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Short Communication

Energy pumping for a larger span of energy

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Abstract

This paper is devoted to energy pumping: exhibiting a multi-degree-of-freedom (dof) nonlinear attachment coupled to a single-dof linear one, energy's area leading to energy pumping phenomenon is increased by involving different nonlinear modes in the process.

1. Introduction

Energy pumping corresponds to control of linear structure by coupling it to adapted passive nonlinear structure [1–5]. However, as described in Ref. [6], energy pumping occurs above a specific value of the initial energy level: so when energy injected is too low, energy transfer from the linear structure to the nonlinear one does not appear. When energy pumping occurs, energy decreases in the linear structure first due to the transient nonlinear efficient phenomenon, then only due to damping dissipation (less efficient) when residual energy is too low in the coupled system. In this letter use of several nonlinear modes is planned in order to increase the span of energy leading to energy pumping activation. An example is given to demonstrate the phenomenon that is also investigated using time–frequency analysis (Hilbert Transform (HT)).

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2. Instantaneous Frequency (IF) analysis

In the previous studies about energy pumping [3,5,7,8], essentially one-degree-of-freedom (dof) nonlinear attachment has been studied as shown in Fig. 1.

The governing equations of such a system are

$$M\ddot{x}_1 + \lambda_1\dot{x}_1 + k_1x_1 + \gamma(x_1 - y_1) = 0,$$
$$m\ddot{y}_1 + \lambda_s\dot{y}_1 + Cy_1^3 + \gamma(y_1 - x_1) = 0. \quad (1)$$

The linear structure is excited by an impulsion, so we consider free oscillations of structures with initial conditions: $x_1(0) = y_1(0) = \dot{y}_1(0) = 0$ and $\dot{x}_1(0) = \sqrt{2h}$ where h is the energy of the system at $t = 0^+$. In this system, energy pumping occurs above a specific value of the energy level. Indeed, by considering system (1) with the following parameters $k_1 = 4000\,\mathrm{N\,m^{-1}}$, $\lambda_1 = 100\,\mathrm{N\,s\,m^{-1}}$, $M = 4000\,\mathrm{kg}$, $\gamma = 1000\,\mathrm{N\,m^{-1}}$, $m = 4000\,\mathrm{kg}$, $\lambda_s = 300\,\mathrm{N\,s\,m^{-1}}$, $C = 600\,\mathrm{N\,m^{-3}}$, energy pumping occurs for $h = 14$ as shown in Fig. 2(c) where resonance of the nonlinear oscillator ($y_1(t)$) occurs which produces attenuation of the response of the linear oscillator ($x_1(t)$) in plot (d). When initial energy is too low, for instance $h = 5$, there is no resonance of the nonlinear attachment as shown in Fig. 2(a) and no energy pumping occurs as shown in Fig. 2(b).

To observe this resonance capture a time–frequency analysis must be performed since resonance between the linear mode and the nonlinear normal mode occurs during the transient responses. That is why we can use the HT and its properties which are often used in nonlinear free vibrations in non-stationary domain, in particular the concept of IF. As a generalization of the definition of frequency, IF is defined as the rate of change of the phase angle at time t of the analytic version of the signal [9]. Given a real signal $s(t)$, the analytic signal $z(t)$ is a complex signal having the actual signal as the real part and the HT of the signal as the imaginary component, namely

$$z(t) = s(t) + \mathrm{j}H[s(t)] = a(t)\mathrm{e}^{\mathrm{j}\phi(t)}, \quad (2)$$

where the amplitude $a(t)$ and the phase $\phi(t)$ are given by

$$a(t) = \sqrt{(s(t))^2 + (H[s(t)])^2} \quad \text{and} \quad \phi(t) = \tan^{-1}\left(\frac{H[s(t)]}{s(t)}\right) \quad (3)$$

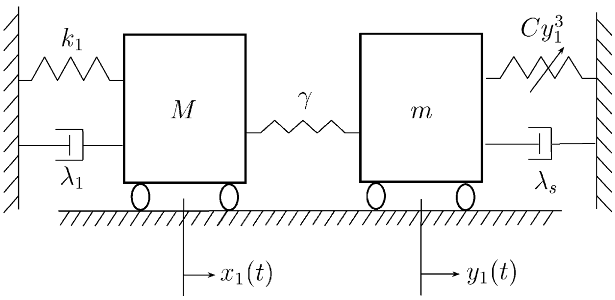

Fig. 1. System considered with one nonlinear normal mode.

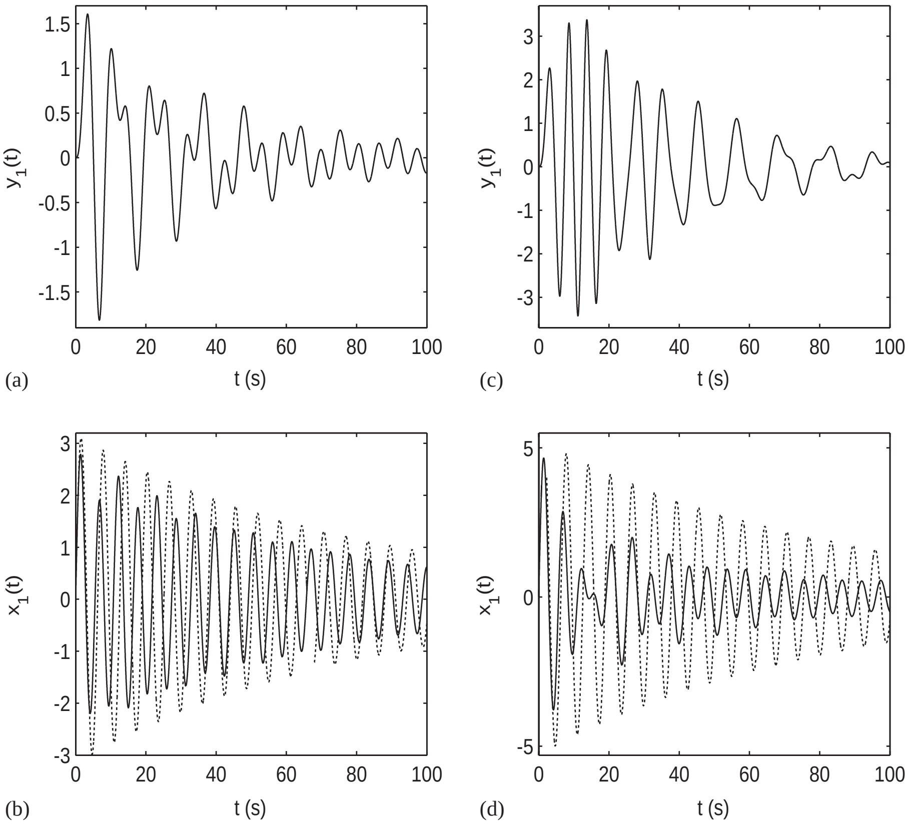

Fig. 2. Resonance capture occurrence above a specific value of initial injected energy h. With $h = 5$, (a) damped free oscillations of nonlinear oscillator $y_1(t)$ and (b) no energy pumping occurs for the linear oscillator $x_1(t)$: dotted line denotes the displacement of the linear oscillator without coupling and the solid line denotes the displacement of the linear oscillator with coupling. With $h = 14$, there is (c) resonance of the nonlinear oscillator $y_1(t)$ and (d) energy pumping occurs: dotted line denotes the displacement of the linear oscillator without coupling and the solid line denotes the displacement of the linear oscillator with coupling.

and the HT is given by the principal value of the following integral:

$$H[s(t)] = \frac{1}{\pi}\int_{-\infty}^{\infty} \frac{s(\tau)}{t-\tau}\,\mathrm{d}\tau. \tag{4}$$

The IF is, by definition

$$f_i(t) = \frac{1}{2\pi}\frac{\mathrm{d}\phi(t)}{\mathrm{d}t}. \tag{5}$$

The IF definition captures the time variation of the frequency accurately, where as when the Fourier domain is used, the results contain a large number of components with different frequencies and the simple nature of the signal is lost. Thus, a frequency analysis can be performed with the calculation of IF. By considering system (1) with the previous values of parameters, we can analyse IF when energy pumping occurs ($h = 14$) as shown in Fig. 3.

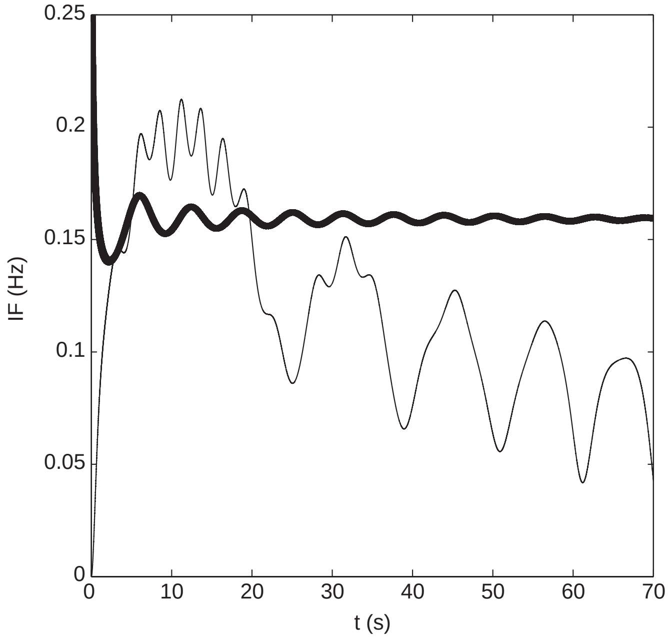

Fig. 3. Resonance capture of the nonlinear normal mode $h = 14$ studied with time–frequency analysis: IF of $y_1(t)$ is denoted by a solid line and the IF of the linear mode is denoted by a thick solid line.

Clearly, when energy pumping occurs, it appears that during 20 s a resonant capture also occurs with the nonlinear oscillator: the IF of $y_1(t)$ crosses the linear frequency and becomes identical to the IF of the linear mode as shown in Fig. 3 (energy transfer occurs). However, this phenomenon takes place for a high level of energy ($h = 14$) since when initial energy is too small ($h = 5$), the phenomenon does not occur. That is why multiple nonlinear normal modes can be used to obtain a larger span of energy.

3. Use of multiple nonlinear normal modes

The use of several nonlinear normal modes is planned by conceiving a nonlinear attachment composed of several nonlinear oscillators. That is why a 4 dof system composed of a linear structure weakly coupled to a nonlinear structure is considered as shown in Fig. 4. The nonlinear system is not linked to the ground. Thus, it can be easily added to the linear structure in practice for real structures.

According to Newton's second law of motion

$$\begin{aligned}
&M\ddot{x}_1 + \lambda_1\dot{x}_1 + k_1x_1 + \gamma(x_1 - y_1) = 0,\\
&m_1\ddot{y}_1 + \lambda_2(\dot{y}_1 - \dot{y}_2) + C_1(y_1 - y_2)^3 + \gamma(y_1 - x_1) = 0,\\
&m_2\ddot{y}_2 + \lambda_2(\dot{y}_2 - \dot{y}_1) + \lambda_3(\dot{y}_2 - \dot{y}_3) + C_1(y_2 - y_1)^3 + C_2(y_2 - y_3)^3 = 0,\\
&m_3\ddot{y}_3 + \lambda_3(\dot{y}_3 - \dot{y}_2) + C_2(y_3 - y_2)^3 = 0.
\end{aligned} \tag{6}$$

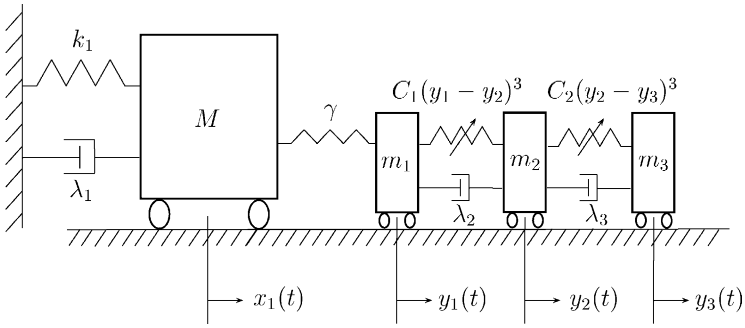

Fig. 4. System considered with multiple nonlinear oscillators.

The linear structure is excited by an impulsion, so we consider free oscillations of structures with initial conditions: $x_1(0) = y_1(0) = \dot{y}_1(0) = y_2(0) = \dot{y}_2(0) = y_3(0) = \dot{y}_3(0) = 0$ and $\dot{x}_1(0) = \sqrt{2h}$, where h is the energy of the system at $t = 0^+$. The decoupled nonlinear structure (attached to the linear one) possesses three nonlinear normal modes [7]: the first mode corresponds to a rigid-body mode of the decoupled nonlinear system; there is an in-phase nonlinear normal mode, $y_2(t) - y_3(t) = y_1(t) - y_2(t)$, and there is an out-of-phase nonlinear normal mode, $y_2(t) - y_3(t) = -(y_1(t) - y_2(t))$. So the nonlinear modal coordinates $w_1(t)$, $w_2(t)$ and $w_3(t)$ which correspond to the nonlinear normal modes of the decoupled nonlinear structure can be introduced

$$\begin{aligned} w_1(t) &= y_1(t) + y_2(t) + y_3(t), \\ w_2(t) &= (y_2(t) - y_3(t)) - (y_1(t) - y_2(t)) = 2y_2(t) - y_1(t) - y_3(t), \\ w_3(t) &= (y_2(t) - y_3(t)) + (y_1(t) - y_2(t)) = y_1(t) - y_3(t). \end{aligned} \tag{7}$$

When coupling is introduced, the system possesses nonlinear normal modes which are perturbations of the precedent nonlinear normal modes. When frequency of one nonlinear normal mode becomes identical to the natural frequency of the linear mode, internal resonances (and bifurcations of nonlinear normal modes) may occur and energy pumping can take place [5]. By designing skillfully the nonlinear oscillators, it is possible to have nonlinear normal modes that cross the linear frequency for different levels of energy. Thus, energy transfer will take place first between the linear mode and with one nonlinear normal mode, and when the energy decreases (due to damping dissipation or if the initial energy injected is low), the energy transfer will still take place with another nonlinear normal mode. To observe this phenomenon, a time–frequency analysis must be performed since resonances between the linear mode and the nonlinear normal modes occur during the transient responses. By considering system (6) with the following parameters $k_1 = 4000\,\mathrm{N\,m^{-1}}$, $\lambda_1 = 100\,\mathrm{N\,s\,m^{-1}}$, $M = 4000\,\mathrm{kg}$, $\gamma = 1000\,\mathrm{N\,m^{-1}}$, $m_1 = m_2 = m_3 = 1333\,\mathrm{kg}$, $\lambda_2 = \lambda_3 = 100\,\mathrm{N\,s\,m^{-1}}$, $C_1 = 600\,\mathrm{N\,m^{-3}}$, $C_2 = 120\,\mathrm{N\,m^{-3}}$, we can analyse IFs of the nonlinear normal modes. The viscous damping ratio of the linear structure is thus 2.5%, which is closed to the damping ratio of structures in Civil Engineering. It should be noted that the total mass added ($3 \times 1333\,\mathrm{kg}$) is the same as in Section 2 ($m = 4000\,\mathrm{kg}$) and damping ratio for nonlinear system is also the same as in Section 2 ($\lambda_2/m_2 = \lambda_3/m_3 = \lambda_s/m$).

First, if we consider a high initial energy $h = 20$, then in this case, as shown in Fig. 5, a resonance capture occurs with the third nonlinear normal mode $w_3(t)$ (the IF of $w_3(t)$ becomes identical to the IF of the linear mode during 40 s as shown in Fig. 5(a)). Thus, there is a resonance of the third nonlinear normal mode as shown in Fig. 5(b) which produces attenuation of linear vibrations as shown in Fig. 5(c). In this case, there is not a resonance capture with the nonlinear normal mode $w_2(t)$ since its IF is not captured by the IF of the linear mode as shown in Fig. 5(a) (the energy injected is too high and the phenomenon of energy pumping is less pronounced).

Secondly, if we consider a lower initial energy $h = 1.6$, then as shown in Fig. 6, a resonance capture occurs with the second nonlinear normal mode $w_2(t)$ (the IF of $w_2(t)$ becomes identical to the IF of the linear mode as shown in Fig. 6(a) during 40 s). Thus, there is resonance of the nonlinear normal mode as shown in Fig. 6(b) which produces attenuation of linear vibrations as shown in Fig. 6(c). In this case, there is not a resonance capture with the nonlinear normal mode $w_3(t)$ since its IF is under the IF of the linear mode as shown in Fig. 6(a) (the energy injected is too low).

It should be noted that it was not possible to obtain energy pumping for this low level of energy in the case of 1 dof nonlinear system as shown in Section 2 with the same parameters. So the use of multiple nonlinear normal modes allows to expand the span of energy where energy pumping occurs. The nonlinear normal modes resonate according to the level of energy present in the system.

Between this large span of energy, there are resonance captures with the two nonlinear normal modes $w_2(t)$ and $w_3(t)$. Indeed, for $h = 5.2$, as shown in Fig. 7, first for $t < 20$ s there is resonance of the nonlinear normal mode $w_3(t)$ as shown in Fig. 7(b) (and so an attenuation of linear vibrations as shown in Fig. 7(d)) since the IF of $w_3(t)$ becomes identical to the linear frequency as shown in Fig. 7(a). But after $t = 20$ s, as energy of the linear oscillator decreases, the resonance capture of $w_3(t)$ is finished. However, owing to the design of multiple nonlinear normal modes, resonance of the nonlinear normal mode $w_2(t)$ occurs as shown in Fig. 7(c) since the energy is lower and the IF of $w_2(t)$ becomes identical in mean to the linear frequency as shown in Fig. 7(a). Thus, the vibrations of the linear oscillator are still attenuated after $t = 20$ s as shown in Fig. 7(d). Thus, energy pumping phenomenon is more efficient.

4. Conclusion

Energy pumping is improved owing to the use of multiple nonlinear normal modes. Thus, the span of energy where the phenomenon occurs is increased. The calculation of instantaneous frequency owing to the Hilbert Transform underlines the phenomena of resonance captures with the different nonlinear normal modes resonating for different levels of energy. The use of multiple nonlinear normal modes allows the occurrence of energy pumping for lower energies. Different nonlinear normal modes resonate in the function of the energy level. The efficiency of energy pumping is thus improved by increasing the span of energy where energy pumping occurs. To understand better the phenomenon and the different captures, an analytical study can be performed using the analytical techniques (modified balance method [6],...) already used for analysing energy pumping.

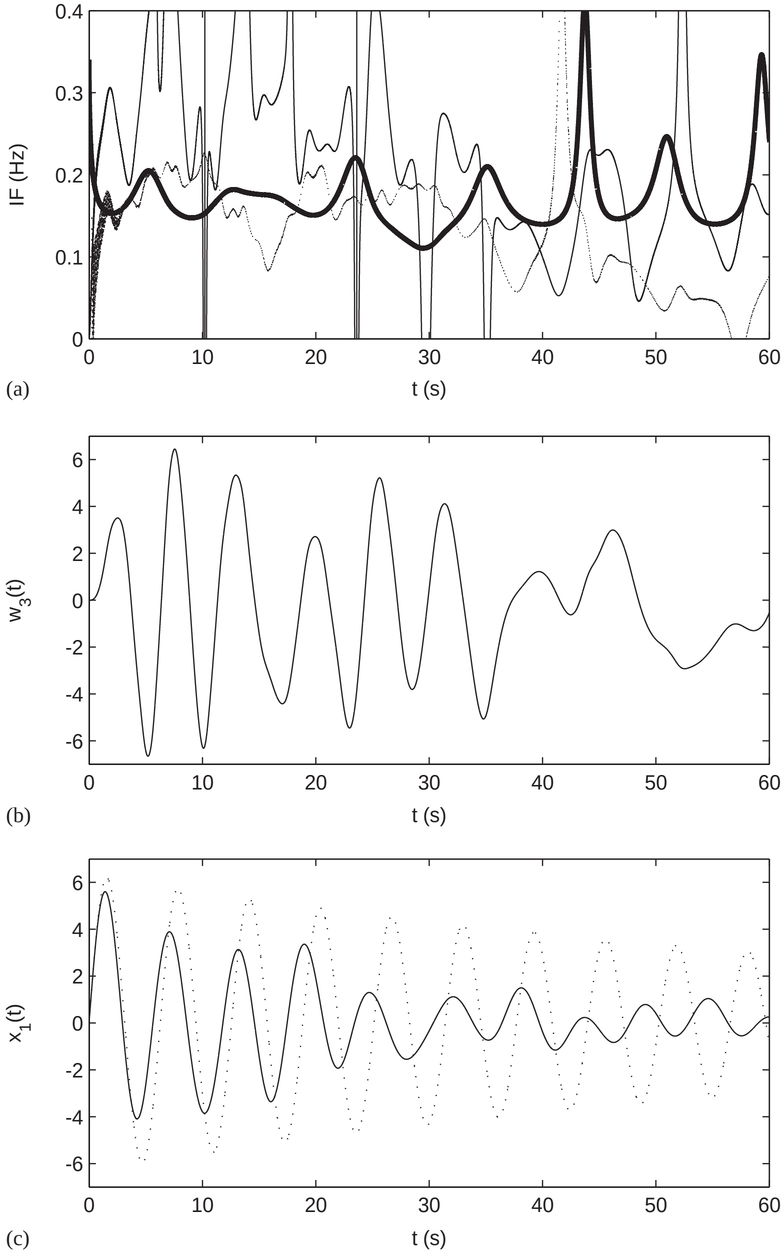

Fig. 5. Resonance capture of the nonlinear normal mode $w_3(t)$ with $h = 20$. (a) Time–frequency analysis is considered: IF of $w_2(t)$ is denoted by a solid line, IF of $w_3(t)$ is denoted by a thin dotted line and IF of the linear oscillator is denoted by a thick solid line. (b) Resonance of $w_3(t)$. (c) Vibrations of the linear oscillator (dotted line denotes the displacement of the linear oscillator without coupling and the solid line denotes the displacement of the linear oscillator with coupling).

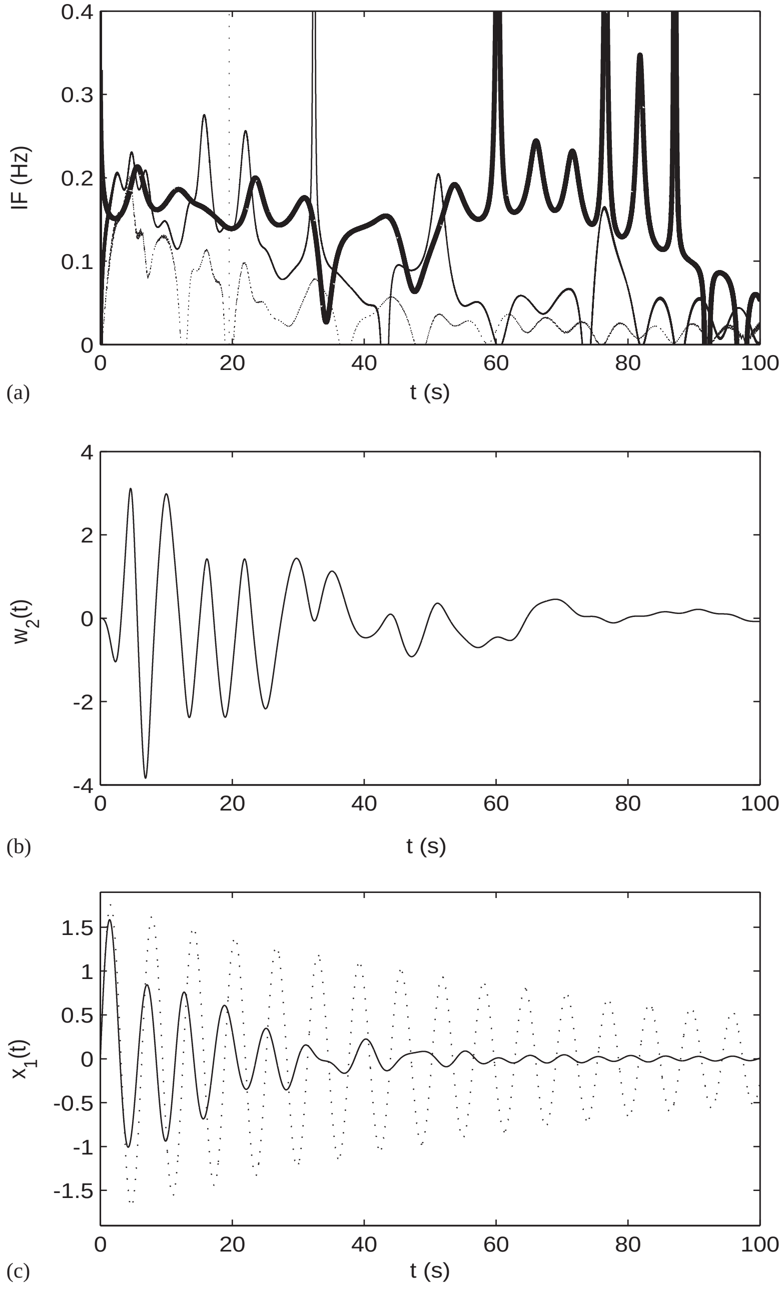

Fig. 6. Resonance capture of the nonlinear normal mode $w_2(t)$ with $h = 1.6$. (a) Time–frequency analysis is considered: IF of $w_2(t)$ is denoted by a solid line, IF of $w_3(t)$ is denoted by a thin dotted line and IF of the linear oscillator is denoted by a thick solid line. (b) Resonance of $w_2(t)$. (c) Vibrations of the linear oscillator (dotted line denotes the displacement of the linear oscillator without coupling and the solid line denotes the displacement of the linear oscillator with coupling).

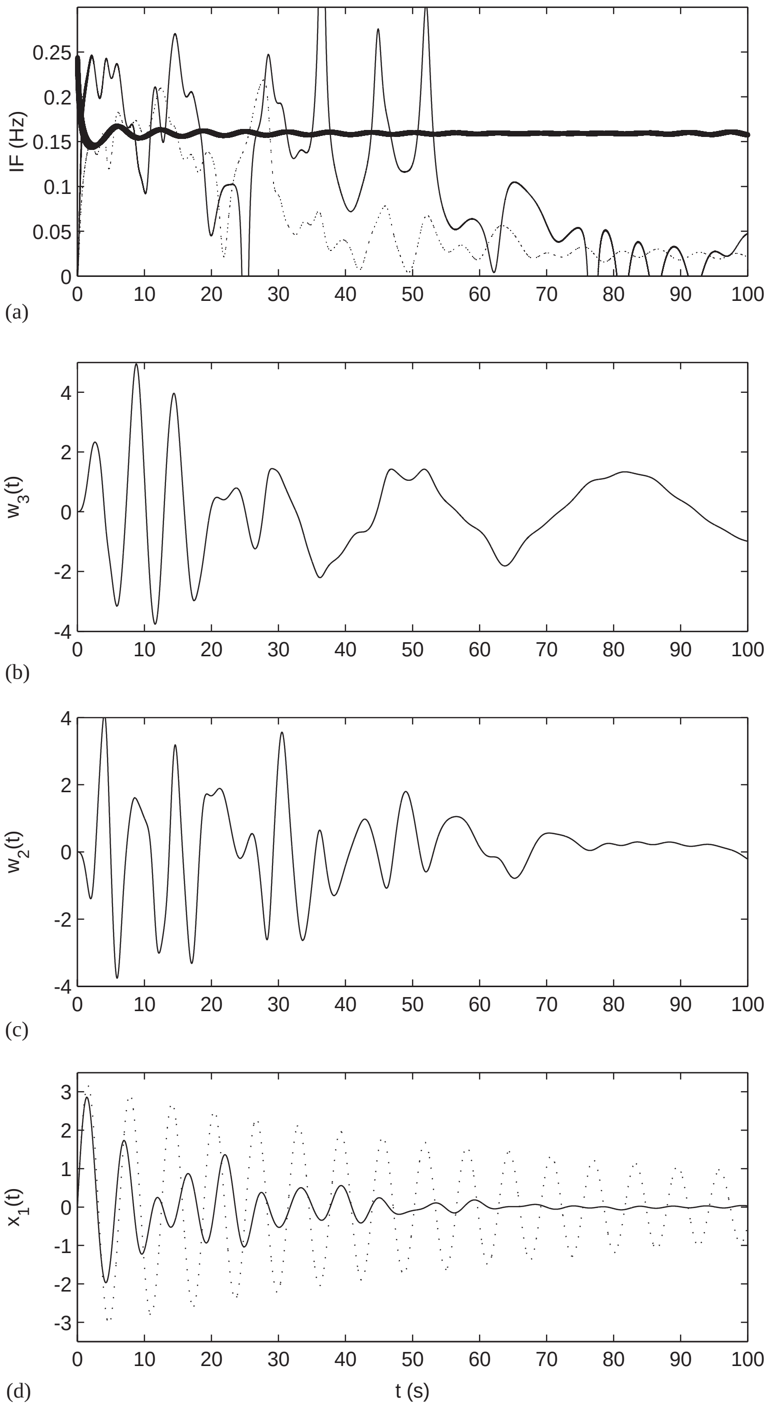

Fig. 7. Resonance capture of the two nonlinear normal modes $w_2(t)$ and $w_3(t)$ with $h = 5.2$. (a) Time–frequency analysis is considered: IF of $w_2(t)$ is denoted by a solid line, IF of $w_3(t)$ is denoted by a thin dotted line and IF of the linear oscillator is denoted by a thick solid line. (b) Resonance of $w_3(t)$. (c) Resonance of $w_2(t)$. (d) Vibrations of the linear oscillator (dotted line denotes the displacement of the linear oscillator without coupling and the solid line denotes the displacement of the linear oscillator with coupling).

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