Available online at www.sciencedirect.com
SCIENCE DIRECT®
ELSEVIER
Journal of Sound and Vibration 284 (2005) 1253–1254
JOURNAL OF SOUND AND VIBRATION
www.elsevier.com/locate/jsvi

Book Review

A.P. Seyranian, A.A. Mailybaev, Multiparameter Stability Theory with Mechanical Applications, World Scientific, Singapore, ISBN 981-238-406-5, 2004 (420 pp., US$86.00, £64.00).

Perturbation Theory has a long tradition. You take a problem that differs from a well-understood problem only through terms involving a small parameter, you expand everything you can as a power series in the parameter, and then you try to learn as much as possible about the behaviour of the problem from the low order terms in the expansion. One difficulty is that this behaviour itself can be sensitive to other arbitrarily small perturbations, and so it makes more sense to consider several parameters simultaneously. Formal multiparameter expansions become very complicated, so that it may be hard to see where things are going, but the major insights of René Thom and of Vladimir Arnold in the 1960s led to the recognition that a given problem typically requires only a finite number of parameters (its *codimension*) in order to describe all its perturbations, up to appropriate coordinate changes. Moreover, certain simple perturbation expressions (*universal unfoldings* or *versal deformations*) suffice to capture the totality of this perturbed behaviour.

These ideas have been slow to transfer from the mathematical to the engineering literature in the West, but in the Russian academic tradition of a more unified scientific culture the transition has been faster. The notion of versal deformation implies a shift in outlook toward the whole theory of perturbations: rather than study directly the behaviour of a perturbed system, we do better first to understand the behaviour of the entire multiparameter class of systems to which it naturally belongs (which may already have been classified and simply needs to be looked up), and then set about determining exactly where the system being studied fits into the wider story. Many are the reinvented wheels that can be avoided by taking this approach.

The book reviewed is an excellent representative both of the mathematical outlook just described and of the close Russian-style interaction between abstract geometrical thinking and specific engineering applications. The stability of an equilibrium (or periodic) state of a system depends in the first instance on the eigenvalues of the system linearized about that state. Therefore, the first step in a general study of stability of equilibria is a study of the typical behaviour of the eigenvalues of a matrix varying with several parameters. Of particular interest will be the *stability boundary*, which is the frontier of the region in parameter space that corresponds to matrices all of whose eigenvalues have negative real parts. This becomes increasingly complicated as the number of parameters increases, the structures occurring unavoidably in typical k-parameter families being precisely those whose codimension is no more than k. The purpose of this book is to take the classification of stability boundaries due to Arnold

doi:10.1016/j.jsv.2005.02.013

and to make it more quantitative, showing how certain important aspects of the geometry of the stability boundary can be calculated explicitly from the system at hand.

After an introductory chapter on stability theory, a long Chapter 2 deals with bifurcation analysis of eigenvalues of matrices depending on several parameters: this is the core of the book on which most of the rest depends. Chapter 3 studies generic structure and local characterization of the stability boundary up to codimension 3, using key ideas of *general position* that pervade the whole approach. The singularities or non-smooth points that typically occur are important to understand, not least for the major influence they exert on the efficacy of numerical methods.

Motivated by ordinary differential equations of order m, the next chapter looks closely at the behaviour of the roots of an mth order polynomial as parameters vary, using classical techniques of the Newton polygon and the Weierstrass preparation theorem. The next few chapters then focus on conservative systems, gyroscopic systems and linear Hamiltonian systems (all closely related) and on mechanical interpretations of some of the theoretical results—including the curious phenomenon of destabilization by damping. Chapter 9 turns to stability of parameter-dependent periodic systems, where it is the bifurcation of Floquet multipliers that is important, with Chapter 10 looking at stability boundaries in this context, and Chapters 11 and 12 studying interactions of parametric excitation and damping in several different specific contexts. The text is clearly written and the mathematics attractively set out with plenty of clear and instructive diagrams: an enjoyable book to read. Much of the work is based on the authors' own publications, and the references naturally give prominence to Russian literature while recent Western papers are less well represented. The English is almost flawless, with very occasional near-misses such as the use of *enforced* for *strengthened* and confusion of (in)definite articles adding a little spice.

The book concludes with some remarks on extending the ideas to nonlinear aspects of bifurcation and to partial differential equations, although literature using methods of singularity theory certainly does exist in both those fields. The increasing mathematical complications that arise can often be tamed through exploitation of *symmetries* that are a natural component of many real-world problems: for an overview of these matters see Ref. [1]. Incorporating symmetries of systems into the general study of stability boundaries would be a valuable extension of the methods in this book.

Reference

[1] M. Golubitsky, I. Stewart, *The Symmetry Perspective*, Birkhäuser, Basel, 2003.

D.R.J. Chillingworth
Department of Mathematics, University of Southampton,
Highfield, Southampton SO17 1BJ, UK
E-mail address: d.r.j.chillingworth@maths.soton.ac.uk