



ELSEVIER

Available online at www.sciencedirect.com

SCIENCE @ DIRECT®

Journal of Sound and Vibration 285 (2005) 585–596

JOURNAL OF
SOUND AND
VIBRATION

www.elsevier.com/locate/jsvi

Generalized analysis of a muffler with any number of interacting ducts

T. Kar, M.L. Munjal*

Facility for Research in Technical Acoustics, Department of Mechanical Engineering, Indian Institute of Science, Bangalore-560 012, India

Received 14 November 2003; received in revised form 26 April 2004; accepted 26 August 2004
Available online 24 November 2004

Abstract

The muffler elements that use perforated elements for acoustic attenuation are common in practice. In typical commercial mufflers perforated elements are used involving two, three, four or more interacting ducts. Analysis of such configurations involves writing down the basic governing equations of mass continuity, momentum balance, etc., and then elimination of velocity variables to obtain the coupled ordinary differential equations in terms of acoustic pressure variables. Mathematical modelling and the consequent analytical derivation of the transmission loss for these multi-duct acoustical elements become increasingly tedious, as just not the number of ducts, but also their relative arrangement along with the boundary conditions dictate the analysis considerably. In the present paper, authors have proposed a generalization and thus an algebraic algorithm to directly produce the system matrix, eliminating the tedium of writing the basic governing equations and elimination of velocity variables. Also, a convenient approach for applying the boundary conditions is outlined here.

© 2004 Elsevier Ltd. All rights reserved.

1. Introduction

Analysis of perforated acoustic elements began with Sullivan and Crocker's [1] analytical prediction for a concentric-tube resonator. Later, Sullivan [2] proposed a segmentation method

*Corresponding author. Tel.: +91 80 22932303; fax: +91 80 23600648.
E-mail address: [munjal@mecheng.iisc.ernet.in](mailto:munj@mecheng.iisc.ernet.in) (M.L. Munjal).

Nomenclature			
f	frequency (Hz)	p_i	acoustic pressure
j	imaginary unit	ζ_{ik}	perforate impedance at the interface of the i th and k th ducts
ρ_0	air density	a_0	speed of sound in air
$\tilde{\rho}_i$	complex dynamic density of the absorbing material in the i th duct	S_i	cross-sectional area of the i th duct
$\tilde{\rho}_{ik}$	mean complex dynamic density between the i th and k th ducts	<i>Subscript</i>	
ρ_i	density perturbation over $\tilde{\rho}_i$ or ρ_0 , in the i th duct	0	air
u_i	particle velocity fluctuation over U_i	i	i th duct
u_{ik}^*	radial particle velocity at the interface of the i th and k th ducts	ik	interface of the i th and k th ducts; the i th row k th column element (of a matrix)

for the same configuration and extended it to three-duct elements, considering lumped admittance at discrete intervals along the perforate boundary, whereas successive branch points are separated by a rigid and uniform duct segment. Such lumped admittance elements separated by rigid tubular segments are described by unique transfer matrices and multiplied successively to arrive at the final form. On the negative side, it was the discretization of an essentially continuous process that kept it at bay.

Jayaraman and Yam [3] gave the first decoupling approach for a closed-form solution for a simple co-axial concentric-tube resonator. It has a limitation in the form of an assumed hypothetical situation of equal flow velocities in the two interacting ducts, which is obviously not a fact. Nevertheless, it may be perceived as the most effective method for any randomly configured acoustic element of parallel ducts with a stationary medium. Later Munjal et al. [4] produced a generalized decoupling method for analyzing the perforated elements. Application of different flow conditions in the distributed parameter method are described by Munjal [5] for both two- and three-duct elements as shown in Fig. 1.

Further attempts have been made to analyze similar acoustical elements for different configurations and flow conditions. In a recent development, Munjal [6] analyzed an axisymmetric pod silencer, making use of the bulk reaction model, where the pod segment may be treated as a three-duct element. Similarly, the wave interaction in four ducts are examined by Munjal [7] for the case of a flush-tube three-pass perforated element muffler.

The commonality among different tubular acoustical elements is the trend of governing differential equations and their affiliated coefficients. The number of coefficients increases with the number of interacting ducts, leading to involved algebraic calculations. It may be noticed that the number increases from eight for a simple concentric-tube configuration to fourteen for the three-duct perforated elements. To make things worse, coefficients vary with configurations of equal number of interacting tubes; i.e., the coefficients for an annular airgap lined duct are different from their counterparts of the reverse-flow or the cross-flow three-duct expansion chambers.

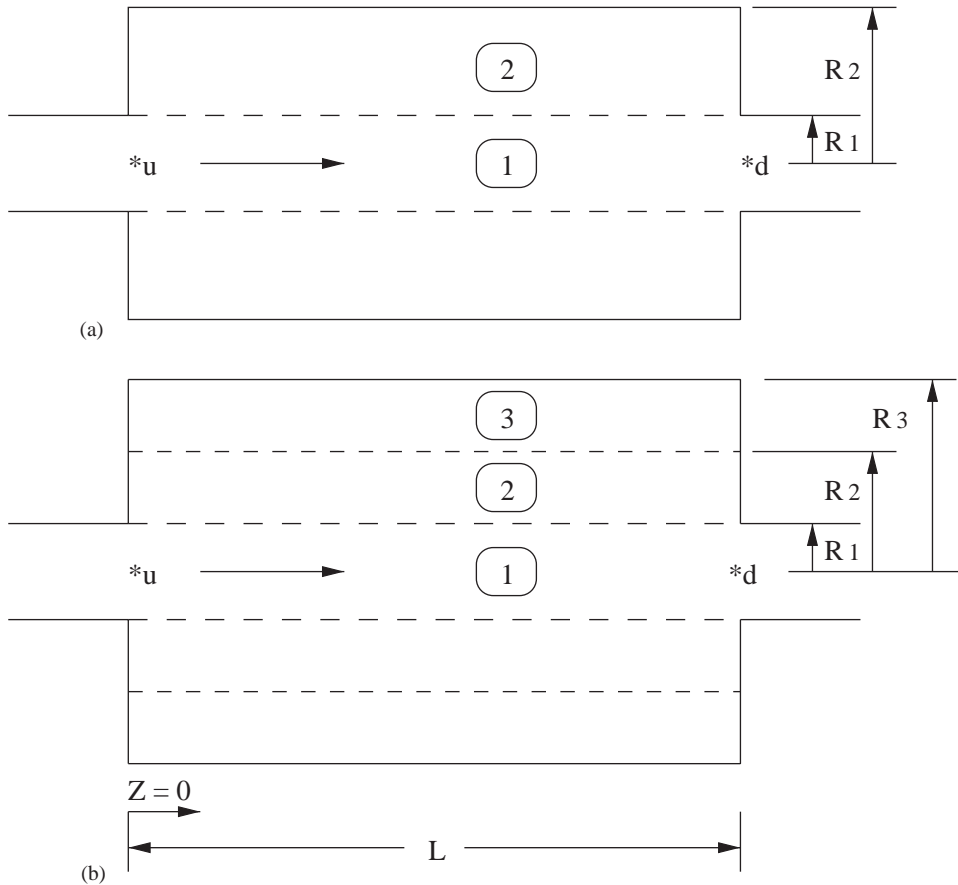


Fig. 1. (a) Two-duct and (b) three-duct concentric-tube resonators.

In the literature [8,9], the mass continuity equations associated with the interacting ducts for a dissipative silencer seem unreasonable, as they fail to satisfy the fundamental principle of the mass conservation between a pair of interacting finite control volumes. In this work, the continuity equation has been modified in order to meet the above criterion by incorporating a mean complex dynamic density $\tilde{\rho}_{ik}$, for the mass interaction between the i th and k th ducts. For a generality, it is rational to assume that, for all the ducts, the dynamic density $\tilde{\rho}_i$, wave number k_i and characteristic impedance Y_i are all complex numbers. This eventually accommodates the corresponding real values for a duct without any absorbing material and hence any non-dissipative configuration.

It would therefore be desirable to develop an algebraic algorithm for modeling a configuration with “ n ” interacting tubes or parallel ducts, which will provide a clarity of thought for completeness, and hence a generality, in a way, to preclude the tedious algebraic manipulations.

2. Methodology: a general algorithm

The following analysis presumes linear plane waves in a uniform, inviscid incompressible mean flow. Specifically, the assumptions made in the formulation of the governing equations of motion for any perforated-tube muffler configuration are as follows:

1. Spatial variations of acoustic pressure and thence density, across the wave fronts in any duct are negligible.
2. Amplitudes of pressure and density perturbations in ducts are negligible compared to their mean values.
3. Grazing flow condition is applied to all ducts.
4. Temperature variation and viscous effect of the medium are negligible.
5. Perforation along the entire length of the tube is uniform.
6. Wall thicknesses of the ducts are negligible as compared to their radii.
7. The porous absorbent material of any i th duct is considered as an equivalent fluid with complex wave number k_i and characteristic impedance Y_i .

The associated sound propagation and attenuation is described next in terms of a set of randomly configured ‘ n ’ parallel tubes, with no limitation on their inter-connectivity. Then, the linearized equation for mass continuity of the i th duct is given by

$$\frac{D\rho_i}{Dt} + \tilde{\rho}_i \frac{\partial u_i}{\partial z} + \sum \frac{P_{ik}}{S_i} u_{ik}^* \tilde{\rho}_{ik} = 0 \quad (1)$$

and the corresponding generalized momentum equation for the same duct becomes

$$\tilde{\rho}_i \frac{Du_i}{Dt} + \frac{\partial p_i}{\partial z} = 0. \quad (2)$$

Here, $\tilde{\rho}_i$ is the time-average component of the density for any tube and ρ_i is the space average of the density perturbation over the corresponding cross-sectional area S_i . Likewise, u_i is the velocity perturbation on U_i , the mean flow velocity in the i th duct. Wave interaction between the i th and k th ducts is caused by u_{ik}^* , the radial particle velocity across their interface, $\tilde{\rho}_{ik}$, the average complex dynamic density, and P_{ik} is the respective interface perimeter.

The fluctuating radial particle velocity through the perforated interface u_{ik}^* , as mentioned in Eq. (1), is a function of the pressure difference across the perforations, i.e., p_i and p_k . Thus,

$$u_{ik}^* = \frac{p_i(z) - p_k(z)}{\rho_0 a_0 \zeta_{ik}}, \quad (3)$$

where ζ_{ik} is the non-dimensional specific acoustic impedance of the perforates at the interface of the i th and k th ducts, and a_0 is the speed of sound in air. The mean complex dynamic density $\tilde{\rho}_{ik}$, at the interface of the i th and k th ducts may be given as

$$\tilde{\rho}_{ik} = 0.5(\tilde{\rho}_i + \tilde{\rho}_k). \quad (4)$$

Assuming an isentropic process in each of the ducts, the pressure and density perturbations in the i th duct are related by

$$p_i = \rho_i a_i^2. \quad (5)$$

Assuming harmonic time dependence of all variables in Eqs. (1) and (2), and using Eqs. (3)–(5), yields a set of “ N ” number of coupled differential equations. Hence, the corresponding wave equation for any i th duct may be given as

$$(1 - M_i^2)p_i'' - [2jk_iM_i + \sum U_iC_{ik}]p_i' + [k_i^2 - \sum j\omega C_{ik}]p_i + [\sum U_iC_{ik}]p_k' + [\sum j\omega C_{ik}]p_k = 0, \tag{6}$$

where the parametric constant C_{ik} is given by

$$C_{ik} = \frac{P_{ik}\tilde{\rho}_{ik}}{S_i\zeta_{ik}\rho_0a_0}. \tag{7}$$

These wave equations may be arranged in a matrix as a set of simultaneous differential equation and may be given by

$$[\mathbf{A}]\{p(z)\} = \{0\}, \tag{8}$$

where

$$\{p(z)\} = [p_1(z) \ p_2(z) \ \dots \ p_n(z)]^T$$

and $[\mathbf{A}]$ is an N th order square matrix, given by

$$[\mathbf{A}] = D^2[I_n] + D[\beta] + [\gamma], \tag{9}$$

where D is the differential operator d/dz . Then, the constituent elements of $[\mathbf{A}]$ may be given as

$$[\mathbf{A}]_{ik} = \begin{cases} D^2 + \beta_{ik}D + \gamma_{ik} & \text{if } i = k, \\ \beta_{ik}D + \gamma_{ik} & \text{if } i \neq k. \end{cases} \tag{10}$$

Coefficients β_{ik} and γ_{ik} are constants, the expressions for which may be predicted heuristically as shown below.

Plane wave propagation in ducts (Eq. (10)) has the system matrix $[\mathbf{A}]$, with normalized coefficients γ_{ik} and β_{ik} , which are parametric constants. Coefficients β_{ik} 's provide the acoustic resistance to particle velocity, and are therefore analogous to damping in a vibrating system. These coefficients vanish for a stationary medium, leaving only γ_{ik} 's. The total number of parametric coefficients (both β_{ik} and γ_{ik}) increases with the number of interacting ducts and stands at ‘ $6n - 4$ ’, whereas ‘ n ’ is the number of interacting tubular ducts. Irrespective of the limits and the boundary conditions (BCs), coefficients of the system matrix can be generalized and may be given as

$$\gamma_{ik} = \frac{j\omega C_{ik}}{1 - M_i^2}, \quad \beta_{ik} = -\frac{jU_i}{k_0} \gamma_{ik} \quad \text{if } i \neq k, \tag{11}$$

$$\sum_{i=1}^n \gamma_{ik} = \frac{k_i^2}{1 - M_i^2}, \quad \sum_{i=1}^n \beta_{ik} = \frac{-2jM_i k_i}{1 - M_i^2}. \tag{12}$$

Thus, the off-diagonal terms of the matrix $[\mathbf{A}]$ are known from Eq. (11), and then making use of Eq. (12), the diagonal terms may be calculated. Consequently, the coupled differential equations may be framed in a matrix form.

Expressions (11) and (12) have been verified for all known two-duct, three-duct and four-duct configurations analyzed by Munjal et al. [4,5,7]. Further applications are given later in Section 3.

Reverting to the matrix equations (8), one can rearrange them as a set of ‘ $2 \times n = m$ ’ simultaneous first-order equations. These may be written as

$$\begin{bmatrix} [-I_n] & D[I_n] \\ D[I_n] & [D\beta + \gamma] \end{bmatrix} \{Y\} = \{0\}, \quad (13)$$

where $[I_n]$ is the n th order identity matrix, and

$$[D\beta + \gamma]_{ik} = D\beta_{ik} + \gamma_{ik}, \quad (14)$$

$$\{Y\}_i = \begin{cases} \frac{dp_i}{dz} & \text{for } i = 1, 2, \dots, n, \\ p_{(i-n)} & \text{for } i = n + 1, n + 2, \dots, m. \end{cases} \quad (15)$$

Eq. (13) can be solved by making use of the standard eigen-matrix method [10]. Finally, the state vectors (pressures and particle velocities) at $z = 0$ and $z = l$ may be rearranged and related by a forward transfer matrix $[\mathbf{T}]$, with elements t_{ik} as follows:

$$\{\Psi\} = [\mathbf{T}]\{\Omega\}, \quad (16)$$

where

$$\{\Psi\} = [p_1, Y_0 u_1, p_2, Y_0 u_2, \dots, p_n, Y_0 u_n]_{z=0}^T, \quad (17)$$

where $Y_0 = \rho_0 a_0$ is the characteristic impedance of the gaseous medium. Similarly, vector $\{\Omega\}$ can be framed corresponding to $z = l$.

In order to determine the transmission loss of the acoustical system, the transfer matrix $[\mathbf{T}]$ needs to be reduced systematically to a second-order square matrix using appropriate BCs [5,7]. Hence, given a system of ‘ n ’ interacting ducts, ‘ $m - 2$ ’ BCs distributed at the two ends may be made use of to derive the desired four pole parameters. BCs for such acoustical elements may be classified as follows:

1. State vectors (acoustic pressures and particle velocities) of two different ducts at the same end may be related in terms of a ‘ 2×2 ’ transfer matrix. For example, state vectors for ducts 1 and 3 of Fig. 2, fused into the end chamber at $Z = L$, are related by the equivalent transfer matrix of the end chamber.
2. Acoustic admittance of a duct at the boundary.

Application of BCs of the first kind may use a methodical approach as shown by Munjal for a configuration with four interacting ducts [7]. Thus, all BCs of the first kind at one end can be applied before considering the ones at the other end. BCs of the second kind can always be given in terms of the admittance α , at the axial boundaries of the ducts. These may be defined as

$$\alpha = \frac{\rho_0 a_0 u_i}{p_i} \quad (18)$$

for the i th duct, and may have different values for the upstream and the downstream ends.

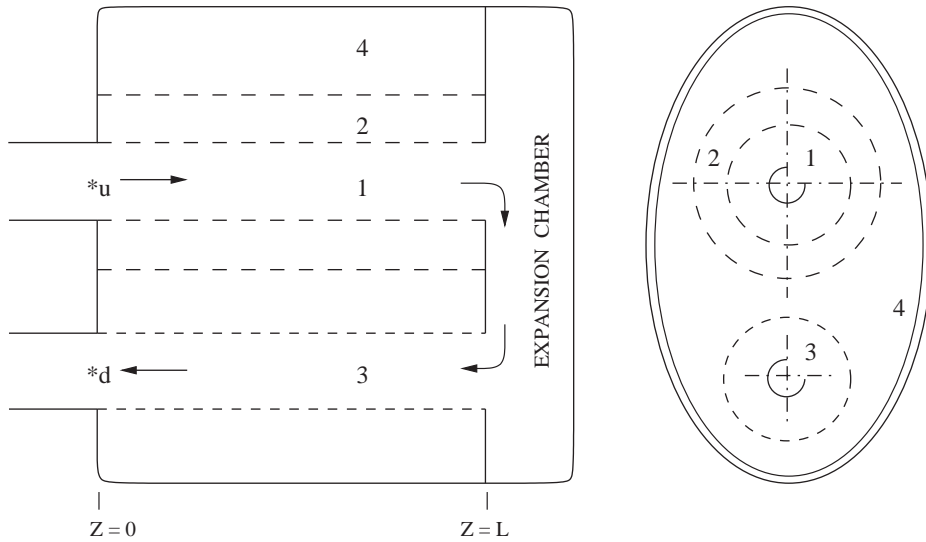


Fig. 2. Four-duct reverse flow muffler configuration.

In case of a rigid end, the particle velocity and hence the admittance becomes zero. Once all the BCs of the first kind are applied, a simple algorithm may be followed to apply the rest of the BCs as follows:

- In any muffler configuration with ‘*n*’ interacting ducts, there will be a few ducts with BCs of the second type applicable at both the ends. Pressure and velocity components of one such duct with known admittances are pushed down to occupy the (*m* – 1)th and *m*th positions, respectively, for both the downstream vector, {Ψ} and upstream vector, {Ω}. Thus, applying those BCs, the number of simultaneous equations will reduce to ‘*m* – 2’ and they may be re-written as

$$\Psi_k = \sum_{i=1}^{m-2} [t_{ki} + \delta_k \{\alpha_u t_{(m-1)i} - t_{mi}\}] \Omega_i, \quad 1 \leq k \leq m - 2, \tag{19}$$

where δ_k is given as

$$\delta_k = \frac{t_{k(m-1)} + \alpha_d t_{km}}{[\{t_{m(m-1)} + \alpha_d t_{mm}\} - \alpha_u \{t_{(m-1)(m-1)} + \alpha_d t_{(m-1)m}\}]}, \tag{20}$$

where α_u and α_d are the upstream admittance and the downstream admittance, respectively.

- Repeating the process successively for all such ducts will leave the truncated transfer matrix operating over a vector of four variables. During this cyclic process one has to ensure that the value of the index ‘*m*’ is reduced by two after every cycle and again the elements of the transfer matrix [T] be replaced by the corresponding ones from Eq. (19).

If, at the final step, the BCs do not relate to the same duct (as for the case of cross flow), the process of reduction may still be applied in a similar fashion, ensuring that the set of state variables relevant to those BCs are placed at the bottom half of both the vectors.

2.1. Acoustic impedance of the perforates

In some applications, fiber-based porous sound-absorbing materials are used in either duct across an interface. One such case arises when different backing materials are used in both the annular cavities in a three-duct concentric-tube resonator of Fig. 1. The perforate impedance for such configuration with a default stationary medium in either of the ducts may be modified and extended from the expression of the acoustic impedance of perforations facing backing material at one side given by Kirby and Cumming [11] and Selamet [9]. The impedance $\zeta_{ik} = \Re + j\Im$ may be given as

$$\zeta_{ik} = \left[6 \times 10^{-3} + jk_0 \left(t_h + 0.375 \left(\frac{k_i Y_i}{k_0 Y_0} + \frac{k_k Y_k}{k_0 Y_0} \right) d_h \right) \right] / \sigma, \quad (21)$$

where k_i and Y_i are complex wave number and characteristic impedance of the acoustic lining with indices i and k representing the ducts across the interface. The corresponding grazing flow perforate impedance is given by [12]

$$\zeta_{ik} = \left[0.00734(1 + 72.23M_i) + j2.27 \times 10^{-3}(1 + 51t_h) \left(1 + \frac{k_k Y_k}{k_0 Y_0} \right) d_h f \right] / \sigma. \quad (22)$$

If the medium in the k th duct is a sound-absorbing material, then $Y_k = Y_w$ and $k_k = k_w$. For any such sound-absorbing material, Y_w and k_w are precisely outlined by the empirical formulae of Delany and Bazley [13], later improved through modifications by Mechel [14]:

$$\frac{Y_w}{Y_0} = \begin{cases} 1 + 0.0485(A)^{0.754} - j0.087(A)^{0.73}, & \text{if } A \leq 60, \\ \frac{0.5(A/\pi) + j1.4}{-1.466 + j0.212A^{0.5}}, & \text{if } A > 60, \end{cases} \quad (23)$$

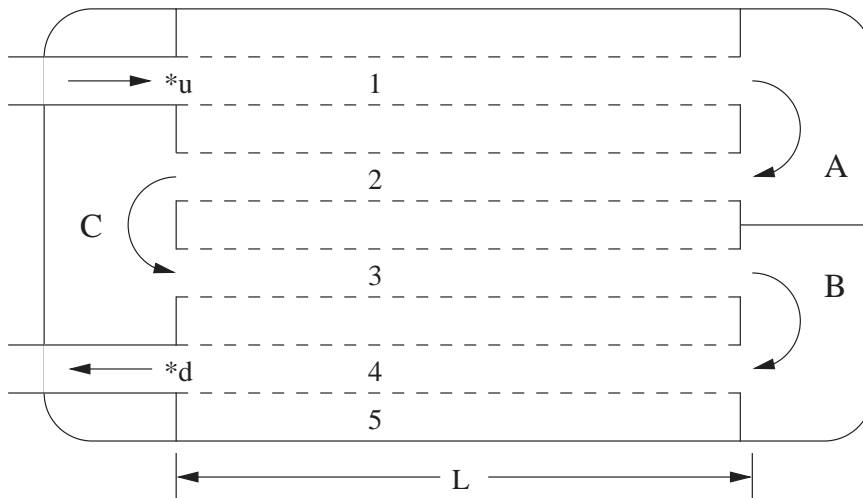
$$\frac{k_w}{k_0} = \begin{cases} 1 + 0.0978(A)^{0.6929} - j0.189(A)^{0.6185}, & \text{if } A \leq 60, \\ -1.466 + j0.212A^{0.5}, & \text{if } A > 60. \end{cases} \quad (24)$$

Here A , the normalized flow resistivity of a λ -deep acoustic lining, is given by $A = E\lambda/Y_0$, whereas E is the flow resistivity of the lining material.

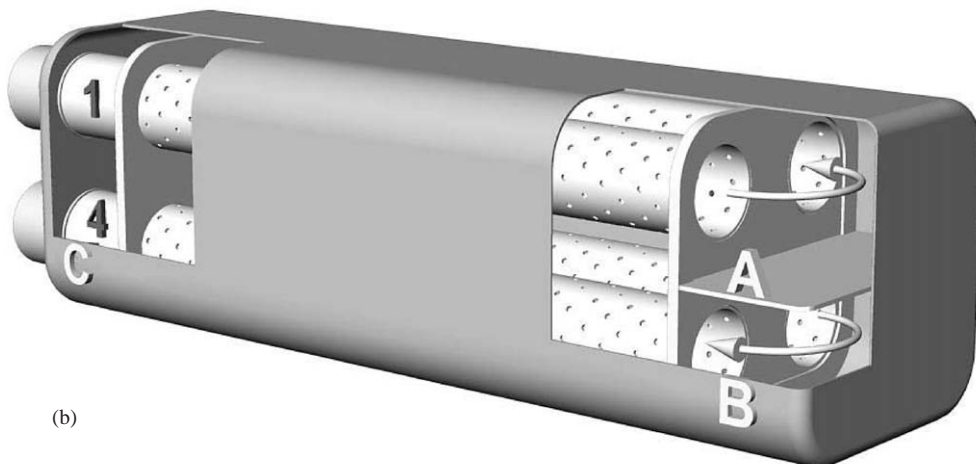
Expressions for complex characteristic impedance and wavenumber in Eqs. (23) and (24) are such as to reduce to Y_0 and k_0 , respectively, for zero flow resistivity (absence of the absorptive material). Still more general and precise formulae may be found in Ref. [15].

3. Applications

In order to demonstrate generality of the generalized algorithm, a few hypothetical muffler configurations shown in Figs. 2–4 are analyzed here. The generalized coefficient matrices are obtained for each of the configurations in accordance with the algorithm enunciated above. For each of the cases given below, the basic task is to construct the coefficient matrix $[\gamma]$ of Eq. (9). With the matrix $[\gamma]$ thus constructed for any given perforated-tube acoustic element, diagonal terms γ_{ii} may be derived by making use of Eq. (12) and so also the matrix $[\beta]$. Once the matrices $[\gamma]$ and $[\beta]$ are generated, solution to the system will follow as described above (Eqs. (13)–(20)).



(a)



(b)

Fig. 3. Five-duct reverse flow concentric-tube resonator. (a) Schematic diagram, (b) the corresponding 3-D solid model.

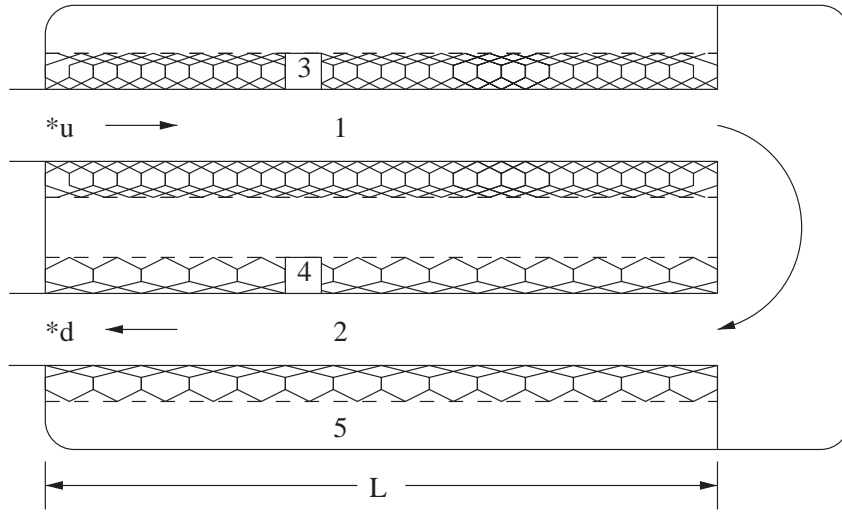


Fig. 4. Five-duct reverse flow concentric-tube resonator.

3.1. Four-duct reverse flow

Fig. 2 is basically a reverse-flow four-duct configuration that comprises all but one perforated tube, arranged in an elliptical shell (duct-4), fused into an expansion chamber at one end. A grazing flow exists in the ducts enumerated 1 and 3 in Fig. 2, whereas the media in the shell and the annular cavity (duct-2) are assumed stationary.

The generalized algorithm is put to use as in Eqs. (11) and (12) to articulate the coefficient matrix $[\gamma]$ of Eq. (9), and it may be given as

$$[\gamma] = \begin{bmatrix} \gamma_{11} & \frac{j8\pi f}{a_0 d_1 \zeta_{12} (1 - M_1^2)} & 0 & 0 \\ \frac{j8\pi f d_1}{a_0 (d_2^2 - d_1^2) \zeta_{12}} & \gamma_{22} & 0 & \frac{j8\pi f d_2}{a_0 (d_2^2 - d_1^2) \zeta_{24}} \\ 0 & 0 & \gamma_{33} & \frac{j8\pi f}{a_0 d_3 \zeta_{34} (1 - M_3^2)} \\ 0 & \frac{j2\pi^2 f d_2}{a_0 S_4 \zeta_{24}} & \frac{j2\pi^2 f d_3}{a_0 S_4 \zeta_{34}} & \gamma_{44} \end{bmatrix}. \tag{25}$$

The diagonal element γ_{ii} may be evaluated readily by means of Eq. (12).

3.2. Five-duct reverse flow (flush-tube four-pass perforated element chamber)

As shown in Fig. 3, a five-duct reverse flow perforated tube resonator consists of a set of four perforated tubes, and an enclosing shell. The flow is set in motion with duct 1, that expands into the expansion chamber (cavity) “A”. As the flow grazes through the rest of the perforated tubes

and the end cavities as shown in the virtual 3-D solid model, sound attenuates due to the combined effect of the impedances offered by the perforated tubes and the sudden area changes. The generalized coefficient matrix $[\gamma]$ for the common perforated portion may be given as

$$[\gamma] = \begin{bmatrix} \gamma_{11} & 0 & 0 & 0 & \frac{j8\pi f}{a_0 d_1 \zeta_{15} (1 - M_1^2)} \\ 0 & \gamma_{22} & 0 & 0 & \frac{j8\pi f}{a_0 d_2 \zeta_{25} (1 - M_2^2)} \\ 0 & 0 & \gamma_{33} & 0 & \frac{j8\pi f}{a_0 d_3 \zeta_{35} (1 - M_3^2)} \\ 0 & 0 & 0 & \gamma_{44} & \frac{j8\pi f}{a_0 d_4 \zeta_{45} (1 - M_4^2)} \\ \frac{j2\pi^2 d_1 f}{a_0 S_5 \zeta_{15}} & \frac{j2\pi^2 d_2 f}{a_0 S_5 \zeta_{25}} & \frac{j2\pi^2 d_3 f}{a_0 S_5 \zeta_{35}} & \frac{j2\pi^2 d_4 f}{a_0 S_5 \zeta_{45}} & \gamma_{55} \end{bmatrix}. \quad (26)$$

As indicated earlier, γ_{ii} may be evaluated readily by means of Eq. (12).

3.3. Five-duct reverse flow (flush-tube two-pass perforated element chamber) with absorptive layers

The configuration in Fig. 4 differs from the four-pass element in as much as fiber-based porous sound-absorbing materials are used in the concentric cavities surrounding the perforated tubes for a better performance in a selected frequency range. Use of the generalized algorithm yields the following coefficient matrix $[\gamma]$:

$$[\gamma] = \begin{bmatrix} \gamma_{11} & 0 & \frac{j8\pi f \tilde{\rho}_{13}}{Y_0 d_1 \zeta_{13} (1 - M_1^2)} & 0 & 0 \\ 0 & \gamma_{22} & 0 & \frac{j8\pi f \tilde{\rho}_{24}}{Y_0 d_2 \zeta_{24} (1 - M_2^2)} & 0 \\ \frac{j2\pi^2 f d_1 \tilde{\rho}_{13}}{Y_0 S_3 \zeta_{13}} & 0 & \gamma_{33} & 0 & \frac{j2\pi^2 f d_3 \tilde{\rho}_{35}}{Y_0 S_3 \zeta_{35}} \\ 0 & \frac{j2\pi^2 f d_2 \tilde{\rho}_{24}}{Y_0 S_4 \zeta_{24}} & 0 & \gamma_{44} & \frac{j2\pi^2 f d_4 \tilde{\rho}_{45}}{Y_0 S_4 \zeta_{45}} \\ 0 & 0 & \frac{j2\pi^2 f d_3 \tilde{\rho}_{35}}{Y_0 S_5 \zeta_{35}} & \frac{j2\pi^2 f d_4 \tilde{\rho}_{45}}{Y_0 S_5 \zeta_{45}} & \gamma_{55} \end{bmatrix}. \quad (27)$$

Again, γ_{ii} may be evaluated by means of Eq. (12). Here it may be noted that $\tilde{\rho}_{13} = \tilde{\rho}_{35}$ and $\tilde{\rho}_{24} = \tilde{\rho}_{45}$ for the above configuration (Fig. 4).

4. Concluding remarks

The generalized algebraic algorithm presented here for perforated element mufflers results in great convenience and minimizes oversights or human errors that often accompany complex

mathematical modeling. Most importantly, the algorithm for applying the BCs holds good for all acoustical configurations. The generalized algorithm presented in this paper applies to dissipative ducts as well as those filled up with the gaseous medium (usually air). However the physical model of Refs. [1–5] implied in the algorithm may not hold exactly for dissipative ducts as shown in Ref. [16], although it has been applied with some success in Refs. [8,9].

Acknowledgements

The authors would like to place on record their appreciation of the financial sponsorship of FRITA by the Department of Science and Technology of the Government of India.

References

- [1] J.W. Sullivan, M.J. Crocker, Analysis of concentric tube resonators having unpartitioned cavities, *Journal of the Acoustical Society of America* 64 (1) (1978) 207–215.
- [2] J.W. Sullivan, A method for modeling perforated tube muffler components—Part I: theory, *Journal of the Acoustical Society of America* 66 (3) (1979) 772–778.
- [3] K. Jayaraman, K. Yam, Decoupling approach to modeling perforated tube muffler components, *Journal of the Acoustical Society of America* 69 (2) (1981) 390–396.
- [4] M.L. Munjal, K.N. Rao, A.D. Sahasrabudhe, Aeroacoustic analysis of perforated muffler components, *Journal of Sound and Vibration* 114 (2) (1987) 173–188.
- [5] M.L. Munjal, *Acoustics of Ducts and Mufflers*, Wiley, New York, 1987.
- [6] M.L. Munjal, Analysis and design of pod silencers, *Journal of Sound and Vibration* 262 (2003) 497–507.
- [7] M.L. Munjal, Analysis of a flush-tube three-pass perforated muffler by means of transfer matrices, *International Journal of Sound and Vibration* 2 (2) (1997) 63–68.
- [8] Chao-Nan Wang, Numerical decoupling analysis of a resonator with absorbent material, *Applied Acoustics* 58 (1999) 109–122.
- [9] A. Selamet, I.J. Lee, Z.L. Ji, N.T. Huff, Acoustic attenuation performance of perforated absorbing silencers, *SAE Noise and Vibration Conference*, Paper 2001-01-1435, 2001.
- [10] K.S. Peat, A numerical decoupling analysis of perforated pipe silencer elements, *Journal of Sound and Vibration* 128 (2) (1988) 199–212.
- [11] R. Kirby, A. Cummings, The impedance of perforated plate subjected to grazing gas flow and backed by porous media, *Journal of Sound and Vibration* 217 (4) (1998) 619–636.
- [12] K.N. Rao, M.L. Munjal, Experimental evaluation of impedance of perforates with grazing flow, *Journal of Sound and Vibration* 108 (2) (1986) 283–295.
- [13] M.E. Delany, B.N. Bazley, Acoustical characteristics of fibrous absorbent material, *Applied Acoustics* 3 (1970) 106–116.
- [14] F.P. Mechel, Extension of low frequencies of the formulae of Delany and Bazley for absorbing materials, *Acustica* 35 (1976) 210–213 (in German).
- [15] F.P. Mechel, *Formulas of Acoustics*, Springer, Berlin, 2002.
- [16] S.N. Panigrahi, M.L. Munjal, Comparison of various methods for analyzing lined circular ducts, *Journal of Sound and Vibration*, in press; doi:10.1016/j.jsv.2004.09.006.