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Green's function for a plane three-dimensional fluid layer at small horizontal distances from the source

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Abstract

The Green's function for a plane three-dimensional layer in its common form is usually determined as an infinite series of the waveguide's normal modes. The slow convergence of the series close to the source, limits the applicability of Green's function to solving sound generation and scattering problems in waveguides, especially if near-field effects are significant. In the present work, a more convenient form of Green's function for such a waveguide is obtained as a sum of two quickly converging series. The first series is a difference between the Green's functions for Helmholtz and Laplace equations, whereas the second series is Green's function for the Laplace equation, determined as a sum of the mirror images of the source in the waveguide boundaries. The behaviour of the obtained function close to the source is investigated. Numerical experiments show significantly better convergence for the obtained function as compared with the Green's function in its common form. It is also shown that the obtained function can be easily calculated at the points directly underneath or above the source, where the terms of the Green's function series in the common form are singular.

0. Introduction

The acoustic field in a medium can be represented as an integral over its sources, which are considered a set of elementary monopoles and dipoles. Such representation is a powerful technique widely used in acoustics for solving problems of sound generation and scattering. Many

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numerical methods are based on such a representation. One of the most common of these methods is the boundary element method (BEM) [1], which allows one to solve complex acoustical problems. Development of methods and models, based on the integral representation of the acoustic field, continues [2].

As the fields of an elementary monopole and dipole in a medium are represented by Green's function for the medium and its spatial derivative, respectively, the ability to calculate values of Green's function accurately and efficiently is crucial for the successful use of many numerical methods. In unbounded media, Green's function for the Helmholtz equation has a simple form [3], although it may require the use of desingularizing procedures at close distances from the source [4].

In waveguides, however, the calculation of Green's function meets substantial difficulties. Commonly a waveguide Green's function is represented as either a series of mirror images of the source in the waveguide walls or a series of the waveguide normal modes [5]. Due to poor convergence of both series, especially near the source, attempts have been made to derive alternative expressions for waveguide Green's functions, which would allow trouble-free numerical implementation. For instance, Linton [6] compared several expressions for the Green's function of the two-dimensional Helmholtz equation in periodic domains and waveguides. In a recent publication [7], another representation for a two-dimensional waveguide Green's function is derived. The author with his co-authors previously suggested a way of obtaining the expression for such a Green's function as a sum of quickly converging series and an asymptotic term [8]. Further development of this technique allowed the author to carry out an investigation of the influence of near-field resonances on the scattered far field [9].

All of the above-mentioned publications, however, deal with two-dimensional waveguides. While two-dimensional models can produce valid results in a number of cases, in most real situations three-dimensional scattering must be considered. For example, solving scattering problems in such areas as sonar technology and geological surveying, require the development of three-dimensional models of scattering.

In three-dimensional waveguides, the problem of slow convergence of the Green's function is commonly solved by truncating the Green's function series at some mode number N. This truncation is often justified by the fact that only a few lowest normal waveguide modes propagate through the waveguide, while all others are evanescent and exponentially decay with increasing distance from the source.

In many cases, however, higher-order evanescent modes play an important role in acoustic scattering. It is known that various types of acoustic and elastic waves may be excited near the elastic scattering surface [10,11], significantly affecting the scattered wave in the far field. For example, such waves may cause multiple pulse echoes [10].

To consider such effects theoretically, it is necessary to develop a self-consistent model, which allows for multiple scattering of the acoustic field by different parts of the scattering object as well as by the waveguide boundaries. As the multiple scattering in the near field is described by the evanescent waveguide modes, considering these modes in the Green's function is crucial for correct prediction of characteristics of the scattered field in a three-dimensional waveguide.

Furthermore, Green's function in its traditional form is not applicable to sound propagation directly underneath or above the source. As every term of the Green's function series has a singularity at zero horizontal distance from the source, calculation of the three-dimensional

Green's function below or above the source is not possible. This significantly restricts the use of integral representation of the acoustic field in solving three-dimensional scattering problems in waveguides.

In the present article, an expression for the efficient calculation of Green's function for a three-dimensional plane fluid layer is obtained. The convergence of the obtained series is further improved at small horizontal distances from the source. It is shown by numerical examples that the obtained series converges significantly quicker than the Green's function in its traditional form. It is demonstrated also that the obtained expression can be evaluated efficiently at the points directly above or underneath the source.

1. Three-dimensional waveguide Green's function

A three-dimensional waveguide in the form of a fluid layer of depth, D, bounded by two infinite plane boundaries, is considered (Fig. 1). The boundaries of the layer are parallel to the x- and y-axis, whereas the z-axis is vertical and directed from the bottom to the top. The layer is filled with a compressible inviscid homogeneous fluid of density, ρ_0, and sound speed, c_0.

A harmonic temporal dependence, $\exp(-i\omega t)$, is assumed, where ω is the angular frequency. Green's function for the layer, $G(x, y, z; x_0, y_0, z_0)$, is proportional to the pressure field of an elementary monopole and satisfies the following equation [3]:

$$(\nabla^2 + \bar{k}^2)G(x, y, z; x_0, y_0, z_0) = \delta(x - x_0)\delta(y - y_0)\delta(z - z_0), \quad \bar{k} = \omega/c_0, \tag{1}$$

where (x, y, z) are the coordinates of the observation point, and (x_0, y_0, z_0) are the coordinates of the source point. The top and bottom boundaries of the layer are assumed acoustically soft and rigid respectively, so that the following boundary conditions are satisfied:

$$G|_{z=D} = 0, \quad \left.\frac{\partial G}{\partial z}\right|_{z=0} = 0. \tag{2}$$

The waveguide considered in this paper is a model of an ocean, where the boundary between water and the atmosphere can be described as soft (zero pressure), and the ocean bottom can be regarded as rigid (zero velocity).

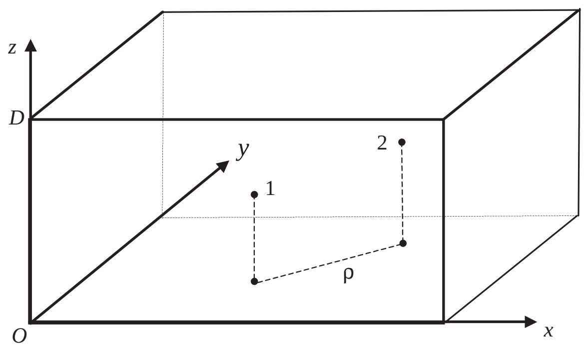

Fig. 1. Geometry of the waveguide. (1) Source point, (x_0, y_0, z_0); (2) observation point, (x, y, z); ρ is the distance between the source and observation points in the (x, y) plane.

In further analysis, all variables and parameters that have the dimension of length are normalized on D/π. With the use of this normalization, the solution of Eq. (1) is commonly written as follows [5]:

$$G_C(\rho, z, z_0) = \sum_{n=1}^{\infty} G_n(\rho, z, z_0), \tag{3}$$

$$G_n(\rho, z, z_0) = \frac{\mathrm{i}}{2\pi}\cos[(n-0.5)z_0]\cos[(n-0.5)z]H_0^{(1)}(g_n\rho), \tag{4}$$

where $H_0^{(1)}(g_n\rho)$ is a Hankel function of the first kind of zero order, ρ is the distance between the source and the observation points in the plane (x, y):

$$\rho = \sqrt{(x-x_0)^2 + (y-y_0)^2}, \tag{5}$$

and g_n are longitudinal wavenumbers, determined by

$$g_n = \sqrt{k^2 - (n-0.5)^2}. \tag{6}$$

It may be noted that the non-dimensional wavenumber, $k = \bar{k}D/\pi$, is the number of acoustic half-wavelengths, which can be fitted into D.

The Green's function in its common form, G_C, is an infinite series of the waveguide normal modes. To be employed in a numerical algorithm, the series needs to be truncated, and in most applications, only modes of orders $n < k + 0.5$ are taken into consideration. Indeed, the amplitude of the lower order modes of the waveguide decreases with distance as $1/\sqrt{\rho}$, whereas the higher modes of orders $n > k + 0.5$ decay exponentially and, therefore, their direct contribution to the far field can be neglected.

The Green's function, truncated in this way, correctly determines the far pressure field of an elementary monopole source. However, if the size of a scattering object is finite, such truncation does not take into account the indirect influence of the exponentially decaying evanescent modes on the far field through boundary conditions on the surface of the object [9]. Due to this influence, obtaining amplitudes of the lower propagating modes in far field requires the evaluation of the Green's function at small intervals between the source and observation points. It can be shown that Eq. (3) is not suitable for this purpose due to its poor convergence at $\rho < 1$. In fact, the Hankel function $H_0^{(1)}(g_n\rho)$ tends to infinity when its argument tends to zero [3], and evaluation of the Green's function in the form of Eq. (3) becomes unfeasible at small ρ. Besides, as every term of Eq. (3) is singular at $\rho = 0$ regardless of the vertical distance, $|z - z_0|$, between the source and observation points, Eq. (3) is not suitable for determining the acoustic field above or underneath the source. Therefore, transformation of G_C to a better converging form becomes necessary for achieving an accurate solution for a scattering problem in a three-dimensional waveguide.

2. Transformation of the Green's function

To transform the Green's function, determined by Eq. (3), into a more convenient form, let an auxiliary function, $\bar{G}(\rho, z, z_0)$, be added to and subtracted from Eq. (3), so that the Green's function takes the following modified form:

$$G_M = \sum_{n=1}^{\infty} G_n + \bar{G} - \bar{G}. \tag{7}$$

It will be convenient to represent the function $\bar{G}$ as Green's function of the Laplace equation, which is the solution of Eq. (1) at $\bar{k} = 0$. Eqs. (3) and (4) lead to the following expression for $\bar{G}$:

$$\bar{G}(\rho, z, z_0) = \sum_{n=1}^{\infty} \bar{G}_n(\rho, z, z_0), \tag{8}$$

$$\bar{G}_n(\rho, z, z_0) = \frac{1}{\pi^2}\cos[(n-0.5)z_0]\cos[(n-0.5)z]K_0[(n-0.5)\rho], \tag{9}$$

where $K_0(x)$ is the modified Hankel function of zero order.

On the other hand, the function $\bar{G}$ can be represented as a series of mirror images of the source in the waveguide boundaries [5]:

$$\bar{G}(\rho, z, z_0) = \frac{1}{4\pi}\sum_{n=1}^{\infty}\{S_n^+(\rho, z+z_0) + S_n^-(\rho, z+z_0) + S_n^+(\rho, z-z_0) + S_n^-(\rho, z-z_0)\}, \tag{10}$$

where

$$S_n^{\pm}(\rho, z) = \frac{1}{\sqrt{(2\pi(2n-1\mp 1)\pm z)^2+\rho^2}} - \frac{1}{\sqrt{(2\pi(2n-1)\pm z)^2+\rho^2}}. \tag{11}$$

It may be noted that Eq. (10) can be derived from Eqs. (8) and (9) by means of known formulas for transformation of Hankel function series [12].

With the use of Eqs. (7)–(11), the modified Green's function for the three-dimensional fluid layer can be written as

$$G_M(\rho, z, z_0) = \tilde{G}(\rho, z, z_0) + \bar{G}(\rho, z, z_0), \tag{12}$$

where

$$\begin{aligned}\tilde{G}(\rho, z, z_0) = &\sum_{n=1}^{\infty}\left(\frac{\mathrm{i}}{2\pi}H_0^{(1)}(g_n\rho) - \frac{1}{\pi^2}K_0\left[\left(n-\tfrac{1}{2}\right)\rho\right]\right)\\ &\times\cos\left[\left(n-\frac{1}{2}\right)z_0\right]\cos\left[\left(n-\tfrac{1}{2}\right)z\right],\end{aligned} \tag{13}$$

and $\bar{G}$ is determined by Eqs. (10) and (11).

Since Hankel functions $H_0^{(1)}(x)$ and $K_0(x)$ are linked as follows:

$$K_0(x) = \tfrac{1}{2}\pi\mathrm{i}H_0^{(1)}(\mathrm{i}x) \tag{14}$$

and

$$g_n = \mathrm{i}\left(n-\tfrac{1}{2}\right) + O(k/n)^2, \quad n \gg k, \tag{15}$$

both terms in the series in Eq. (13) tend to each other as n increases. This leads to the convergence of the series in Eq. (13) at $\rho > 0$. The series can be said to describe the contribution of the lower order waveguide modes. On the other hand, the function $\bar{G}$ does not depend on the wavenumber k and determines the contribution of higher waveguide modes up to infinite order. The convergence of the function $\bar{G}$ can be proved by the fact that the terms of Eq. (11) tend to zero as $1/n^2$ at large $n \gg \rho/4\pi$. Thus, the modified Green's function G_M converges at all non-zero horizontal distances ρ between the source and observation points.

3. Modified Green's function at zero horizontal distance from the source

Since both Hankel functions, $H_0^{(1)}(g_n\rho)$ and $K_0[(n-\frac{1}{2})\rho]$, are singular at $\rho = 0$, efficient calculation of the Green's function at $\rho \to 0$ requires its further transformation. At $x \ll 1$ the functions $H_0^{(1)}(x)$ and $K_0(x)$ can be approximated as follows [13]:

$$H_0^{(1)}(x) = 1 + \frac{2\mathrm{i}}{\pi}\left(C + \ln\frac{x}{2}\right) + O(x^2 \ln x), \tag{16}$$

$$K_0(x) = -\left(C + \ln\frac{x}{2}\right) + O(x^2 \ln x), \quad C = 0.57721\ldots . \tag{17}$$

At $\rho \to 0$ Eqs. (16) and (17) can be substituted into Eq. (13), which can be shown to take the following form:

$$\tilde{G}(\rho, z, z_0) = \sum_{n=1}^{\infty} \cos\left[\left(n-\tfrac{1}{2}\right)z_0\right] \cos\left[\left(n-\tfrac{1}{2}\right)z\right] \times \begin{cases} \left[\dfrac{\mathrm{i}}{2\pi} H\left(k+\tfrac{1}{2}-n\right) + \dfrac{1}{\pi^2}\ln\dfrac{n-\frac{1}{2}}{|g_n|}\right], & \rho = 0 \\ \left[\dfrac{\mathrm{i}}{2\pi} H_0^{(1)}(g_n\rho) - \dfrac{1}{\pi^2} K_0\left[\left(n-\tfrac{1}{2}\right)\rho\right]\right], & \rho > 0, \end{cases} \tag{18a, b}$$

where the Heaviside step function, $H(x)$, is determined by [14]

$$H(x) = \begin{cases} 0, & x < 0, \\ 1, & x \geqslant 0. \end{cases} \tag{19}$$

Eq. (18a) determines the Green's function for the waveguide directly underneath or above the source.

4. Improving convergence of the modified Green's function

Although the series in Eq. (18) converges, the convergence of a series, containing logarithm, is slow. However, the convergence of this particular series can be improved by applying a technique, which is similar to the technique used in the derivation of Eq. (12).

It can be shown that the logarithmic term in Eq. (18) at $n \gg k$ can be approximated as

$$\ln \frac{n-\frac{1}{2}}{|g_n|} = \frac{k^2}{2(n-\frac{1}{2})^2} + O\left(\frac{k}{n}\right)^4, \quad n \gg k. \tag{20}$$

With the use of known summation formulas [15], the following equation can be obtained:

$$\frac{1}{2\pi^2}\sum_{n=1}^{\infty}\frac{1}{(n-\frac{1}{2})^2}\cos\left[\left(n-\tfrac{1}{2}\right)z\right]\cos\left[\left(n-\tfrac{1}{2}\right)z_0\right] = \frac{1}{4}\left(1-\frac{1}{2\pi}(z+z_0)-\frac{1}{2\pi}|z-z_0|\right). \tag{21}$$

If Eq. (21) is added to and subtracted from Eqs. (13) and (18), the function $\tilde{G}$ can be determined as follows.

$$\tilde{G}(\rho, z, z_0)$$

$$= \begin{cases} \sum\limits_{n=1}^{\infty}\cos\left[\left(n-\tfrac{1}{2}\right)z_0\right]\cos\left[\left(n-\tfrac{1}{2}\right)z\right]\left[\dfrac{\mathrm{i}}{2\pi}H\left(k+\tfrac{1}{2}-n\right)+\dfrac{1}{\pi^2}\ln\dfrac{n-\frac{1}{2}}{|g_n|}-\dfrac{k^2}{2\pi^2}\dfrac{1}{(n-\frac{1}{2})^2}\right] \\ \quad +\dfrac{k^2}{4}\left(1-\dfrac{1}{2\pi}(z+z_0)-\dfrac{1}{2\pi}|z-z_0|\right) \quad \text{if } \rho=0, \\ \sum\limits_{n=1}^{\infty}\cos\left[\left(n-\tfrac{1}{2}\right)z_0\right]\cos\left[\left(n-\tfrac{1}{2}\right)z\right]\left[\dfrac{\mathrm{i}}{2\pi}H_0^{(1)}(g_n\rho)-\dfrac{1}{\pi^2}K_0\left[\left(n-\tfrac{1}{2}\right)\rho\right]\right. \\ \left. -\dfrac{k^2}{2\pi^2}\dfrac{1}{(n-\frac{1}{2})^2}\right]+\dfrac{k^2}{4}\left(1-\dfrac{1}{2\pi}(z+z_0)-\dfrac{1}{2\pi}|z-z_0|\right) \quad \text{if } 0<\rho\ll D, \\ \sum\limits_{n=1}^{\infty}\cos\left[\left(n-\tfrac{1}{2}\right)z_0\right]\cos\left[\left(n-\tfrac{1}{2}\right)z\right]\left[\dfrac{\mathrm{i}}{2\pi}H_0^{(1)}(g_n\rho)-\dfrac{1}{\pi^2}K_0\left[\left(n-\tfrac{1}{2}\right)\rho\right]\right] \quad \text{otherwise.} \end{cases} \tag{22a, b, c}$$

Eqs. (10)–(12) and (22) represent the contribution of this paper. As all the series in these equations converge quickly, they allow one to calculate the three-dimensional waveguide Green's function at arbitrary distances from the source. The convergence of the series is demonstrated by numerical examples in the next section.

5. Numerical results

Figs. 2–7 show results of calculation of the Green's function in its common form, G_C, and in its modified form, G_M. Calculations are carried out at the horizontal distances from the source, ρ, smaller than the waveguide depth, D, for two values of the non-dimensional wavenumber, $k=5$ and $k=50$.

The criterion used for determining the convergence of the series can be formulated as follows. A series, determined by

$$S = \sum_{n=1}^{\infty} S_n \tag{23}$$

is considered to have converged at some $n = N$, if the condition

$$|\mathrm{Re}(S_N)| \leqslant \varepsilon \left| \mathrm{Re}\left(\sum_{n=1}^{N} S_n \right) \right|, \quad 0 < \varepsilon \ll 1 \tag{24}$$

is satisfied for three sequential terms S_{N-2}, S_{N-1}, and S_N. To achieve the convergence of the imaginary part of the series, it is sufficient to take into consideration any number of terms, which is not less than $k + \frac{1}{2}$.

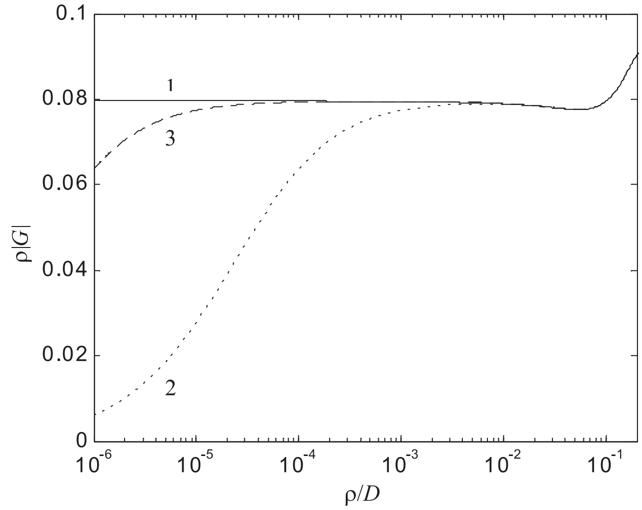

Fig. 2. Absolute value of the Green's function multiplied by the horizontal distance, ρ, from the source versus the normalized ρ; $z = z_0 = D/2$; $k = 5$: (1) G_M, $\varepsilon = 10^{-4}$; (2) G_C, $\varepsilon = 10^{-4}$; (3) G_C, $\varepsilon = 10^{-6}$.

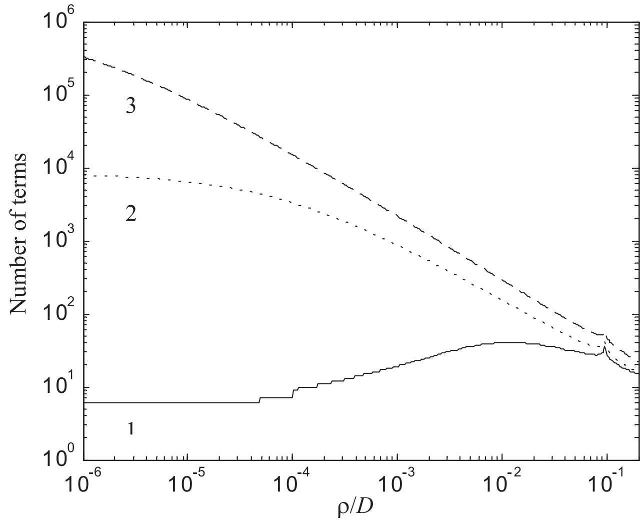

Fig. 3. Number of terms necessary to achieve convergence of the Green's function series versus the normalized ρ; $z = z_0 = D/2$, $k = 5$: (1) $\tilde{G}$, $\varepsilon = 10^{-4}$; (2) G_C, $\varepsilon = 10^{-4}$; (3) G_C, $\varepsilon = 10^{-6}$.

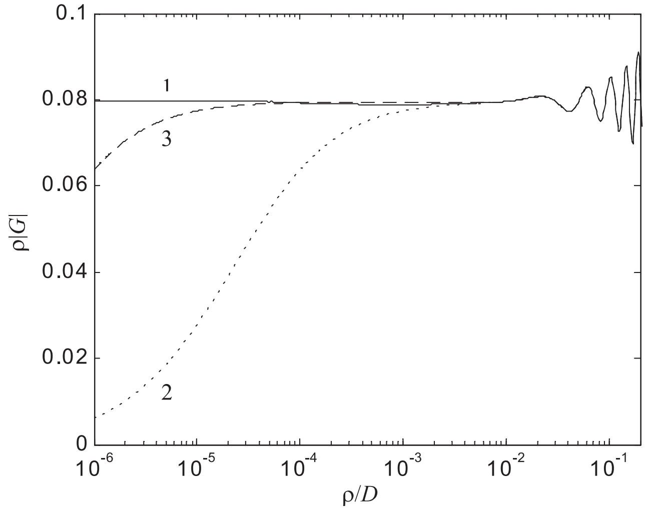

Fig. 4. Absolute value of the Green's function multiplied by the horizontal distance, ρ, from the source versus the normalized ρ; $z = z_0 = D/2$; $k = 50$: (1) G_M, $\varepsilon = 10^{-4}$; (2) G_C, $\varepsilon = 10^{-4}$; (3) G_C, $\varepsilon = 10^{-6}$.

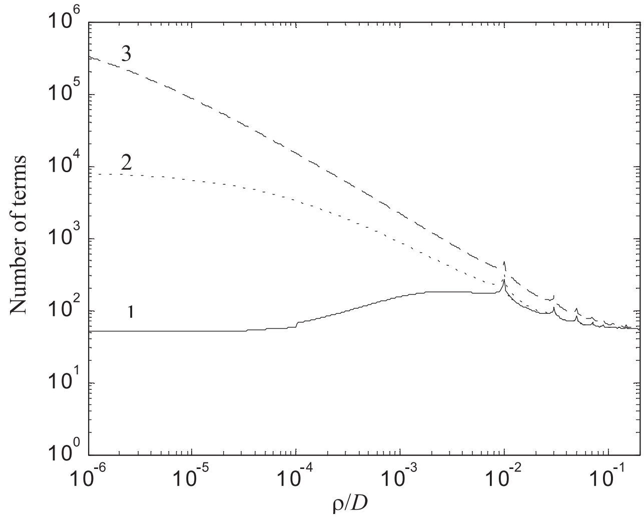

Fig. 5. Number of terms necessary to achieve convergence of the Green's function series versus the normalized ρ; $z = z_0 = D/2$; $k = 50$: (1) $\tilde{G}$, $\varepsilon = 10^{-4}$; (2) G_C, $\varepsilon = 10^{-4}$; (3) G_C, $\varepsilon = 10^{-6}$.

Eq. (22b) is used for the calculation of the modified Green's function at $0 < \rho < 10^{-4}D$, whereas Eq. (22c) is utilized for $\rho > 10^{-4}D$. The choice of the boundary between the areas of application of the two equations affects only the number of terms required to achieve convergence of the series and does not influence the convergence by itself.

Figs. 2 and 4 show the absolute value of the Green's function in its common, G_C, and modified, G_M, forms versus the horizontal distance from the source, ρ, normalized on the waveguide depth,

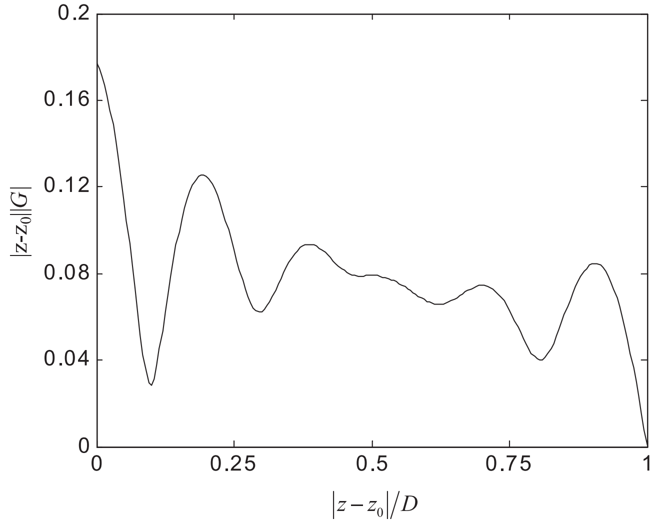

Fig. 6. Absolute value of the modified Green's function G_M multiplied by the vertical distance, $|z-z_0|$, from the source versus the normalized $|z-z_0|$; $\varepsilon = 10^{-4}$; $\rho = 0,\ k = 5$.

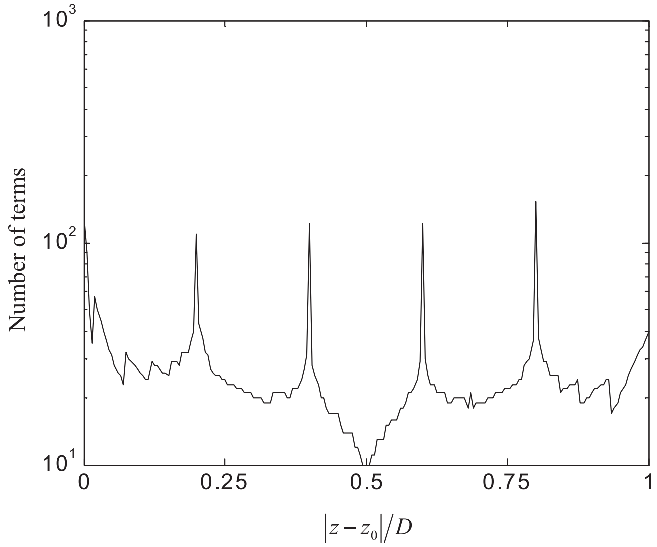

Fig. 7. Number of terms necessary to achieve convergence of the function $\tilde{G}$ versus the normalized $|z-z_0|$; $\varepsilon = 10^{-4},\ \rho = 0;\ k = 5$.

D, for $k = 5$ and 50, respectively. For better clarity of the graphs, the sharp rise of the Green's function at $\rho \to 0$ is eliminated by multiplying the Green's function by ρ. Figs. 3 and 5 show the number of terms necessary to achieve the convergence of the series G_C and $\tilde{G}$ as determined by Eq. (24). The convergence of the series $\bar{G}$, determined by Eqs. (10) and (11), is considered further in this section.

At $\rho \to 0$ and $z = z_0$ the Green's function for the waveguide is expected to depend on ρ in the same way as the Green's function in an unbounded fluid. It can be seen easily from Fig. 2, that only the modified Green's function (curve 1) tends to the expected value of $1/(4\pi\rho)$ at $\rho \to 0$. This is achieved by summation of only a small number of terms of the series (Fig. 3, curve 1). At the same time, the calculation of the common Green's function, G_C, with the same $\varepsilon = 10^{-4}$ (Fig. 2, curve 2), does not converge to the correct value at $\rho/D < 10^{-2}$ even though the number of terms approaches 10^4. Further decrease of the convergence error ε to 10^{-6} (Fig. 2, curve 3) improves the result only partially. It can be seen that the calculated Green's function significantly diverges from the required asymptotic behaviour at $\rho/D < 10^{-4}$, and even this imperfect result requires hundreds of thousands of terms (Fig. 3, curve 3).

Figs. 4 and 5, corresponding to $k = 50$, show that the convergence of the series does not depend significantly on the wavenumber k. The discrepancy between the common and modified Green's functions is of the same order, and the number of terms required for the convergence of the common Green's function is equally large. The larger number of terms, needed for the convergence of the modified Green's function, can be explained by the requirement that the number of terms should not be less than $k + \frac{1}{2}$.

Figs. 6 and 7 demonstrate a significant property of the modified Green's function. They represent, respectively, the Green's function and the number of terms in the series versus the normalised vertical distance from the source at zero horizontal distance, ρ. Whereas the calculation of the Green's function by means of the traditional equation (3) is not possible at $\rho = 0$ due to the singularity in every term of the series, Figs. 6 and 7 show, that the Green's function in its modified form (Eq. (12)) can be easily calculated directly above or underneath the source. The required number of terms is no more than 30 at most points, and only near the points where $\mathrm{Re}(G_M)$ is zero, does the number of terms reach the order of 10^2.

The numerical experiments also confirm the convergence of the series $\bar{G}$, determined by Eqs. (10) and (11). The number of terms, needed to achieve convergence varied from just 4 at $\rho/D = 10^{-6}$ to 17 at $\rho/D = 1$. The increase in the number of terms with the distance ρ can be explained by the fact that the series $\bar{G}$ is convergent only at $n \gg \rho/4\pi$. At large distances, all the series continue to converge, but the number of terms in the series $\bar{G}$ increases.

6. Conclusions

In the present work, a modified Green's function for a three-dimensional fluid layer is derived. The modified Green's function consists of two series. One of the series is the difference between the Green's function of the Helmholtz equation and the Green's function of the corresponding Laplace equation, written as a series of the waveguide normal modes. The second series is the Greens' function of the Laplace equation, written as a series of mirror images of the source in the waveguide walls. The modified Green's function is transformed further to improve its convergence at small horizontal distances, ρ, from the source.

The numerical experiments show that the modified Green's function can be calculated much more efficiently than the Green's function in its common form if ρ is smaller than the waveguide depth. The number of terms, needed to achieve the convergence of the modified Green's function, is no more than several dozens at any ρ within the considered range. On the other hand, despite taking into account hundreds of thousands of terms, the calculation of the Green's function in its common form did not produce correct results at small ρ.

An important advantage of the obtained Green's function is that it is possible to calculate its values at $\rho = 0$, where terms of the common Green's function series are singular. This property, confirmed by numerical experiments, allows one to calculate the Green's function directly above or underneath the source.

In summary, the modified Green's function, obtained in the present work, overcomes the shortcomings of the traditional form of the Green's function at horizontal distances from the source smaller than the waveguide depth. The ability to calculate values of the Green's function near the source in three-dimensional waveguides opens an opportunity to develop self-consistent models of scattering, which will properly take into account the influence of near-field acoustic phenomena on the scattered far field.

Acknowledgements

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