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Short Communication

Sound transmission loss of unbounded panels in bending vibration considering transverse shear deformation

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1. Introduction

Information on sound transmission loss characteristics of panels is essential for the design of acoustic enclosures. Transmission loss characteristics of infinite limp panels and thin plates are widely discussed in Refs. [1–3]. Transmission loss of limp panels increases with frequency and the transmission loss depends on the mass per unit area of the panel. In the case of structural panels having stiffness, the transmission loss becomes very low at specific frequencies called coincidence frequencies. The transmission loss characteristics of such panels are theoretically obtained based on models that assume that the panels behave like thin plates. In such models the deformation of the transverse plane due to transverse shear stress is neglected. In the case of thick panels, transverse shear deformation can become important.

Sound transmission loss of sandwich panels is extensively studied by many researchers. Watters and Kurtze [4] obtained the transmission loss of sandwich panels considering the core as a spacer. Using a variational approach Dym and Lang [5] introduced the effect of core stiffness in the sound transmission characteristics. Extensional stiffness of the core introduces symmetric modes. Moore and Lyon [6] presented an approximate expression for the associated coincidence frequency called double wall resonance frequency. Apart from the symmetric coincidence, two types of antisymmetric coincidence occur, one depending on the bending rigidity of the panel and the other depending on the shear stiffness of the core. Moore and Lyon [6] have given the expressions for the speed of both the types of antisymmetric wave motions. In these expressions

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both the motions are uncoupled. At very high frequencies the bending wave coincidence occurs depending on the bending rigidity of the individual face sheet. Considering all the above types of wave motions Moore and Lyon [6] have presented the transmission loss of panels.

It is noticed that the above model for the antisymmetric wave motion involving bending of the panel does not consider the deformation of the transverse plane due to the transverse shear stresses. Although core shear flexibility is included in these studies, it represents shear waves and it does not include the transverse shear effects on the bending waves. Accordingly the coincidence frequency and the transmission loss characteristics derived do not include this effect. Renji et al. [7] derived an expression for the coincidence frequency of the bending waves considering the transverse shear effects. In this work, transmission loss characteristics of unbounded panels in bending vibration considering transverse shear deformation are investigated. The expression derived in this study directly relates the transmission loss to the critical frequency and the transverse shear stiffness and it is expressed in the same form as that for a thin plate model.

2. Expression for transmission loss

Consider a panel with an incident acoustic field. Sound power transmission coefficient is defined as the ratio of the intensity of the transmitted sound to the intensity of the incident sound. If I_i is the intensity of the incident sound wave and I_t is the intensity of the transmitted sound wave, the sound power transmission coefficient τ is defined by

$$\tau = I_t/I_i. \tag{1}$$

Accordingly, sound transmission loss is defined by

$$\mathrm{TL} = 10\,\log(1/\tau). \tag{2}$$

Differential equation governing the bending vibration of a thin plate kept in x–y plane subjected to an external force of q per unit area is

$$\nabla^4 w + (c/D)\partial w/\partial t + (\rho/D)\partial^2 w/\partial t^2 = q, \tag{3}$$

where c is the damping coefficient and w is the displacement of the plate in the z direction, that is normal to the plate. The plate has a flexural rigidity of D and mass per unit area of ρ.

The shear deformation of the transverse plane is neglected while using Eq. (3). Considering the deformation of the transverse plane of the plate due to the transverse shear stresses, the differential equation becomes

$$\nabla^4 w + (c/D)\partial w/\partial t + (\rho/D)\partial^2 w/\partial t^2 - (\rho/N)\{\partial^2(\nabla^2 w)/\partial t^2\} = q, \tag{4}$$

where N is the shear rigidity of the plate. In the above equation the shear deformation is represented by Mindlin's plate theory. If G is the shear modulus of the material of the plate, the shear rigidity of the plate having thickness t is Gt/k. The value of the parameter k is 1.20 for a rectangular cross section. For a honeycomb panel, N is given by $G_c t_c\{1 + (t_f/t_c)\}^2$ where the suffix c represents the core and f represents the face sheet. When N is very large the shear flexibility is very low and the effect of transverse shear deformation is negligible.

Consider a panel of infinite extent with an incident acoustic field, having wavenumber k, as shown in Fig. 1. A part of the incident energy is reflected and the remaining energy sets the panel into vibration. The panel radiates sound on either side. Here the sound radiated to the same side of the incident sound is termed as the radiated sound and the sound radiated to the other side is called the transmitted sound. Let the acoustic pressure of the incident, the reflected, the radiated and the transmitted sound be represented by p_i, p_r, $p_{\rm rad}$ and p_t, respectively. Assuming harmonic variations, these pressures can be represented as

$$\begin{aligned} p_i &= A_i \mathrm{e}^{\mathrm{j}(\omega t - kz\cos\theta + kx\sin\theta)}, \\ p_r &= A_r \mathrm{e}^{\mathrm{j}(\omega t + kz\cos\theta + kx\sin\theta)}, \\ p_{\rm rad} &= A_{\rm rad} \mathrm{e}^{\mathrm{j}(\omega t + kz\cos\theta + kx\sin\theta)}, \\ p_t &= A_t \mathrm{e}^{\mathrm{j}(\omega t - kz\cos\theta + kx\sin\theta)}, \end{aligned} \tag{5}$$

where A with the appropriate suffixes represent the amplitudes. If the medium present on either sides of the panel is the same, the sound power transmission coefficient of the panel is given by

$$\tau = |A_t|^2/|A_i|^2. \tag{6}$$

The incident wave produces a trace wave in the panel and this forced wave has a wavelength of $\lambda/\sin\theta$ and wavenumber of $k\sin\theta$. The displacement of the panel normal to the x–y plane, which is due to the forced wave in the panel, becomes

$$w = A\mathrm{e}^{\mathrm{j}(\omega t + kx\sin\theta)}, \tag{7}$$

where A is the amplitude of the displacement of the panel. If the speed of the acoustic wave in the medium is denoted by c and its density is denoted by ρ_a, the amplitude of the particle velocity in the transmitted wave is $A_t/\rho_a c$. Satisfying the continuity of the particle velocity at the transmitted

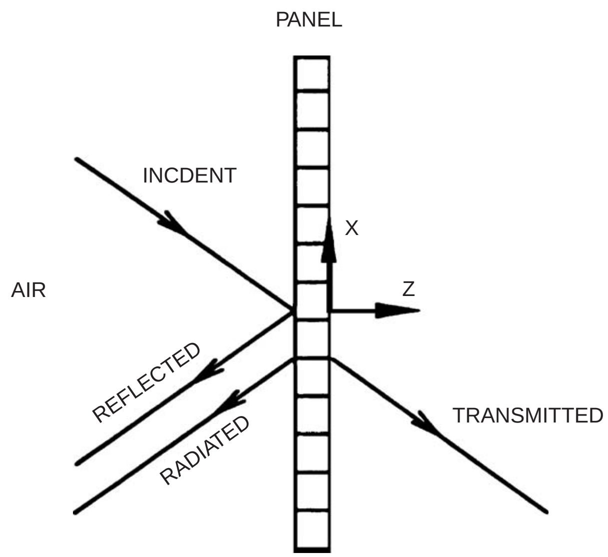

Fig. 1. Panel with incident acoustic field.

side, the amplitude of the displacement of the plate is related to the amplitude of the transmitted sound wave by the expression

$$A = [\{A_t/\rho_a c\} \cos \theta]/(\mathrm{j}\omega). \tag{8}$$

The net acoustic pressure is

$$q = p_i + p_r + p_{\mathrm{rad}} - p_t. \tag{9}$$

Since the medium present on both sides of the plate has the same properties, the sound power radiated to both sides of the panel are equal and hence $A_t = -A_{\mathrm{rad}}$. The requirement of the continuity of the particle velocity necessitates that $A_i = A_r$. Using the above results and substituting Eq. (5) in Eq. (9), the external force on the plate due to acoustic excitation becomes

$$q = 2(A_i - A_t)\mathrm{e}^{\mathrm{j}\omega t}. \tag{10}$$

If η_d is the loss factor, it can be written in terms of complex stiffness as

$$(c/D)\partial w/\partial t = \mathrm{j}\eta_d \nabla^4 w. \tag{11}$$

Substituting Eqs. (7), (10) and (11) by Eq. (4) yields

$$D\{k^4 \sin^4 \theta + \mathrm{j}\eta_d k^4 \sin^4 \theta - (\rho\omega^2/D) - (\rho\omega^2/N)k^2 \sin^2 \theta\}A = 2(A_i - A_t). \tag{12}$$

Expressing the amplitude of the displacement of the plate in terms of the amplitude of the transmitted sound, i.e., using Eq. (8) in Eq. (12), the amplitudes of transmitted and incident sound waves are related by

$$D\{k^4 \sin^4 \theta + \mathrm{j}\eta_d k^4 \sin^4 \theta - (\rho\omega^2/D) - (\rho\omega^2/N)k^2 \sin^2 \theta\}A_t \cos \theta/(\mathrm{j}\omega\rho_a c) = 2(A_i - A_t). \tag{13}$$

If we use the relation

$$f_{c,t}^2 = c^4\rho/(4\pi^2 D), \tag{14}$$

from Eqs. (13) and (6) the expression for sound power transmission coefficient of the panel becomes

$$\tau^{-1} = \{1 + \eta_d a \cos \theta \sin^4 \theta (f/f_{c,t})^2\}^2 + a^2 \cos^2 \theta \{1 - (f/f_{c,t})^2 \sin^4 \theta [1 - c^2\rho/(N \sin^2 \theta)]\}^2, \tag{15}$$

where $a = \rho\omega/2\rho_a c$. The parameter $f_{c,t}$ is the critical frequency of the panel without considering the transverse shear deformation. Eq. (15) gives the sound power transmission coefficient of an infinite panel considering transverse shear deformation.

Expressions for the sound transmission loss of thin plates and limp panels can be derived from Eq. (15) and it can be seen that they are the same as what are existing in Refs. [1,2]. Expression for the sound power transmission coefficient of a thin plate can be derived by setting N very large in Eq. (15) as

$$\tau^{-1} = \{1 + \eta_d a \cos \theta \sin^4 \theta (f/f_{c,t})^2\}^2 + a^2 \cos^2 \theta \{1 - (f/f_{c,t})^2 \sin^4 \theta\}^2. \tag{16}$$

The critical frequency of a limp panel is infinite. Hence, by taking $f_{c,t}$ in Eq. (15) to ∞, the expression for the sound power transmission coefficient of a limp panel can be derived as

$$\tau^{-1} = 1 + a^2 \cos^2 \theta. \tag{17}$$

The frequency at which the sound power transmission coefficient is the maximum can be determined by differentiating Eq. (15) with respect to the frequency. If no dissipation in the panel is assumed, this frequency is

$$f^2 = f_{c,t}^2/\{1 - [(c^2\rho/N)/\sin^2\theta]\}/\sin^4\theta. \tag{18}$$

When a panel is excited acoustically, the frequency at which the speed of the forced bending wave in the panel is equal to the speed of the free bending wave in the panel is called the coincidence frequency. It is expected that sound power transmission coefficient is very high at the coincidence frequency of the panel. An expression for estimating the coincidence frequency of a panel considering the transverse shear deformation is derived in an earlier study [7]. But the expression for the coincidence frequency is derived without considering the sound power transmission coefficient of the panel. Instead it is derived by equating the wavelength of the trace wave in the panel to the wavelength of the free bending wave in the panel. From the present results it can be seen that the frequency at which the sound power transmission coefficient is the maximum is the coincidence frequency of the panel, denoted by f_{co}. Same expression for coincidence frequency is obtained from both the approaches. This confirms the correctness of the expression for the sound power transmission coefficient derived here.

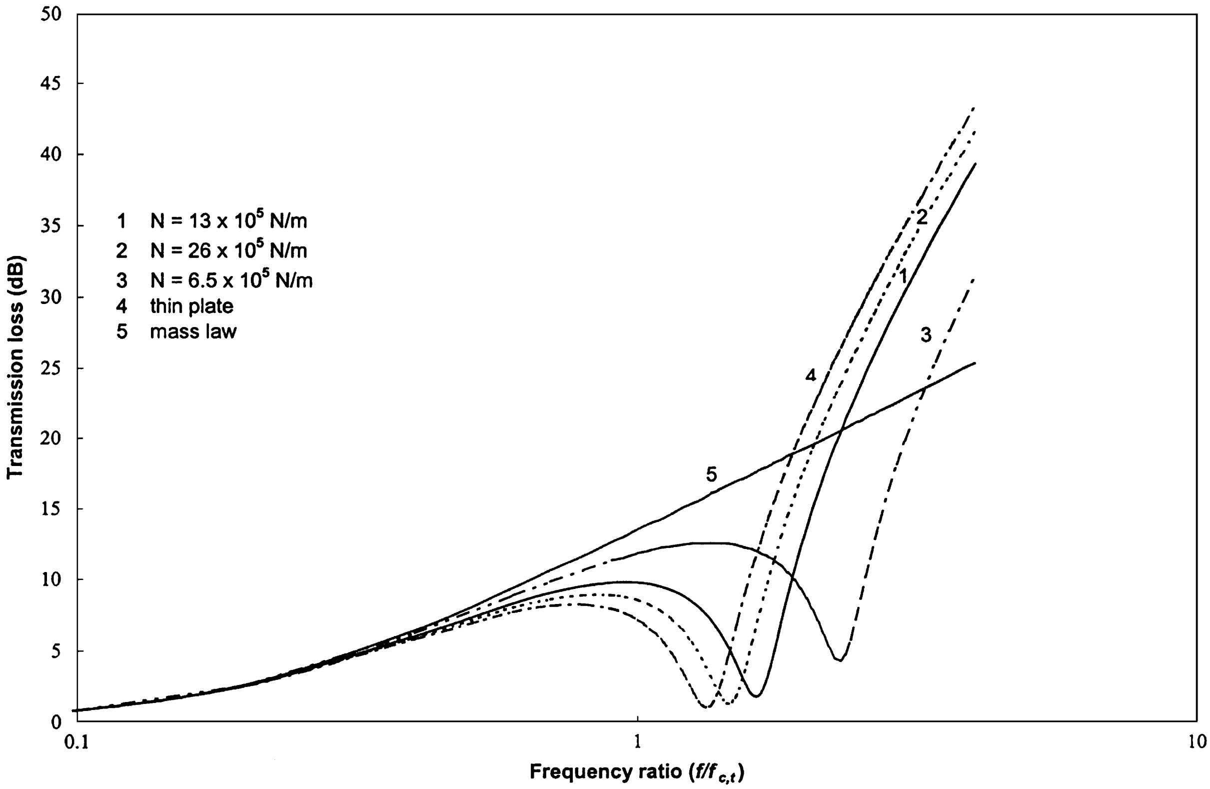

Fig. 2. Oblique incidence transmission loss of panels.

At very low frequencies the panel behaves like a limp panel. At coincidence frequency the sound power transmission coefficient is given by

$$\tau^{-1} = \{1 + \eta_d a \cos\theta \sin^4\theta (f/f_{c,t})^2\}^2, \tag{19}$$

which is also true for a thin plate. For a particular value of the parameter a and angle of incidence, the sound power transmission coefficient at the coincidence frequency is very much dependent on the loss factor and it is independent of the shear flexibility. It is to be noted that the coincidence frequency gets altered due to the shear flexibility and correspondingly the parameter a. This can have an effect on the sound power transmission coefficient. At very high frequencies compared to the coincidence frequency, the expression for the sound power transmission coefficient becomes

$$\tau^{-1} = a^2 \cos^2\theta (f/f_{\mathrm{co}})^4. \tag{20}$$

The loss factor has no effect on the sound power transmission coefficient at higher frequencies. The term $\{a^2\cos^2\theta\}^{-1}$ is the sound power transmission coefficient of the limp panels at high frequencies. Since at higher frequencies f/f_{co} is more than unity, the panel transmits less sound compared to what is estimated using the limp panel model.

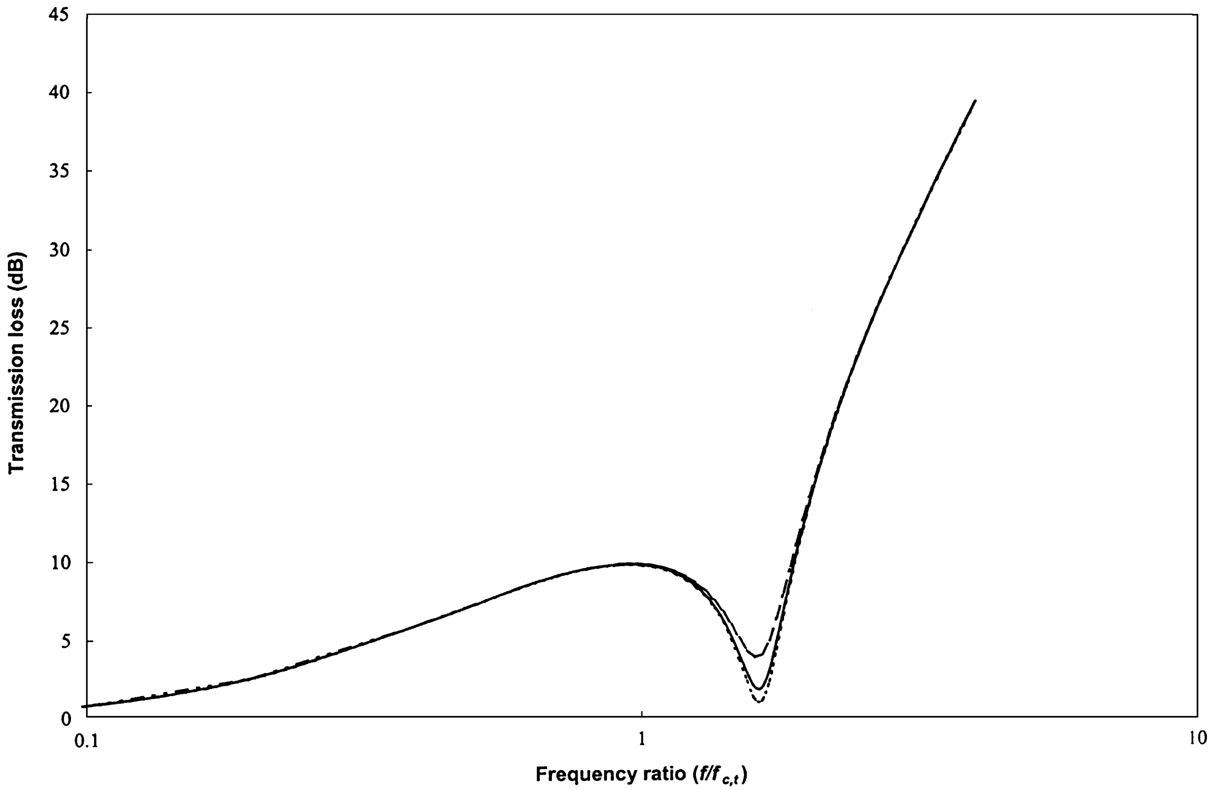

Fig. 3. Oblique incidence transmission loss of panels for various values of loss factors: —, 0.02; - - -, 0.01, – — –, 0.05.

3. Transmission loss characteristics

To get a better insight into the transmission loss characteristics, transmission loss of panels for different structural parameters are presented. Consider an infinite panel having flexural rigidity of 5000 N m and mass per unit area of 2.76 kg/m^2. The panel has a shear rigidity (N) of 13×10^5 N/m. The speed of sound in air is assumed to be 343 m/s and the characteristics impedance of the air is considered to be 415 Rayl. The critical frequency of the panel without considering the transverse shear deformation is estimated to be 440 Hz and that considering the transverse shear deformation is 508 Hz. Let the angle of incidence be 60°. The coincidence frequency of the panel without considering the transverse shear deformation is estimated to be 587 Hz and that considering the transverse shear deformation is 718 Hz. Sound transmission loss of the panel at various frequencies, for various values of $f/f_{c,t}$, for a loss factor of 0.02 is shown in Fig. 2. In Fig. 3, the transmission loss characteristics of the panel for three different values of loss factors are shown. For this the shear rigidity of the panel is considered to be 13×10^5 N/m.

The results show that at low frequencies the panel behaves like limp panel. The shear flexibility and the loss factor has no effect on the transmission loss characteristics.

The TL is the minimum at the coincidence frequency given by Eq. (18). Comparison with the TL estimated using the thin plate model, given in Fig. 2, shows that the effect of shear flexibility is

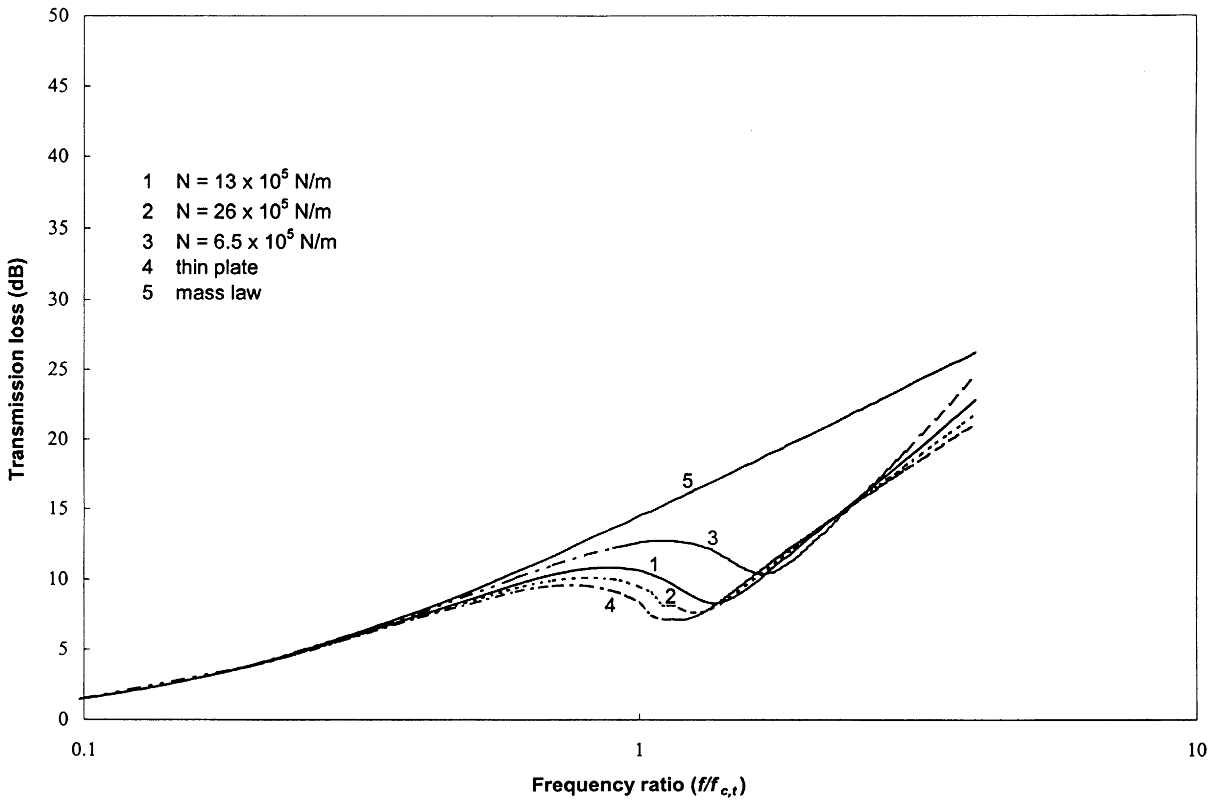

Fig. 4. Field incidence transmission loss of panels.

to increase the coincidence frequency. At coincidence frequency the TL is controlled by the loss factor (Fig. 3) and it can be improved by increasing the loss factor as in the case of thin panels.

At very high frequencies the TL is higher than that estimated by the limp panel model. Comparison with the TL estimated using the thin plate model shows that the effect of the shear flexibility is to decrease the TL at a particular frequency. But it should be noted that the shear flexibility does not directly reduce the transmission loss but it increases the coincidence frequency and consequently the transmission loss at a particular frequency is reduced.

Eq. (15) gives the sound power transmission coefficient for oblique incidence. When the acoustic field is reverberant, the sound power transmission coefficient is defined in a statistical sense. The random incidence sound power transmission coefficient, τ_r, is defined by the integral [3]

$$\tau_r = 2.09 \int_0^{1.36} \tau \sin\theta \cos\theta \, d\theta. \tag{21}$$

The angle of incidence is normally considered to be in the range 0–78° and the corresponding transmission loss is called field incidence transmission loss. Field incidence TL of the panels for different values of shear rigidity and loss factor of 0.02 are given in Fig. 4. Results obtained for limp panel model and thin plate model are also shown in Fig. 4. The dip seen in the transmission loss characteristics is not as sharp as in the case of oblique incidence. Effect of loss factor on transmission loss values is shown in Fig. 5. At higher frequencies the transmission loss is improved

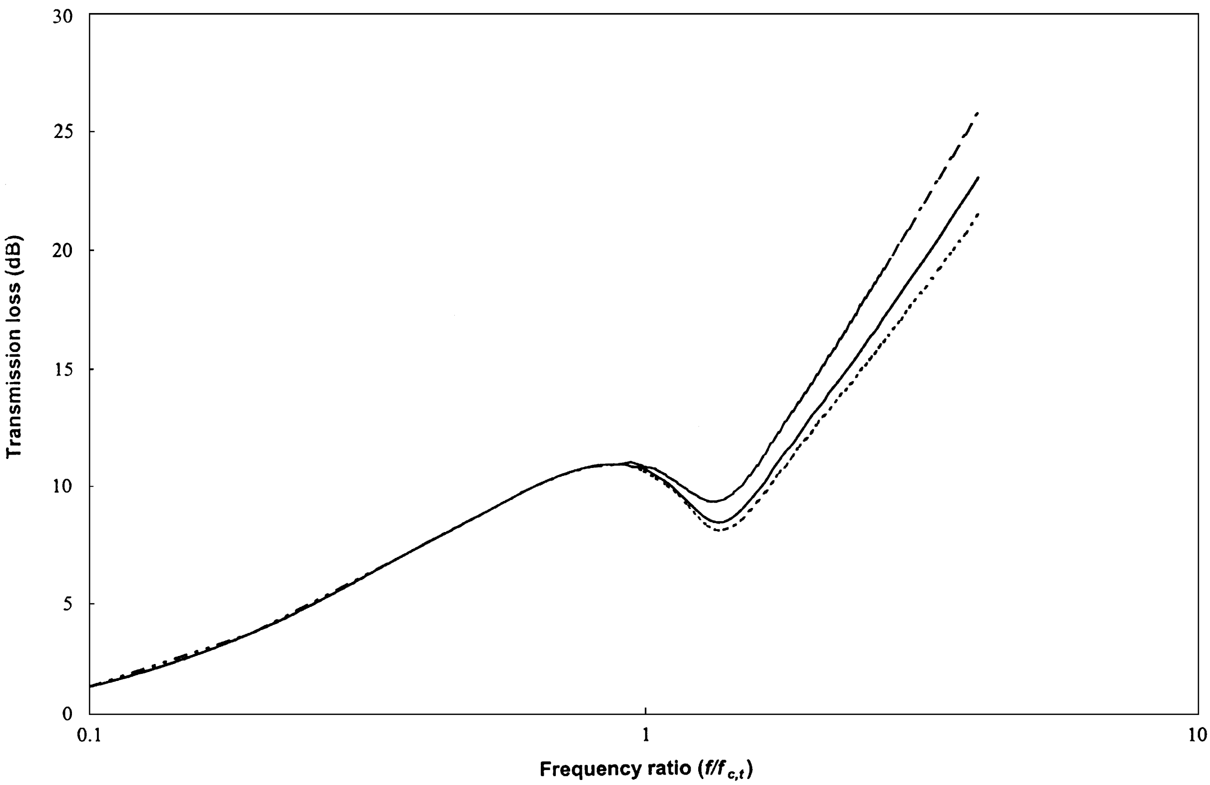

Fig. 5. Field incidence transmission loss of panels for various values of loss factors: —, 0.02; - - -, 0.01, – — –, 0.05.

due to loss factor. It is to be remembered that the loss factor has no significant effect on transmission loss at higher frequencies when the acoustic field is at a particular angle.

4. Conclusions

An expression for the sound power transmission coefficient of an unbounded panel in bending vibration involving antisymmetric motion is derived considering the transverse shear deformation. The most important effect of the shear flexibility is the increase in the frequency at which the transmission loss is the minimum. Consequently, at higher frequencies the transmission loss values are reduced. The transmission loss can be improved by increasing the loss factor. But it is effective only at frequencies near the coincidence frequency as in the case of thin panels. The loss factor is effective even at higher frequencies if the acoustic field is diffused.

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