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Journal of Sound and Vibration 284 (2005) 494–502

JOURNAL OF
SOUND AND
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Short Communication

Radiation force acting on an absorbing cylinder placed in an incident plane progressive acoustic field

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Received 21 January 2004; received in revised form 20 July 2004; accepted 20 September 2004
Available online 24 December 2004

Abstract

The radiation force function Y_p for two viscoelastic polymeric cylinders suspended in an inviscid fluid in a plane incident sound (or ultrasound) field is calculated. The theory is modified to include the effect of hysteresis-type of compressional and shear waves' absorption in the cylinder material. This type of absorption is typical for a wide range of polymeric materials. The fluid-loading and temperature effects on the shape of the radiation force function are also considered. The results of numerical calculations indicate how damping, the fluid surrounding the cylinder, and temperature influence the frequency dependence of the material's scattering properties.

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1. Introduction

Many theoretical and experimental papers dealing with the acoustic radiation force and its temperature dependence on spheres [1–8] exist in the literature. Considerable attention has been directed particularly to the measurement of the radiation force intensity over a small region of the field on a suspended target, in order to calibrate high-frequency transducers. Awatani [9] developed the theory of acoustic radiation force experienced by a rigid cylinder and later Hasegawa et al. [10] expanded his work for a lossless elastic cylinder placed in a plane progressive sound field.

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doi:10.1016/j.jsv.2004.09.025

However, absorption of ultrasound waves by the material itself has been usually ignored in most previous studies and very little is known about the influence of temperature on an absorbing cylinder placed in a plane incident ultrasound field. Anson and Chivers [11] studied the temperature effect on the radiation force function curves for absorbing spheres in a very limited frequency range.

The goal of this letter is to extend the work done by Hasegawa et al. [10] to include the effect of hysteresis-type of absorption and study the influence of the ambient temperature on radiation force function curves for polymeric cylinders in a large-frequency bandwidth. The development of the theory for absorbing cylinders is necessary for medical applications since the acoustic radiation force can be used in the assessment of bone osteoporosis in which polymeric phantoms are currently used as samples for this purpose [12]. It is difficult to include all types of absorption in solids since all the mechanisms involved are frequency-dependent. Nevertheless, in this work the sound absorption was considered of constant amplitude as a function of frequency, describing the behaviour of many polymeric materials. Numerical calculations of the radiation force function Y_p are performed for two polymeric materials indicating how the penetration of the ultrasound waves into the target are affected by variations in the cylinder material scattering properties. A comparison of Y_p curves with the calculations of non-absorbent materials is presented to clearly demonstrate the effect of absorption and the radiation force experienced by a lucite cylinder immersed in a high-density fluid is also studied. In addition, the temperature influence on the radiation force function curves is considered.

2. Scattering of an incident plane progressive wave by an absorbing cylinder

Here, it is assumed that a plane wave is incident upon an infinite cylinder and can be described either by a velocity potential or pressure. The fluid in which the incident wave travels has a density ρ_0 and a sound speed c_0. The fluid medium is assumed to be non-viscous, so that acoustic streaming can be neglected. Using the cylindrical coordinates (r, θ, z) the pressure of the incident wave can be written as [13] :

$$P_i = P_0 \sum_{n=0}^{\infty} \varepsilon_n (-\mathrm{i})^n J_n(k_0 r) \cos(n\theta) \tag{1}$$

where P_0 is the amplitude, ε_n is the Neumann factor which is defined by $\varepsilon_0 = 1$, and $\varepsilon_j = 2$, $j = 1, \dots n$, $J_n(x)$ is the Bessel function of the first kind of order n and argument x, and k_0 is the wave number in the fluid medium. For convenience, the time factor $\exp(\mathrm{i}\omega t)$ is not explicitly written in Eq. (1).

The scattered wave is represented by [13]:

$$P_s = P_0 \sum_{n=0}^{\infty} \varepsilon_n (-\mathrm{i})^n d_n H_n^{(2)}(k_0 r) \cos(n\theta) \tag{2}$$

where ${H_n}^{(2)}$ is the Hankel function of the second kind and the factors d_n are the scattering coefficients [10] given by

$$d_n = -\left[\frac{F_n J_n(x) - x J'_n(x)}{F_n H_n^{(2)}(x) - x H_n^{(2)'}(x)}\right], \tag{3}$$

$x = k_0 a$ and the coefficients F_n correspond to $\tan \Phi_n$ [13] and are given by

$$F_n = \frac{\rho_0}{\rho^*} \frac{\tilde{x}_2^2}{2} \left[\frac{A_n(\tilde{x}_1) - B_n(\tilde{x}_2)}{C_n(\tilde{x}_1) - D_n(\tilde{x}_2)} \right] \tag{4}$$

with ρ_0 and ρ^* the mass densities of the fluid medium and cylinder material, respectively, $\tilde{x}_1 = x_1(1 - \mathrm{i}\gamma_1)$, $\tilde{x}_2 = x_2(1 - \mathrm{i}\gamma_2)$, $x_1 = (\omega/c_1)a$, $x_2 = (\omega/c_2)a$, with c_1 and c_2 the velocities of longitudinal and shear waves in the cylindrical material, and γ_1 and γ_2 the normalized longitudinal and shear absorptions, respectively [14]. (An erratum that in the coefficient F_n given by Eq. (6) in Ref. [10], the term x_2^2 is missed. The corrected coefficient F_n is given by Eq. (4).)

$$A_n(\tilde{x}_1) = \frac{\Lambda_n(\tilde{x}_1)}{\Lambda_n(\tilde{x}_1) + 1}, \quad B_n(\tilde{x}_2) = \frac{n^2}{\Lambda_n(\tilde{x}_2) + n^2 - \frac{1}{2}\tilde{x}_2}$$

$$C_n(\tilde{x}_1) = \frac{\Lambda_n(\tilde{x}_1) + n^2 - \frac{1}{2}\tilde{x}_2}{\Lambda_n(\tilde{x}_1) + 1}, \quad D_n(\tilde{x}_2) = \frac{n^2(\Lambda_n(\tilde{x}_2) + 1)}{\Lambda_n(\tilde{x}_2) + n^2 - \frac{1}{2}\tilde{x}_2} \tag{5}$$

with $\Lambda_n(\tilde{x}_i) = -\tilde{x}_i J'_n(\tilde{x}_i)/J_n(\tilde{x}_i)$; $i = 0, 1, 2$ where $J'_n(x)$ is the first order derivative of Bessel function of the first kind.

Absorption is included by the standard method of introducing complex wave numbers into the theory [15]. Incorporating complex wave numbers into the acoustic scattering theory holds only for *linear* viscoelasticity. Here, it is assumed that the normalized absorption coefficients of compressional and shear waves are constant quantities independent of frequency.

3. Acoustic radiation force function

The value of the radiation force per unit length can be expressed by [10]

$$F = \overline{E} Y_p S_c \tag{6}$$

where $\bar{E}$ is the mean energy density of the incident wave, Y_p is the radiation force function that depends on the scattering and absorption properties of the target and is the radiation force per unit cross-section and unit energy density, and $S_c = 2a$ is the cross-sectional area for a unit-length cylinder.

For cylinders, the acoustic radiation force function Y_p is expressed as [10]

$$Y_p = -\frac{2}{x} \sum_{n=0}^{\infty} [\alpha_n + \alpha_{n+1} + 2(\alpha_n \alpha_{n+1} + \beta_n \beta_{n+1})] \tag{7}$$

where the coefficient α_n, β_n are the real and imaginary parts of the scattering coefficients d_n, respectively, defined in Eq. (3).

4. Numerical results and discussion

The Y_p relationship was evaluated numerically by the use of Eq. (7). As initial test the Y_p curve for a brass cylinder was computed in the absence of absorption. Excellent agreement was found between this curve and Fig. 2 in Ref. [10]. The two polymeric cylindrical materials, for which graphical results are shown in the present letter, are listed in Table 1. These polymers are assumed to be immersed in water with a sound velocity 1500 m/s and a density 10^3 kg/m^3. The curves are plotted as a function of the size parameter k_0a. In addition, for ease of comparison, the same scales on the graphs have been used. The present results cover the range $0 \leqslant k_0a \leqslant 40$ in a k_0a step of 0.004. The importance of the statement of the k_0a step used in the calculations is to be noticed since the resonance phenomena encountered are very sharp and inadequate sampling in the size parameter may lead to false Y_p curves.

The results of the calculation of Y_p are dependent to a greater or less extent on the mechanical properties of the materials, as it has been mentioned above. For this reason, Table 1 lists properties of the materials used in the calculations, such as densities, longitudinal and shear wave velocities at room temperature and their absorption coefficients [11]. These materials are classified in Group I as given by Chivers and Anson [16] and the dependence of the general form of the Y_p curve on the different material parameters is evaluated.

The results are given in Figs. 1 and 2 for polymethylmethacrylate (PMMA) known as lucite and polyethylene cylinders, with and without the inclusion of absorption, respectively. Maxima and minima in Y_p curves for these materials correspond to resonance frequencies of normal vibrational modes of the cylinder. As expected, the decrease in peak values of Y_p curves in the case of absorption should undoubtedly be attributed to the damping due to ultrasound absorption by the cylinder material due to internal losses. It is a noteworthy fact that the effect of absorption is relatively small for the lucite cylinder compared to the polyethylene one, which has greater absorption coefficients. It is also shown in the figures that absorption affects the Y_p amplitude values. As the frequency increases, sound (or ultrasound) absorption increases, as it is well known. This effect is shown in the figures at high k_0a values where a damping of all peaks appears clearly. Another important observation is the radiation force enhancement for absorbing cylinders at high frequency. This enhancement is mostly due to sound absorption by the viscoelastic material; when absorption is strong (at high frequency), the sound-energy density in the fluid in the scattering area is small compared to the non-absorption case. Hence, the net force per cross-section of the cylinder in the direction of the incident waves is expected to be greater for an absorbing cylinder which is confirmed by these results.

Table 1
Material parameters used in the numerical calculations

Group I							
Material	Mass density (10^3 kg/m^3)	Compressional velocity c_1 (m/s)	Shear velocity c_2 (m/s)	Normalized longitudinal absorption γ_1	Normalized shear absorption γ_2	$-\frac{dc_1}{dT}$ (m/s °C)	$-\frac{dc_2}{dT}$ (m/s °C)
Lucite	1.191	2690	1340	0.0035	0.0053	2.5	2
Polyethylene	0.957	2430	950	0.0073	0.022	9.6	6.8

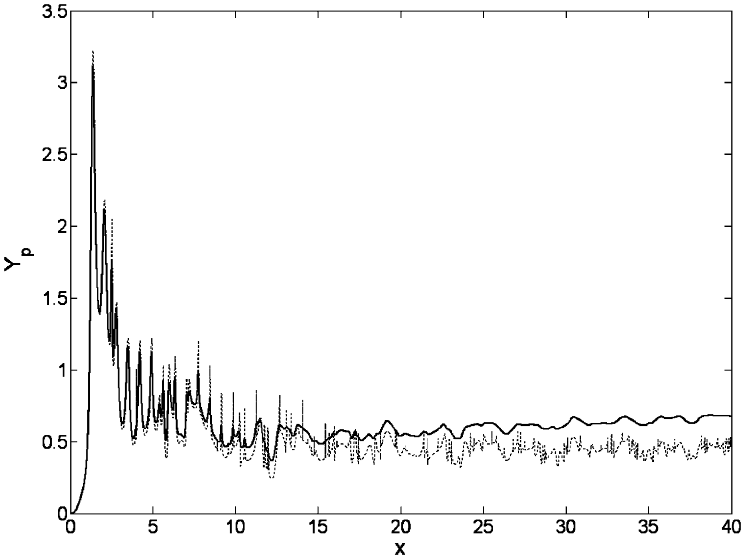

Fig. 1. Y_p curves for a lucite cylinder immersed in water without (dashed line) and with the effect of absorption (solid bold line).

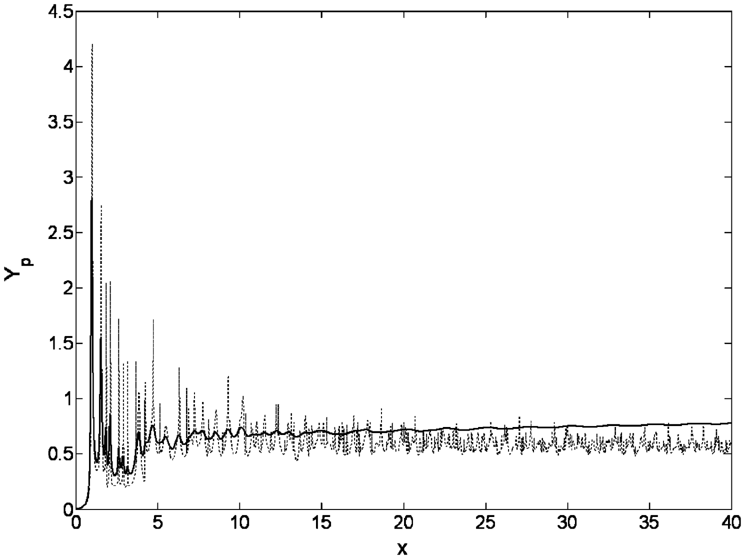

Fig. 2. Same as Fig. 1 but for a polyethylene cylinder.

Fig. 3 shows additional calculation of the radiation force function curve for a lucite cylinder immersed in a low-viscosity and high-density fluid (in this case mercury) with and without the effect of absorption. This computation was done in order to study the fluid-loading effect on the acoustic radiation force. The fluid density is $\rho_{\mathrm{Hg}} = 13579.04\,\mathrm{kg/m^3}$ in which the sound velocity is

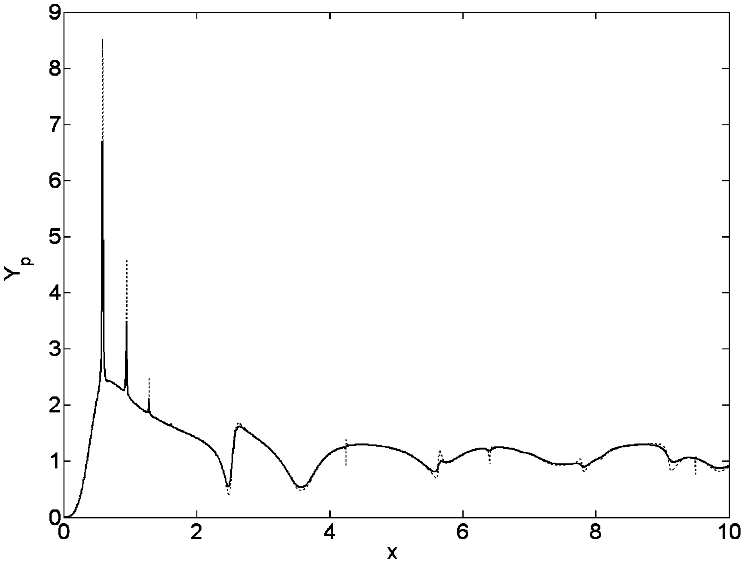

Fig. 3. Y_p curves for a lucite cylinder immersed in mercury without (dashed line) and with the effect of absorption (solid bold line). The high-resonance peak appears at $x_0 = 0.6$. One notices also that the acoustic radiation force is not greatly affected by sound absorption.

$c_{\mathrm{Hg}} = 1407\,\mathrm{m/s}$ [17]. (Here the viscosity of the fluid surrounding the cylinder is neglected.) As it was discussed in a previous work [18], the fluid-loading effect produces interactions between various resonance modes that can have a significant effect on the frequency response of the cylinder. This is clearly observed in Fig. 3 where a "giant resonance" peak appears at low k_0a values ($k_0a = 0.6$) and confirms the results of the work done on the backscattered sound from absorbing cylinders [18].

The effect of changing the temperature (T) of the fluid surrounding the cylinder is also considered. Here, it was assumed that the absorbing cylinders were immersed in water and the heat transfer was established for a long period of time, so there are no temperature variations when the acoustic radiation force is measured.

As it is well known, temperature affects the sound velocity in the medium. Therefore, the radiation force is very sensitive to variations of the sound speeds versus temperature. Thus, $x_0 = x_0(T)$, $\tilde{x}_1 = \tilde{x}_1(T)$ and $\tilde{x}_2 = \tilde{x}_2(T)$. The thermal coefficients (temperature derivatives of the sound speeds) of the surrounding fluid (in this case water) and those of the two wave (longitudinal and shear) velocities inside the polymeric material, were taken from Refs. [19,20], respectively, and are listed in the last two columns of Table 1. The dependence of the general form of the Y_p curve on the different material parameters is empirically analysed for three different temperature values (30 °C/50 °C/70 °C) for lucite and polyethylene materials with the inclusion of absorption. Variation of fluid and material densities versus temperature were neglected.

Figs. 4 and 5 show the effect of changing the temperature on the Y_p curves for lucite and polyethylene material, respectively; absorption is included. An increase of 20 °C in temperature induces amplitude (relative) variation by approximately 3.4% for lucite and up to −20% for polyethylene cylinder, whose absorption coefficients are greater that lucite. One notices also the

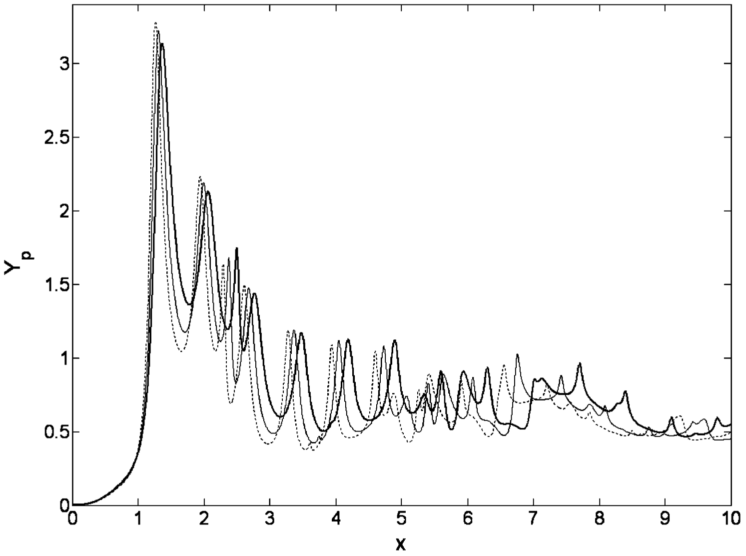

Fig. 4. Y_p curves for an absorbent lucite cylinder immersed in heated water; $T_1 = 30\,^{\circ}\mathrm{C}$ (solid bold line), $T_2 = 50\,^{\circ}\mathrm{C}$ (solid line), $T_3 = 70\,^{\circ}\mathrm{C}$ (dashed line).

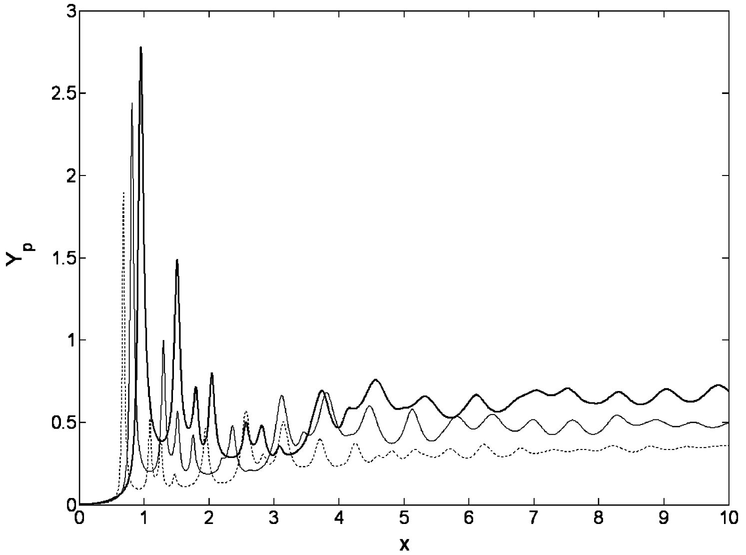

Fig. 5. Same as Fig. 4 but for a polyethylene cylinder.

k_0a-shift in the resonance peaks as observed in all the Y_p curves due to the effect of temperature. A detailed discussion concerning the behaviour of radiation force at high frequency as a function of temperature is complex since the effects of the different parameter variations are extremely dependent upon each other, and is outside the scope of the present letter. Here, Y_p curves shown in Figs. 4 and 5 were plotted in the range $0 \leqslant k_0 a \leqslant 10$ in which the variations of the resonance peaks are more significant.

5. Conclusion

The size parameter dependency of the acoustic radiation force function for absorbing cylinders placed in a plane incident ultrasound field is analysed for two polymeric materials with emphasis on the effect of temperature in a large-size parameter bandwidth. The results of numerical calculations are presented indicating how the surrounding fluid, damping and temperature affect the frequency dependence of the cylinder material's scattering properties. This study is also helpful in assessment of bone in osteoporosis by acoustic radiation force, since tubular polymeric phantoms are currently used as models for this purpose.

Acknowledgements

The author is indebted to the editor and anonymous reviewer for careful reading of the manuscript and for suggestions to improve its presentation.

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