{p}\} + \Delta t\{\dot{U}_{1y}^{p}\} - \frac{\Delta t^2}{2} m_y M_{U1}^{-1}\{C_{i1}(\{U_i^{p}\} - \{u_i^{p}\}) + k_{12i1}\{u_i^{p}\} + k_{13i1}\{U_i^{p}\}\}, \tag{33}$$

$$u_{1y}^{p+1} = u_{s1y}^{p+1}. \tag{34}$$

(3) The normal velocities of node 1 on the interface between the fluid saturated porous medium and elastic single-phase medium:

$$\{\dot{u}_{1x}^{p+1}\} = \{\dot{u}_{1x}^{p}\} - \frac{\Delta t}{2} \frac{1}{M_{s1} + M_{u1} + M_{U1}} \times m_x\{k_{1i1}(\{u_{si}^{p}\} + \{u_{si}^{p+1}\}) + k_{11i1}(\{u_i^{p}\} + \{u_i^{p+1}\}) + k_{12i1}(\{U_i^{p}\} + \{U_i^{p+1}\}) + k_{12i1}(\{u_i^{p}\} + \{u_i^{p+1}\}) + k_{13i1}(\{U_i^{p}\} + \{U_i^{p+1}\})\}, \tag{35}$$

$$\dot{U}_{1x}^{p+1} = \dot{u}_{1x}^{p+1} = \dot{u}_{s1x}^{p+1}. \tag{36}$$

(4) The tangential displacements of node 1 on the interface between the fluid saturated porous medium and elastic single-phase media:

$$\{\dot{u}_{1y}^{p+1}\} = \{\dot{u}_{1y}^{p}\} - m_y \frac{1}{M_{u1} + M_{s1}} \left\{ C_{i1}\{(\{u_i^{p+1}\} - \{u_i^p\}) - (\{U_i^{p+1}\} - \{U_i^p\})\} + \frac{\Delta t}{2}\{k_{11i1}(\{u_i^{p+1}\} + \{u_i^p\}) + k_{12i1}(\{U_i^{p+1}\} + \{U_i^p\}) + k_{3i1}(\{u_{si}^{p+1}\} + \{u_{si}^p\})\} \right\}, \tag{37}$$

$$\{\dot{U}_{1y}^{p+1}\} = \{\dot{U}_{1y}^{p}\} - m_y M_{U1}^{-1} \left\{ C_{i1}\{(\{U_i^{p+1}\} - \{U_i^p\}) - (\{u_i^{p+1}\} - \{u_i^p\})\} + \frac{\Delta t}{2}\{k_{12i1}(\{u_i^{p+1}\} + \{u_i^p\}) + k_{13i1}(\{U_i^{p+1}\} + \{U_i^p\})\} \right\}, \tag{38}$$

$$\dot{u}_{1y}^{p+1} = \dot{u}_{s1y}^{p+1}. \tag{39}$$

3.3. Explicit finite element expressions of nodes on boundary joint nodes between ideal fluid medium and elastic single-phase media

3.3.1. Continuous conditions on joint nodes

Ref. [6] gave the continuous conditions on the joint line between ideal fluid medium and elastic single-phase medium.

(1) The movements are continuous in the normal direction of the interface

$$\{u_{fx}\} = \{u_{sx}\}. \tag{40}$$

(2) The global stress is continuous in the normal direction of the interface

$$\sigma_{fx} = \sigma_{sx}. \tag{41}$$

(3) The stress in the tangential direction of the interface is zero

$$\tau_{fxy} = \tau_{sxy} = 0. \tag{42}$$

In Eqs. (41) and (42), σ_{fx} and τ_{fxy} are the normal and tangential stress of the ideal fluid medium separately.

3.3.2. Displacement explicit finite element expressions of nodes on the joint line

Based on the explicit finite element expressions for the dynamic analysis of the elastic single-phase medium and ideal fluid medium, and the continuous conditions on the interface, the displacement explicit finite element expressions for the dynamic analysis of the nodes on the joint line can be developed.

(1) The normal displacements of node 1 on the interface between the elastic single-phase medium and ideal fluid medium:

$$\{u_{s1x'}^{p+1}\} = \{u_{s1x}^{p}\} + \Delta t\{\dot{u}_{s1x}^{p}\} - \frac{\Delta t^2}{2} m_x \frac{1}{M_{s1} + M_{f1}} (k_{1i1}\{u_{si}^{p}\} + k_{3i1}\{u_{fi}^{p}\}), \tag{43}$$

$$u_{f1x}^{p+1} = u_{s1x}^{p+1}. \tag{44}$$

(2) The tangential displacements of node 1 on the interface between the elastic single-phase medium and ideal fluid medium:

$$\{u_{s1y}^{p+1}\} = \{u_{s1y}^{p}\} + \Delta t\{\dot{u}_{s1y}^{p}\} - \frac{\Delta t^2}{2} m_y M_{s1}^{-1} k_{1i1}\{u_{si}^{p}\}, \tag{45}$$

$$\{u_{f1y}^{p+1}\} = \{u_{f1y}^{p}\} + \Delta t\{\dot{u}_{f1y}^{p}\} - \frac{\Delta t^2}{2} m_y M_{f1}^{-1} k_{3i1}\{u_{fi}^{p}\}. \tag{46}$$

(3) The normal velocities of node 1 on the interface between the elastic single-phase medium and ideal fluid medium:

$$\{\dot{u}_{s1x}^{p+1}\} = \{\dot{u}_{s1x}^{p}\} - \frac{\Delta t}{2} \frac{1}{M_{s1} + M_{f1}} m_x \{k_{1i1}(\{u_{si}^{p}\} + \{u_{si}^{p+1}\}) + k_{3i1}(\{u_{fi}^{p}\} + \{u_{fi}^{p+1}\})\}, \tag{47}$$

$$\dot{u}_{f1x}^{p+1} = \dot{u}_{s1x}^{p+1}. \tag{48}$$

(4) The tangential displacements of node 1 on the interface between the elastic single-phase medium and ideal fluid medium:

$$\{\dot{u}_{s1y}^{p+1}\} = \{\dot{u}_{s1y}^{p}\} - \frac{\Delta t}{2} m_y M_{s1}^{-1} k_{1i1}(\{u_{si}^{p+1}\} + \{u_{si}^{p}\}), \tag{49}$$

$$\{\dot{u}_{f1y}^{p+1}\} = \{\dot{u}_{f1y}^{p}\} - \frac{\Delta t}{2} m_y M_{f1}^{-1} k_{3i1}(\{u_{fi}^{p+1}\} + \{u_{fi}^{p}\}). \tag{50}$$

3.4. Some general remarks

Now the displacement and velocity explicit finite element expressions for dynamic analysis of the node on the joint line among the three media are given. The displacement and velocity expressions of nodes in the interfaces of any two media among the three media can be proposed by using a similar method. Then the different responses of each node corresponding to different time steps can be calculated according to the numerical order of nodes by using the calculating steps given by Ref. [5].

4. Application of this method to the analysis of the dynamic response of dams

Sediments will come into being beside the feet of dams after reservoirs run for a period of time. The presence of sediments will change the characteristic of dynamic response of dams. Knowledge on this is essential for the anti-seismic design of new dams and the assessment of earthquake safety of existing dams. The analyses of the dynamic response of dams are a very complex problem. To solve, this problem needs a complete mode of seismic response analysis of dam–water–sediment–foundation system. The model involves the dynamic response of solid medium, the fluid saturated porous medium, fluid medium, and the dynamic interactions among them. At the same time the case of complex medium and complex geometrical shapes near dams and the wave propagation out of the semi-infinite foundation should be considered, too. Therefore, an effective numerical method must be provided for the dynamic analysis of this large complex system.

In this section, considering the sediment as the fluid saturated porous medium, making use of the explicit finite element method proposed above and introducing the artificial boundaries [10,11], a model used to analyze the dynamic responses of three-media coupled systems is set. This model cannot only be used to consider the dynamic dam–water–sediment–rock interaction strictly, but also be used to analyze the seismic response of the dams with inclined surface. And in this method, the dynamic interactions among structure, water, sediment and foundation are considered strictly. Applying these numerical methods, the influence of sediment's thickness on the vertical displacement at the top point of dams during earthquakes is analyzed mainly.

4.1. The computation model and the material properties

The computation model and mesh division are shown in Fig. 3. In Fig. 3, 1 represents rock; 2 represents dam; 3 represents sediment; and 4 represents water. To satisfy the requirements of precision and stability, the edge lengths of an element are 20 m in x-direction and 10 m in y-direction. Quadrangle elements and triangle elements with constant strain are used in the grids. The left and right boundaries of the rock are artificial boundaries. The material properties of five media are shown in Tables 1 and 2. In these media, the foundation rock and the dam are considered as elastic single-phase media. The sediment is considered as fluid saturated porous media. The water is considered as ideal fluid. The incidence wave field is a vertical sine wave (P wave) whose amplitude is 1.

In Table 1, N and A are similar to Lame's constants μ and λ of the solid phase of the sediment. ρ_s and ρ_f are the mass densities of the solid phase and liquid phase of the sediment. R and Q are the same as those in Biot dynamic equations. n represents the porosity of the sediment. K represents the coefficient of permeability. In Table 2, ρ, λ and μ represent the mass density and Lame constants of the corresponding material, respectively. In Table 2, K_L represents the compressibility coefficient of the water.

4.2. Result and analysis

The different responses of the vertical displacement at the top point of dams, corresponding to the different frequencies of the input vertical sine wave while other conditions are kept unchanged, are shown in Figs. 4–6. These figures show that the sediments will change the formant and the

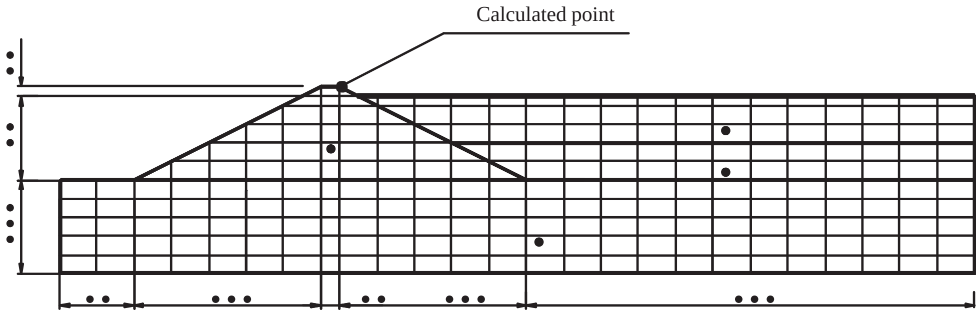

Fig. 3. Calculating model and mesh dividing.

Table 1
Material properties of sediment

A (Pa)	N (Pa)	n	ρ_s (kg/m^3)	ρ_f (kg/m^3)	Q (Pa)	R (Pa)	K (cm/s)
5.5×10^8	7.7×10^6	0.60	2640	1000	0.8×10^9	1.2×10^9	0.001

Table 2
Material properties of the dam, foundation rock, and water

Dam			Foundation rock			Water	
ρ (kg/m^3)	λ (Pa)	μ (Pa)	ρ (kg/m^3)	λ (Pa)	μ (Pa)	ρ (kg/m^3)	K_L (Pa)
2700	4.2×10^9	1.85×10^9	2800	14.4×10^9	4.4×10^9	1000	2.0×10^9

resonance frequency of the dams. With the increase of the sediment's thickness, the resonance frequency of the dams is decreased and the vertical displacement at the top point of dams is decreased too.

5. Closing remark

In the paper, an explicit finite element method for dynamic response analysis with fluid saturated porous media has been developed. The method is applied to analyze the seismic response of a reservoir with the consideration of the dynamic interactions among water, dam, sediment and foundation rock. The vertical displacement at the top point of dams is calculated and some conclusions are given. From this conclusion, we know that the sediment's thickness will affect the dynamic response of the dams.

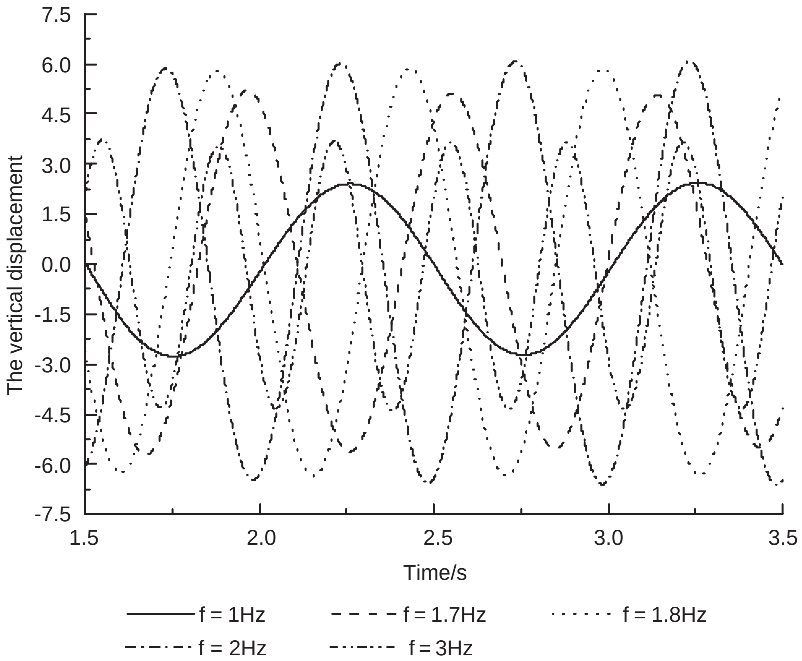

Fig. 4. The vertical displacement at the top point of the dam corresponding to different frequencies when the sediment's thickness = 10 m.

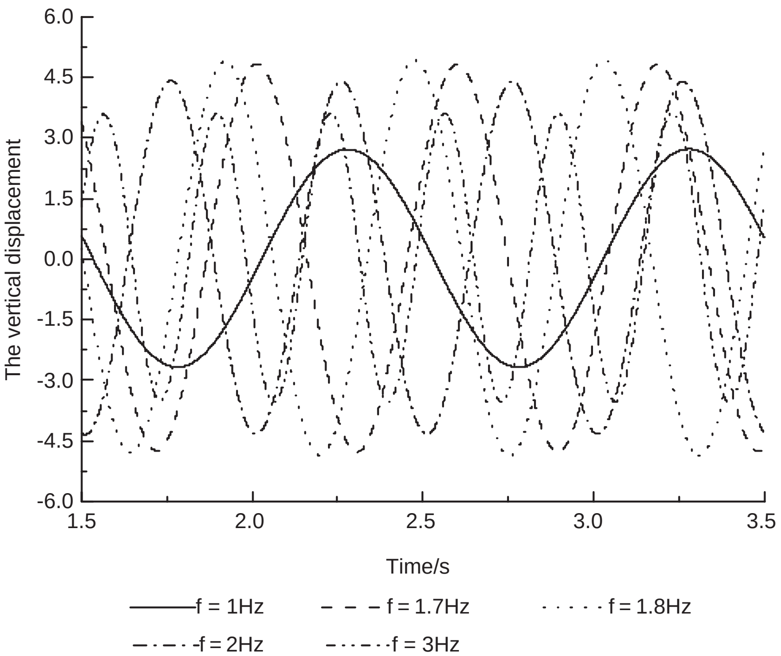

Fig. 5. The vertical displacement at the top point of the dam corresponding to different frequencies when the sediment's thickness = 20 m.

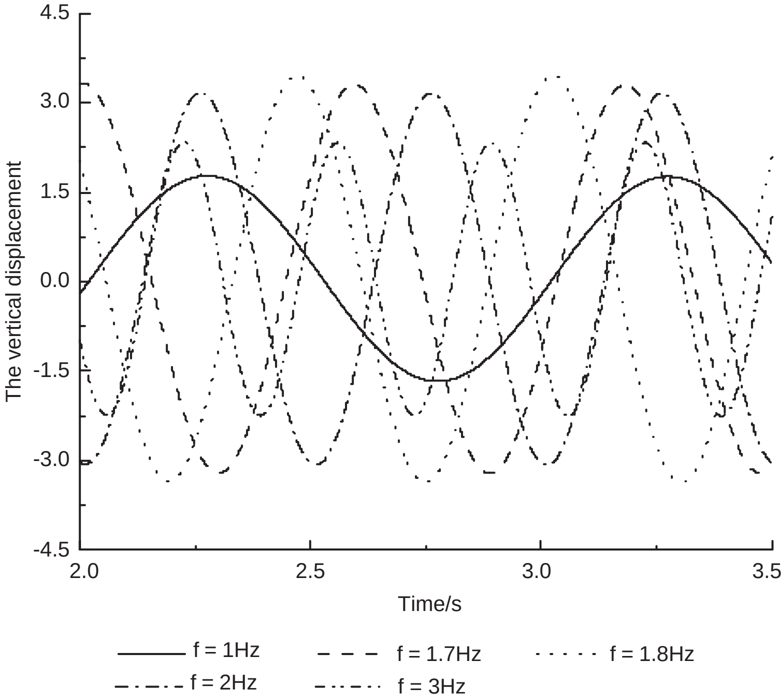

Fig. 6. The vertical displacement at the top point of the dam corresponding to different frequencies when the sediment's thickness = 40 m.

It also shows that computational effort and memory requirement can be reduced considerably by using the method developed in the paper. If the method is developed to deal with nonlinear problems, it will be more powerful. Of course, no method is perfect; each method must have its advantages and disadvantages. The main disadvantage of the proposed method in the paper is the problem of numerical calculating instability. Nonlinear problems, numerical calculating instability problems and other practical applications cannot be discussed in this paper due to the limitation of the length of the paper. They will be presented in other papers.

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