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Damage localization in ambient vibration by constructing proportional flexibility matrix

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Abstract

Damage localization approaches based on changes of flexibilities constitute an important technique for damage detection. However, the unavailability of flexibility matrix with output-only data makes flexibility-based approaches not really applicable in the very important cases of ambient vibrations. An algorithm is presented to construct a proportional flexibility matrix (PFM) from a set of arbitrarily scaled tested modal shapes and modal frequencies. The constructed PFM is just within a scalar multiplier to the real flexibility matrix, and the scalar multiplier is theoretically the first modal mass, which is undetermined before the mode is properly scaled. Instead of real flexibilities, the PFMs are incorporated into the damage locating vectors (DLV) method for damage localizations in ambient vibrations. PFMs for the pre- and post-damaged structure need to be comparable before being integrated into the DLV procedure. This requirement is guaranteed when there is at least one reference degree with unchanged mass after damage. Two numerical examples show that a small number of measured modes can produce PFMs with sufficient accuracy to correctly locate the damages by the DLV method from output-only data.

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1. Introduction

Structural health monitoring (SHM) is emerging as a frontier in the field of civil structural engineering as a technology to enhance safety of key infrastructures [1], like high-rise buildings, bridges, dams and offshore structures, whose failures pose great impact on economy and society. In the range of SHM, damage detection based on changes of dynamic properties of structures is one of the challenges, and flexibility-based damage localization methods are among the most developed techniques [2–5]. Flexibility has the advantage that it can be constructed by truncated modes at sensor locations without loss of much accuracy. For the numerous flexibility-based damage detection methods, damage locating vectors (DLV) method proposed by Bernal [5] for damage localizations is the most recently proposed method with mathematical rigor and easiness to operate.

However, only in the condition when at least one sensor–actuator pair exists, the flexibility can be assembled from the measured modal parameters [6]. For the prevailing ambient vibration for civil structures, in which the loads and their locations are unknown or not exactly known, construction of flexibility matrix is still an unsolved problem. We may use the analytical model to compute mass-normalized modal shapes from arbitrarily scaled ones, but for a real complex structure, it is not an easy work to set up an analytical model that is consistent with its real model. Therefore, flexibility matrices are not readily available for ambient vibrations in practice.

An approach is proposed to assemble a proportional flexibility matrix (PFM) from arbitrarily scaled modes and modal frequencies with output only data. The DLV method for damage localizations is extended to the ambient vibrations by integrating the proposed PFMs into the frames of DLV method. The remainder of this paper is organized as follows. The DLV method is introduced first. The algorithm to construct PFM is then presented. PFMs of pre- and post-damaged structures are properly scaled to employ the DLV method to make damage localizations. Finally, two numerical examples, which are damage localizations of a 7-dofs mass–spring system and a 53-dofs truss structure, are given to demonstrate the capability of the presented approach in output-only cases.

2. Damage locating vectors (DLV) method

The DLV method, developed by Bernal [5], is a general approach to extract spatial information for damage localization from changes in measured flexibility. The fundamental idea of the DLV approach is that the vectors that span the null-space of change in flexibility (between the pre- and post-damage states) induce no stress in the damaged elements (small in the presence of truncations and approximation) when they are treated as static loads on the structure.

First a special set of vectors, designated DLVs, is determined by singular value decomposition (SVD) to the incremental flexibility. Secondly, the internal forces in every element of the undamaged structure with the DLVs as loads are calculated. The elements having negligible internal forces can be identified as the elements that are possibly damaged.

The procedures of the DLV localization can be summarized as follows:

In the first place, flexibility matrices at sensor locations are assembled from measured data for the cases before and after damage, denoted $\mathbf{F_U}$ and $\mathbf{F_D}$, respectively. Then the change in flexibilities is computed as

$$\mathbf{F}_{\boldsymbol{\Delta}} = \mathbf{F}_{\mathbf{U}} - \mathbf{F}_{\mathbf{D}}. \tag{1}$$

Through SVD to $\mathbf{F}_{\boldsymbol{\Delta}}$ the DLVs are obtained, namely

$$\mathbf{F}_{\boldsymbol{\Delta}} = \mathbf{USV}^{\mathbf{T}} = [\mathbf{U}_1 \ \ \mathbf{U}_0]\begin{bmatrix} \mathbf{s}_1 & \mathbf{0} \\ \mathbf{0} & \mathbf{0} \end{bmatrix}\begin{bmatrix} \mathbf{V}_1^{\mathbf{T}} \\ \mathbf{V}_0^{\mathbf{T}} \end{bmatrix}. \tag{2}$$

For ideal conditions the DLVs are simply $\mathbf{V}_0$ that are the columns of the right singular matrix $\mathbf{V}$ associated with the null space. But in practical applications the singular values corresponding to $\mathbf{V}_0$ are never equal to zero due to noise and computational errors. To select the DLVs from $\mathbf{V}$, an index svn was proposed by Bernal [5] and defined as

$$\text{svn}_i = \sqrt{\frac{s_i c_i^2}{\max_k (s_k c_k^2)}} \quad \text{for} \ \ i = 1 : m, \tag{3}$$

where m is the number of columns in $\mathbf{V}$; s_i is the ith singular value of the matrix $\mathbf{F}_{\boldsymbol{\Delta}}$; c_i is a constant that is used to normalize the maximum characterizing stress in undamaged structure element, which is induced by the static load $c_i V_i$, to have a value of one. The vector V_i can be taken as a DLV if $\text{svn}_i \leqslant 0.20$.

Once the set of DLVs have been obtained, the damage localization can be carried out as such that each of the DLVs is applied to the undamaged structure, and the characterizing stress in each structural element is computed. Then for each DLV vector, the normalized stress $\bar{\sigma}_j$ in the jth element is defined as the characterizing stress σ_j normalized by the largest characterizing stress over all the elements of its kind:

$$\bar{\sigma}_j = \frac{\sigma_j}{\max_k (\sigma_k)}. \tag{4}$$

In order to introduce additional robustness into the technique, the information from multiple DLVs should be combined. The vector of weighted-average stress indices for each of the DLVs, WSI, need to be calculated to select the potentially damaged elements:

$$\text{WSI} = \frac{\sum_{i=1}^{\text{ndlv}} \{\bar{\sigma}_j\}_i / \overline{\text{svn}}_i}{\text{ndlv}}, \tag{5}$$

in which

$$\overline{\text{svn}_i} = \max(\text{svn}_i, 0.015),$$

where ndlv is the number of DLVs and $\{\bar{\sigma}_j\}_i$ is the vector of $\bar{\sigma}_j$ values for the ith DLV. Then the elements having $\text{WSI} < 1$ are generally treated as damaged elements.

The DLV method provides an approach with mathematical rigor for damage localization, and what is more appealing is that it is effective when operated with multiple damage scenarios, a truncated modal basis and an arbitrary number of sensors, while keeping the calculation to a low level. However, in the condition of ambient vibrations, which prevail in civil structures, the flexibility cannot be assembled from arbitrarily scaled modes. The following part is to propose an algorithm for creation of proportional flexibility matrix with arbitrarily scaled modes, and extends the DLV method to output only cases with the proposed PFMs.

3. Construction of proportional flexibility matrix

The structural flexibility matrix **F** can be assembled from a set of mass-normalized modal shape $\bar{\varphi}_i$ and circular modal frequencies $\bar{\omega}_i$ as

$$\mathbf{F} = \sum_{i=1}^{N} \frac{1}{\bar{\omega}_i^2} \bar{\varphi}_i \bar{\varphi}_i^{\mathrm{T}}, \tag{6}$$

where N is the number modes of the structure. In practice, rarely are all N modes identified from the vibration data, but flexibility can be estimated with sufficient accuracy from truncated low modes because it is inversely proportional to the square of modal frequencies. However, in the condition of ambient vibration, where arbitrary-scaled test modal shapes rather than mass normalized ones are presented, assembling flexibility matrix via Eq. (6) is not feasible.

The usually available arbitrary scaled test modal shape φ_i can be written as

$$\varphi_i = r_i \bar{\varphi}_i, \tag{7}$$

in which r_i is the mass normalization factor for ith mode. Substituting Eq. (7) into Eq. (6), one can get

$$\mathbf{F} = \sum_{i=1}^{N} \frac{1}{(r_i \bar{\omega}_i)^2} \varphi_i \varphi_i^{\mathrm{T}} = \sum_{i=1}^{N} \frac{1}{\omega_i^2} \varphi_i \varphi_i^{\mathrm{T}}, \tag{8}$$

in which

$$\omega_i = r_i \bar{\omega}_i. \tag{9}$$

Comparing Eq. (8) with Eq. (6), φ_i is the ith mass normalized modal shape with respect to modal frequency ω_i.

The flexibility matrices in Eqs. (6) and (8) can be taken as those of two different structures, denoted as real structure and "dummy structure", respectively. Obviously the two flexibility matrices are the same, and that means the two structures have identical stiffness matrices. But these structures have different mass matrices and modal frequencies. The identities and differences between the two structures are identified in Table 1.

The dummy structure can be created by scaling and redistributing the mass of the real structure in such a way that the arbitrarily scaled modal shapes of the real structure are mass normalized to the obtained mass matrix of the dummy structure, while keeping the modal shapes unchanged. The scaling and redistributing of mass will not alter the stiffness distribution of the real structure, and thus guarantee that the dummy and real structures have the same stiffness matrices.

The above description for creation of dummy structure can be realized by computing the mass matrix of the dummy structure $\mathbf{M_r}$ through solving the following equations:

$$\mathbf{\Phi^T M_r \Phi} = \mathbf{I}, \tag{10}$$

Table 1
Identities and differences between the real and dummy structures

Structures	Stiffness matrices	Mass matrices	Modal shapes	Modal frequencies
Real Dummy	Identical	Different	Identical, but for dummy structure, the modal shapes are mass normalized	Different, but relationship (9) between those of two structures exists

in which $\mathbf{\Phi}$ is the arbitrarily scaled modal matrix of the real structure with φ_i as its modal vector. $\mathbf{I}$ is the identity matrix. The elements in $\mathbf{M_r}$ are unknowns to be solved. Usually there are more unknowns than the equations in Eq. (10), and a least-square solution is pursued by Moore–Penrose inverse operation.

For the real and dummy structures, two eigenequations are, respectively,

$$(\mathbf{K} - \bar{\omega}_i^2\mathbf{M}_0)\varphi_i = 0 \quad \text{for real structure,} \tag{11}$$

$$(\mathbf{K} - \omega_i^2\mathbf{M_r})\varphi_i = 0 \quad \text{for dummy structure,} \tag{12}$$

where $\mathbf{M}_0$ is the mass matrix for the real structure, and $\mathbf{K}$ is the stiffness matrix for both real and dummy structures. $\mathbf{M}_0$ and $\mathbf{K}$ in above equations are unknown.

Substituting Eq. (12) into Eq. (11), the following equation is obtained:

$$\left(\mathbf{M_r} - \frac{1}{r_i^2}\mathbf{M}_0\right)\varphi_i = 0, \tag{13}$$

where r_i and the elements in $\mathbf{M}_0$ are unknowns to be solved. Obviously, the number of unknowns is much more than that of equations that Eq. (13) provides. To reduce the unknowns, $\mathbf{M}_0$ can be reasonably assumed diagonal. Then the number of unknowns in Eq. (13) is only one more than the number of equations for every given modal vector φ_i.

If the diagonal matrix $\mathbf{M}_0$ is normalized in a certain way, like making a designated diagonal element to be one, i.e.

$$\mathbf{M}_0 = \frac{1}{r_M}\,\bar{\mathbf{M}}_0, \tag{14}$$

where $\bar{\mathbf{M}}_0$ is the normalized matrix with one diagonal element to have the value of one by dividing $\mathbf{M}_0$ by $1/r_M$. Eq. (13) is rewritten as follows with the substitution of Eq. (14) in it:

$$\left(\mathbf{M_r} - \frac{1}{\eta_i}\bar{\mathbf{M}}_0\right)\varphi_i = 0, \tag{15}$$

where

$$\eta_i = r_i^2 r_M. \tag{16}$$

Then a unique solution for η_i and $\bar{\mathbf{M}}_0$ is obtained for each modal vector φ_i by solving Eq. (15).

Although η_i is known by the above procedure, the mass normalization factor r_i cannot be identified in Eq. (16). However, the ratio between r_i^2 and r_1^2 can be evaluated as that of η_i and η_1,

$$\gamma_i = \frac{r_i^2}{r_1^2} = \frac{\eta_i}{\eta_1}. \tag{17}$$

Then flexibility matrix in Eq. (8) can be assembled as

$$\mathbf{F} = \frac{1}{r_1^2}\sum_{i=1}^{N}\frac{1}{\gamma_i\bar{\omega}_i^2}\varphi_i\varphi_i^{\mathrm{T}} = \frac{1}{r_1^2}\mathbf{F_P} \tag{18}$$

and the PFM $\mathbf{F_P}$ is obtained as

$$\mathbf{F_P} = \sum_{i=1}^{N}\frac{1}{\gamma_i\bar{\omega}_i^2}\varphi_i\varphi_i^{\mathrm{T}}, \tag{19}$$

which is within a scalar of r_1^2 to the real flexibility matrix $\mathbf{F}$. r_1^2 is actually the first modal mass of the real structure, and it is undetermined because of the arbitrarily scaled test modal shapes. In whatever way the modes are scaled, $\mathbf{F_P}$ in Eq. (19) is proportional to the real flexibility matrix $\mathbf{F}$, and the scalar multiplier between $\mathbf{F}$ and $\mathbf{F_P}$ depends on the scale of the modes. An example to illustrate the construction of PFM for a 7-dofs system is provided later.

4. Damage locating vectors method with PFMs

PFMs for the pre- and post-damaged structures, denoted as $\mathbf{F_{PU}}$ and $\mathbf{F_{PD}}$, respectively, can be constructed by the procedure presented above. Then $\mathbf{F_{PU}}$ and $\mathbf{F_{PD}}$ take the places of $\mathbf{F_U}$ and $\mathbf{F_D}$ in Eq. (1), and the flexibility difference is rewritten as

$$\mathbf{F_\Delta} = \frac{1}{r_{1U}^2}\left(\mathbf{F_{PU}} - \frac{r_{1U}^2}{r_{1D}^2}\mathbf{F_{PD}}\right), \tag{20}$$

in which r_{1U}^2 and r_{1D}^2 are the first modal masses for the pre- and post-damaged structures, respectively. The bracketed part in Eq. (20) is named proportional flexibility difference $\mathbf{F_{P\Delta}}$, which is

$$\mathbf{F_{P\Delta}} = \mathbf{F_{PU}} - \frac{r_{1U}^2}{r_{1D}^2}\mathbf{F_{PD}}. \tag{21}$$

As shown in Eq. (20), $\mathbf{F_{P\Delta}}$ is within a scalar of r_{1U}^2 to $\mathbf{F_\Delta}$, then the null-space of $\mathbf{F_{P\Delta}}$ is the same as $\mathbf{F_\Delta}$. Therefore the DLVs, which induce zero stresses in potentially damaged members of structures, can be extracted through SVD to $\mathbf{F_{P\Delta}}$ instead of $\mathbf{F_\Delta}$.

The proportional flexibility difference $\mathbf{F_{P\Delta}}$ is undetermined until the ratio between r_{1U}^2 and r_{1D}^2 has been established. With r_{1U}^2 or r_{1D}^2 not being identified in Eq. (16) in mind, the ratio between r_{1U}^2 and r_{1D}^2 cannot be calculated directly. But if the r_M s for the pre- and post-damaged structures are the same, then r_{1U}^2/r_{1D}^2 can be replaced with η_{1U}/η_{1D}, in which the numerator and denominator are obtained by solving Eq. (15) with the first modal vectors for the pre- and post-damaged structures. This is true when there exists in the mass matrix at least one element that does not change in the pre- and post-damaged states, and the normalizations of mass matrices $\mathbf{M}_0$ of both pre- and post-damaged structures through Eq. (14) are made on those unchanged elements.

At this point, $\mathbf{F}_{\mathbf{P}\Delta}$ is determined and the DLV method outlined in the foregoing part can be fully implemented on $\mathbf{F}_{\mathbf{P}\Delta}$.

5. Examples

Two examples are examined, which are multi-damage localizations of a 7-dofs mass–spring system and 14-bay planar truss structure, respectively.

5.1. Example 1

The 7-dofs mass–spring system as shown in Fig. 1 was picked up from [7] $m_1 = 2\,\mathrm{kg}$, $m_2 = 5\,\mathrm{kg}$, $m_3 = 4\,\mathrm{kg}$, $m_4 = 4\,\mathrm{kg}$, $m_5 = 3\,\mathrm{kg}$, $m_6 = 2\,\mathrm{kg}$, $m_7 = 1\,\mathrm{kg}$. The stiffness of each spring is $10^4\,\mathrm{N/m}$.

The structure is excited by independent band-limited white noises at m_1 and m_5. Accelerations are measured, and 5% RMS noise is added to all the outputs. Classical damping for each mode with critical damping ratio of 1% is modeled. The damage is simulated as a reduction by 30% of k_{24} and by 50% of k_{67}. To simulate the ambient vibrations, inputs are not collected.

In this example the ERA [8] and NExT [9] are used to perform the modal identification without using the input information. The identified first three modes for both the intact and damaged structures are shown in Table 2.

In this example the first three modes are used in constructing PFM. Presuming $\mathbf{M}_\mathbf{r}$ is symmetric, a group of nine linear equations is formulated to solve 28 unknowns by Moore–Penrose inverse operation in Eq. (10). Then $\mathbf{M}_0$ is normalized by letting the first element to be one, and r_M in

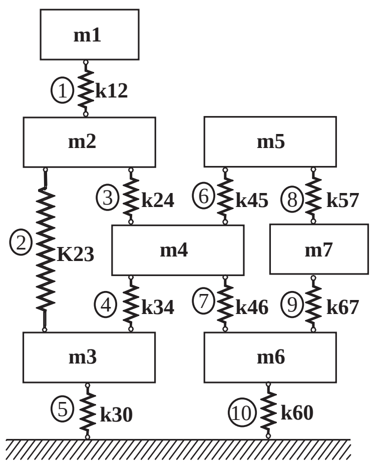

Fig. 1. 7-dofs spring–mass system.

Table 2
Identified modal parameters of 7-dofs system

	Undamaged			Damaged		
	1st mode	2nd mode	3rd mode	1st mode	2nd mode	3rd mode
Frequency (Hz)	4.17	8.24	12.26	4.09	7.62	12.06
Modal vector	1.00	1.00	1.00	1.00	1.00	1.00
	0.87	0.45	−0.20	0.88	0.53	−0.19
	0.54	0.07	−0.67	0.54	0.08	−0.74
	0.66	−0.32	−0.26	0.63	−0.37	−0.32
	0.72	−0.99	0.40	0.75	−1.05	0.48
	0.44	−0.47	−0.05	0.39	−0.43	−0.06
	0.60	−0.84	0.31	0.65	−1.00	0.49

Eq. (14) is the first element of $\mathbf{M}_0$. For each modal vector φ_i, $i = 1$–3, Eq. (15) is solved for η_i and $\bar{\mathbf{M}}_0$. For pre-damaged structure, $\eta_i (i = 1, 2, 3)$ are 4.66, 3.46 and 2.44, respectively, and for damaged structure are 4.58, 3.40 and 2.66.

Then the $\eta_i (i = 1, 2, 3)$ for both pre-damaged structure and damaged structure are incorporated into Eqs. (17) and (18) to compute the $\mathbf{F_{PU}}$ and $\mathbf{F_{PD}}$. The multipliers between elements in the PFM and those in flexibility matrix are shown in Fig. 2. It is shown that all the element multipliers are evenly scattered around 11 with less than 8% deviation. Therefore, the elements multipliers can be the scalar multiplier between the PFM and flexibility matrix.

Applying the SVD to the proportional flexibility difference $\mathbf{F_{P\Delta}}$, one finds there is one DLV. The WSI coefficients for all elements are presented in Table 3 and plotted in Fig. 3. As can be seen in Table 3, the set with WSI = 1.6858 and 2.5461 contain the elements, k_{24} and k_{67}, that are actually damaged. It is obvious from Fig. 3 that the damaged members are successfully located when only outputs are measured.

5.2. Example 2

A more real structure consists of 28 nodes and 53 members as shown in Fig. 4. The total length of the structure is 5.56 m with 0.40 m in each bay, and the height of the structure is 0.40 m. The members are steel bars with tube cross section with an inner diameter of 3.1 mm and an outer diameter of 17.0 mm. The elastic modulus of the material is $1.999 \times 10^{11}\,\mathrm{N/m^2}$, and the mass density is $7827\,\mathrm{kg/m^3}$. The members are connected at pinned joints. There are two supports at the ends of the structure: a pin support at the left end and a vertical roller support at the right end. The structure totals 53 dofs. One percent classical damping for every mode is modeled for this structure.

Damages in the structure is modeled by loss of sections in member 3, 17, 27, 50 and 53, and their reductions of section are by 70%, 90%, 90%, 70% and 90%, respectively. The excitations used to generate the vibration data is random white noise applied along the vertical direction at node 4, and the output is taken to be accelerations.

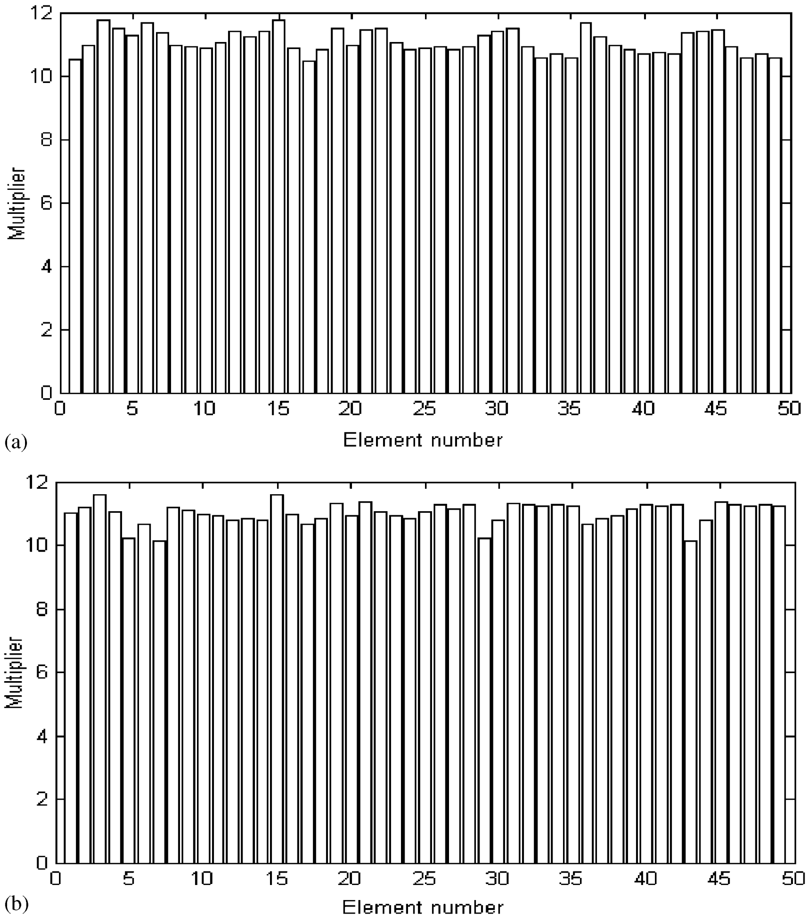

Fig. 2. Multipliers between elements in the PFM and those in flexibility matrix: (a) Undamaged structure; (b) Damaged structure.

Table 3
The WSI Indices for 7-dofs system

Element	WSI	Element	WSI
1	52.2593	6	15.2243
2	60.2081	7	66.6667
3[a]	1.6858	8	48.8963
4	61.8939	9[a]	2.5461
5	40.3163	10	42.5429

[a]Indicates damaged elements.

Acceleration responses from 13 measured degrees are used by the stochastic subspace identification (SSI) to perform the modal parameter identification. The identified first eight modes, as well as the analytical ones are shown in Table 4.

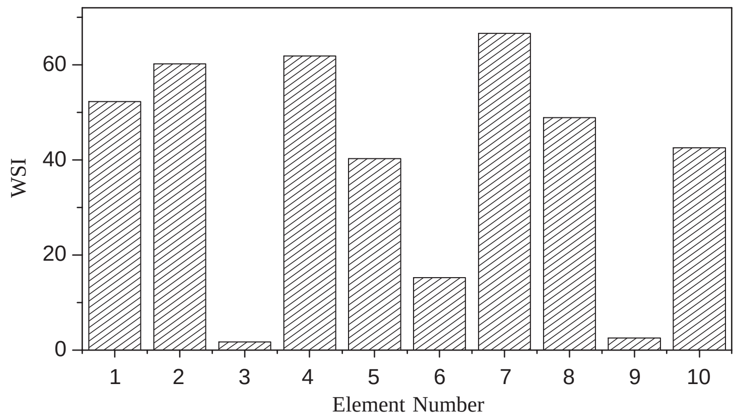

Fig. 3. WSI Indices for 7-dofs system (element 3 and 9 are damaged).

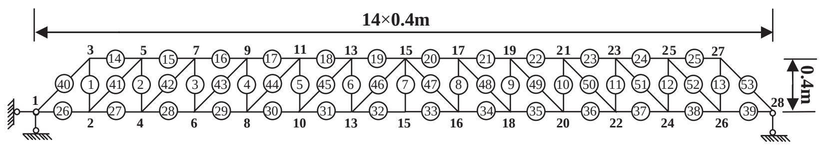

Fig. 4. The 14 bays truss.

Table 4
Analytical and identified natural frequencies of 14 bays truss

Modes	Analytical results		Identified results	
	Undamaged	Damaged	Undamaged	Damaged
1	8.7497	6.9374	8.7876	6.8875
2	29.5650	20.4471	29.5644	20.4286
3	43.3810	26.8073	43.3975	26.8026
4	59.0027	45.1092	58.9927	45.1030
5	90.4837	71.9382	90.4516	71.9344
6	119.6018	102.4855	119.5438	102.4426
7	124.2574	115.5995	124.1895	115.5516
8	150.7169	121.5042	150.5921	121.4425

The identified first eight modes are used to construct the PFMs both for the intact and damaged structures. The axial force distribution and the WSI coefficients for the truss members are calculated as index for locating damages. As can be seen in Table 5, the set with WSI = 2.1578, 1.2116, 0.7928, 2.1238 and 0.5653 contains the truss members that are actually damaged (truss member 3, 17, 27, 50, 53). The results shown in Fig. 5 also demonstrate that DLV method incorporated with PFMs successfully detects the multi-damage locations in this example.

Table 5
The WSI indices for 14 bays truss

Element	WSI	Element	WSI	Element	WSI
1	12.1469	19	14.3285	37	12.3934
2	11.8434	20	17.4407	38	12.3921
3[a]	2.1578	21	14.8924	39	13.3738
4	11.7236	22	18.4564	40	11.7141
5	8.4295	23	19.6447	41	7.9080
6	7.6756	24	13.0013	42	17.7020
7	11.3544	25	17.0868	43	13.6868
8	10.2909	26	13.8530	44	17.4265
9	13.7601	27[a]	0.7928	45	10.6663
10	8.3478	28	16.5072	46	10.5446
11	12.3740	29	19.0341	47	12.9813
12	14.1215	30	11.7603	48	7.2067
13	19.7494	31	20.5268	49	11.5803
14	13.6537	32	15.2066	50[a]	2.1238
15	18.4759	33	13.9287	51	15.9898
16	17.8310	34	12.0985	52	16.0632
17[a]	1.2116	35	17.9911	53[a]	0.5653
18	15.6092	36	13.2462		

[a]Indicates damaged elements.

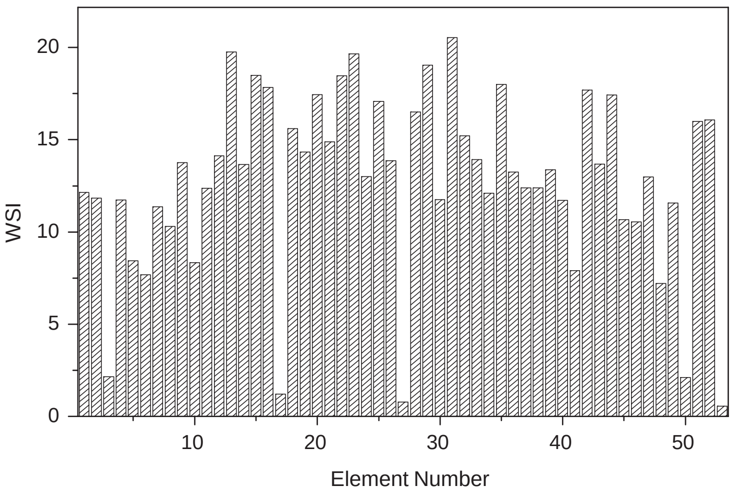

Fig. 5. WSI Indices for 14 bays truss (truss member 3, 17, 27, 50 and 53 are damaged).

6. Conclusions

A procedure to construct proportional flexibility matrix is presented with only output data where flexibility matrix is not achievable. With the introduction of dummy structure in relevancy with the real structure, the PFM is constructed within a scalar to the real flexibility matrix, and the scalar is shown to be the first modal mass. The proposed PFMs are integrated into available damage detection techniques, such as DLV method for damage localization. The comparability of PFMs for the pre- and post-damaged structures is guaranteed when the mass does not change significantly after damages. The proposed method is examined by two examples, and the multi-damages scenarios are successfully identified for a simple 7-dofs spring–mass system and a 14-bay planar truss structure. Due to simplifications, approximate solutions and computation errors, the in-proportionality of PFM to real flexibility is within the range such that DLV method can make correct damage localization. In both examples, the two-damage scenario and five-damage scenario are successfully identified by DLV method integrated with PFMs with only output data.

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