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Anticontrol and synchronization of chaos for an autonomous rotational machine system with a hexagonal centrifugal governor

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Abstract

Anticontrol and synchronization of chaos for an autonomous rotational machine system with a hexagonal centrifugal governor are studied in the paper. Two different procedures for the design of the controller are proposed to anticontrol the governor system effectively. Finally, five methods are studied for chaos synchronization.
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1. Introduction

Anticontrol and synchronization of chaos have received great attention for many research activities in recent years [1–7]. Sometimes, chaos is not only useful but actually important. For example, chaos is desirable in many applications of liquid mixing while the required energy is minimized. For this purpose, making a non-chaotic dynamical system chaotic is called ‘‘anticontrol of chaos’’. Besides, secure communication and information processing are proposed as potential applications of synchronization of chaotic systems.

In previous researches, most of them were concentrated to several well-known systems, such as Lorenz system, Rössler system and Chua’s circuits system, etc. In this paper, an autonomous hexagonal centrifugal governor system is studied. It plays an important role in many rotational

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machines such as diesel engine, steam engine and so on. Two different procedures, linear and nonlinear controllers with certain feedback gain are proposed to anticontrol, i.e. to chaotify, the governor system. Four methods, linear feedback, nonlinear feedback, adaptive feedback, backstepping design and parameter evaluation from time sequences approaches are also discussed for synchronization of two coupled chaotic system.

2. Equations of motion

The rotational machine with centrifugal governor is depicted in Fig. 1. Some basic assumptions for the system are

(1) neglecting the mass of the rods and the sleeve;
(2) viscous damping in rod bearing of the fly-ball is presented by damping constant c.

From Fig. 1, the kinetic and potential energies of the system are written as follows:

$$T = 2 \times \left\{ \tfrac{1}{2} m \left[(r + l\ \sin\phi)^2 \eta^2 + l^2 \dot{\phi}^2 \right] \right\} = m\eta^2 (r + l\ \sin\phi)^2 + ml^2 \dot{\phi}^2,$$

$$V = 2kl^2(1 - \cos\phi)^2 + 2mgl(1 - \cos\phi),$$

where l, m, r and ϕ represent the length of the rod, the mass of fly ball, the distance between the rotational axis and the suspension joint, and the angle between the rotational axis and the rod. It is easy to obtain the Lagrangian

$$L = T - V = m\eta^2 (r + l\ \sin\phi)^2 + ml^2 \dot{\phi}^2 - 2kl^2 (1 - \cos\phi)^2 - 2mgl(1 - \cos\phi).$$

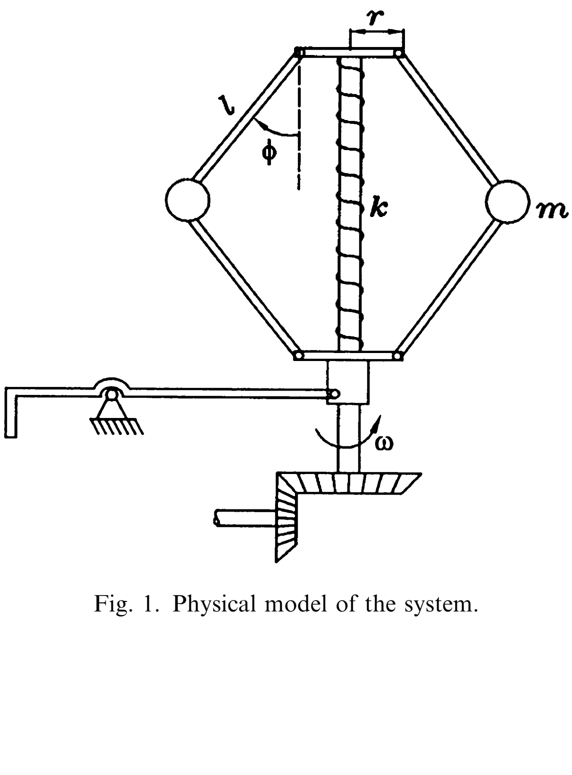

Fig. 1. Physical model of the system.

Then using Lagrange equation, the equation of motion for the governor can be derived as follows:

$$2\left[ml^2\ddot{\phi} - mrl\eta^2 \cos\phi - (2k + m\eta^2)l^2 \sin\phi \cos\phi + (2kl + mg)l \sin\phi\right] = -c\dot{\phi}, \tag{2.1}$$

where c is the damping coefficient.

For the rotational machine the net torque is the difference between the torque Q produced by the engine and the load torque Q_L, which is available for angular acceleration. That is,

$$J\frac{\mathrm{d}\omega}{\mathrm{d}t} = Q - Q_L, \tag{2.2}$$

where J is the moment of inertia of the machine. As the angle ϕ varies, the position of control valve which admits the fuel is also varied. The dynamical equation (2.2) can be written in the form [8]

$$J\dot{\omega} = \gamma\cos\phi - \beta, \tag{2.3}$$

where $\gamma > 0$ is a proportionality constant and β is an equivalent torque of the load. Eq. (2.3) is the second differential equation of motion for the system.

Usually, the governor is geared directly to the output shaft such that its speed of rotation is proportional to the engine speed, i.e. $\eta = n\omega$. The operation of the fly-ball governor can be briefly described as follows. At first, set the speed of engine at ω_0. If the speed of engine drops down, the centrifugal force acting on the fly-ball would decrease, thus the control valve of fuel will open wider. When more fuel is supplied, the speed of the engine increases until equilibrium is again reached. Similarly, if the speed rises up, the fuel supply is reduced and the speed decreases until ω_0 is recovered.

By changing the time scale $\tau = \Omega_n t$, Eqs. (2.1) and (2.3) can be written in nondimensional form

$$\begin{aligned}
&\dot{\phi} = \varphi,\\
&\dot{\varphi} = d\omega^2\cos\phi + (e + p\omega^2)\sin\phi\cos\phi - \sin\phi - b\varphi,\\
&\dot{\omega} = q\cos\phi - F,
\end{aligned} \tag{2.4}$$

where

$$q = \frac{\gamma}{J\Omega_n},\quad F = \frac{\beta}{J\Omega_n},\quad d = \frac{n^2 mr}{2kl + mg},\quad e = \frac{2kl}{2kl + mg},$$
$$p = \frac{n^2 ml}{2kl + mg},\quad b = \frac{c}{2ml^2\Omega_n},\quad \Omega_n = \sqrt{\frac{2kl + mg}{ml}},$$

and the dot presented the derivative with respect to τ, φ is $\mathrm{d}\phi/\mathrm{d}\tau$. Hence, the dynamics of the system of rotational machine with centrifugal governor is described by a three-dimensional autonomous system. Denoting $\phi = x, \dot{\phi} = y, \omega = z$, Eq. (2.4) is rewritten in the form

$$\begin{aligned}
&\dot{x} = y,\\
&\dot{y} = \mathrm{d}z^2\cos x + \tfrac{1}{2}(e + pz^2)\sin 2x - \sin x - by,\\
&\dot{z} = q\cos x - F.
\end{aligned} \tag{2.5}$$

3. Anticontrol of chaos

Anticontrol of chaos is making a nonchaotic dynamical system chaotic. This implies that the regular behaviors will be destroyed and replaced by chaotic behavior. In the real world, chaotic behavior is important. Examples include liquid mixing, human heartbeat regulations, resonance prevention in mechanical systems and secure communication [3]. In this section, Eqs. (2.5) are modified by the addition of a linear and a nonlinear feedbacks to chaotify the system, respectively.

3.1. Adding a linear feedback

The state equations of the centrifugal governor system with a linear feedback controller are represented as

$$\begin{aligned}\dot{x} &= y + a_1 x,\\ \dot{y} &= dz^2\cos x + \tfrac{1}{2}(e + pz^2)\sin 2x - \sin x - by + a_2 y,\\ \dot{z} &= q\cos x - F + a_3 z.\end{aligned} \tag{3.1}$$

Here a_1, a_2, a_3 are feedback gains and the values of parameters d, e, p, q, F, b are given as 0.008, 0.8, 0.04, 3, 2, 0.4, respectively.

By numerical integration method, the phase portrait of the system, Eq. (3.1), is plotted in Fig. 2 for $a_1 = a_2 = a_3 = 0$. Clearly, the motion is periodic. But Eq. (3.1) exhibits both strange attractors and limit cycles for certain choices of a_1, a_2 and a_3. For example, when $a_1 = 0.2$, $a_2 = -0.1$, $a_3 = -0.1$, one can observe a chaotic attractor as depicted in Fig. 3. By simulation results, for certain interval of parameters, the maximum Lypunov exponent of the system is positive, i.e., the system exhibits the strange attractor, we defined that chaotic region. Inspired by the consideration of chaotification, we found regions of specific feedback gains for which this system is chaotic as shown in Fig. 4.

3.2. Adding a nonlinear feedback

For our purpose, the nonlinear feedback controller, $\varepsilon x|x|$, is added to the right-hand side of the first equation of Eq. (2.5). Then the system equations are represented as

$$\begin{aligned}\dot{x} &= y + \varepsilon x|x|,\\ \dot{y} &= dz^2\cos x + \tfrac{1}{2}(e + pz^2)\sin 2x - \sin x - by,\\ \dot{z} &= q\cos x - F.\end{aligned} \tag{3.2}$$

System (3.2) is obtained for which certain value of ε (for example, $0.008 \leqslant \varepsilon \leqslant 0.032$) has strange attractor by numerical solution. As illustrated in Fig. 5, chaotic motion is observed from system (3.2) with $\varepsilon = 0.01$.

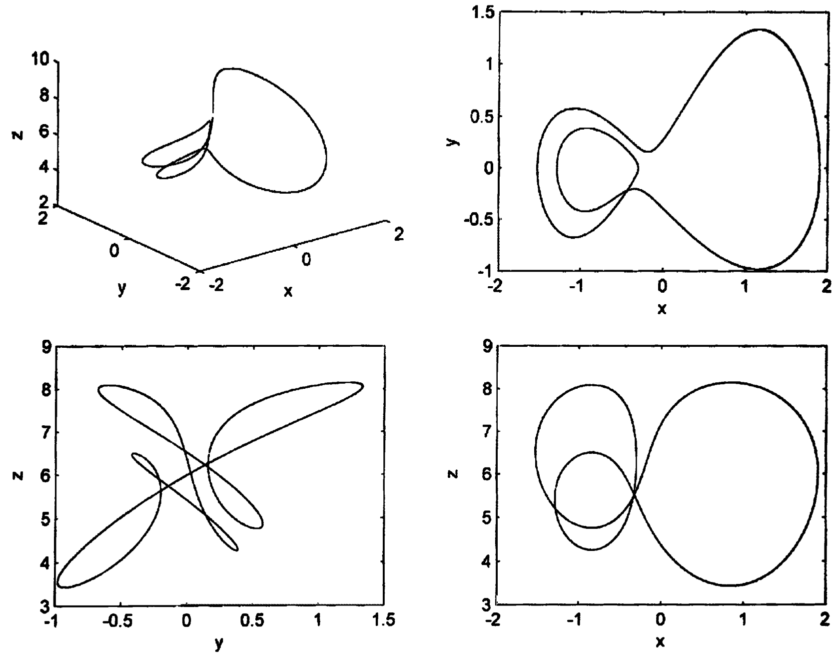

Fig. 2. Phase portrait of uncontrolled system ($a_1 = a_2 = a_3 = 0$).

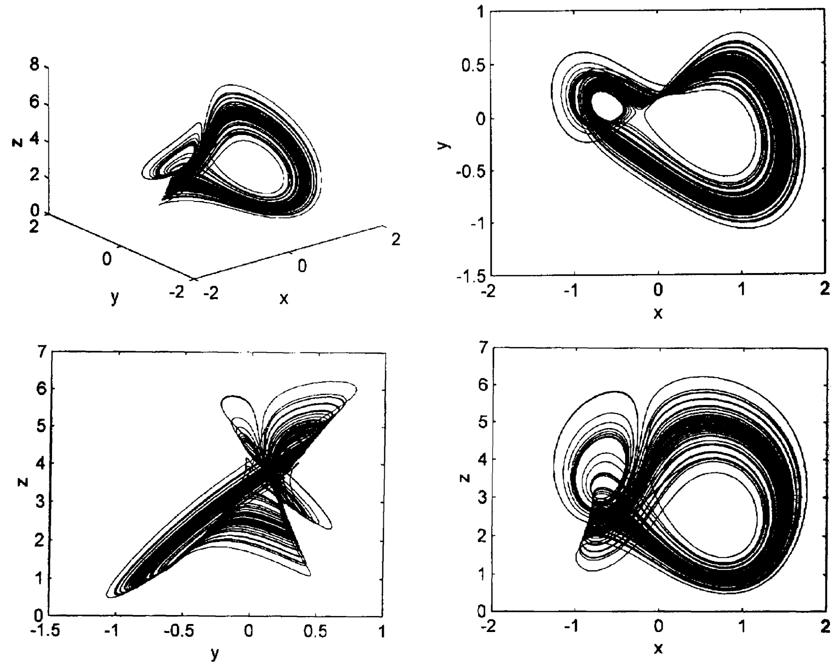

Fig. 3. Phase portrait of controlled system with $a_1 = 0.2$, $a_2 = -0.1$, $a_3 = -0.1$.

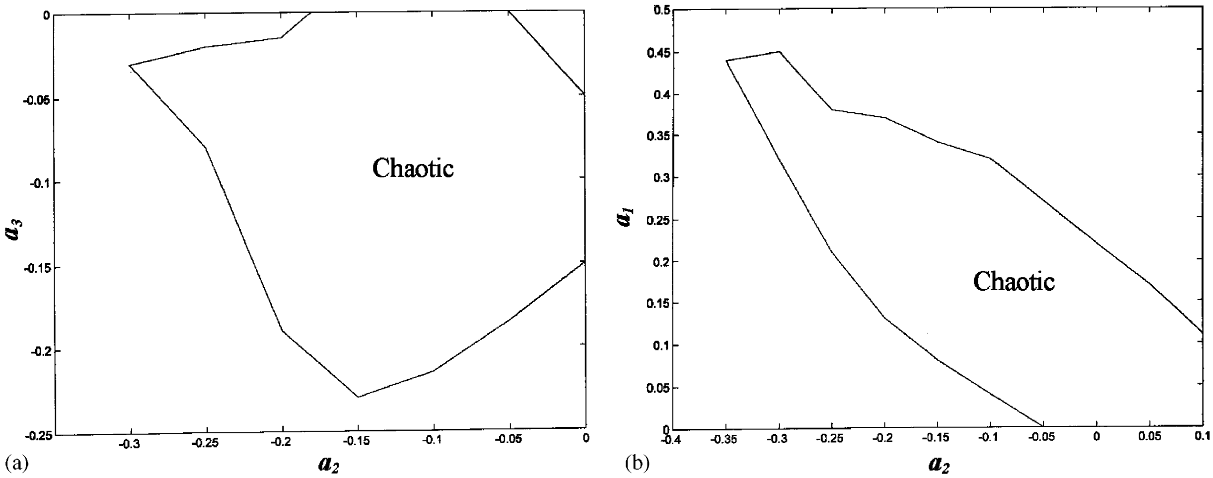

Fig. 4. Parameter diagram of (a) a_2 versus a_3 for $a_1=0.2$, (b) a_2 versus a_1 for $a_3=-0.1$.

4. Chaos synchronization

A characteristic property of chaotic dynamics is the sensitive dependence on initial condition. Different initial conditions may cause entirely different trajectories for the system. However, Pecora and Carroll [4] showed that synchronization can be achieved for the chaotic systems. This interest phenomenon plays a significant role in the chaotic dynamics of communication signals and may be applied to the real-time recovery of signals that have been masked in a strange attractor and thus to encode communication. Other applications of synchronization of chaos also have expectative potential [2]. A natural way to develop synchronization for chaotic systems is through system decomposition. Chaotic system (3.1) is decomposed into two subsystems as follows:

Drive system:

$$
\begin{aligned}
\dot{x}_1 &= y_1 + 0.2x_1,\\
\dot{y}_1 &= dz_1^2\cos x_1 + \tfrac{1}{2}\left(e + pz_1^2\right)\sin 2x_1 - \sin x_1 - by_1 - 0.1y_1,\\
\dot{z}_1 &= q\cos x_1 - F - 0.1z_1.
\end{aligned}
\tag{4.1}
$$

Response system:

$$
\begin{aligned}
\dot{x}_2 &= y_2 + 0.2x_2,\\
\dot{y}_2 &= dz_2^2\cos x_2 + \tfrac{1}{2}\left(e + pz_2^2\right)\sin 2x_2 - \sin x_2 - by_2 - 0.1y_2,\\
\dot{z}_2 &= q\cos x_2 - F - 0.1z_2.
\end{aligned}
\tag{4.2}
$$

In the following, linear feedback, nonlinear feedback, adaptive feedback and backstepping design approaches are discussed.

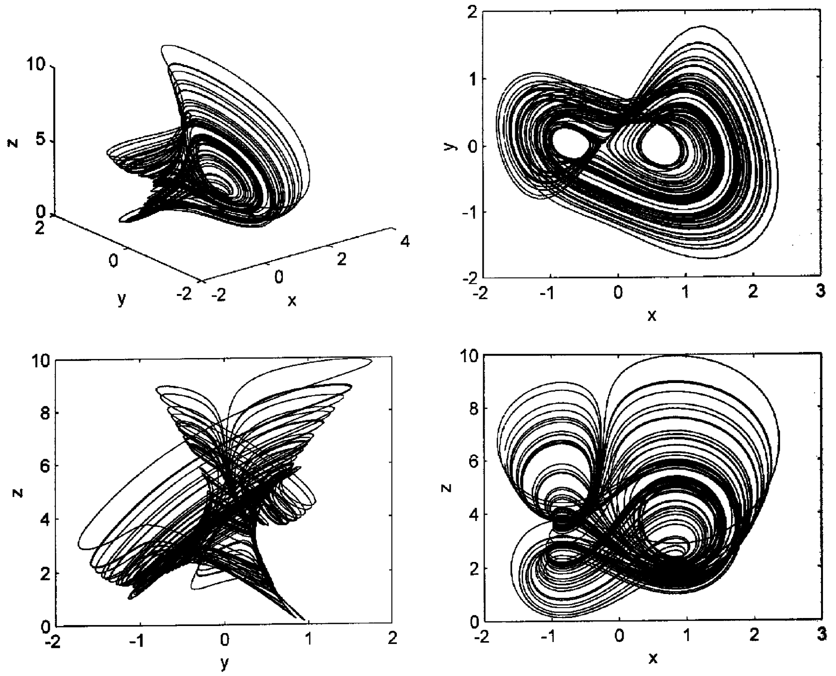

Fig. 5. Phase portrait of controlled system with $\varepsilon = 0.01$.

4.1. Linear feedback synchronization

In this approach, the error between the output of the identical drive and response is used as the control signal. For the unidirectional case, where only first equation of response system (4.2) is combined with a linear feedback, while the equations of drive remain the same [4]:

$$\begin{aligned}\dot{x}_2 &= y_2 + 0.2x_2 + k_1(x_1 - x_2),\\ \dot{y}_2 &= dz_2^2 \cos x_2 + \tfrac{1}{2}(e + pz_2^2)\sin 2x_2 - \sin x_2 - by_2 - 0.1y_2\\ \dot{z}_2 &= q\cos x_2 - F - 0.1z_2,\end{aligned} \tag{4.3}$$

where k_1 is the constant feedback gain. With $k_1 = 0.2$, the synchronization errors, $e_x = x_2 - x_1$, $e_y = y_2 - y_1$, and $e_z = z_2 - z_1$, are shown in Fig. 6. In this case, $k_1 = 0.15$ is a critical value, below which no complete synchronization occurs.

4.2. Nonlinear feedback synchronization

The chaotic response system (4.2) by adding nonlinear coupling term are written as

$$\begin{aligned}\dot{x}_2 &= y_2 + 0.2x_2 + k_2\sin(x_1 - x_2),\\ \dot{y}_2 &= dz_2^2 \cos x_2 + \tfrac{1}{2}(e + pz_2^2)\sin 2x_2 - \sin x_2 - by_2 - 0.1y_2,\\ \dot{z}_2 &= q\cos x_2 - F - 0.1z_2.\end{aligned} \tag{4.4}$$

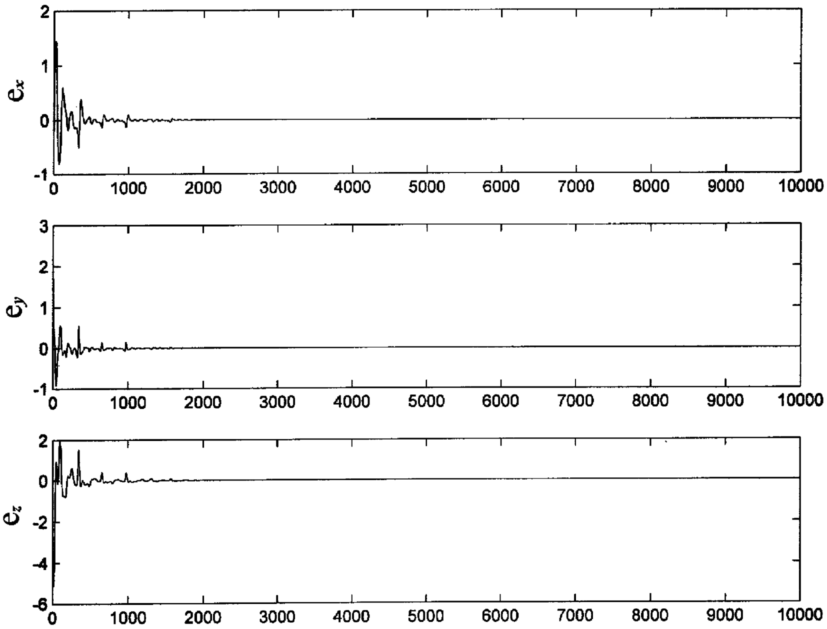

Fig. 6. Chaos synchronization via a linear feedback with $k_1 = 0.2$.

With $k_2 = 0.2$, the synchronization errors are shown in Fig. 7. In this case, $k_2 = 0.15$ is a critical value, below which no completely synchronization occurs.

4.3. Adaptive feedback synchronization

Some adaptive control strategies can direct a chaotic trajectory to stable orbits but not unstable ones. However, it is possible to combine the feedback method for chaos synchronization [2].

For response system (4.2), the linear feedback (4.3) is replaced by

$$\begin{aligned} \dot{x}_2 &= y_2 + 0.2x_2 + k_3(x_1 - x_2), \\ \dot{y}_2 &= dz_2^2 \cos x_2 + \tfrac{1}{2}\left(e + pz_2^2\right) \sin 2x_2 - \sin x_2 - by_2 - 0.1y_2, \\ \dot{z}_2 &= q \cos x_2 - F - 0.1z_2, \\ \dot{q} &= k_4(y_1 - y_2). \end{aligned} \tag{4.5}$$

where the system parameter q is used as an adjustable function for adaptation, and k_4 is a constant adaptive control gain to be determined in the design. Using this method, the response can be synchronized by the chaotic drive, as shown in Fig. 8.

4.4. Backstepping design

Backstepping design is a recursive procedure that combines the choice of a Lyapunov function for selecting a proper controller in chaos synchronization [9]. The drive system is expressed as

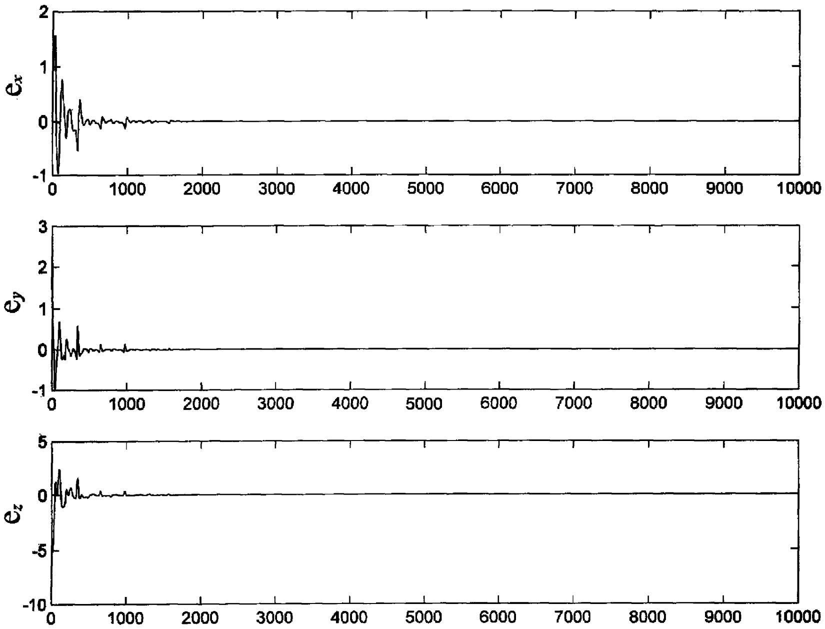

Fig. 7. Chaos synchronization via a nonlinear feedback with $k_2 = 0.2$.

Eq. (4.1) and a controller u is added to the right-hand side of the second equation of response system (4.2). Let the state errors between the response system and drive system be

$$e_x = x_2 - x_1, \quad e_y = y_2 - y_1, \quad e_z = z_2 - z_1; \tag{4.6}$$

then error system can be derived as

$$\begin{aligned} \dot{e}_x &= e_y + 0.2e_x, \\ \dot{e}_y &= d\left[\cos x_2(e_z^2 + 2z_1e_z) + \frac{z_1^2}{2}(e_x^2 + 2x_1e_x)\right] + (e-1)e_x \\ &\quad + \frac{p}{2}\left[\sin 2x_2(e_z^2 + 2z_1e_z) + 2z_1^2e_x\right] - (b+0.1)e_y + u + \text{h.o.t.}, \\ \dot{e}_z &= \frac{q}{2}e_x - 0.1e_z. \end{aligned} \tag{4.7}$$

If system (4.7) did not have u, it would have an equilibrium point (0,0,0). The problem of synchronization between drive and response system can be transformed into how to find a control law u for stabilizing the error variables of system (4.7) at the origin.

First we consider the stability of system as follows:

$$\dot{e}_z = \frac{q}{2}e_x - 0.1e_z, \tag{4.8}$$

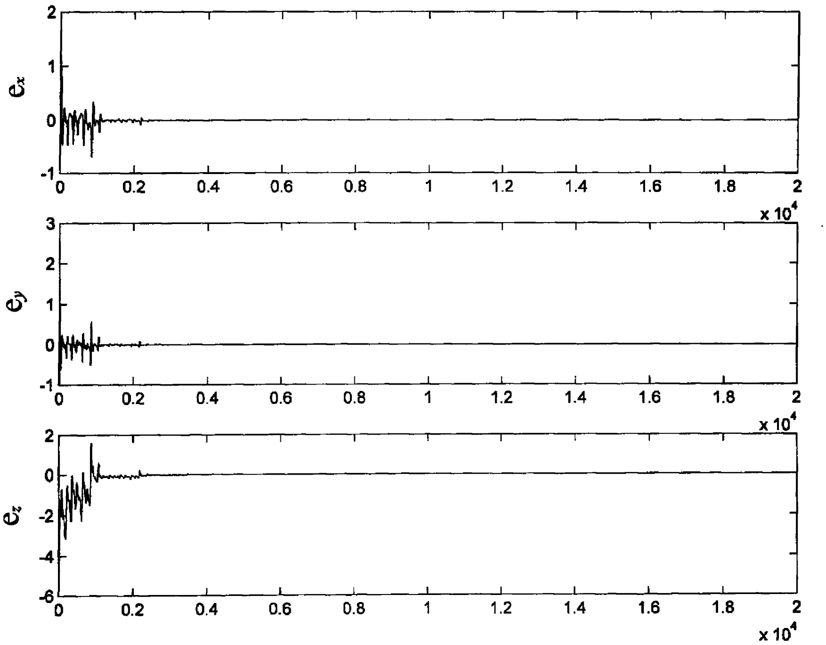

Fig. 8. Chaos synchronization via adaptive feedback with $k_3 = 0.5$ and $k_4 = 0.2$.

where e_x is regarded as a controller and it makes system (4.8) asymptotically stable. Choose Lyapunov function $V_1(e_z) = e_z^2/2$. The derivative of V_1 is

$$\dot{V}_1 = -0.1e_z^2 + \frac{q}{2}e_z e_x. \tag{4.9}$$

Assume controller $e_x = \alpha_1(e_z)$ and $\alpha_1(e_z) = 0$, then

$$\dot{V}_1 = -0.1e_z^2 < 0 \tag{4.10}$$

makes system (4.8) asymptotically stable. Function $\alpha_1(e_z)$ is an estimative function when e_x is considered as a controller. The error between e_x and $\alpha_1(e_z)$ is

$$w_2 = e_x - \alpha_1(e_z). \tag{4.11}$$

Study (e_z, w_2) system

$$\begin{aligned} \dot{e}_z &= \frac{q}{2}w_2 - 0.1e_z, \\ \dot{w}_2 &= e_y + 0.2w_2. \end{aligned} \tag{4.12}$$

Consider $e_y = \alpha_2(e_z, w_2)$ as a controller in system (4.12). Choose Lyapunov function $V_2(e_z, w_2) = V_1(e_z) + w_2^2/2$. The derivative of V_2 is

$$\dot{V}_2 = -0.1e_z^2 + 0.2w_2^2 + \frac{q}{2}w_2 e_z + w_2 e_y. \tag{4.13}$$

If $\alpha_2(e_z, w_2) = -\frac{q}{2}e_z - w_2$, then

$$\dot{V}_2 = -0.1e_z^2 - 0.8w_2^2 < 0 \tag{4.14}$$

makes system (4.12) asymptotically stable. Define the error variable w_3 as

$$w_3 = e_y - \alpha_2(e_z, w_2). \tag{4.15}$$

Study full dimension (e_z, w_2, w_3) system

$$\begin{aligned}
\dot{e}_z &= \frac{q}{2}w_2 - 0.1e_z,\\
\dot{w}_2 &= w_3 + \alpha_2 + 0.2w_2,\\
\dot{w}_3 &= d\left[\cos x_2(e_z^2 + 2z_1e_z) + \frac{z_1^2}{2}(w_2^2 + 2x_1w_2)\right] + (e-1)w_2 - (b+0.1)(w_3+\alpha_2)\\
&\quad + \frac{p}{2}\left[\sin 2x_2(e_z^2 + 2z_1e_z) + 2z_1^2w_2\right] - \frac{d\alpha_2}{dt} + u + \text{h.o.t.},
\end{aligned} \tag{4.16}$$

where

$$\frac{d\alpha_2}{dt} = -\frac{q}{2}\left[\frac{q}{2}w_2 - 0.1e_z\right] - (w_3 + \alpha_2 + 0.2w_2).$$

Choose Lyapunov function $V_3(e_z, w_2, w_3) = V_2(e_z, w_2) + w_3^2/2$. The derivative of V_3 is

$$\begin{aligned}
\dot{V}_3 = &-0.1e_z^2 - 0.8w_2^2 + w_3\left[2z_1e_z\left(q\ \cos\ x_2 + \frac{p}{2}\sin\ 2x_2\right) + ew_2\right.\\
&\left. - (b+0.1)(w_3+\alpha_2) + \frac{dz_1^2}{2}(w_2^2 + 2x_1w_2) + pz_1^2w_2 - \frac{d\alpha_2}{dt} + u\right] + \text{h.o.t.}
\end{aligned} \tag{4.17}$$

Let

$$\begin{aligned}
u = (b+0.1)\alpha_2 - 2z_1e_z\left(q\ \cos x_2 + \frac{p}{2}\ \sin 2x_2\right) - ew_2\\
-\frac{dz_1^2}{2}\left(w_2^2 + 2x_1w_2\right) - pz_1^2w_2 + \frac{d\alpha_2}{dt}
\end{aligned}$$

then

$$\dot{V}_3 = -0.1e_z^2 - 0.8w_2^2 - (b+0.1)w_3^2 + \text{h.o.t.}$$

is quadric negative definite. Whereas x_1, z_1 and x_2 are bounded, we can conclude that the equilibrium point (0,0,0) of error system (4.7) is locally asymptotically stable. For proper initial errors between drive and response systems, the initial errors will converge to zero and synchronization between two chaotic subsystems will be achieved. The numerical results with initial condition ($x_1(0) = 1.42$, $y_1(0) = -2.1$, $z_1(0) = 5.55$, $x_2(0) = 1.4$, $y_2(0) = -2$, $z_2(0) = 5.5$) are shown in Fig. 9.

4.5. Parameter evaluation from time sequences

In this section, we study another method to estimate the parameter of chaotic system by a random optimization method for chaos synchronization [10]. For system (4.3), q is the unknown parameter and k_1 is a coupling constant. The difference of the two time sequences is calculated as

$$U = \int_0^T |x_2 - x_1|^2 \, dt,$$

the integral time T is larger than a typical period of the chaotic oscillation in the governor system and the parameter q is randomly modified as

$$q' = q + \zeta,$$

where ζ is a random number. We can obtain a time sequence $x_2'(t)$ by numerical simulation of Eq. (4.3) with the modified parameter q'. Then the difference of the two time sequences is calculated as

$$U' = \int_0^T |x_2' - x_1|^2 \, dt.$$

If the difference U' is smaller than U, the parameter is changed from q to q'. On the other hand, if the difference U' is larger than U, the parameter is unchanged and kept to be q. The obtained parameter value is expected to be the desired parameter until the difference U becomes zero, i.e., complete chaos synchronization is achieved.

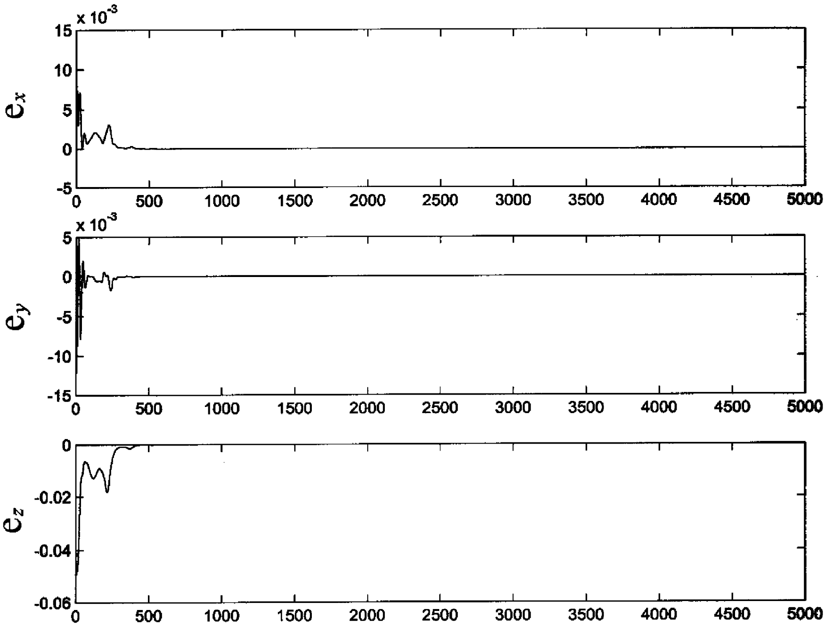

Fig. 9. Chaos synchronization via backstepping design.

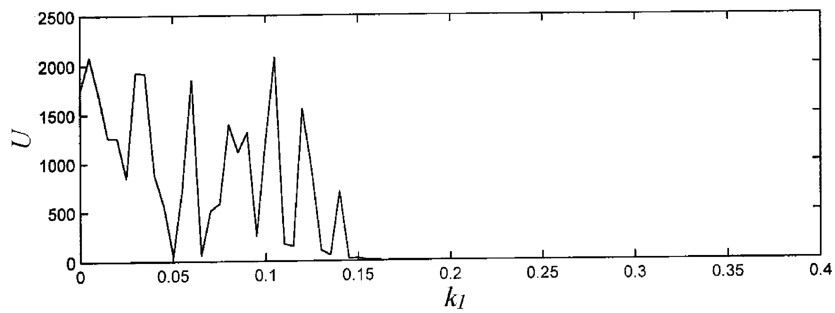

Fig. 10. The minimum value of U with respect to the coupling constant k_1.

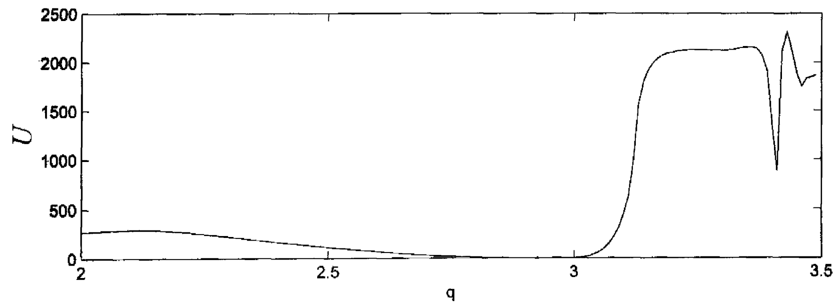

Fig. 11. The difference U versus the parameter q for $k_1 = 0.2$.

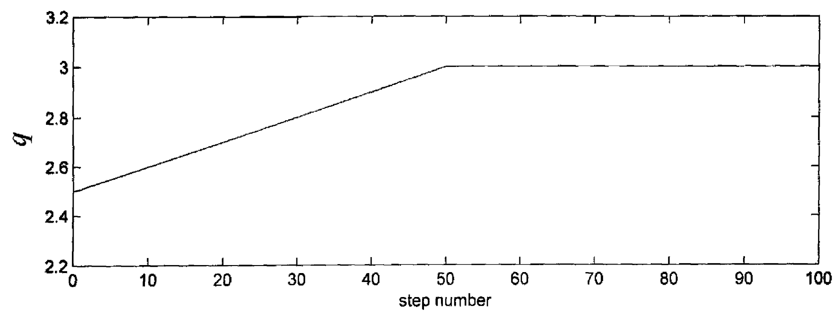

Fig. 12. Time evolution of the parameter q by the random optimization process.

Fig. 10 displays the minimum value of U with respect to the coupling constant k_1. The numerical simulation shows that the complete chaos synchronization $x_2(t) = x_1(t)$ occurs for $k_1 > 0.15$. This result is the same as that in Section 4.1. The difference U as a function of the parameter q for $k_1 = 0.2$ is shown in Fig. 11. The complete chaos synchronization is attained when the value of U takes a minimum value 0 at $q = 3$. Time evolution of the parameter q by the random optimization process is shown in Fig. 12.

5. Conclusions

The problems of anticontrol and synchronization of chaos for an autonomous rotational machine system with a hexagonal centrifugal governor has been discussed in this paper. For anticontrol of chaos, two different procedures to design the controller have been presented. The periodic motion of the system disappeared and was replaced by chaotic motion effectively by adding a linear and a nonlinear feedback term, respectively.

Synchronization of two chaotic oscillators is also studied in this paper. For two identical chaotic systems, increase of coupling strength leads to the occurrence of complete synchronization. Chaos synchronization of the autonomous governor system has been presented by adding linear feedback term, adding sinusoidal term and adaptive feedback methods. Chaos synchronization is also attained by a recursive procedure, backstepping design that combines the choice of a Lyapunov function for selecting a proper controller. Finally, the parameter of chaotic system is estimated from time sequences for chaos synchronization is studied. In this paper, the theoretical study of the governor system for anticontrol and synchronization of chaos had been proposed. The knowledge of anticontrol tells us the various conditions for steadily running chaotic process, prevent it from going into some catastrophic event that is known to occur whenever the chaotic orbit wanders into some particular regions of state space. As for synchronization, our study affords a more complex model for secure communication than the Lorenz system and Rössler system to obtain better security.

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