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Short Communication

Effect of skewness on fatigue life with mean stress correction

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1. Introduction

Most loadings in nature can be modeled as Gaussian processes in light of the central limit theorem. However, structural responses can be highly non-Gaussian. For example, the response of compliant offshore structures can be non-Gaussian even under Gaussian wave loadings [1]. This paper will study the influence of non-normality on the fatigue life of structures.

The non-normality is commonly characterized in terms of two parameters, namely, skewness and kurtosis. Lutes and colleagues found out that kurtosis can significantly influence the rate of fatigue damage accumulation [2]. For narrowband symmetrical non-Gaussian processes, a closed form expression of the fatigue life prediction including the effect of kurtosis can be derived [3–5]. For broadband non-Gaussian processes, the effect of kurtosis on fatigue life can be studied only via Monte Carlo simulation. It has been shown that the mean damage accumulation rate increases when the kurtosis increases. Sarkani confirmed this general trend with fatigue experiments of welded joints [6].

The effect of skewness on fatigue life is less studied in the literature, even though the effect of skewness can also be significant [7]. Yu and colleagues discovered that the mean damage accumulation rate decreases with an increase of skewness. They assumed that fatigue damage only depends on stress ranges of the stress cycle and the mean stress is ignored. For slightly skewed non-Gaussian processes, the mean values of the stress cycle are small and the assumption is valid. However, for highly skewed non-Gaussian processes, the mean stresses of the stress cycle are no longer small. It is known that the mean stress can either increase or decrease fatigue damages [8,9].

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Hence, it is appropriate to include the mean stress when we study the effect of skewness on fatigue life. This paper will contribute to the fatigue knowledge in this regard. Since the spectrum of the stress also influences the fatigue life, we shall consider the same pre-specified power spectral density function of the stress.

The remainder of the paper is organized as follows. In Section 2, we review the method to simulate non-Gaussian processes and the S–N curve method for fatigue analysis. Some common mean stress correction formulas are discussed. In Section 3, we study the effect of skewness on fatigue life with mean stress correction for both narrowband and broadband stress processes. In Section 4, we conclude the paper with some remarks.

2. Preliminaries

2.1. Simulation of non-Gaussian processes

It can be shown that an arbitrary second-order random process $X(t)$ can be represented in a mean square convergent series as [10,11]

$$X_0 \equiv \frac{X-\mu_X}{\sigma_X} = g[U(t)] = k\left[U + \sum_{n=3}^{N} a_n H_{n-1}(U)\right], \tag{1}$$

where $U(t)$ is a standard zero mean unit variance Gaussian process, μ_X and σ_X are the mean and standard deviation of $X(t)$, respectively. k is a scaling factor ensuring that $X_0(t)$ has unit variance. $H_n(u) = (-1)^n e^{u^2/2} \frac{d^n}{du^n} e^{-u^2/2}$ is the nth order Hermite polynomial. Winterstein has found that the first four moments are sufficient to capture a great deal of non-Gaussian characteristics of the structural response [4]. For this reason, we have chosen $N = 4$ in this paper. Eq. (1) becomes

$$X_0(t) = k\{U(t) + a_3[U(t)^2 - 1] + a_4[U(t)^3 - 3U(t)]\}, \tag{2}$$

where the coefficients can be determined by matching the moments of $X_0(t)$ up to fourth order,

$$\begin{aligned} 1 &= k^2(1 + 2a_3^2 + 6a_4^2), \\ \alpha_3 &= k^3(108a_3a_4^2 + 8a_3^3 + 36a_3a_4 + 6a_3), \\ \alpha_4 &= k^4(3348a_4^4 + 2232a_3^2a_4^2 + 1296a_4^3 + 576a_3^2a_4 \\ &\quad + 60a_3^4 + 252a_4^2 + 24a_4 + 60a_3^2 + 3), \end{aligned} \tag{3}$$

where α_3 and α_4 are the skewness and kurtosis of $X_0(t)$.

In the numerical examples presented later, we consider the following spectral density function of $U(t)$:

$$\Phi_{UU}(\omega) = \frac{\Phi_1}{(\omega_1^2 - \omega^2)^2 + (2\varsigma_1\omega_1\omega)^2} + \frac{\Phi_2}{(\omega_2^2 - \omega^2)^2 + (2\varsigma_2\omega_2\omega)^2}. \tag{4}$$

Since $U(t)$ has a unit variance, the following equation holds:

$$\frac{\pi\Phi_1}{2\varsigma_1\omega_1^3} + \frac{\pi\Phi_2}{2\varsigma_2\omega_2^3} = 1. \tag{5}$$

The corresponding correlation function can be calculated as

$$R_{UU}(t) = \sum_{i=1}^{2} \sigma_{ui}^2 \mathrm{e}^{-\varsigma_i \omega_i |t|} \left(\cos \omega_{di} t + \frac{\varsigma_i}{\sqrt{1-\varsigma_i^2}} \sin \omega_{di} |t| \right), \tag{6}$$

where $\sigma_{ui} = \pi \Phi_i / \left(2\varsigma_i \omega_i^3\right)$, $\omega_{di} = \omega_i \sqrt{1-\varsigma_i^2}$, $i = 1, 2$.

We will vary the parameters in Eq. (4) to adjust the bandwidth of the stress. After numerically solving Eq. (3) for the coefficients k, a_3 and a_4 in terms of α_3 and α_4, we use Eq. (2) to transform the Gaussian processes into non-Gaussian processes with the predetermined α_3 and α_4. Note that the non-Gaussian stress processes studied in this paper are zero-mean, asymmetrically distributed.

2.2. Fatigue prediction

The fatigue property of the material can be characterized by the S–N curve defined as

$$N_F s^b = K, \tag{7}$$

where s is the stress amplitude, N_F is the number of cycles to failure. K and b are material constants. For the stress cycle with a large mean, the effect of the mean stress has to be considered in the fatigue life prediction. An equivalent stress amplitude including the effect of the mean stress on fatigue life can be developed according to one of the following common correction formulas [8,12]:

$$\begin{aligned} &\text{Goodman:} \quad \frac{s}{s_{\mathrm{eq}}} + \frac{s_m}{s_f} = 1, \\ &\text{Gerber:} \quad \frac{s}{s_{\mathrm{eq}}} + \left(\frac{s_m}{s_f}\right)^2 = 1, \\ &\text{Soderberg:} \quad \frac{s}{s_{\mathrm{eq}}} + \frac{s_m}{s_y} = 1, \end{aligned} \tag{8}$$

where s_{eq} is the equivalent stress amplitude, s_m is the mean stress, s_f is a material constant related to the true fracture strength and s_y is the material yielding strength. Kihl and Sarkani found out that the Goodman correction can provide reasonable and conservative fatigue prediction with both tensile and compressive mean stresses [9]. This paper will adopt the Goodman correction to account for the mean stress in the fatigue analysis.

With the Goodman correction, Eq. (7) becomes

$$N_F = K \left(\frac{s_f s}{s_f - s_m} \right)^{-b}. \tag{9}$$

For high cycle fatigue, the Palmgren–Miner linear damage accumulation rule can be applied [8]. Denote the joint probability density function of the stress amplitude and the mean stress as $\rho(s, s_m)$. Then, the average damage is given by

$$D = N_c \int_{-\infty}^{\infty} \int_0^{\infty} \frac{\rho(s, s_m)}{N_F} \, \mathrm{d}s \, \mathrm{d}s_m = N_c E\left(\frac{1}{N_F}\right) = \frac{N_c}{K} E\left[\left(\frac{s_f s}{s_f - s_m} \right)^b \right], \tag{10}$$

where N_c is the number of cycles in the stress process from the rainflow counting scheme [13]. $E[\cdot]$ denotes the mathematical expectation.

The mean fatigue life in cycles N_r is reached when $D = 1$,

$$N_r = \frac{K}{E\left[\left(\frac{s_f s}{s_f - s_m}\right)^b\right]}. \tag{11}$$

Because fatigue is a random process, a sufficiently large number of the stress samples should be used to count the damage events in order to calculate the fatigue life.

3. Effect of skewness

3.1. Narrowband process

Assume that $U(t)$ is narrowband. For narrowband processes, a peak in $U(t)$ or $X(t)$ is followed by a trough with approximately the same level as the peak S_p and $g(S_p)$. Thus, the magnitude of a rainflow cycle in $X(t)$ can be obtained as

$$S = \tfrac{1}{2}[g(S_p) - g(-S_p)] = k\sigma_x[S_p + a_4(S_p^3 - 3S_p)] \tag{12}$$

and the mean stress is

$$S_m = \tfrac{1}{2}[g(S_p) + g(-S_p)] = k\sigma_x a_3(S_p^2 - 1), \tag{13}$$

where S_p follows the Rayleigh distribution with $E[S_p^2] = 2$. Assume that $|a_3| \ll 1$ and $a_4 \ll 1$. Substituting S and S_m into Eq. (10), we obtain

$$D_{nb} = \frac{N_c}{K}(2\sqrt{2}\sigma_x)^b \Gamma\left(\frac{b}{2} + 1\right)\gamma_{nb}, \tag{14}$$

where $\Gamma(\cdot)$ is the Gamma function and γ_{nb} is a correction factor that reflects the effects of non-normality due to kurtosis and skewness given by

$$\gamma_{nb} = k^b\left[1 + b(b-1)a_4 + b(b+1)ka_3\frac{\sigma_x}{s_f}\right]. \tag{15}$$

When the mean stress correction is ignored, γ_{nb} is reduced to that derived by Winterstein including only the effect of kurtosis [3]. The first part on the right-hand side of Eq. (14) is the fatigue damage under narrowband Gaussian process.

In the numerical examples considered here and below, the stress process $X(t)$ is assumed to be zero mean asymmetrically distributed. The standard deviation of the process is assumed to be 6.5 ksi. The skewness of the processes will be varied to study its effect on fatigue life. For each stress process, 30 time history samples are simulated and each sample has 2^{15} points. The simulated stress samples are then put through the rainflow counting procedure to collect the amplitude and mean of the stress cycle. The number of rainflow cycles collected is generally more than 100,000. The material constant used in the fatigue analysis are $b = 3.21$, $K = 3.6308 \times 10^9$ ksi, $s_f = 98.5$ ksi [9].

The non-normality correction factor γ_{nb} as a function of skewness is shown in Fig. 1. It can be seen from the figure that γ_{nb} decreases when the absolute value of skewness increases. Since the expected fatigue damage is proportional to the non-normality correction factor, the fatigue life will increase with skewness. This general trend is consistent with the observation by Yu and colleagues [7]. They also observed that the non-normality correction factor follows the same trend when the sign of skewness is changed. Note that the mean stress is ignored in their study. When the mean stress is included in the fatigue calculation, we observe that γ_{nb} decreases faster with negative skewness than positive skewness due to the effect of mean stress on the fatigue damage.

It can be seen that Eq. (15) agrees with the simulation results very well up to kurtosis $\alpha_4 = 7$. When $\alpha_4 = 10$, Eq. (15) gives unconservative prediction of the fatigue damage. When $\alpha_4 < 10$, a_3 and a_4 are of order 10^{-2} satisfying the assumption $|a_3| \ll 1$ and $a_4 \ll 1$. When $\alpha_4 = 10$, a_4 is of order 10^{-1}, which violates the assumption that $a_4 \ll 1$. In this case, the integral in Eq. (10) should be used to calculate the fatigue damage.

3.2. Broadband process

For broadband non-Gaussian processes, no analytical expressions are available for the magnitude and mean of the rainflow stress cycles. Monte Carlo simulations are usually resorted to in fatigue analysis. Extensive numerical simulation has been done to study the effect of skewness on fatigue life.

Fig. 2 shows the numerical results of the effect of skewness on fatigue life. When the skewness is in the range of −0.5 to 1, the fatigue life curve appears flat. This suggests that the effect of skewness on fatigue life is not significant in this range. When the skewness is larger than 1 or less than −0.5, the increasing magnitude of skewness will result in higher fatigue life. The fatigue curve is asymmetrical with respect to skewness. The fatigue life increases faster with the negative skewness.

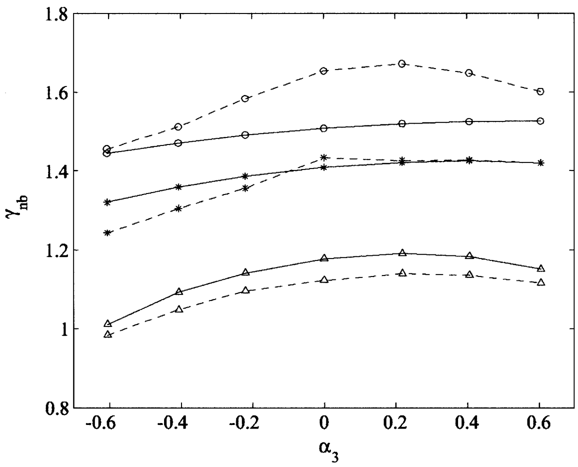

Fig. 1. The non-normality correction factor as a function of skewness. Solid line: Eq. (15). Dashed line: simulations. $\triangle$, $\alpha_4 = 4$; $*$, $\alpha_4 = 7$; $\circ$, $\alpha_4 = 10$.

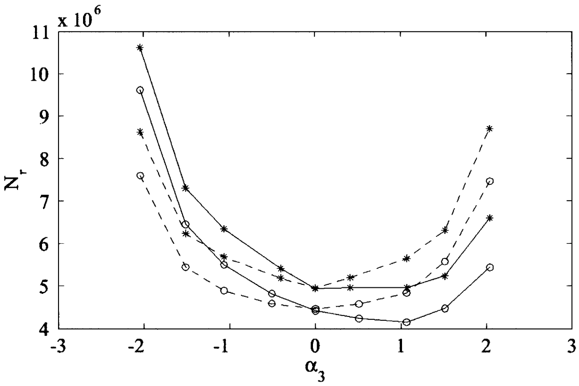

Fig. 2. The effect of skewness on fatigue life. Solid line: prediction with mean stress correction. Dashed line: prediction without mean stress correction. ∗, narrowband; ○, broadband.

Also shown in Fig. 2 are the fatigue predictions without mean stress correction. It can be seen that when the skewness is near zero, the fatigue life without the mean stress correction is of the same order as that calculated with the mean stress correction. This also suggests that the effect of small skewness on fatigue life can be ignored. For a larger positive skewness, the fatigue life prediction without mean stress correction becomes unconservative. However, for a larger negative skewness, it becomes conservative.

4. Concluding remarks

The effect of skewness on fatigue life with Goodman's mean stress correction is studied. For narrowband non-Gaussian processes, an analytical first-order approximation of the non-normality correction factor is derived as a function of both kurtosis and skewness. For broadband non-Gaussian processes, it is shown that the effect of skewness on fatigue life is asymmetrical. The increase of fatigue life with respect to skewness is faster when the skewness is negative than when it is positive.

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