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Book Review

J.D. Achenbach, Reciprocity in Elastodynamics, Cambridge Monographs on Mechanics, Cambridge University Press, Cambridge, ISBN 0-521-81734-X, 2003 (X + 255pp., £45.00).

Reciprocity is a fundamental property of acoustic and elastodynamic fields. For point sources, the reciprocity principle states that certain field quantities are symmetric with respect to interchange of source and receiver positions. Exactly which field quantity is symmetric is determined by the type of a point source. For distributed sources, as well as for waves incident from outside the spatial domain considered, the reciprocity principle takes the form of an integral relation between two elastodynamic fields, which is linear with respect to the amplitude of each of the fields. To the student of waves, the appeal of reciprocity lies in its intuitive nature, a promise of reformulating a given problem into a more easily solvable one, and the fact that it is a readily verifiable, exact property of a field in a complicated environment where no other exact property, aside from energy conservation, is known. Arguably, the reciprocity principle is the single most general, elegant, and helpful theorem in the theory of linear mechanical waves.

The reciprocity principle was established by H.L. von Helmholtz, J.C. Maxwell, Lord Rayleigh, E. Betti, and H. Lamb in the nineteenth century, and was studied, extended and applied to various problems by S. Ballantine, L.M. Lyamshev, B.A. Auld and numerous other researchers in the twentieth century. Despite its venerable history, the reciprocity principle continues to find new and efficient applications both as a means in the theoretical study and the computer simulation of wave phenomena, and as a conceptual foundation of innovative measurement techniques in mechanical engineering and remote sensing of the environment. Publication of *Reciprocity in Elastodynamics* by Professor J.D. Achenbach marks an important stage in our understanding of reciprocity in mechanical waves. As with all good books, *Reciprocity in Elastodynamics* not only summarizes insights achieved to date, but at the same time opens a few promising avenues for future research.

There are three distinctive features of the author's approach to reciprocity that shape this book. First, the author does not stop at the traditional interpretation of reciprocity as a relation between two possible elastodynamic fields generated by respective sets of sources in the *same* solid, but he extends the notion of reciprocity to a similar relation between two elastodynamic fields and their sources in solids with *distinct* parameters. (Other authors refer to such relations as *generalized* or *extended* reciprocity relations.) Second, the author uses extensively the concept of a *virtual wave*, that is, a judiciously chosen auxiliary solution, to obtain analytically the actual field generated by a source from a reciprocity relation between the sought-for field and the virtual wave. Third, the author emphasizes problems and applications that are of primary importance in ultrasonic

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medical imaging and nondestructive testing of materials over those that are prominent in seismics and environmental acoustics.

There are 14 chapters in the book. Introductory Chapter 1 is followed by Chapters 2 (*Some Elastodynamic Theory*) and 3 (*Wave Motion in an Unbounded Elastic Solid*), which provide a background in elastodynamic theory that is necessary to make the presentation self-contained. A useful representation of a wave that is standing in vertical direction and can propagate in two horizontal dimensions is presented in Chapter 7 (*Wave Motion Guided by a Carrier Wave*). The reciprocity principle is formulated and proven for waves in systems of increasing complexity in Chapters 4 (*Reciprocity in Acoustics*), 5 (*Reciprocity in One-Dimensional Elastodynamics*), and 6 (*Reciprocity in Two- and Three-Dimensional Elastodynamics*). The use of the reciprocity theorem in conjunction with a virtual wave is first illustrated in Chapter 8 (*Computation of Surface Waves by Reciprocity Considerations*) by calculating, without the traditional use of integral transforms, amplitudes of surface waves generated by line and point loads in a homogeneous solid half-space with a plane boundary. The reciprocity theorem is further used to derive orthogonality relations for normal modes supported by a homogeneous elastic layer with two parallel, free boundaries in Chapter 9 (*Reciprocity Considerations for an Elastic Layer*) and amplitudes of normal modes generated in such a layer by line and point loads in Chapter 10 (*Forced Motion of an Elastic Layer*). In Chapter 11 (*Integral Representations and Integral Equations*), the reciprocity theorem is used to derive the elastodynamic counterpart of what is known in acoustics and optics as the Kirchhoff–Helmholtz integral theorem. The result is applied to study scattering by cracks and flaws in an unbounded elastic solid. The study is continued in Chapter 12 (*Scattering in Waveguides and Bounded Bodies*), where defects located either on the surface or within an elastic layer or a half-space are considered. Chapter 13 (*Reciprocity for Coupled Acousto-Elastic Systems*) considers fluid–solid systems where sound waves in a fluid interact with waves in a solid. A different kind of coupling is addressed in Chapter 14 (*Reciprocity for Piezoelectric Systems*), where the reciprocity theorem is extended to coupled electromagnetic-mechanical waves in a piezoelectric solid.

Although some recent research results obtained by the author and his co-workers are included and comprise an integral part of the book, this is not a full-fledged research monograph on reciprocity in elastodynamics but rather an exceptionally well-written introduction to the subject. The introductory nature of the book is evident just from its bibliography. Among its 85 entries there are numerous books on the theory of elasticity and waves in elastic solids and fluids as well as classic papers on reciprocity. The majority of the more recent research papers that are included are those that are directly used in the text. If an attempt was made to survey all the research on reciprocity of mechanical waves and its applications, the list of references would easily double or treble.

The author does an admirable job of demonstrating actual and potential applications of reciprocity, especially in ultrasonics of solids. Representative of this is an analysis of reciprocity in piezoelectric systems (Chapter 14). Reciprocity relations are established for a system comprising a scatterer immersed in a fluid or a solid and probed by piezoelectric transducers. The analysis includes mechanical waves and electromagnetic processes in the transducers. The relations are further used as a tool in the nondestructive evaluation of materials in terms of quantities measured at the electric terminals of the transducers. In addition to revealing the power of reciprocity considerations, the author also makes their limitations very clear. He demonstrates, by

a number of examples, where and how the reciprocity considerations can be supplemented by other analytical methods such as perturbation techniques, in order to arrive at an explicit solution of a given problem.

No work of this size is flawless, and this book is not an exception. Problems sometimes arise when the author intends to illustrate the validity or application of the reciprocity theorem by analyzing an explicit solution of a specific problem. A specific example of this is the solution to the problem of forced vibrations of a beam derived in Section 5.4, which, although reciprocal, is incorrect and fails to reproduce amplification of the beam vibrations at a resonance. In the case of sound diffraction by rigid baffles (Sections 13.5 and 13.7), solutions are reproduced from a monograph of Junger and Feit [M.C. Junger and F. Feit, *Sound, Structures and Their Interaction*, MIT Press, Cambridge, MA, 1972] without fully explaining the notation. Consulting the pages in the monograph which the author cites does not help because the reader is directed to unrelated sections. More importantly, the reciprocity of coupled acousto-elastic systems and its applications in ocean seismology and structural acoustics are not given the attention they deserve. Suffice to say that no publications on the subject after 1985 are cited in the book.

There are elegant and important corollaries of the reciprocity principle that are not discussed or even mentioned in the book, such as symmetries of plane-wave reflection and transmission coefficients from an arbitrary layered solid and of mode scattering matrices in irregular 2-D waveguides. Also missing is the discussion of the application of reciprocity to derivation of variational derivatives of mode propagation constants in layered waveguides (or of phase velocities of surface waves) with respect to the elastic parameters of the solid, its density and position of interfaces. Surprisingly, there is no discussion of the fundamental issue of the connection between the reciprocity principle and wave energy or wave action conservation.

The book discusses neither physical phenomena that break reciprocity nor subtle information about the environment, which can be gleaned from observations of reciprocity breaking. Over the past few decades, a significant body of research has been devoted to reciprocity-breaking physical phenomena and observational schemes that allow one to retrieve information about medium non-stationarity from measurements of non-reciprocity of various field quantities. An example of such an approach is acoustic tomography of ocean currents by the method of reciprocal transmissions. This method is capable of measuring flow velocities that are orders of magnitude less than uncertainties in the sound speed in water and shear and compressional wave speeds in an ocean bottom.

One may hope that some of these fascinating subjects will be covered in future editions of the book.

This book is a delight to read. No matter how complicated a particular reasoning or derivation may be, every step is explained with insight and clarity that make the material easily accessible and enjoyable for beginners and experts alike. High-quality copyediting, printing, and page layout by Cambridge University Press complement this impression. If there has ever been a page-turner written on elastodynamics, this is the one.

Reciprocity in Elastodynamics is highly recommended both as a central text or supplemental reading for a course at the graduate level, and as a valuable reference that belongs to the bookshelves of practitioners of ultrasonic medical imaging and non-destructive evaluation of

materials, seismology, exploratory geophysics, ocean engineering, and structural and environmental acoustics.

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