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Short Communication

Natural frequencies of orthotropic, monoclinic and hexagonal plates by a meshless method

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Abstract

The collocation method with multiquadrics basis functions and a first-order shear deformation theory are used to find natural flexural frequencies of a square plate with various material symmetries and subjected to different boundary conditions. Computed results are found to agree well with the literature values obtained by the solution of the three-dimensional elasticity equations using the finite element method.

1. Introduction

Batra et al. [1] recently used the three-dimensional linear elasticity equations and the finite element method (FEM) to find the first 10 frequencies of free vibration of thick square plates made of orthotropic, trigonal, monoclinic, hexagonal and triclinic materials under different boundary conditions at the edges. The domain of study was divided into a $40 \times 40 \times 4$ mesh of uniform 20-node brick elements with four elements in the thickness direction and the consistent mass matrix was employed. Frequencies computed by the FEM are upper bounds of their

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analytical counterparts. Computed results were found to match well with the analytical solution of Srinivas and Rao [2] for a simply supported orthotropic plate. Batra et al.'s [1] analysis also captured frequencies of in-plane pure distortional modes of vibrations of a simply supported plate. As pointed out by Batra and Aimmanee [3] some of these modes are absent in Srinivas and Rao's solution as well as in several subsequent works [4–7].

Here we use the collocation method with multiquadrics basis functions and the first-order shear deformation theory (FSDT) to find natural frequencies of square plates of various aspect ratios, different material symmetries, and under different boundary conditions at the edges. An advantage of this method over the FEM is that the discretization of the domain into brick elements and the element connectivity are not needed. The present method requires only coordinates of nodes on the midsurface of the plate. Thus the input required for the present meshless method and the effort required to prepare the input are considerably less than that needed for the FEM. Because of the FSDT used, frequencies of very thick plates can not be accurately computed. Furthermore, not all through-the-thickness modes of vibration can be captured. Qian et al. [8,9] employed the meshless local Petrov–Galerkin method (MLPG) to analyze free and forced vibrations of thick homogeneous and functionally graded plates with the higher-order shear and normal deformable plate theory of Batra and Vidoli [10]. The computed frequencies were found to match well with those obtained analytically.

Details of the collocation method with multiquadrics and its application to the analysis of plate problems are given in Refs. [11–17]. The shape parameter, c, in the expression for multiquadrics (e.g., see Eq. (2.3) of Ref. [17]) is set equal to six times the distance between two consecutive nodes, and the shear correction factor is taken to equal $\frac{5}{6}$.

2. Results

A schematic sketch of the problem studied, dimensions of the plate, and the location of the rectangular Cartesian coordinate axes used to describe deformations of the plate are given in Fig. 1. Displacements of a point along the x-, y- and z-axis are denoted by u, v, and w, respectively.

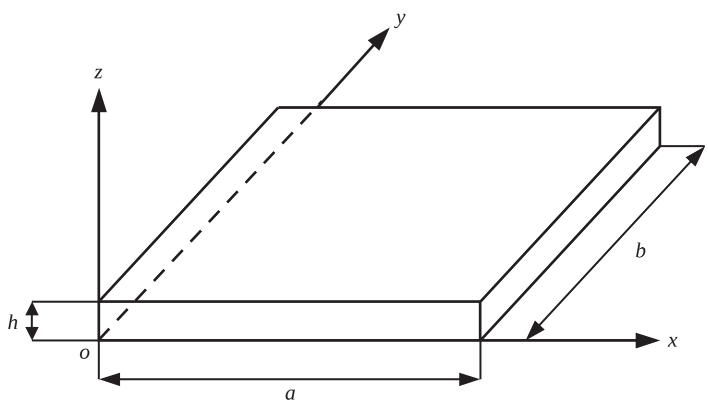

Fig. 1. Schematic sketch of the problem studied.

Table 1
For different aspect ratios, the first 10 non-dimensional natural frequencies of a SSSS orthotropic square plate

N	Method	$h=0.1$	$h=0.2$	$h=0.3$	$h=0.4$	$h=0.5$
1	Batra et al. [1]	0.0477*	0.1721*	0.3407*	0.5304*	0.7295*
		(0.0474)	(0.1694)	(0.3320)	(0.5134)	(0.7034)
	Present 7×7	0.0479	0.1739	0.3434	0.5331	0.7312
	11×11	0.0477	0.1728	0.3414	0.5305	0.7281
	15×15	0.0477	0.1725	0.3410	0.5300	0.7273
2	Batra et al. [1]	0.1021*	0.3221	0.4832	0.6443	0.8054
		(0.1033)	[0.3222]	[0.4833]	[0.6444]	[0.8055]
	Present 7×7	0.1048				
	11×11	0.1033				
	15×15	0.1031				
3	Batra et al. [1]	0.1227*	0.3221	0.4832	0.6443	0.8054
		(0.1188)	[0.3222]	[0.4833]	[0.6444]	[0.8055]
	Present 7×7	0.1258				
	11×11	0.1235				
	15×15	0.1232				
4	Batra et al. [1]	0.1611	0.3372*	0.6198*	0.8666	1.0823
		[0.1611]	(0.3476)		(0.8667)	(1.0824)
	Present 7×7		0.3406	0.6207		
	11×11		0.3391	0.6201		
	15×15		0.3387	0.6195		
5	Batra et al. [1]	0.1611	0.4012*	0.6504	0.9158*	1.2144*
		[0.1611]	(0.3707)	(0.6504)		
	Present 7×7		0.4044		0.9116	1.2031
	11×11		0.4025		0.9119	1.2044
	15×15		0.4018		0.9113	1.2039
6	Batra et al. [1]	0.1721*	0.4338	0.7318*	1.0756*	1.4214*
		[0.1694]	(0.4338)			
	Present 7×7	0.1756		0.7325	1.0714	1.4100
	11×11	0.1732		0.7317	1.0720	1.4122
	15×15	0.1728		0.7310	1.0714	1.4112
7	Batra et al. [1]	0.1828*	0.5304*	0.9324*	1.2886	1.4924
		(0.1888)	(0.5134)			
	Present 7×7	0.1853	0.5349	0.9334		
	11×11	0.1851	0.5315	0.9300		
	15×15	0.1728	0.5305	0.9288		
8	Batra et al. [1]	0.2169	0.5508*	0.9566*	1.2886	1.6107
		(0.2170)				[1.6110]
	Present 7×7		0.5469	0.9396		
	11×11		0.5516	0.9507		
	15×15		0.5517	0.9514		
9	Batra et al. [1]	0.2327*	0.6443	0.9664	1.3409*	1.6107
			[0.6444]	[0.9666]		[1.6110]
	Present 7×7	0.2341			1.3323	
	11×11	0.2339			1.3317	
	15×15	0.2335			1.3303	
10	Batra et al. [1]	0.2459*	0.6443	0.9664	1.3668*	1.7119
		(0.2475)	[0.6444]	[0.9666]		
	Present 7×7	0.2519			1.3345	
	11×11	0.2480			1.3501	
	15×15	0.2473			1.3514	

Exact frequencies from Ref. [2] are listed in parentheses, and those from Ref. [3] in square brackets. Bending frequencies are marked with *.

Boundary conditions for a simply supported (S), clamped (C) and a free (F) edge are given below:

$$
\begin{aligned}
&w = 0,\ M_{xx} = 0,\ M_{xy} = 0 \text{ on a simply supported (S) edge } x = \text{constant},\\
&w = 0,\ M_{yy} = 0,\ M_{yx} = 0 \text{ on a simply supported (S) edge } y = \text{constant},\\
&w = 0,\ \theta_x = 0,\ \theta_y = 0 \text{ on a clamped (C) edge } x = \text{constant or } y = \text{constant},\\
&Q_x = 0,\ M_{xx} = 0,\ M_{xy} = 0 \text{ on a free (F) edge } x = \text{constant},\\
&Q_y = 0,\ M_{yy} = 0,\ M_{yx} = 0 \text{ on a free (F) edge } y = \text{constant}.
\end{aligned} \tag{1}
$$

Here M_{xx}, M_{xy} and Q_x represent, respectively, the normal bending moment, the twisting moment and the shear force on a plate edge $x =$ const.; θ_x and θ_y represent rotations about the y- and the x-axis, respectively.

Values of material parameters used and the non-dimensionalization of frequencies are the same as those in Ref. [1]. Tables 1–9 compare the first 10 frequencies computed by the present method with those given in Ref. [1]. In Table 1 we have listed frequencies computed with the present method by using 7×7, 11×11 and 15×15 collocation points distributed uniformly on the plate's midsurface. Here we consider a unit square plate ($a = b = 1$, see Fig. 1). For $h = 0.1$, 0.2, 0.3, 0.4 and 0.5, the 7×7 collocation points give the first frequency within 4% of its value for the analytical solution. With an increase in the number of collocation points from 7×7 to 11×11 and then to 15×15, the presently computed first frequency approaches its analytical value from above; the maximum difference between the first frequency obtained from the analytical and the numerical solutions equals 3.4% for $h = 0.5$. The FE solution of Batra et al. [1]

Table 2
For different aspect ratios, the first 10 non-dimensional natural frequencies of a SCSC orthotropic square plate

N	Method	$h = 0.1$	$h = 0.2$	$h = 0.3$	$h = 0.4$	$h = 0.5$
1	Batra et al. [1]	0.0614*	0.2041*	0.3800*	0.5699*	0.7666*
	Present	0.0617	0.2040	0.3779	0.5648	0.7575
2	Batra et al. [1]	0.1281*	0.3221	0.4832	0.6443	0.8054
	Present	0.1288				
3	Batra et al. [1]	0.1283*	0.3823*	0.6639*	0.9531*	1.2452*
	Present	0.1289	0.3802	0.6555	0.9359	1.2173
4	Batra et al. [1]	0.1611	0.4096*	0.7394*	1.0089	1.2610
	Present		0.4098	0.7374		
5	Batra et al. [1]	0.1869*	0.5045	0.7568	1.0821*	1.4272*
	Present	0.1875			1.0758	1.4145
6	Batra et al. [1]	0.2138*	0.5493*	0.9486*	1.2886	1.6107
	Present	0.2144	0.5472	0.9402		
7	Batra et al. [1]	0.2351*	0.5904*	0.9664	1.3108	1.6383
	Present	0.2361	0.5846			
8	Batra et al. [1]	0.2522	0.6443	0.9831	1.3551*	1.7623*
	Present				1.3371	1.7318
9	Batra et al. [1]	0.2667*	0.6554	0.9889*	1.3923*	1.7967*
	Present	0.2670		0.9713	1.3592	1.7455
10	Batra et al. [1]	0.2831*	0.6922*	1.0923	1.4559	1.8099*
	Present	0.2840	0.6915			1.8166

Bending frequencies are marked with *.

Table 3
For different aspect ratios, the first 10 non-dimensional natural frequencies of a CCCC orthotropic square plate

N	Method	$h = 0.1$	$h = 0.2$	$h = 0.3$	$h = 0.4$	$h = 0.5$
1	Batra et al. [1]	0.0804*	0.2563*	0.4593*	0.6674*	0.8755*
	Present	0.0808	0.2556	0.4551	0.6573	0.8575
2	Batra et al. [1]	0.1379*	0.4053*	0.6943*	0.9850*	1.2749*
	Present	0.1387	0.4030	0.6846	0.9646	1.2416
3	Batra et al. [1]	0.1650*	0.4770*	0.8097*	1.0886	1.3606
	Present	0.1650	0.4731	0.7972	0.9646	1.2416
4	Batra et al. [1]	0.2120*	0.5442	0.8164	1.1441*	1.4788*
	Present	0.2123			1.1195	1.4395
5	Batra et al. [1]	0.2193*	0.5921*	0.9930*	1.3631	1.7040
	Present	0.2200	0.5870	0.9766	1.1195	1.4395
6	Batra et al. [1]	0.2721	0.6011*	1.0015*	1.3965	1.7937
	Present		0.5951	0.9823	1.3646	
7	Batra et al. [1]	0.2775*	0.6814	1.0222	1.4058*	1.8010*
	Present	0.2766			1.3703	1.7504
8	Batra et al. [1]	0.2830*	0.7178	1.0765	1.4351	1.8121*
	Present	0.2833				1.7563
9	Batra et al. [1]	0.3145*	0.7469*	1.2354*	1.6683*	1.8740*
	Present	0.3140	0.7379	1.2118	1.6801	1.8981
10	Batra et al. [1]	0.3175*	0.7561*	1.2466*	1.7282*	2.2113
	Present	0.3171	0.7478	1.2226	1.6855	

Bending frequencies are marked with *.

Table 4
For different aspect ratios, the first 10 non-dimensional natural frequencies of a SSSS monoclinic square plate

N	Method	$h = 0.1$	$h = 0.2$	$h = 0.3$	$h = 0.4$	$h = 0.5$
1	Batra et al. [1]	0.0527	0.1972	0.4058	0.6545	0.9036
	Present	0.0527	0.1989	0.4107	0.6638	
2	Batra et al. [1]	0.1241	0.3627	0.5439	0.7251	0.9064
	Present	0.1279				
3	Batra et al. [1]	0.1424	0.3628	0.5441	0.7253	0.9299
	Present	0.1434				0.9426
4	Batra et al. [1]	0.1814	0.4441	0.8745	1.2999	1.6280
	Present		0.4574	0.8988	1.3505	
5	Batra et al. [1]	0.1814	0.4780	0.8887	1.3494	1.7819
	Present		0.4838	0.9043	1.3989	1.8062
6	Batra et al. [1]	0.1971	0.6539	0.9979	1.3587	1.7939
	Present	0.1992	0.6651			
7	Batra et al. [1]	0.2423	0.6662	1.0855	1.4467	1.8064
	Present	0.2526				
8	Batra et al. [1]	0.2782	0.7245	1.0865	1.4472	1.8810
	Present	0.2817				1.9280
9	Batra et al. [1]	0.3004	0.7249	1.2129	1.7281	2.1418
	Present	0.3079		1.2393		
10	Batra et al. [1]	0.3211	0.8124	1.3003	1.8056	2.2511
	Present	0.3247	0.8403		1.8566	

Bending frequencies are marked with *.

Table 5
For different aspect ratios, the first 10 non-dimensional natural frequencies of a CCCC monoclinic square plate

N	Method	$h = 0.1$	$h = 0.2$	$h = 0.3$	$h = 0.4$	$h = 0.5$
1	Batra et al. [1]	0.0993*	0.3382*	0.6405*	0.9694*	1.3091*
	Present	0.1012	0.3435	0.6481	0.9771	1.3150
2	Batra et al. [1]	0.1835*	0.6012*	1.0465*	1.4010	1.7518
	Present	0.1894	0.6059	1.0519		
3	Batra et al. [1]	0.2005*	0.6061*	1.0501	1.5028*	1.9593*
	Present	0.2025	0.6185		1.5079	1.9656
4	Batra et al. [1]	0.2633*	0.6994	1.1119*	1.6295*	2.0888
	Present	0.2680		1.1274	1.6480	
5	Batra et al. [1]	0.3133*	0.8000*	1.2602	1.6766	2.1274*
	Present	0.3240	0.8105			2.1529
6	Batra et al. [1]	0.3393*	0.8408	1.3865	1.8479	2.3086
	Present	0.3424				
7	Batra et al. [1]	0.3492	0.9244	1.4035*	2.0116*	2.6005*
	Present			1.4185	2.0329	2.6346
8	Batra et al. [1]	0.3746*	0.9336*	1.5608*	2.1029	2.6146
	Present	0.3839	0.9395	1.5702		
9	Batra et al. [1]	0.3875*	0.9751*	1.5828	2.1942*	2.8229*
	Present	0.3923	0.9941		2.2110	2.8558
10	Batra et al. [1]	0.4210	1.0557*	1.7128*	2.4169*	2.8245*
	Present		1.0937	1.7413	2.4895	3.0409

Bending frequencies are marked with *.

Table 6
For different aspect ratios, the first 10 non-dimensional natural frequencies of a SCSC monoclinic square plate

N	Method	$h = 0.1$	$h = 0.2$	$h = 0.3$	$h = 0.4$	$h = 0.5$
1	Batra et al. [1]	0.0830*	0.2779*	0.5185*	0.7253	0.9064
	Present	0.0835	0.2803	0.5230		
2	Batra et al. [1]	0.1397*	0.3628	0.5441	0.7796*	1.0536*
	Present	0.1437			0.7860	1.0618
3	Batra et al. [1]	0.1814	0.4780*	0.9150	1.3798	1.7197*
	Present		0.4918			1.8354
4	Batra et al. [1]	0.1924*	0.5706*	0.9836*	1.4014*	1.8064
	Present	0.1936	0.5732	0.9408	1.4073	
5	Batra et al. [1]	0.2345*	0.6948	1.0406*	1.4099*	1.8363*
	Present	0.2370		0.9861	1.4406	1.9675
6	Batra et al. [1]	0.2504*	0.7131*	1.0865	1.4472	1.9126*
	Present	0.2609	0.7226			2.5044
7	Batra et al. [1]	0.3246*	0.7249	1.2285	1.6341	2.0353
	Present	0.3368				
8	Batra et al. [1]	0.3343*	0.8197	1.2669*	1.8527*	2.3649
	Present	0.3368		1.2861	1.8854	
9	Batra et al. [1]	0.3478	0.8273*	1.4279	1.8995	2.4506*
	Present		0.8562			2.7643
10	Batra et al. [1]	0.3626	0.9132*	1.5261*	2.0471	2.5429*
	Present		0.9195	1.5288		2.9318

Bending frequencies are marked with *.

Table 7
For different aspect ratios, the first 10 non-dimensional natural frequencies of a SSSS hexagonal square plate

N	Method	$h=0.1$	$h=0.2$	$h=0.3$	$h=0.4$	$h=0.5$
1	Batra et al. [1]	0.0555*	0.2076*	0.4264*	0.6857*	0.9681*
	Present	0.0552	0.2080	0.4285	0.6907	0.9776
2	Batra et al. [1]	0.1340*	0.4230	0.6343	0.8453	1.0558
	Present	0.1345				
3	Batra et al. [1]	0.1340*	0.4230	0.6343	0.8453	1.0558
	Present	0.1345				
4	Batra et al. [1]	0.2076*	0.4662*	0.8940*	1.1935	1.4898
	Present	0.2083	0.4696	0.9035		
5	Batra et al. [1]	0.2116	0.4662*	0.8940*	1.3599*	1.8379*
	Present		0.4696	0.9036	1.3804	1.8756
6	Batra et al. [1]	0.2116	0.5979	0.8961	1.3599*	1.8379*
	Present				1.3804	1.8757
7	Batra et al. [1]	0.2543*	0.6855*	1.2624*	1.6834	2.0983
	Present	0.2562	0.6917	1.2807		
8	Batra et al. [1]	0.2543*	0.8165*	1.2654	1.6834	2.0983
	Present	0.2563	0.8253			
9	Batra et al. [1]	0.2991	0.8165*	1.2654	1.7656	2.2003
	Present		0.8256			
10	Batra et al. [1]	0.3214*	0.8449	1.3273	1.8659*	2.3407
	Present	0.3236			1.9059	

Bending frequencies are marked with *.

Table 8
For different aspect ratios, the first 10 non-dimensional natural frequencies of a CCCC hexagonal square plate

N	Method	$h=0.1$	$h=0.2$	$h=0.3$	$h=0.4$	$h=0.5$
1	Batra et al. [1]	0.0968*	0.3325*	0.6305*	0.9510*	1.2778*
	Present	0.0970	0.3330	0.6304	0.9494	1.2741
2	Batra et al. [1]	0.1878*	0.5960*	1.0639*	1.4856	1.8530
	Present	0.1887	0.5970	1.0649		
3	Batra et al. [1]	0.1878*	0.5960*	1.0639*	1.4856	1.8530
	Present	0.1887	0.5970	1.0649		
4	Batra et al. [1]	0.2660*	0.7449	1.1160	1.5370*	1.9980*
	Present	0.2677			1.5386	2.0027
5	Batra et al. [1]	0.3157*	0.7449	1.1160	1.5370*	1.9980*
	Present	0.3169			1.5386	2.0027
6	Batra et al. [1]	0.3183*	0.8081*	1.4089*	1.9088	2.3772
	Present	0.3196	0.8111	1.4154		
7	Batra et al. [1]	0.3727	0.9280*	1.4357	2.0105*	2.5981*
	Present		0.9300		2.0252	2.6287
8	Batra et al. [1]	0.3727	0.9405*	1.5847*	2.1974	2.7334
	Present		0.9427	1.5911		
9	Batra et al. [1]	0.3840*	0.9591	1.6125*	2.2343*	2.8658*
	Present	0.3864		1.6189	2.2528	2.9085
10	Batra et al. [1]	0.3840*	1.1045*	1.6542	2.2810*	2.9352*
	Present	0.3864	1.1105		2.2988	2.9759

Bending frequencies are marked with *.

Table 9
For different aspect ratios, the first 10 non-dimensional natural frequencies of a SCSC hexagonal square plate

N	Method	$h=0.1$	$h=0.2$	$h=0.3$	$h=0.4$	$h=0.5$
1	Batra et al. [1]	0.0789*	0.2768*	0.5343*	0.8178*	1.0558
	Present	0.0791	0.2772	0.5349	0.8186	
2	Batra et al. [1]	0.1459*	0.4230	0.6343	0.8453	1.1128*
	Present	0.1468			1.1142	
3	Batra et al. [1]	0.1788*	0.4941*	0.9298*	1.3984*	1.7787
	Present	0.1793	0.4975	0.9384	1.4162	
4	Batra et al. [1]	0.2116	0.5717*	1.0261*	1.4258*	1.8310
	Present		0.5724	1.0273	1.4942	
5	Batra et al. [1]	0.2386*	0.7148	1.0710	1.4679*	1.8767*
	Present	0.2399			1.9603	1.9095
6	Batra et al. [1]	0.2607*	0.7360	1.1027	1.4911*	1.9513*
	Present	0.2631			2.2171	1.9597
7	Batra et al. [1]	0.3118*	0.7475*	1.2654*	1.6834	2.0983
	Present	0.3126	0.7519	1.3446		
8	Batra et al. [1]	0.3412*	0.8285*	1.3326*	1.9334*	2.4603
	Present	0.3442	0.8375	1.5124	2.2535	
9	Batra et al. [1]	0.3576	0.8449	1.4865	1.9760	2.5297*
	Present				2.5794	
10	Batra et al. [1]	0.3664*	0.9221*	1.4883*	2.1022	2.5356
	Present	0.3682	0.9239	1.5877		

Bending frequencies are marked with *.

has an error of 3.7% in the first frequency for $h=0.5$. Whereas for $h=0.2$, 0.3, 0.4 and 0.5, the three-dimensional analysis by the FEM can capture the second mode of vibration corresponding to pure distortional deformations, the present method misses it. The present collocation method does not give frequencies of any of the pure distortional modes. This is because the in-plane displacement components that are uniform through the plate thickness have been neglected. Vel and Batra [18] considered these and thus captured some of the pure distortional modes of vibration. However, the present method computes reasonably accurately frequencies of the first few flexural modes of vibration of a simply supported orthotropic square plate. Whereas a frequency computed with the FE solution of 3-D elasticity equations is an upper bound for the corresponding frequency from the analytical solution, this need not be the case for the frequenies obtained from the present method. Qian et al. [8] also found that a frequency computed with the MLPG method did not necessarily exceed that obtained from the analytical solution. Batra and Aimmanee [19] have used the FEM and the mixed higher-order shear and normal deformable plate theory [7,10] to analyze vibrations of a thick isotropic plate.

In Tables 2–9, in order to save space, frequencies computed only by using 15×15 collocation points are listed.

3. Conclusions

It is shown that the collocation method with multiquadratics basis function and the first-order shear deformation theory can successfully compute flexural modes of vibration of orthotropic, monoclinic, and hexagonal plates. Computational effort required with this approach is considerably less than that needed with the analysis of the three-dimensional elasticity equations by the finite element method. The present method is truly meshless and computationally less expensive than the meshless local Petrov–Galerkin (MLPG) method employed by Qian et al. [8,9].

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