Available online at www.sciencedirect.com
SCIENCE DIRECT
ELSEVIER
Journal of Sound and Vibration 284 (2005) 1033–1050
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

A robust singular value decomposition for damage detection under changing operating conditions and structural uncertainties

S. Vanlanduit*, E. Parloo, B. Cauberghe, P. Guillaume, P. Verboven

Department of Mechanical Engineering (WERK), Vrije Universiteit Brussel, Pleinlaan 2, B-1050 Brussels, Belgium

Received 31 October 2003; received in revised form 19 July 2004; accepted 26 July 2004
Available online 18 November 2004

Abstract

A technique is proposed to detect damage in structures from measurements taken under different conditions (i.e. different operational excitation levels, geometrical uncertainties and surface treatments of the structure). The method is based on a robust singular value decomposition (RSVD) which will be introduced in this article. Using the RSVD the distance of an observation to the subspace spanned by the intact measurements can be computed. Furthermore, from statistics, a threshold can be determined to automatically decide, based on the observation's distance to the subspace, if the observation comes from a damaged or intact sample. The proposed RSVD method is compared with an existing method based on the classical least-squares (LS) SVD. The damage detection method is validated on an aluminium beam with different damage scenarios (a saw cut and a fatigue crack), under several conditions (different beams with small dimensional changes, beams covered with damping material and different operating levels).

1. Introduction

In recent decades many techniques have been proposed for detecting damage based on changes of modal parameters. Unfortunately, the modal parameter changes due to varying operating conditions and structural uncertainties (e.g. due to inter-product variability) are often much more

*Corresponding author. Tel.: +32 2 629 2805; fax: +32 2 629 2865.
E-mail address: steve.vanlanduit@vub.ac.be (S. Vanlanduit).

doi:10.1016/j.jsv.2004.07.016

important, and hence they mask the information on the damage state of the structure. Therefore, most vibration-based damage detection techniques only give good results in well-controlled laboratory conditions.

During the last decades many techniques were proposed to detect damage based on changes of modal parameters (see the literature survey in Ref. [1]). A common property of these methods—based on changes in modal parameters—is that they are very sensitive to changes in: (a) the environment, (b) the operational conditions, and (c) structural uncertainties. Recently, a few publications appeared that aim at removing the influence of these variations. In Ref. [2] a technique is proposed to eliminate the influence of operational conditions. The technique is based on a singular value decomposition (SVD) of a frequency response function (FRF) matrix $\mathbf{H} = [H_1, \ldots, H_N]$, where $H_1, \ldots, H_N$ are FRFs at N conditions. In Ref. [3] another matrix decomposition (principal component analysis or PCA) of a matrix of estimated features (i.e. resonance frequencies) is used to eliminate the influence of environmental changes. Both authors have shown that matrix decompositions can help in removing the influence of varying conditions of the structure/measurement. However, when damaged features (FRFs or estimated parameters) are included in the matrix $\mathbf{H}$, the decomposition largely deviates from the one based on merely features from intact structures. This is true since damaged features are outliers, as was recognized by Worden in Ref. [4] (in the paper [4] the fact that a damaged feature is an outlier with respect to the features of intact samples is used to separate damaged and intact observations). This means that existing SVD damage detection techniques [5,3] only work well when the SVD of the data matrix from intact observations can be computed (in general, however, it is not known beforehand if an observation is intact or damaged).

The purpose of this article is to present a robust SVD which computes the SVD of the intact structures from a set of observations of both intact and damaged structures, which are possibly measured in different conditions. In order to guarantee the success of the proposed technique, it is assumed that there are at least as many intact as damaged observations (FRFs, response spectra or estimated parameters) are present in the observation matrix $\mathbf{H}$ which has to be decomposed.

In the next section the theory behind the method will be described. Firstly, in Section 2.2 a review of damage detection using the least-squares SVD will be given. Next, in Section 2.3, a somewhat more robust iterative SVD (ISVD) is introduced. The proposed robust SVD (RSVD) is described in Section 2.4. All three methods are validated on an extensive experimental case study of an aluminium beam measured under various conditions. The results of the experimental validation can be found in Section 4. Finally conclusions will be drawn in Section 5.

2. Theory

2.1. Motivation of SVD-based damage detection techniques

Depending on the material type of a structure and the loading scenario, different types of damage can be present in a structure: (fatigue) cracks, delaminations, wear, etc. From a structural behaviour point of view, damage is characterized by a local reduction in the stiffness near the location where the damage is present. This local stiffness reduction effect can be used to separate damage with changes due to environmental conditions. Indeed, the latter usually introduces a global change in the stiffness and/or damping of the structure (this is certainly the case when the structure is made out of one material).

Using a sensitivity analysis, the derivative of the ith resonance frequency ω_i with respect to a change in the parameter u can be determined [5]:

$$\frac{\delta\omega_i}{\delta u} = \frac{1}{2m_i}\psi_i^t\left(\frac{1}{\omega_i}\frac{\delta\mathbf{K}}{\delta u}\right)\psi_i, \tag{1}$$

with ψ_i the ith mode shape and m_i the ith modal mass. For a uniform stiffness change $\delta\mathbf{K}$ can be written as $\delta\mathbf{K} = q\mathbf{K}$ with q the percentage of stiffness reduction. This means that Eq. (1) can be re-written as

$$\frac{\delta\omega_i}{\delta u} = \frac{q}{2m_i\omega_i}\psi_i^t\mathbf{K}\psi_i = \frac{qk_i}{2m_i\omega_i} = \frac{q\omega_i}{2} \tag{2}$$

because $k_i/m_i = \omega_i^2$, with k_i the modal stiffness.

Eq. (2) clearly shows that a global change in stiffness results in a linear change in resonance frequency. For local changes this is generally not the case. This fact will be used to separate damage (local stiffness reduction and nonlinear resonance frequency change) from environmental changes (global stiffness reduction and linear change). An efficient technique to detect nonlinear changes is to compute a linear subspace from intact observations (using a singular value decomposition) and calculate the distance of the damaged observation to the linear subspace. In the next sections several algorithms to perform this task will be described.

2.2. Classical least-squares SVD

Assume that $\mathbf{H} = [H_1, \ldots, H_N]$ is a matrix of structural observation features taken from N experimental conditions (both N_d-damaged and N_u-undamaged, where it is assumed that $N_u \leqslant N_d$). As will be shown below, the features can be FRFs, response spectra or estimated modal parameters of the structure. Then, based on the classical least-squares SVD, an automatic classification between damaged and intact observations can be made as follows:

Algorithm. Least-squares SVD-based damage detection.

(1) Compute the SVD of the M by N matrix $\mathbf{H}$: $\mathbf{H} = \mathbf{USV}^H$ with $\mathbf{S} = \mathrm{diag}(\sigma_1, \ldots, \sigma_{\min(N,M)})$.
(2) Put all singular values $\sigma_{L+1}, \ldots, \sigma_{\min(N,M)}$ below the noise level equal to zero: $\mathbf{S_1} = \mathrm{diag}(\sigma_1, \ldots, \sigma_L, 0, \ldots, 0)$.
(3) Re-synthesize the singular vectors to a rank L matrix: $\mathbf{H}_1 = \mathbf{US}_1\mathbf{V}^H$
(4) Compute the residual matrix $\mathbf{E_1}$: $\mathbf{E_1} = \mathbf{H} - \mathbf{H_1}$.
(5) Estimate the standard deviation s of the residuals: $s = [1/(MN-1)]\sum_{i=1}^{M}\sum_{j=1}^{N}(\mathbf{H}(i,j) - \mathbf{H_1}(i,j))^2$.
(6) For each observation j (i.e. each column j) estimate the standard deviation $s_j : s_j = [1/(M-1)]\sum_{i=1}^{M}(\mathbf{H}(i,j) - \mathbf{H_1}(i,j))^2$.
(7) Now, if observation j (column j) can be spanned by the subspacecomputed using the SVD, the relative distance d_j^{SVD} defined by $d_j^{\mathrm{SVD}} = s_j/s$ should—for each j—obey a χ^2 distribution with mean $m = 1$ and standard deviation $\sigma = \sqrt{2M}$ [6]. The $\alpha = 95\%$ confidence level of the χ^2

distribution gives the threshold T for the distance d_j^{SVD} to distinguish between intact and damaged samples: when $d_j^{\mathrm{SVD}} > T$ with $T = \sqrt{(M-1)\chi^2_{(100-\alpha)/2(M-1)}}$ the structure is damaged (for values $M > 30$ a maximal value of $M = 30$ is taken in order to make sure that the threshold is not too tight).

For Step (2) of the least-squares SVD algorithm described above, a subdivision should be made based on the actual feature that is used in the matrix $\mathbf{H}$:

- When FRFs or response spectra (displacements, velocities or outputs) are taken as the columns of $\mathbf{H}$, the number of non-zeros singular values (L) is equal to $\min(N_m, N_o)$, with N_m the number of modes in the frequency band and N_o the number of measured spectra (outputs) for each observation (this is the case because the rank of an FRF matrix of an L-dof system is equal to L).
- When on the other hand estimated parameters (as for instance the resonance frequencies of the structure) are taken as features, there is only one non-zero singular value. Indeed, for environmental changes or small global structural variabilities (in material properties, surface treatment), resonance frequencies change linearly, and thus all observations can be spanned by one single singular vector (remark that for localized damage on the other hand, a nonlinear shift in resonance frequencies occurs).

The problem with the least-squares SVD technique is that it is very sensitive to outliers in the measurements. This is illustrated in Fig. 1 for a data matrix $\mathbf{H} = \begin{bmatrix} 1 & \cdots & 16 \\ 1 & \cdots & 16 \end{bmatrix}$, where $\mathbf{H}(16, 2)$ is put equal to 0 to introduce an outlier. The result of the SVD decomposition in Fig. 1 clearly shows that the least-squares solution is influenced to an important extent by the single outlier.

In the next section a more robust SVD-based damage detection method will be introduced to reduce the influence of outliers (i.e. damaged features).

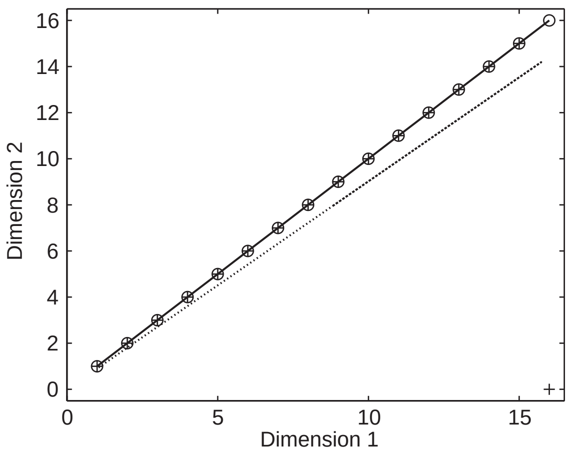

Fig. 1. Illustration of the effect of an outlier on the least-squares SVD. ‘o’: data without outlier, ‘+’: data with outlier, full line: least-squares fit of the data without outliers, dotted line: least-squares fit of the data with the outlier.

2.3. Iterative SVD

The first approach to reduce the influence of outliers is to use a two-step iteration approach:

- Use the classical least-squares SVD algorithm from the previous section.
- Eliminate observations with an outlying distance d_j^{SVD}.
- Compute an SVD of the remaining observations.

In detail the method contains the following steps:

Iterative SVD (ISVD)-based damage detection.

(1) Steps (1)–(7) are equal to those of the least-squares SVD presented in the previous section.
(2) Next, median of the observation distances is computed: $d_{\mathrm{med}} = \mathrm{median}(d_1^{\mathrm{SVD}}, \ldots, d_N^{\mathrm{SVD}})$.
(3) The columns $j_1, \ldots, j_{N/2}$ which have a distance $d_j^{\mathrm{SVD}} > d_{\mathrm{med}}$ are removed from the data matrix:
$$\mathbf{H_2} = \mathbf{H}(:, \{1, \ldots, N\}/\{j_1, \ldots, j_{N/2}\})$$
(4) The SVD of the reduced data matrix is computed: $\mathbf{H_2} = \mathbf{U_2 S_2 V_2}^H$ with $\mathbf{S_2} = \mathrm{diag}\,(\sigma_1, \ldots, \sigma_{\min(N/2,M)})$.
(5) Compute the right singular vectors: $\mathbf{V_3} = \mathbf{U_2}^H \mathbf{S}_2^{-1}\mathbf{H}$.
(6) Re-synthesize the singular value decomposition: $\mathbf{H_3} = \mathbf{U_2 S_3 V_3}^H$, with $\mathbf{S_3}(i,i) = \mathbf{S_2}(i,i)$ for $i = 1, \ldots, L$ and $\mathbf{S_3}(i,i) = 0$ for $i > L$.
(7) Compute the residual matrix $\mathbf{E_3}$: $\mathbf{E_3} = \mathbf{H} - \mathbf{H_3}$.
(8) Estimate the standard deviation s of the residuals using the median absolute deviation (MAD) as a robust scale estimator [7]: $s = \mathrm{MAD}(\mathbf{E_3})$ with $\mathrm{MAD}(x) = 1.4826 * \mathrm{median}(|x - \mathrm{median}(x)|)$ (remark that this is necessary because half of the observation could be outliers, which would lead to a large increase of the classical sample variance).
(9) For each observation j (i.e. each column j) estimate the standard deviation s_j: $s_j = [1/(M-1)]\sum_{i=1}^{M}(\mathbf{H}(i,j) - \mathbf{H_3}(i,j))^2$.
(10) Now, if observation j can be spanned by the L-dimensional subspace computed using the SVD, the residual for each j the distance $d_j^{\mathrm{ISVD}} = s_j/s$ should obey a χ^2 distribution with mean $m = 1$ and standard deviation $\sigma = \sqrt{2M}$. The $\alpha = 95\%$ confidencelevel of the χ^2 distributiongives the threshold $T = \sqrt{(M-1)\chi^2_{(100-\alpha)/2(M-1)}}$ for the distance d_j^{ISVD} to distinguish between intact and damaged samples: when $d_j^{\mathrm{ISVD}} > T$ the sample corresponding to observation j is considered damaged.

2.4. Robust SVD

The robust SVD method (RSVD) that is proposed in this section is the SVD counterpart of the so-called least trimmed squares procedure which was developed for linear regression in Ref. [8] (later on, a faster variant of the algorithm was implemented in Ref. [9]).

The proposed RSVD method is conceptually very simple:

- Take a random subset of half of the columns in the data matrix.
- Compute the SVD of these columns.

- Extend the right singular matrix to all the columns in the data matrix.
- Calculate the residual between the original data matrix and the re-synthesized matrix based on the first L singular vectors corresponding to the combination of columns.
- Compute the RMS value of the 50% smallest distances $d_{k_1}, \ldots, d_{k_{(N/2)}}$ as the cost function κ.
- The SVD of the combination of columns with the smallest cost function κ is taken as the RSVD solution.

In more detail, the following steps have to be performed:

Robust SVD damage detection.

(1) Construct a matrix $\mathbf{C}$ within the rows of all combinations of $N/2$ observations (column numbers of the $\mathbf{H}$ matrix) out of the total of N observations.
(2) For each of these $N!/(N!/2)^2$ combinations $i_1, \ldots, i_{N/2}$—i.e. rows of $\mathbf{C}$—(or 250 randomly selected combinations in case $N > 10$):

  (a) Remove columns $i_1, \ldots, i_{N/2}$ from the data matrix $\mathbf{H}: \mathbf{H_4} = \mathbf{H}(:, \{1, \ldots, N\}/\{i_1, \ldots, i_{N/2}\})$.
  (b) Compute the SVD of $\mathbf{H_4}$: $\mathbf{H_4} = \mathbf{U_4 S_4 V_4^{\mathit{H}}}$ with $\mathbf{S_4} = \mathrm{diag}\,(\sigma_1, \ldots, \sigma_{\min(N/2,M)})$.
  (c) Compute the extended right singular vectors: $\mathbf{V_5} = \mathbf{U_4^{\mathit{H}} S_{4_2}^{-1} H}$.
  (d) Re-synthesize the singular value decomposition: $\mathbf{H_5} = \mathbf{U_4 S_5 V_5^{\mathit{H}}}$, with $\mathbf{S_5}(i,i) = \mathbf{S_4}(i,i)$ for $i = 1, \ldots, L$ and $\mathbf{S_5}(i,i) = 0$ for $i > L$.
  (e) Compute the residual $\mathbf{E_5}$: $\mathbf{E_5} = \mathbf{H} - \mathbf{H_5}$.
  (f) Calculate the distances $d_j = s_j/s$(as in the ISVD algorithm).
  (g) Evaluate the cost function κ: $\kappa = \sum_{k=1}^{N/2} |d_{k:N}|^2$.

(3) The SVD of the combination of columns with the smallest cost function κ is taken as the RSVD solution.
(4) The residual $\mathbf{E_6} = \mathbf{H_6} - \mathbf{H}$ between the data $\mathbf{H}$ and the re-synthesized RSVD $\mathbf{H_6}$ is used to compute the distances $d_j^{\mathrm{RSVD}} = s_j/s$ where $s = \mathrm{MAD}(\mathbf{E_3})$ and $s_j = [1/(M-1)]\sum_{i=1}^{M} (\mathbf{H}(i,j) - \mathbf{H_3}(i,j))^2$.
(5) The test statistic $T = \sqrt{(M-1)\chi^2_{(100-\alpha)/2(M-1)}}$ is used to classify the intact and damaged observations: $d_j > T$ for a damaged structure.

3. Simulation results

In order to quantify the sensitivity of the proposed technique a simulation is performed in this section. Twelve cases of a 10 degree-of-freedom (dof) system (with mass 1 kg and stiffness 3000 N/m) are considered:

- In the first 6 cases a uniform reduction of the stiffness is applied to all 10 dofs (Case 1: 0.1%, Case 2: 0.5%, Case 3: 1%, Case 4: 2%, Case 5: 5% and Case 6: 7.5%). This is used to simulate the global influence of environmental changes.

- In the last 6 cases a reduction is applied to only one of the links (six different randomly chosen link locations) between the dofs: (Case 7: 1%, Case 8: 5%, Case 9: 10%, Case 10: 20%, Case 11: 50% and Case 12: 75%). This simulates the presence of a crack between two dofs.

The results of the three SVD methods when applied to the simulation data are shown in Fig. 2. The classical least-squares SVD incorrectly classifies the two smallest damage cases of 1% and 5% stiffness reduction (see Fig. 2a). Moreover, the distances of the intact observations are very close to the threshold. Although the ISVD gives better results (see Fig. 2b) the smallest damage case (1% stiffness reduction) is still not detected. The RSVD finally gives a very clear separation between damaged and intact observations, even for the smallest damage case (see Fig. 2c).

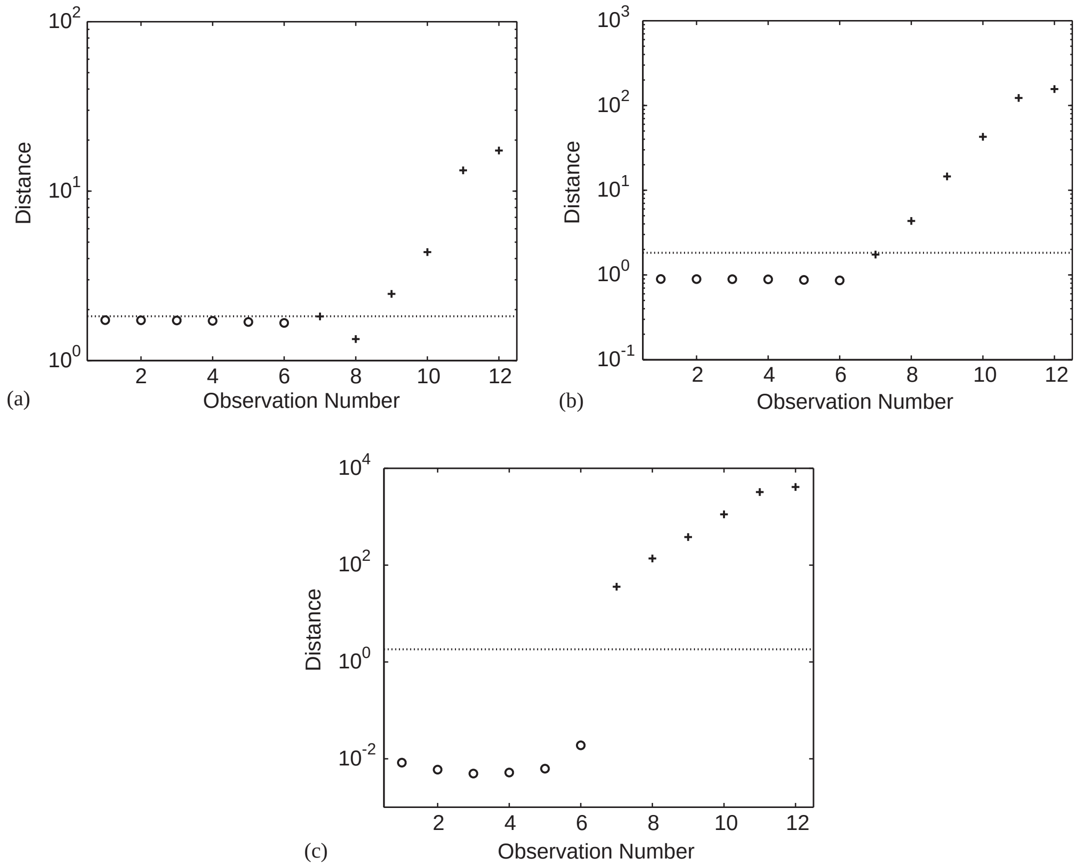

Fig. 2. Relative distances of 12 observation cases (six damaged indicated by '+' and six intact indicated by 'o'): (a) d_j^{SVD}; (b) d_j^{ISVD}; and (c) d_j^{RSVD} from SVD, iterative SVD and robust SVD, respectively.

4. Experimental results

4.1. Set-up

In this paper, nine different experimental cases are considered in order to be able to compare the sensitivity of the proposed damage detection technique (see Section 2) in the presence of structural uncertainties with respect to:

- the structures' geometry (product variability);
- the surface treatment and boundary conditions.

The following cases are tested:

Case 1: Aluminium beam 1 (dimensions 400 × 5 mm), intact.
Case 2: Beam from Case 1 covered with plastic tape (see Fig. 3a).
Case 3: Beam from Case 1 covered with acoustic damping material (see Fig. 3b).
Case 4: Aluminium beam 2 (same dimensions as beam 1 with a small variability), intact.
Case 5: Aluminium beam 3 (same dimensions as beam 1 with a small variability), intact.
Case 6: Beam from Case 4 with 30% through saw cut in the middle.
Case 7: Beam from Case 4 with 50% through saw cut in the middle (see Fig. 3c).

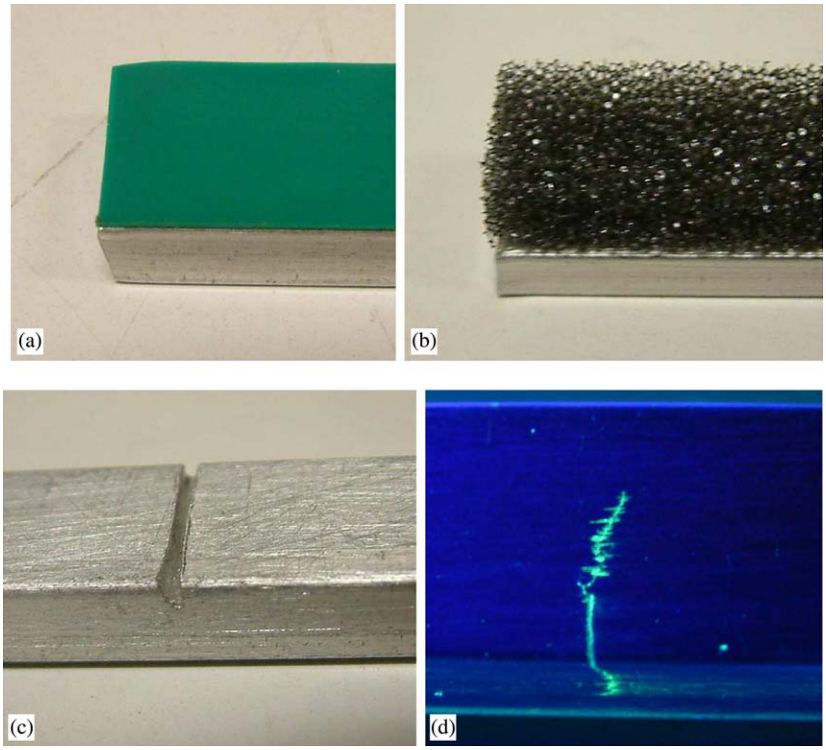

Fig. 3. Beams with different environmental conditions and damage scenarios: (a) covered with plastic tape; (b) covered with acoustic damping material; (c) saw cut; and (d) fatigue crack (made visible with liquid penetrant using UV light).

Case 8: Beam from Case 5 with small fatigue crack.
Case 9: Beam from Case 5 with large fatigue crack (see Fig. 3d).

Remark that the first five cases represent an intact realization of the structure, while the last four cases are damaged. The results are given in Section 4.3.

Moreover, in order to evaluate the sensitivity to changing operating conditions, each of Cases 1–9 was measured using four operating conditions corresponding to excitation levels 0.5, 1, 2 and 4 V. In Section 4.2 it will be shown that it is possible to eliminate the influence of the operating conditions in the damage detection process.

The beams in the 36 experiments (nine cases and four excitation levels) were supported using thin nylon threads (see Fig. 4). A loudspeaker was used to excite the beams with a periodic chirp (0.5, 1, 2 and 4 V amplitudes) in the 100–4000 Hz frequency range with a 1.25 Hz resolution. A Polytec scanning laser vibrometer was used to measure the response velocity at 25 equidistant locations of the beams in the different experiments.

4.2. Damage detection with changing operating conditions

In first instance, the following velocity spectra where used as inputs for the proposed damage detection method (see Fig. 5):

- Case 5 with excitation levels 0.5, 1, 2 and 4 V
- Case 8 with excitation levels 2 and 4 V
- Case 9 with excitation levels 2 and 4 V

The results discussed below will show that the method is insensitive to operating conditions (i.e. excitation levels).

The key of the proposed method is based on the fact that the velocity spectra form a subspace spanned by a set of N_m vectors as follows from modal analysis theory [5,10] (with the number of

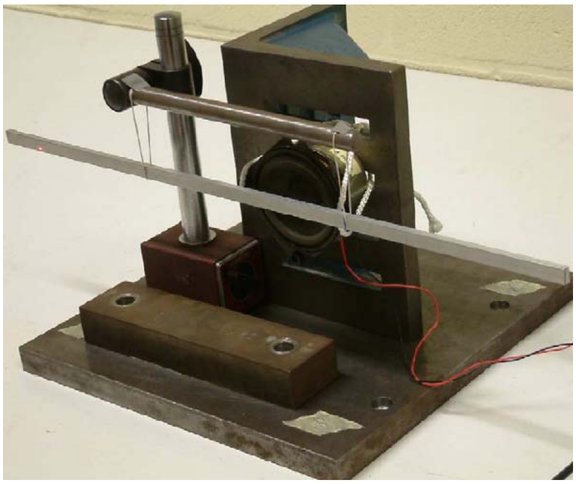

Fig. 4. Measurement set-up for the aluminium beam experiment.

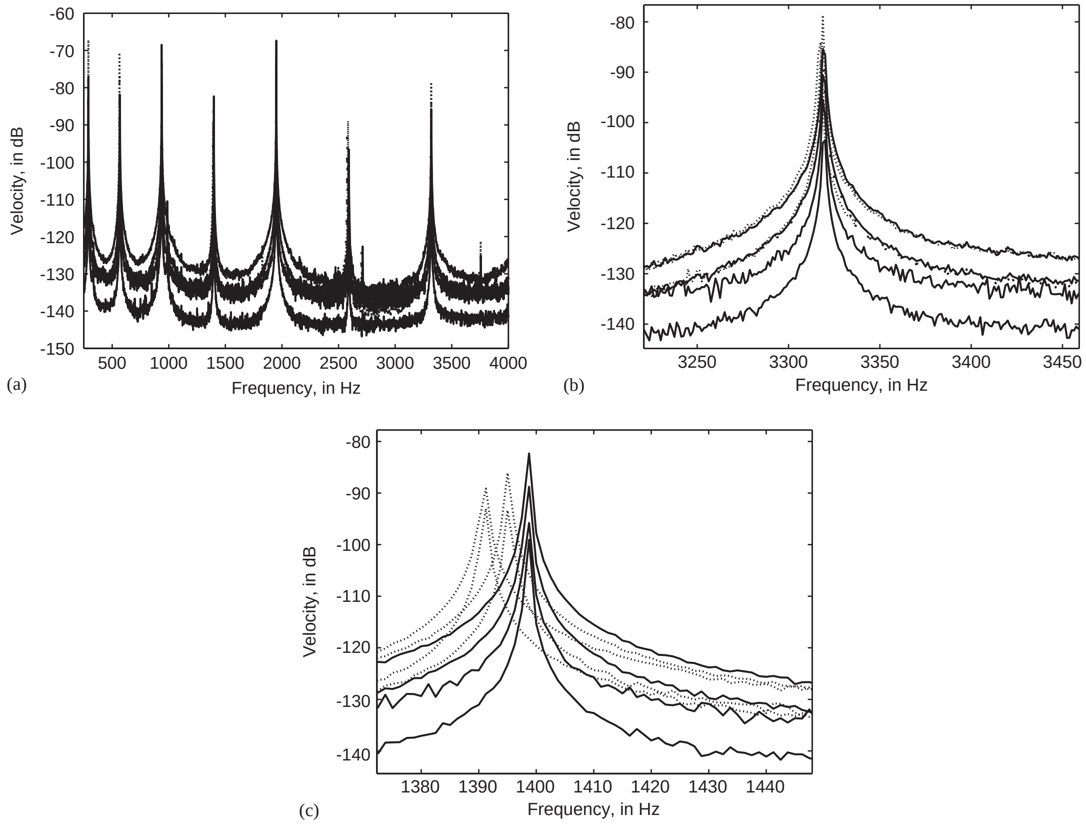

Fig. 5. (a) Velocity spectra of the intact (full line) and fatigue cracked (dotted line) under different operating conditions (Cases 5, 8 and 9); (b) and (c) zooms around an even and odd mode of the figure in (a).

modes $N_m = 7$ for the beam as can be seen from Fig. 5). When the operational condition changes, the subspace does not change: indeed, from Fig. 5b it can be seen that the responses are linearly related. On the other hand, when damage occurs, the subspace changes (see the dotted lines in Fig. 5c). When the subspace is computed from the matrix **H** containing a combination of velocity spectra from intact and damaged velocity spectra and the classical SVD approach is used (Section 2.2), the result is that the number of non-zero singular values is larger than the number of modes (indeed in Fig. 6a there is only 12 dB difference between the first singular value and the 8th singular value, which should be at the noise level if only intact samples are included in the data matrix).

When using the ISVD from Section 2.3 (i.e. when computing an SVD of the measurements with a distance below the threshold in Fig. 7 as described in Section 2.3) there is already 20 dB between the first singular value and the 8th one (Fig. 6b). For the RSVD solution (see Section 2.4) the difference is more than 33 dB (Fig. 6c). This means that in the RSVD case the subspace of the original intact data (which should contain seven modes and therefore the rank is 7) is found back, while in the other cases (SVD and ISVD) a mixture between intact and damaged subspaces is found. In fact the damaged spectra are outliers (with respect to the intact spectra) which attract

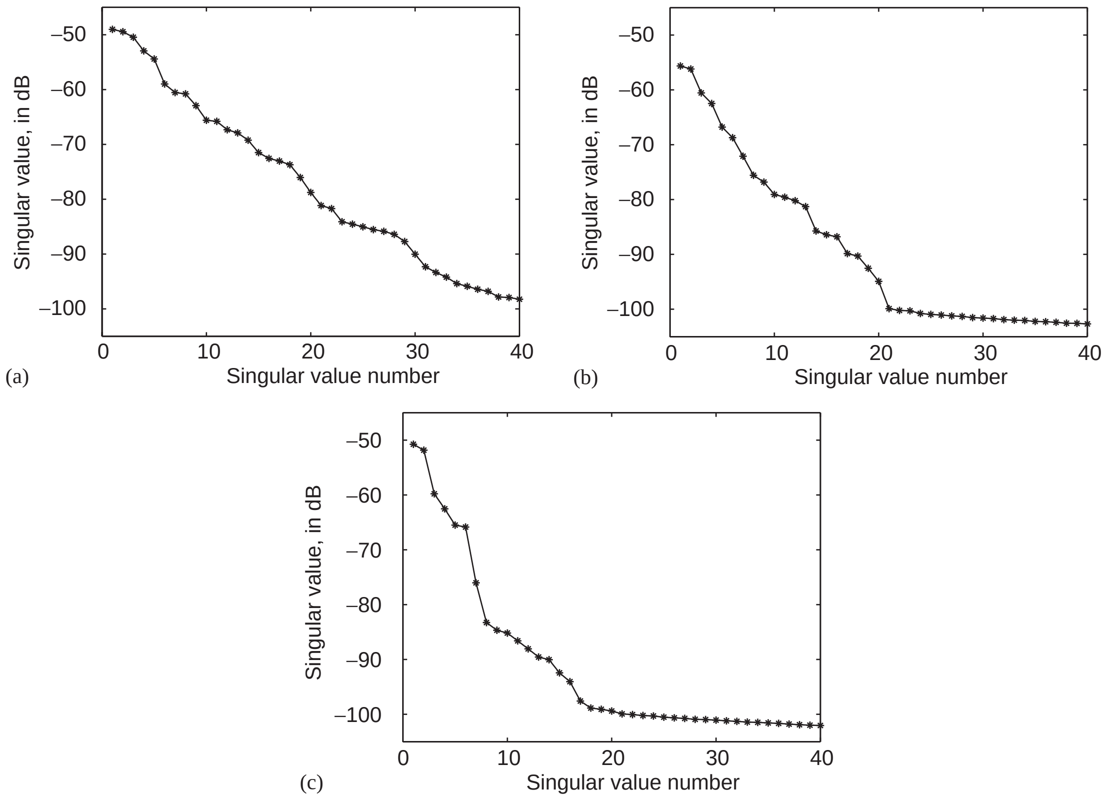

Fig. 6. Singular values of the different SVD techniques when applied on the velocity spectra of Cases 5, 8 and 9: (a) least-squares SVD (Section 2.2); (b) iterative SVD (Section 2.3); and (c) robust SVD (Section 2.3).

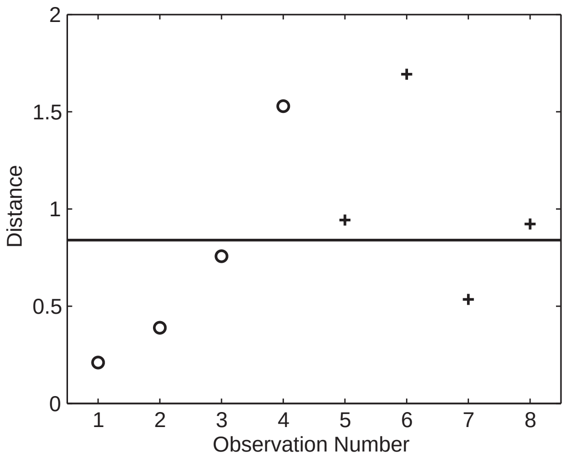

Fig. 7. Distances d_{SVD} of the eight observations at different operating conditions to the LS subspace ('o': intact, '+': damaged). Observations above the full line (median of the distances) are discarded in the iterative SVD approach (Section 2.3).

the subspace of the intact beam towards one of the subspaces of the damaged beam as was illustrated before in Section 2. The result of this erroneously computed subspace is that the distance of the observations to the LS subspace cannot be used to distinguish between intact and damaged samples. In fact, as can be seen from Fig. 8a, three damaged observations have a scaled distance d_j^{SVD} that is smaller than the 95% confidence limit 1.34 (for $M > 30T = \sqrt{(29)\chi^2_{0.05}(29)} = 1.34$) and are thus considered as intact, while one intact sample (observation 4) is incorrectly classified as damaged. For the iterative SVD the distances lead to better results as is shown in Fig. 8b. However, the third and the fourth observations are still incorrectly classified as being damaged. The RSVD distances give a clear and correct classification. From Fig. 9b it can be seen that the cost function evaluated for different combinations of observations increases when the number of damaged observations in the combination increases (this assures that the combination in Fig. 9a corresponding to the minimal cost function contains no damaged observations).

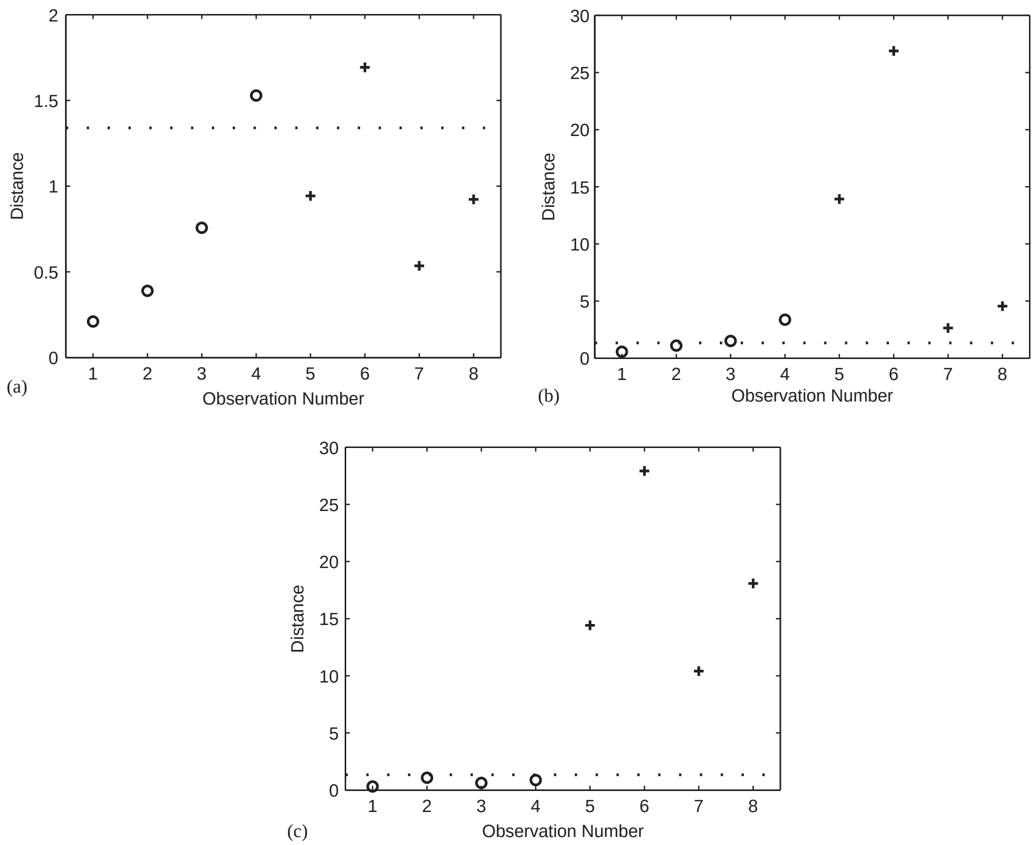

Fig. 8. Relative distances for eight observations in Cases 5, 8 and 9: (a) d_j^{SVD}; (b) d_j^{ISVD}; and (c) d_j^{RSVD} from SVD, iterative SVD and robust SVD, respectively.

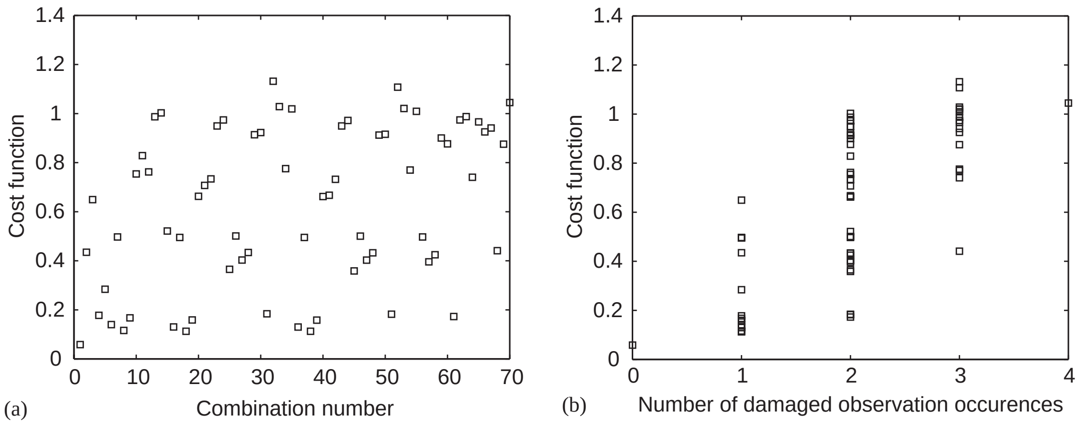

Fig. 9. (a) Cost function κ for different combinations of observations throughout the RSVD algorithm; (b) cost function κ in function of the number of damaged observations in each of the combinations throughout the RSVD algorithm.

4.3. Damage detection with structural uncertainties

While the subspace remains invariant under changing operating conditions (excitation levels) it does change when the conditions of the structure is perturbed. Therefore, the approach illustrated before on the experiment in Section 4.2 cannot be used straightforwardly. Instead, the fact that a global structural changes gives rise to a linear change in the resonance frequency, while localized damage leads to nonlinear changes is used to classify between damaged and intact observations. Therefore, the resonance frequencies of the 36 experiments (nine cases and four operating conditions) are extracted from the mobilities (see Fig. 10) using a fast-stabilizing frequency domain estimator [11]. As a reference, the mode table of the beam in Case 1 with 1V excitation level is given in Table 1, while four of the identified mode shapes are shown in Fig. 11. When calculating the relative resonance frequency shift of Cases 1–9 (excitation level at 2 V) with respect to Case 1 (excitation level 1 V) it can be seen (Fig. 12) that the undamaged beams (full line) undergo a linear resonance frequency shift (up to 1.5%) while the damaged samples (dotted line) result in a nonlinear change.

As was the case in the previous section, the subspace computed with the least-squares SVD is incorrect. Indeed, the difference between the first and the second singular value is 40 dB (see Fig. 13a), while the difference for the iterative SVD (Fig. 13b) and robust SVD (Fig. 13c) is more than 60 dB.

The distance d_j^{SVD} of the observations to the least-squares SVD subspace do not allow a good distinction between damaged and intact samples (see Fig. 15a). Two of the four damaged samples (observations 6 and 8) are classified as intact, and one observation from an intact beam falls outside the 95% confidence limit $T\,(T = \sqrt{(6)\chi^2_{0.05}(6)} = 2.2)$. Moreover, the remaining four intact observations $(1, 2, 4$ and $5)$ have a value very close to the threshold. When discarding all observations above the median of the distances (see the full line in Fig. 14, the classification is improved). Indeed, from Fig. 15b it is clear that Observation 6 is now classified correctly and that

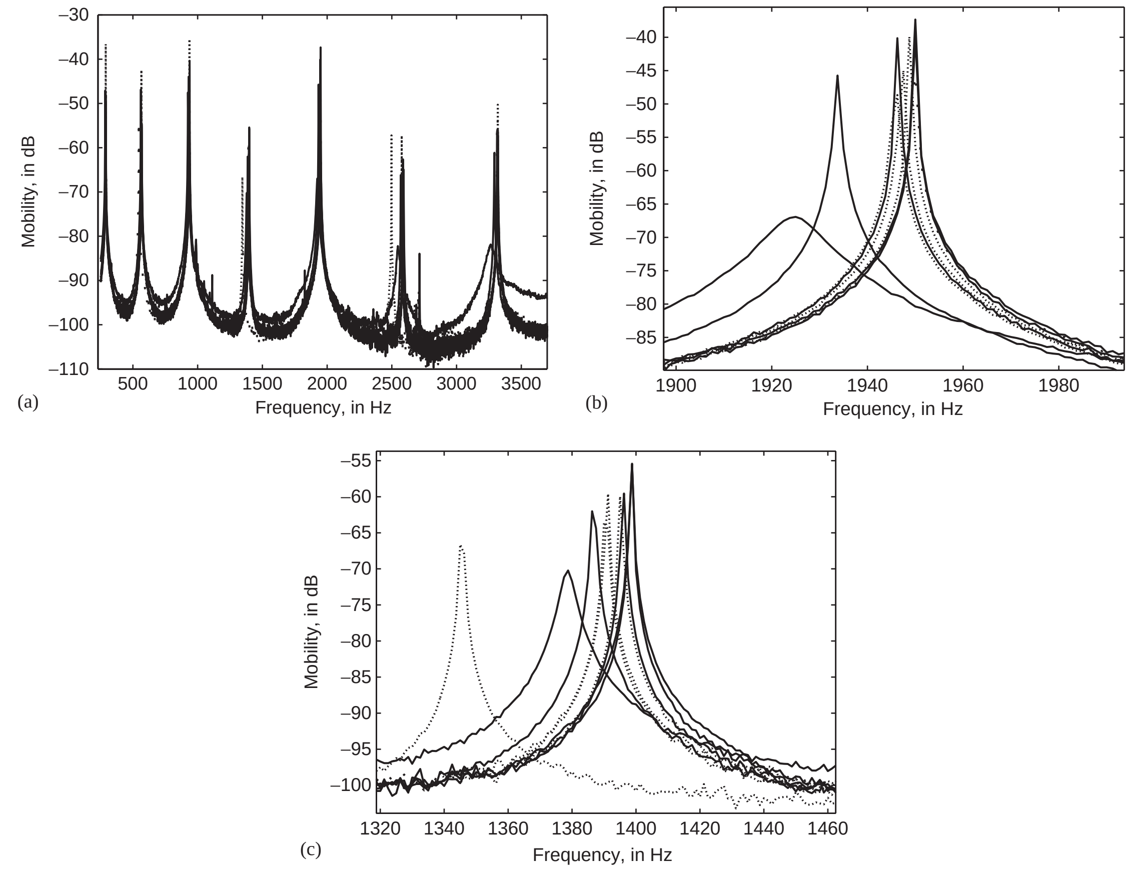

Fig. 10. (a) Mobilities of the intact (full line) and damaged (dotted line) Cases 1–9 representing structural variabilities; (b) and (c) zooms around an even and odd mode of the figure in (a).

Table 1
Mode table of the undamaged aluminium beam

Mode number	ω_r (Hz)	ξ_r (%)
2	290.9	0.15
3	568.1	0.01
4	937.7	0.01
5	1398.5	0.02
6	1949.9	0.00
7	2590.7	0.00
8	3319.3	0.01

the distance of damaged observations increases (from about 15 to about 45). In the robust SVD solution the four damaged beams are identified correctly (although the intact observation 3 is still classified as damaged). Again, the inclusion of damaged samples in the combination of

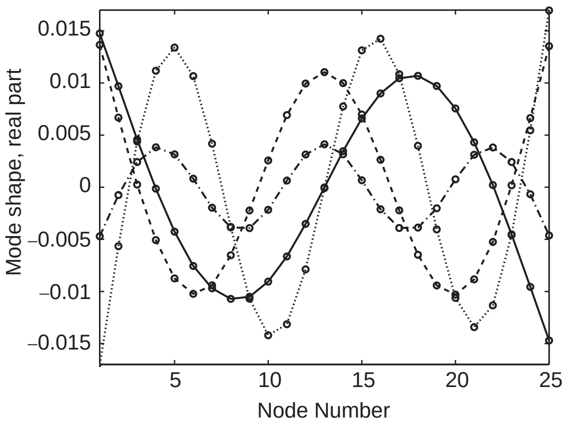

Fig. 11. Second, third, fourth and fifth mode shape of the aluminium beam.

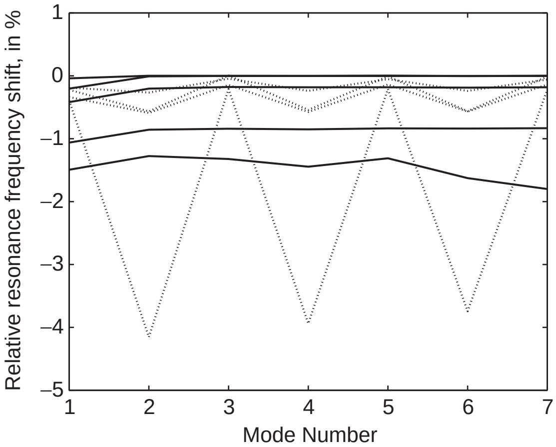

Fig. 12. Relative resonance frequency shift of mode 2–7 of Case 1–9 using 2 V operating level with respect to Case 1 with 1 V operating level.

observations which is used to compute the subspace clearly leads to an increase of the cost function (Fig. 16). This allows one to determine the subspace of the undamaged structure, even if different operational conditions and structural uncertainties are present (the minimum of the cost function corresponds to the combination of observations 1, 2, 4 and 5).

5. Conclusions

In this article, a technique was proposed to detect damage in structures from measurements taken under different conditions (i.e. different operational excitation levels, geometrical

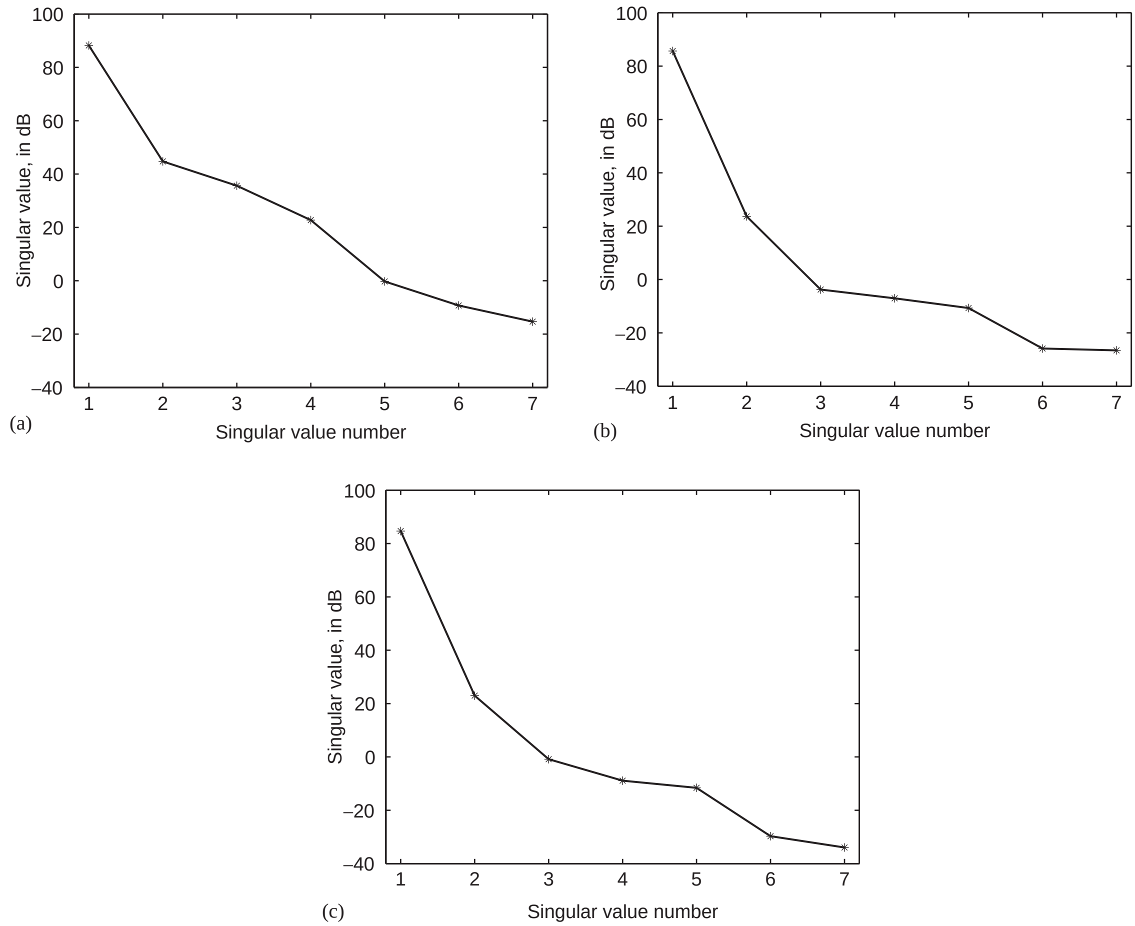

Fig. 13. Singular values of the different SVD techniques when applied on the resonance frequencies of Cases 1–9: (a) least-squares SVD (Section 2.2); (b) iterative SVD (Section 2.3); and (c) robust SVD (Section 2.3).

uncertainties and surface treatments of the structure). From the experimental results in the article it can be concluded that the classical least-squares SVD approach gives incorrect decomposition (and hence an incorrect classification between damaged and intact samples) when both damaged and intact measurements are used to compute the subspace. In general this is the case, since in practice it is not known in advance which observation is damaged or intact (when, however, a set of healthy observations is available and known, the SVD can be applied to this 'clean' data set and consequently the results of the classical SVD are comparable to those of the robust SVD). It was shown that the iterative SVD slightly improved the classification results. However, in order to obtain reliable results, the introduced robust SVD should be used.

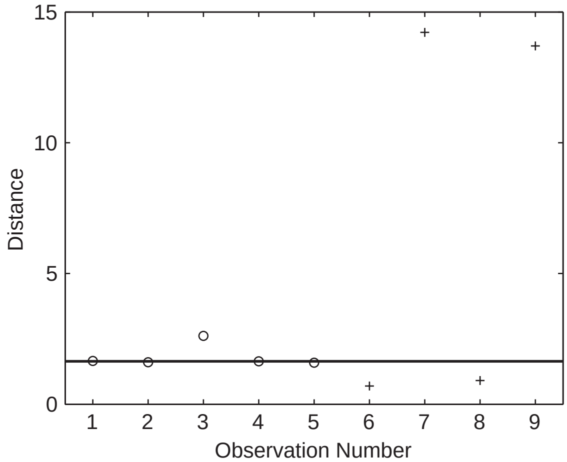

Fig. 14. Distances d_{SVD} of the nine observations at different boundary conditions (Cases 1–9) to the LS subspace ('o': intact, '+': damaged). Observations above the full line (median of the distances) are discarded in the iterative SVD approach (Section 2.3).

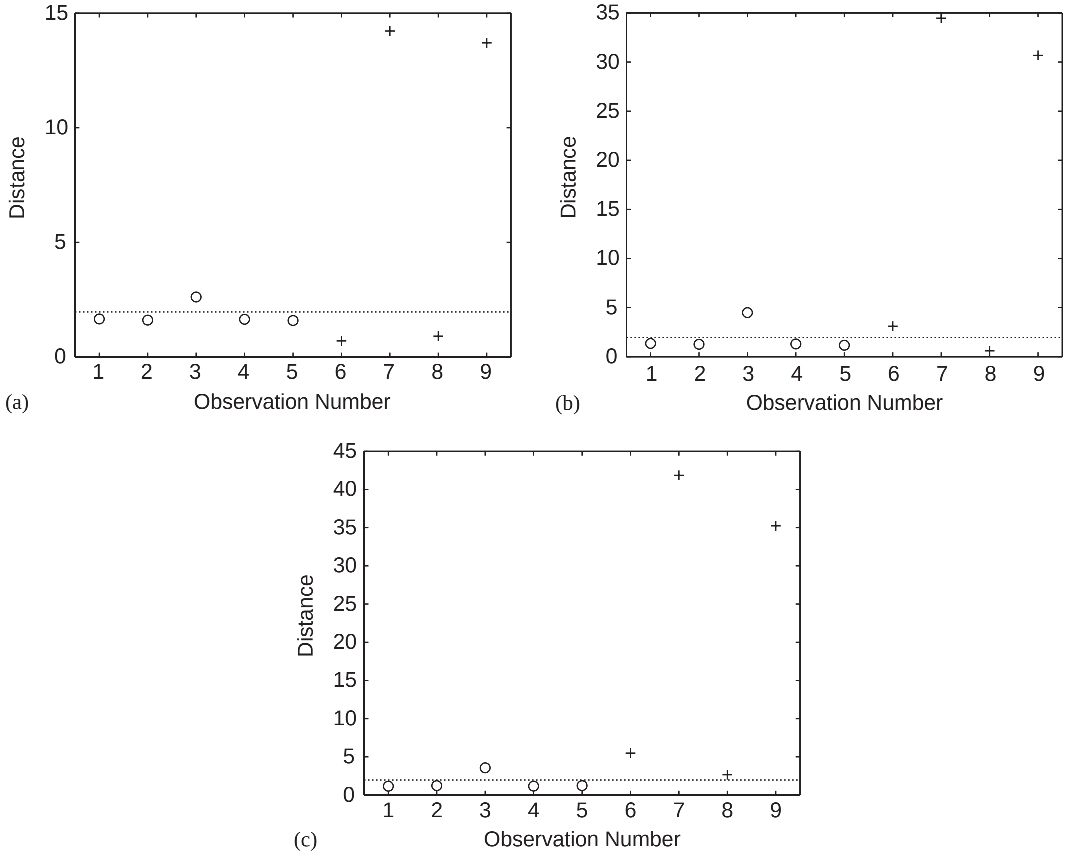

Fig. 15. Relative distances of nine observation cases (each measured in four operating conditions and grouped accordingly): (a) d_j^{SVD}; (b) d_j^{ISVD}; and (c) d_j^{RSVD} from SVD, iterative SVD and robust SVD, respectively.

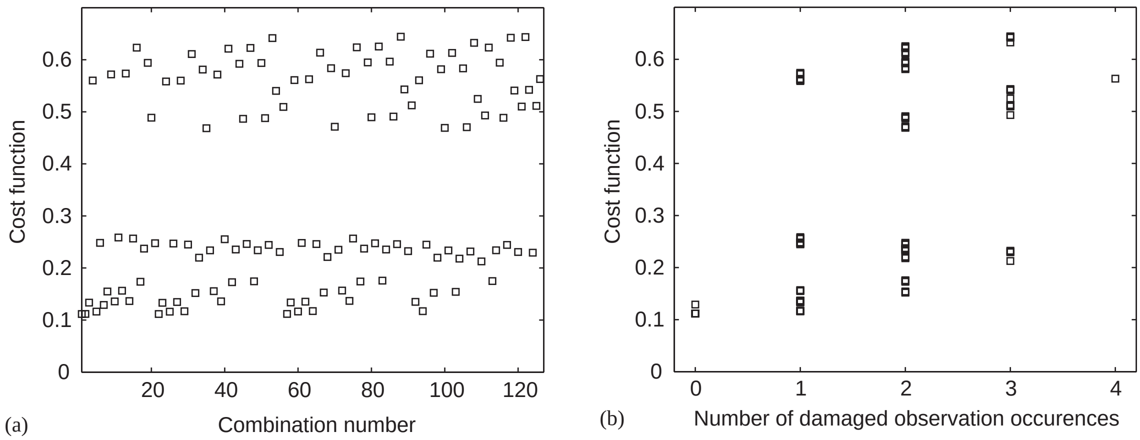

Fig. 16. (a) Cost function κ for different combinations of observations throughout the RSVD algorithm; (b) cost function κ in function of the number of damaged observations in each of the combinations throughout the RSVD algorithm.

Acknowledgements

The financial support of the Fund for Scientific Research (FWO Vlaanderen), the Concerted Research Action "OPTIMech" of the Flemish Community, the Research Council (OZR) of the Vrije Universiteit Brussel (VUB) are gratefully acknowledged. The first author holds a Postdoctoral Research Grant of the Fund for Scientific Research—Flanders (Belgium) (F.W.O.—Vlaanderen) at the mechanical engineering department of the Vrije Universiteit Brussel.

References

[1] S.W. Doebling, C.R. Farrar, M.B. Prime, D.W. Shevitz, Damage identification and health monitoring of structural and mechanical systems from changes in their vibration characteristics: a literature review, Technical Report LA-13070-MS, Los Alamos National Laboratory, 1996.
[2] R. Ruotolo, C. Surace, Using svd to detect damage in structures with different operating conditions, *Journal of Sound and Vibration* 226 (3) (1999) 425–439.
[3] J. Kullaa, Elimination of environment influences from damage-sensitive features in a structural health monitoring system, in: *Proceedings of the 1st European Workshop on Structural Health Monitoring*, 2002, pp. 742–749.
[4] K. Worden, G. Manson, N.R.J. Fieller, Damage detection using outlier analysis, *Journal of Sound and Vibration* 229 (2000) 647–667.
[5] W. Heylen, S. Lammens, P. Sas, Modal Analysis Theory and Testing, PMA, KU Leuven, Leuven, Belgium, 1995.
[6] P.G. Hoel, *Introduction to Mathematical Statistics*, Wiley, New York, 1971.
[7] J. Huber, *Robust Statistics*, Wiley, New York, 1981.
[8] P.J. Rousseeuw, Least median of squares regression, *Journal of the American Statistical Association* 79 (1984) 871–880.
[9] P.J. Rousseeuw, K. Van Driesen, Computing lts regression for large data sets, Technical Report, University of Antwerp, see http://win-www.uia.ac.be/u/statis, 1999.
[10] N.M.M. Maia, J.M.M. Silva (Eds.), *Theoretical and Experimental Modal Analysis*, Research Studies Press, Baldock, UK, 1997.
[11] H. Van der Auweraer, P. Guillaume, P. Verboven, S. Vanlanduit, Application of a fast-stabilizing frequency domain parameter estimation method, *Journal on Dynamic Systems, Measurement and Control* 123 (2001) 651–658.