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Short Communication

On chaotic vibrations of a non-ideal system with two degrees of freedom: 1 : 2 resonance and Sommerfeld effect

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1. Introduction

The excitations of the vibratory system studied here are considered to be limited, that is, on one hand, the real capacity of a particular energy source, which is defined by its characteristic; on the other, it is limited by the dependence of the motion of the oscillatory system on the motion of the energy source (note that this connection is expressed by coupling between the differential equations of motion and of the energy source). Since the influence of a non-ideal energy source on a vibrating system depends on the state of its motion, it is impossible to express this action as an explicit function of time. Thus an oscillating system with a non-ideal source must be considered as autonomous.

We also know that the jump phenomenon and the increase in power required by a source operating near resonance are a manifestation of a non-ideal energy source and are referred to as Sommerfeld effect. One of the problems often faced by designers is how to drive the system

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through resonance and avoid the energy sink described by the Sommerfeld effect. For complete details on non-ideal system, see [1–5].

In this note we will analyze the nonlinear dynamics of a non-ideal vibration problem shown in Fig. 1. It consists of a mass m_1 supported by a linear elastic spring with coefficient of elasticity k_1 and a linear damper with viscous damping coefficient c_1. On the block of mass m_1 a non-ideal motor is placed, with a driving rotor of moment of inertia J and an eccentric mass m_0 situated at a distance r from the axis of rotation. By means of a linear spring with coefficient of elasticity k_2 and a damper with coefficient of damping c_2, a body of mass m_2 is placed on the mass m_1. The other parameters are: L is the torque generated by the DC motor of limited power supply, H is the resisting torque, which is ignored from now on, and U is the voltage of the DC motor.

In the present study we consider this vibrating problem operating near internal resonance 1 : 2. Before, we studied the particular case of internal resonance 1 : 1 [6]. We also mention that the minimization of the time of passage through resonance by using a synthesis of control based on Tikhonov's regularization was applied in this problem [7].

Here, we take the Lagrange equations of motion, which may be written in adimensional form as before (see Refs. [6,7]):

$$\chi_1'' = -\chi_1 - \eta_1\chi_1' + \mu(\chi_2 - \chi_1) + \eta_2(\chi_2' - \chi_1') + \overline{\omega}^2\cos\overline{\varphi} + \overline{\omega}'\sin\overline{\varphi},$$

$$\chi_2'' = -\theta^2(\chi_2 - \chi_1) - \frac{\theta^2}{\mu}\eta_2(\chi_2' - \chi_1'),$$

$$\overline{\omega}' = \lambda + \rho\chi_1''\sin\overline{\varphi}, \quad \lambda' = -\alpha\lambda - \beta\overline{\omega} + u, \tag{1}$$

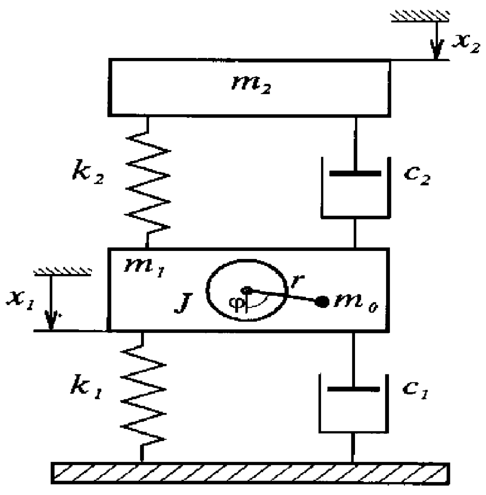

Fig. 1. Non-ideal vibration problem with two degrees of freedom consisting of two blocks coupled with springs and dampers. The DC motor with limited power supply and eccentric mass play the part of the non-ideal perturbation source.

Table 1
Possible 1 : 2 resonance's

Physical parameters	Resonance 1 : 2(A)	Resonance 1 : 2(B)
m_0	0.05	0.05
m_1	1.0	1.0
m_2	0.25	0.35
k_1	0.13	0.07
k_2	0.01	0.04
c_1	0.0	0.0
c_2	0.01	0.01
J	0.001	0.001
r	0.1	0.1
$\overline{\omega}_1$	0.5269	0.7975
$\overline{\omega}_2$	1.0488	1.6007

$$\eta_1 = c_1/\sqrt{k_1 m_1}, \quad \eta_2 = c_2/\sqrt{k_1 m_1}, \quad \mu = k_2/k_1, \quad \theta^2 = \mu m_1/m_2,$$
$$\rho = m_0^2 r/J m_1, \quad \alpha = a\sqrt{\tfrac{m_1}{k_1}}, \quad \beta = b\frac{m_1}{J k_1}, \tag{2}$$

where χ_1 and χ_2 are dimensionless coordinates of the blocks 1 and 2; τ is dimensionless time; $\overline{\varphi}(\tau) = \overline{\omega}\tau + \overline{\delta}$ is the angular motion of the mass relative to τ; $\overline{\omega}$ is the rotor frequency relative to τ and λ is the dimensionless torque; u is the dimensionless voltage; and a and b are parameters related to the DC motor.

Note that if we choose suitably the physical parameters of the considered problem it is possible to obtain the natural frequencies $\overline{\omega}_1$ and $\overline{\omega}_2$ in resonance 1 : 2. The adopted frequencies $\overline{\omega}_1$ and $\overline{\omega}_2$ of the non-ideal vibrating problem defined by Fig. 1 and Eqs. (1) are given by Table 1. Next we will present some numerical results.

2. Numerical simulations results

We carried out a number of numerical simulations and obtain some results that we will present next through Figs. 2–6.

In Fig. 2(a) we show the behavior of the maximum amplitude of blocks 1 and 2 when we vary the rotor frequency $\overline{\omega}$ continuously. It is possible to observe the Sommerfeld effect when we got the maximum value during the passage through resonance at $\overline{\omega}_2 = 1.0488$. On the other hand, in the resonant case 1 : 2(B) (Fig. 2(b)), this effect occurs in the passage through resonance at a smaller value of natural frequency ($\overline{\omega}_1 = 0.7975$).

In vibrating nonlinear systems, the Sommerfeld effect is an important irregular sink of irregular vibrations. In Figs. 3–5, the higher pattern of vibration is shown (upper maximum amplitude contour), of χ_1 before, during and after passage through resonances. The vibrations patterns are similar to those obtained by Ref. [1]. Figs. 3–5 correspond to (a) the 1 : 2(A) resonance and (b) to the resonance 1 : 2(B).

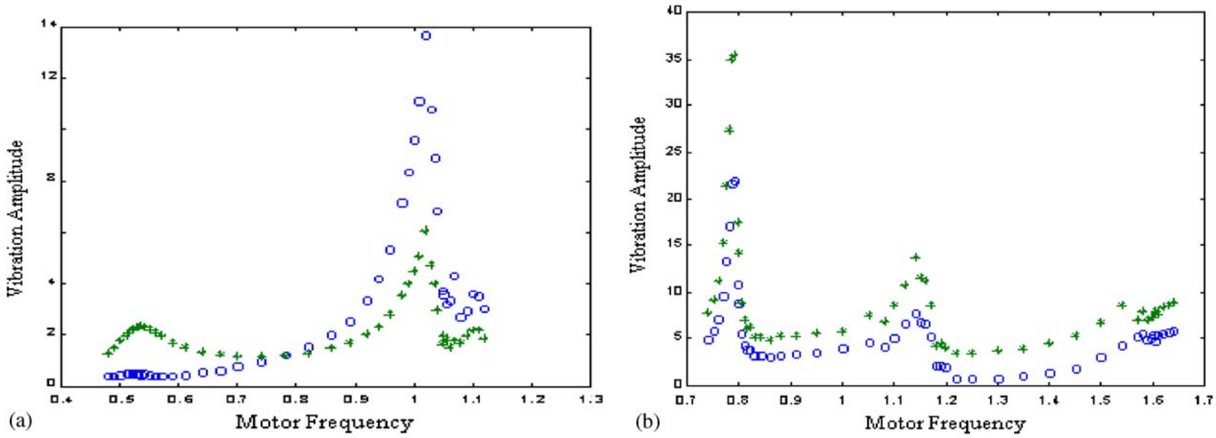

Fig. 2. Passage to the resonances and the Sommerfeld effect. The symbols 'o' and '$*$' correspond to the maximum amplitude of χ_1 and χ_2, respectively. (a) Corresponds to the internal resonance 1 : 2(A). The Sommerfeld effect occurs on the passage $\overline{\omega} = \overline{\omega}_2 = 1.0488$. (b) Corresponds to the internal resonance 1 : 2(B). The Sommerfeld effect occurs on the passage $\overline{\omega} = \overline{\omega}_1 = 0.7975$.

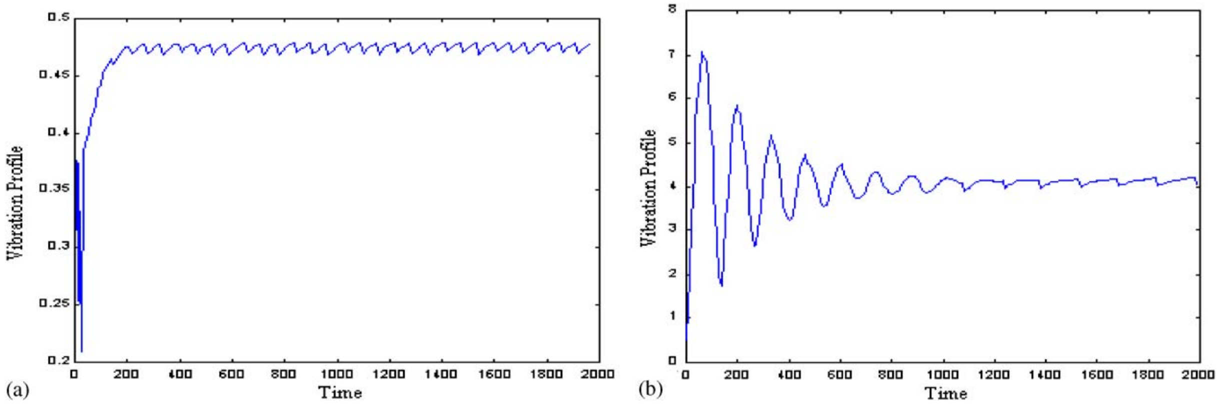

Fig. 3. Top profile of vibrations χ_1, before the Sommerfeld effect. (a) Corresponds to the passage $\overline{\omega} = \overline{\omega}_1 = 0.5269$ of resonance 1 : 2(A). (b) Corresponds to $\overline{\omega} = 0.75$ for resonance 1 : 2(B).

3. Conclusions

We analyzed the vibrations of a nonlinear and non-ideal vibrating system with two degrees of freedom (Fig. 1), when the frequencies of vibration are in internal resonance 1 : 2. We take into account values of Table 1 and seek for the Sommerfeld effect in the internal resonance 1 : 2. In the case of resonance 1 : 2(A), this effect appears for $\overline{\omega} = \overline{\omega}_2 = 1.0488$ (Fig. 2(a)). On the other hand, for resonance 1 : 2(B) this effect occurs when $\overline{\omega} = \overline{\omega}_1 = 0.7975$ (Fig. 2(b)).

Here, we seek irregular (chaotic) vibrations, i.e., when one increases the frequency of the rotor $\overline{\omega}$, the vibrations become more irregular than before. The Sommerfeld effect apparently

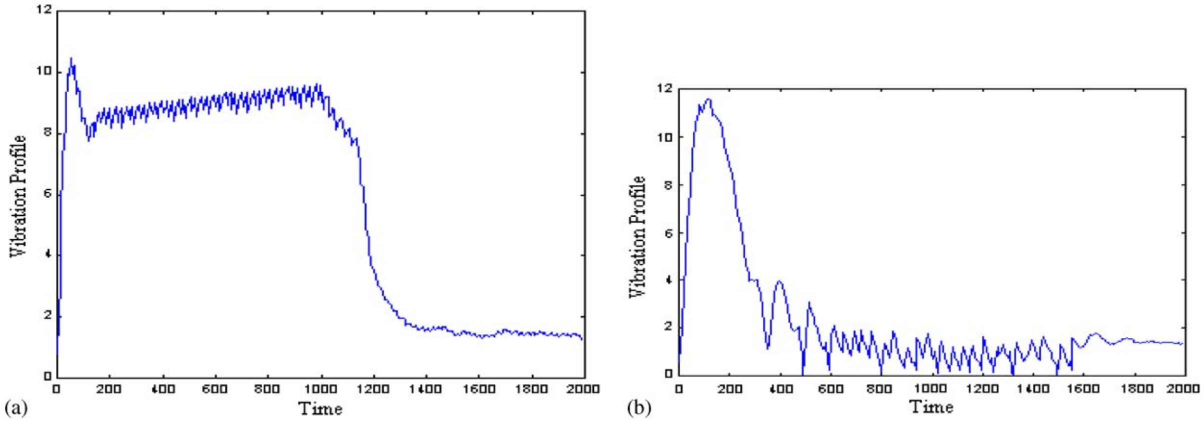

Fig. 4. Top profile of vibrations χ_1. (a) Shows the profile to $\overline{\omega} = 1.0$ (immediately before the Sommerfeld effect) in resonance 1 : 2(A). (b) Corresponds to the passage $\overline{\omega} = \overline{\omega}_1 = 0.7975$ of resonance 1 : 2(B).

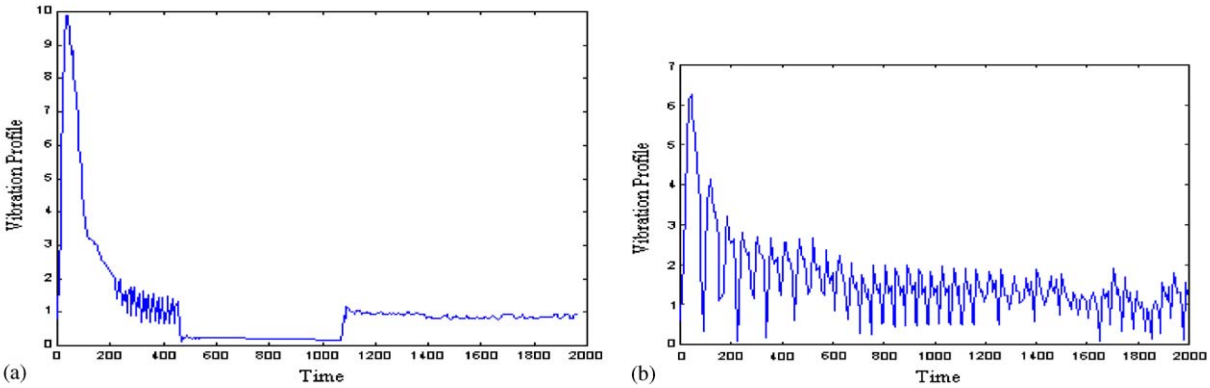

Fig. 5. Top profile of vibrations χ_1. (a) The profile in the passage $\overline{\omega} = \overline{\omega}_2 = 1.0488$ in the case 1 : 2(A). (b) Shows the profile to $\overline{\omega} = 0.84$ (after to the Sommerfeld effect).

contributes to transform a regular vibration to an irregular (chaotic) one. It is known that this kind of effect causes the vibrations to be always chaotic [5].

Figs. 2(a)–6(a) show the results of resonance 1 : 2(A) and Figs. 2(b)–6(b) for resonance 1 : 2(B). In case B, the vibrations become more irregular than case A and present a discreet jump phenomenon when $\overline{\omega} \approx 1.15$.

We also mention that the nonlinearity of this vibrating system is of the periodic kind due to the interaction between both perturbation (DC motor) and the vibrating system. The complex behavior of one vibrating system increases when the nonlinearity become strong as in the cases when we take into account quadratic and cubic terms.

Finally, it should be mentioned that in Fig. 7 we show the passage through resonance for the particular case of resonance 1 : 2(A) considering a cubic term, and we can observe the jump phenomenon.

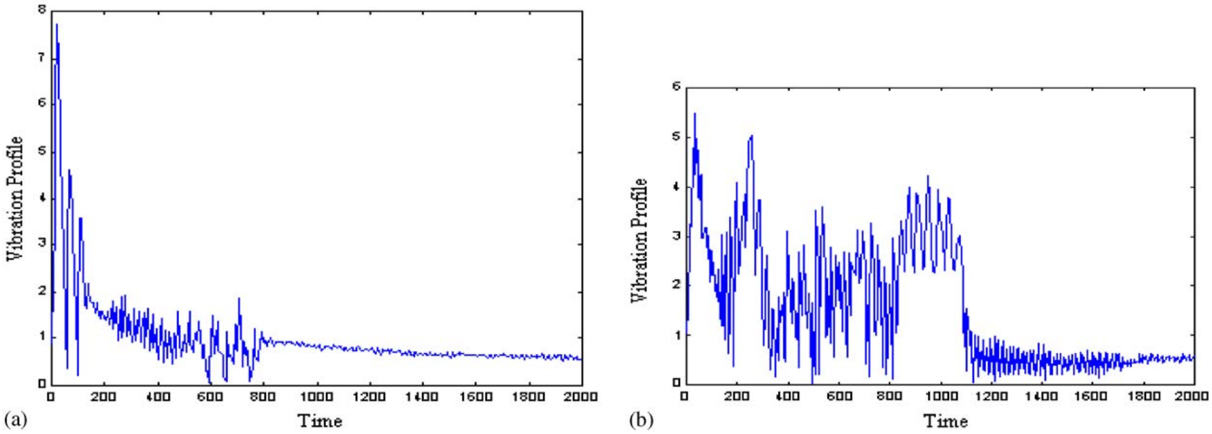

Fig. 6. Top profile of vibrations χ_1. (a) Corresponds to $\overline{\varpi} = 1.1$ of resonance 1 : 2(A). (b) Corresponds to $\overline{\varpi} = \overline{\varpi}_2 = 1.6007$ of resonance 1 : 2(B).

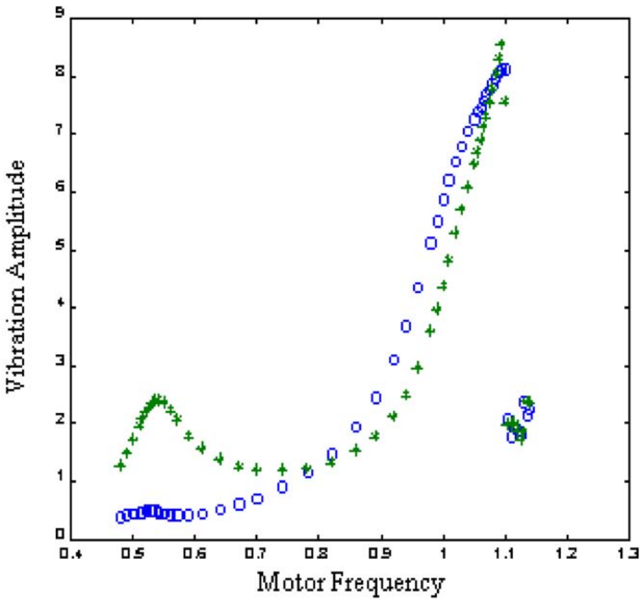

Fig. 7. Passage to resonance and Sommerfeld effect (key as in Fig. 2).

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