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Simplified models and experimental verification for coupled transmission tower–line system to seismic excitations

Hong-Nan Li[a,b,*], Wen-Long Shi[b], Guo-Xin Wang[a], Lian-Guang Jia[b]

[a]*Department of Civil Engineering, Dalian University of Technology, 2 Linggong Road, Ganjingzi District, Dalian 116024, PR China*
[b]*Department of Civil Engineering, Shenyang Architectural and Civil Engineering Institute, Shenyang 110015, PR China*

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Abstract

The simplified computational models of high-voltage transmission tower–line system under out-of-plane and in-plane vibrations are presented due to seismic excitations in this paper. The equations of motion are derived and the computer program is compiled to obtain the earthquake responses of the coupled system. To verify the rationality of the proposed approaches, the shaking-table experiments of the coupled system of transmission lines and their supporting towers are carried out and the results indicate that the errors of theoretical and testing results of systemic seismic responses are within the acceptable arrange in engineering area. Based on these studies, a simplified analysis method is proposed to make the seismic response calculation of coupled tower–conductor system faster and more effective.

1. Introduction

As one of the most important components in electrical transmission systems in the electrical engineering, the high-voltage transmission tower is a kind of significant lifeline structure. Until now, the most research attentions on it have been paid on the actions of static load, impulsive load, equivalent static wind load, etc. [1–4] and only a few of them have dealt with dynamic load.

*Corresponding author. Department of Civil Engineering, Dalian University of Technology, 2 Linggong Road, Ganjingzi District, Dalian 116024, PR China. Tel.: +86 411 4708504; fax: +86 411 4708501.
E-mail address: hnli@dlut.edu.cn (H.-N. Li).

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"The Design Regulations on 110–500 kV Overhead Transmission Line "(DL/T 5092-1999)" [5] and other relevant codes do not provide the approaches on how to consider the effects of lines on aseismic analysis of transmission tower system. Of course, it may not be necessary to consider these effects on the case of dynamic responses of short-span transmission tower system. However, the mass of conductors may have considerable effects on the dynamic responses of long-span transmission towers.

In the past one or two decades, researchers have done some dynamic analyses and experiments on the transmission tower system. Qu and Miao [6] obtained the seismic response of towers with a height of 45 and 70 m by using the spectral method and time-history method, respectively, without considering the actions of cables. Ozono et al. [7,8] suggested two simplified models to investigate the characteristics of coupled tower–line system on the basis of experimental results, in which the conductor was simplified as no-mass spring. And the rationality and feasibility of the models were discussed in theory. Yet, they did not do further research on how to simplify the lines into no-mass springs and how to determine the spring rigidity. Also, it was unknown if the calculated frequencies and modes coincided with the actual ones. Kempner et al. [9,10] analyzed the vibrational properties and roles of insulators in the dynamic responses of high-voltage transmission tower system by experimental and theoretical analyses. After studying the interaction between insulators and conductors, Long [11] concluded that the effects of conductors might not be considered in the seismic response analysis of transmission towers because the acting force on towers from conductor vibration is much less than that of static loads, in which the line span of the coupled system they studied was not large. However, some other researchers had quite different suggestions on these conclusions. Hu and Ma [12] utilized three kinds of models, including truss element, cable element and pre-stressed truss element, to study the three-dimensional dynamic characteristics of the coupled tower–conductor system and suggested that the effects of lines and insulators on the vibrations of transmission towers should not be neglected.

Noteworthy contributions to the related study of transmission towers include some work that have developed effective approaches to deal with the actual problems. Ghobarah et al. [13] investigated the effect of multiple support excitations on the responses of overhead power transmission lines and modeled the transmission towers by space truss elements and the cable by straight two node elements, in which the system was subjected to spatially incoherent seismic ground motions. Deng et al. [14] carried out an experimental study on modeling Jiangyin long-span transmission tower to assess the safety of the towers. McClure and Lapointe [15] put the emphasis on capturing the salient features of the propagation of shock loads in a line section, such as those induced by the sudden failure of components or sudden ice-shedding effects on the conductors. A number of results on reducing the vibration of transmission system by setting TMD, MTMD and VED on the structures were also developed well recently [16].

It has been known from the above state-of-art review that only a few researches are related with the seismic problems of transmission tower–line system and have further shown that studies on dynamic performances of tower structures under the excitation of earthquakes are very necessary. The authors of this paper have completed a number of research works on seismic problems of a coupled system of long-span transmission towers [17–21]. This paper firstly puts forward generally simplified computational models and derives the equations of motion for the coupled system of transmission tower–conductors. In order to examine the validity of the presented theoretical models and calculating method, the shaking-table tests of the coupled system of transmission lines and their supporting towers are carried out especially, and the results indicate that the errors between theoretical and testing results of systemic seismic responses are within the acceptable arrange in engineering area. Based on these studies, a simplified analysis approach is proposed to make the seismic response calculation of coupled tower–conductor system faster and more effective. Finally, some significant conclusions are obtained.

2. Computational models and their equations of motion

2.1. Out-of-plane vibration

Based on Refs. [18,19], the transmission lines and their supporting towers can be modeled as the lumped mass system. Thus, one-span conductor in the case of out-of-plane vibration could be taken as catenaries represented by a cluster of masses. And the simplified computational model of the elastic–gravity coupled vibration system composed of transmission conductors and their supporting towers under out-of-plane and in-plane seismic excitations, respectively, can be further developed and shown in Fig. 1.

Through some derivations, the equation of motion for the coupled system of transmission conductors and their supporting towers to the out-of-plane seismic excitation (shown in Fig. 1(b)) could be obtained as follows (without the damping case):

$$[M]_{(h)}\{\ddot{x}\}_{(h)} + [K]_{(h)}\{x\}_{(h)} = -[M]_{(h)}\{E\}\ddot{x}_g(t) \tag{1}$$

where $\{x\}_{(h)}$ is the displacement vector of system, $\{E\}$ means the unit vector, $\ddot{x}_g(t)$ implies the lateral ground acceleration, $[M]_{(h)}$ and $[K]_{(h)}$ are the mass and rigidity matrices of system, respectively, in which the mass matrix is shown as

$$[M]_{(h)} = [[M]_{\text{line}} \ \ [M]_{\text{tower}}]^{\text{T}}_{(n\times N_x+N_y)\times(n\times N_x+N_y)} \tag{2}$$

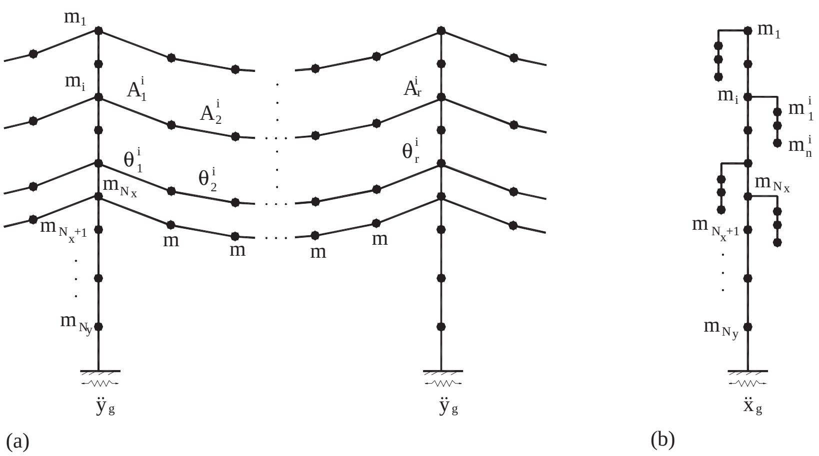

Fig. 1. Computational model of transmission tower–line system. (a) In-plane, (b) out-of-plane.

where

$$[M]_{\text{line}} = \text{diag}[[M]^1_{\text{line}}, [M]^2_{\text{line}}, \ldots, [M]^{N_x}_{\text{line}}]_{(n\times N_x)\times(n\times N_x)}$$

$$[M]_{\text{tower}} = \text{diag}[m_1, m_2, \ldots, m_{N_y}]_{N_y\times N_y}$$

and the rigidity matrix is expressed as

$$[K]_{(h)} = \begin{bmatrix} [K]_{\text{line}} & [K]_{\text{coupling}} \\ [K]^{\text{T}}_{\text{coupling}} & [K]_{\text{tower}} \end{bmatrix} \tag{3}$$

where

$$[K]_{\text{line}} = \text{diag}[[K]^1_{\text{line}}, [K]^2_{\text{line}}, \ldots, [K]^{N_x}_{\text{line}}]_{(n\times N_x)\times(n\times N_x)}$$

$$[K]_{\text{coupling}} = \begin{bmatrix} [K]^1_{\text{line}} & 0 & 0 & 0 & \vdots & 0 \\ 0 & [K]^2_{\text{line}} & 0 & 0 & \vdots & 0 \\ 0 & 0 & \ddots & 0 & \vdots & 0 \\ 0 & 0 & 0 & [K]^{N_x}_{\text{line}} & \vdots & 0 \end{bmatrix}_{(n+N_x)\times N_y}$$

$$[K]_{\text{tower}} = \begin{bmatrix} k'_{11} & k_{12} & \cdots & k_{1N_x} & k_{1(N_x+1)} & \cdots & k_{1N_y} \\ k_{21} & k'_{22} & \cdots & k_{2N_x} & k_{2(N_x+1)} & \cdots & k_{2N_y} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ k_{N_x1} & k_{N_x2} & \cdots & k'_{N_xN_x} & k_{N_x(N_x+1)} & \cdots & k_{N_xN_y} \\ k_{(N_x+1)1} & k_{(N_x+1)2} & \cdots & k_{(N_x+1)N_x-+} & k'_{(N_x+1)(N_x+1)} & \cdots & k_{(N_x+1)N_y} \\ \cdots & \cdots & \cdots & \cdots & \cdots & \cdots & \cdots \\ k_{N_y1} & k_{N_y2} & \cdots & k_{N_yN_x} & k_{N_y(N_x+1)} & \cdots & k'_{N_yN_y} \end{bmatrix}$$

where N_x is the layer number of lines and N_y represents the lumped mass number of simplified tower shelf. Thus, two cases for the mass and stiffness matrices in Eq. (1) are discussed below.

Case 1: If each conductor is represented by $2n$ (even) lumped masses with the same mass, m, then

$$[M]^1_{\text{line}} = [M]^2_{\text{line}} = \cdots = [M]^{N_x}_{\text{line}} = \text{diag}[2m, 2m, \ldots, 2m]_{(n\times n)}$$

$$[K]^1_{\text{line}} = [K]^2_{\text{line}} = \cdots = [K]^{N_x}_{\text{line}} = \left[0, \ldots, 0, -\frac{2nmg}{h_1}\right]^{\text{T}}_{n\times 1}$$

$$[K]^1_{\text{line}} = [K]^2_{\text{line}} = \cdots = [K]^{N_x}_{\text{line}} = g\begin{bmatrix} \frac{2m}{2^{1-n}h_1} & -\frac{2m}{2^{1-n}h_1} & 0 & 0 & 0 \\ -\frac{2m}{2^{1-n}h_1} & \frac{2m}{2^{1-n}h_1}+\frac{4m}{2^{2-n}h_1} & -\frac{4m}{2^{2-n}h_1} & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\frac{2(i-1)m}{2^{i-1-n}h_1} & \frac{2(i-1)m}{2^{i-1-n}h_1}+\frac{2im}{2^{i-n}h_1} & -\frac{2im}{2^{i-n}h_1} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & -\frac{2(n-2)m}{2^{-2}h_1} & \frac{2(n-2)m}{2^{-2}h_1}+\frac{2(n-1)m}{2^{-1}h_1} & -\frac{2(n-1)m}{2^{-1}h_1} \\ 0 & 0 & 0 & -\frac{2(n-1)m}{2^{-1}h_1} & \frac{2(n-1)m}{2^{-1}h_1}+\frac{2nm}{h_1} \end{bmatrix}$$

$$k'_{ii} = k_{ii} + \frac{2nmg}{h_1}$$

where h_1 denotes the distance from the upper mass of each conductor to the suspended chain end and k_{ii} is the rigidity factor of the tower.

Case 2: If each conductor is treated as $2n-1$ (odd) lumped masses with the same mass, m, then

$$[M]^1_{\text{line}} = [M]^2_{\text{line}} = \cdots = [M]^{N_x}_{\text{line}} = \text{diag}[m, 2m, \ldots, 2m]_{(n\times n)}$$

$$[K]^1_{\text{tower}} = [K]^2_{\text{tower}} = \cdots = [K]^{N_x}_{\text{tower}} = \left[0, \ldots, 0, -\frac{(2n-1)m}{h_1}\right]^{\mathrm{T}}_{(n\times 1)}$$

$$[K]^1_{\text{line}} = [K]^2_{\text{line}} = \cdots = [K]^{N_x}_{\text{line}} = g\begin{bmatrix} \frac{m}{2^{1-n}h_1} & -\frac{m}{2^{1-n}h_1} & 0 & 0 & 0 \\ -\frac{m}{2^{1-n}h_1} & \frac{m}{2^{1-n}h_1}+\frac{3m}{2^{2-n}h_1} & -\frac{3m}{2^{2-n}h_1} & 0 & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & -\frac{(2i-3)m}{2^{i-1-n}h_1} & \frac{(2i-3)m}{2^{i-1-n}h_1}+\frac{(2i-1)m}{2^{i-n}h_1} & \frac{(2i-1)m}{2^{i-n}h_1} & 0 \\ \cdots & \cdots & \cdots & \cdots & \cdots \\ 0 & 0 & -\frac{(2n-5)m}{2^{-2}h_1} & \frac{(2n-5)m}{2^{-2}h_1}+\frac{(2n-3)m}{2^{-1}h_1} & -\frac{(2n-3)m}{2^{-1}h_1} \\ 0 & 0 & 0 & -\frac{(2n-3)m}{2^{-1}h_1} & \frac{(2n-3)m}{2^{-1}h_1}+\frac{(2n-1)m}{h_1} \end{bmatrix}$$

$$k'_{ii} = k_{ii} + \frac{(2n-1)mg}{h_1}.$$

2.2. In-plane vibration

On the other hand, the equation of motion for the coupled system of transmission conductors and their supporting towers to the in-plane vibration seismic excitation (shown in Fig. 1(a)) is derived as follows (without the damping case):

$$[M]_{(z)}\{\ddot{x}\}_{(z)} + [K]_{(z)}\{x\}_{(z)} = -[M]_{(z)}\{E\}\ddot{y}_g(t) \tag{4}$$

in which $\{x\}_{(z)}$ implies the displacement vector of system, $\{E\}$ is the unit vector, $\ddot{y}_g(t)$ denotes the longitudinal ground acceleration, $[M]_{(z)}$ and $[K]_{(z)}$ are the mass and rigidity matrices of system. Now, there are also two cases to be discussed below for the displacement vector, mass and stiffness matrices in Eq. (4).

Case 1: If each conductor could be simplified as $2n$ (even) masses with the mass, m, then

$$\{x\}_{(z)} = [\theta_2^1, \ldots, \theta_{2n}^1, \ldots, \theta_2^{N_x}, \ldots, \theta_{2n}^{N_x}, u_1, u_2, \ldots, u_{N_y}]^{\mathrm{T}}_{[(2n-1)\times N_x+N_y]\times 1} \tag{5}$$

where θ_k^i is the kth element angular displacement of conductor in the ith layer line, u_i represents the in-plane displacement of the tower at the ith mass and the mass matrix is given by

$$[M]_{(z)} = \begin{bmatrix} [M]_{\text{line}} & [M]_{\text{couple}} \\ [M]^{\mathrm{T}}_{\text{couple}} & [M]_{\text{tower}} \end{bmatrix}_{[(2n-1)\times N_x+N_y]\times[(2n-1)\times N_x+N_y]} \tag{6}$$

where

$$[M]_{\text{line}} = \text{diag}[[M]^1_{\text{line}}, [M]^2_{\text{line}}, \ldots, [M]^{N_x}_{\text{line}}]_{[(2n-1)\times N_x]\times[(2n-1)\times N_x]}$$

$$[M]_{\text{tower}} = \text{diag}[2nm + m_1, \ldots, 2nm + m_{N_x}, m_{N_x+1}, \ldots, m_{N_y}]_{(N_y\times N_y)}$$

$$[M]_{\text{couple}} = \begin{bmatrix} [M]^1_{t\text{couple}} & 0 & 0 & 0 & 0 \\ 0 & [M]^2_{\text{couple}} & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & [M]^{N_x}_{t\text{couple}} & 0 \end{bmatrix}_{[(2n-1)\times N_x]\times N_y}$$

in which,

$$[M]^1_{\text{line}} = [M]^2_{\text{line}} = \cdots = [M]^{N_x}_{\text{line}} = (M_{ij})_{(2n-1)\times(2n-1)}$$

$$M_{ii} = c_1 \times [(i-1)a_i^2 + (2n+1-i)a^2_{2n+2-i}] \times m$$

$$M_{ij} = \begin{cases} m[c_1(i-1)a_ia_j + c_2(j-i)a_{2n+2-i}a_j + c_1(2n+1-j)a_{2n+2-i}a_{2n+2-j}] & (j>i) \\ M_{ji} & (j<i) \end{cases}$$

$$c_1 = \frac{1}{4}(h_1^2 + l^2), \quad c_2 = \frac{1}{4}(h_1^2 - l^2)$$
$$a_i = 1 + 2^{1-i}, \qquad a_{n+1} = 1, \quad a_{n+i} = 1 - 2^{i-1-n}$$

where $i,j = 1, 2, \ldots, 2n$ and l is the horizontal distance between the lumped masses of the conductors.

$$[M]^1_{\text{couple}} = [M]^2_{\text{couple}} = \cdots = [M]^{N_x}_{\text{couple}} = [M^2_{\text{couple}}, M^3_{\text{couple}}, \ldots, M^{2n}_{\text{couple}}]^{\mathrm{T}}_{(2n-1)\times 1}$$

$$M^i_{\text{couple}} = \frac{m}{2}[(i-1)a_i + (2n+1-i)a_{2n+2-i}] \quad (i = 2, 3, \ldots, 2n)$$

The rigidity matrix in Eq. (4) is given by

$$[K]_{(z)} = \begin{bmatrix} [K]_{\text{line}} & [0] \\ [0]^{\mathrm{T}} & [K]_{\text{tower}} \end{bmatrix}_{[(2n-1)\times N_x+N_y]\times[(2n-1)\times N_x+N_y]} \tag{7}$$

where

$$[K]_{\text{line}} = \text{diag}[[K]^1_{\text{line}}, [K]^2_{\text{line}}, \ldots, [K]^{N_x}_{\text{line}}]_{[(2n-1)\times N_x]\times[(2n-1)\times N_x]}$$

$$[K]^1_{\text{line}} = [K]^2_{\text{line}} = \cdots [K]^{N_x}_{\text{line}} = (k_{ij})_{(2n-1)\times(2n-1)}$$

$$k_{ii} = \begin{cases} \frac{1}{4}nmgh_1(a_i^2 + a_{2n+2-i}^2) + 2mgh_1(n+1-i)2^{-i} & (i = 2, 3, \ldots, n+1) \\ \frac{1}{4}nmgh_1(a_i^2 + a_{2n+2-i}^2) + 2mgh_1(i-n-1)2^{i-2n-2} & (i = n+2, \ldots, 2n) \end{cases}$$

$$k_{ij} = \frac{1}{4}nmgh_1(a_i a_j + a_{2n+2-i}a_{2n+2-j}) \quad (i,j = 2, 3, \ldots, 2n; i \neq j).$$

The $[K]_{\text{tower}}$ in Eq. (7) is the rigidity matrix of tower, i.e., $[K]_{\text{tower}} = (k_{ij})_{(N_y\times N_y)}$.

Case 2: Also if each conductor could be modeled as $2n$ (even) masses with the mass, m, Eq. (5) is replaced by

$$\{x\}_{(z)} = [\theta^1_2, \ldots, \theta^1_{2n}, \ldots, \theta^{N_x}_2, \ldots, \theta^{N_x}_{2n}, u_1, u_2, \ldots, u_{N_y}]^{\mathrm{T}}_{[(2n-2)\times N_x+N_y]\times 1}. \tag{8}$$

And

$$[M]_{(z)} = \begin{bmatrix} [M]_{\text{line}} & [M]_{\text{couple}} \\ [M]^{\mathrm{T}}_{\text{couple}} & [M]_{\text{tower}} \end{bmatrix}_{[(2n-2)\times N_x+N_y]\times[(2n-2)\times N_x+N_y]} \tag{9}$$

where

$$[M]_{\text{line}} = \text{diag}[[M]^1_{\text{line}}, [M]^2_{\text{line}}, \ldots, [M]^{N_x}_{\text{line}}]_{[(2n-2)\times N_x]\times[(2n-2)\times N_x]}$$

$$[M]_{\text{tower}} = \text{diag}[(2n-1)m + m_1, \ldots, (2n-1)m + m_{N_x}, m_{N_x+1}, \ldots, m_{N_y}]_{(N_y\times N_y)}$$

$$[M]_{\text{couple}} = \begin{bmatrix} [M]^1_{\text{couple}} & 0 & 0 & 0 & 0 \\ 0 & [M]^2_{\text{couple}} & 0 & 0 & 0 \\ 0 & 0 & \ddots & 0 & 0 \\ 0 & 0 & 0 & [M]^{N_x}_{\text{couple}} & 0 \end{bmatrix}_{[(2n-2)\times N_x]\times N_y}$$

where

$$[M]^1_{\text{line}} = [M]^2_{\text{line}} = \cdots = [M]^{N_x}_{\text{line}} = (M_{ij})_{(2n-2)\times(2n-2)}$$

$$M_{ii} = c_1[(i-1)a_i^2 + (2n-i)a^2_{2n+1-i}]m$$

$$M_{ij} = \begin{cases} m[c_1(i-1)a_ia_j + c_2(j-i)a_{2n+1-i}a_j \\ \quad +c_1(2n-j)a_{2n+1-i}a_{2n+1-j}] & (j>i) \\ M_{ji} & (j<i) \end{cases} \quad (i,\ j = 2,3,\ldots 2n-1)$$

$$[M]^1_{\text{couple}} = [M]^2_{\text{couple}} = \cdots = [M]^{N_x}_{\text{couple}} = [M^2_{\text{couple}}, M^3_{\text{couple}}, \ldots, M^{2n-1}_{\text{couple}}]^{\mathrm{T}}_{(2n-2)\times 1}$$

$$M^i_{\text{couple}} = \frac{m}{2}[(i-1)a_i + (2n-i)a_{2n+1-i}] \quad (i = 2,3,\ldots,2n-1).$$

The rigidity matrix in Eq. (4) is defined as

$$[K]_{(z)} = \begin{bmatrix} [K]_{\text{line}} & [0] \\ [0]^{\mathrm{T}} & [K]_{\text{tower}} \end{bmatrix}_{[(2n-2)N_x+N_y]\times[(2n-2)N_x+N_y]} \tag{10}$$

where

$$[K]_{\text{tower}} = (k_{ij})_{(N_y\times N_y)}$$
$$[K]_{\text{line}} = \text{diag}[[K]^1_{\text{line}}, [K]^2_{\text{line}}, \ldots, [K]^{N_x}_{\text{line}}]_{[(2n-2)N_x]\times[(2n-2)N_x]}$$
$$[K]^1_{\text{line}} = [K]^2_{\text{line}} = \cdots [K]^{N_x}_{\text{line}} = (k_{ij})_{(2n-2)\times(2n-2)}$$

in which

$$k_{ii} = \begin{cases} \frac{1}{4}nmgh_1(a_i^2 + a^2_{2n+1-i}) + 2mgh_1(n+1-i)2^{-i} & (i<n) \\ \frac{1}{4}nmgh_1(a_i^2 + a^2_{2n+1-i}) + 2mgh_1(i-n-1)2^{i-2n-2} & (i\geqslant n) \end{cases}$$

$$k_{ij} = \frac{1}{4}nmgh_1(a_ia_j + a_{2n+1-i}a_{2n+1-j}) \quad (i,j = 2,3,\ldots,2n-1; i\neq j).$$

3. State equation of system

The time-history method with Rayleigh damping is adopted to calculate the seismic responses of the transmission tower system by compiling the program based on the Matlab.

If the damping is taken into account in the equation of motion of the system, Eqs. (1) and (4) could be written in a uniform manner

$$[M]\{\ddot{x}\} + [C]\{\dot{x}\} + [K]\{x\} = -[M]\{E\}\ddot{a}_g \tag{11}$$

where $[C]$ is the damping matrix of system and $\ddot{a}_g$ means the out-of-plane or in-plane ground accelerations.

Eq. (11) can be rewritten as

$$\{\ddot{x}\} = -[M]^{-1}[C]\{\dot{x}\} - [M]^{-1}[K]\{x\} - [M]^{-1}[M]\{E\}\ddot{a}_g. \tag{12}$$

Let $\{z\} = \left\{ \begin{smallmatrix} x \\ \dot{x} \end{smallmatrix} \right\}$, then

$$\{\dot{z}\} = \begin{Bmatrix} \dot{x} \\ \ddot{x} \end{Bmatrix} = \begin{bmatrix} 0 & E \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix} \begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} + \begin{Bmatrix} 0 \\ -\{E\} \end{Bmatrix} \ddot{a}_g \tag{13}$$

i.e.

$$\{\dot{z}\} = [A_1]\{z\} + [B_1]\{v\} = [A_1]\begin{Bmatrix} x \\ \dot{x} \end{Bmatrix} + [B_1]\ddot{a}_g \tag{14}$$

where

$$[A_1] = \begin{bmatrix} 0 & E \\ -[M]^{-1}[K] & -[M]^{-1}[C] \end{bmatrix}$$

$$[B_1] = \begin{Bmatrix} 0 \\ -\{E\} \end{Bmatrix}.$$

4. Model experiments

To verify the rationality of the computational model presented above, the model tests are carried out and their results are compared with the numerical results from the above theoretical approaches. The setup of the testing model is shown in Fig. 2 and its photograph is given in Fig. 3.

Due to the limitation of shaking-table size (6 m × 6 m), the small-scaled model was designed for the coupled tower–line system so that it can be installed on the table, in which the transmission tower was modeled by steel bar with 10 mm in diameter and the steel chains were used for the conductors based on the dynamic characteristics of the long-span transmission tower and the scales of conductor span with the geometrical size and weight of the tower. The model system consists of three towers and two layer lines that are with two conductors for each layer (Figs. 2 and 3). Two steel boxes were used as lumped masses attached to the top and middle of tower so that the acceleration sensors could be installed in them. Two steel solid blocks were attached to

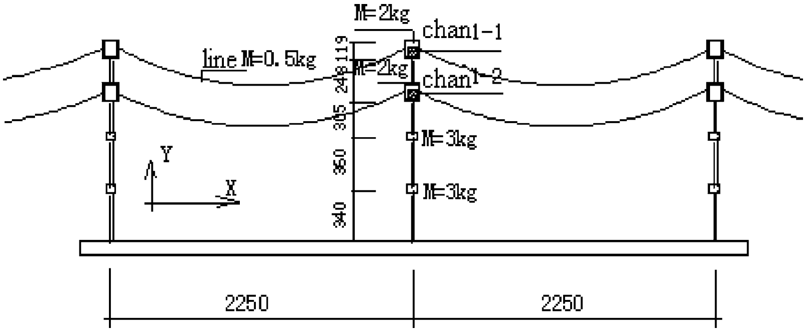

Fig. 2. Testing model (unit: mm).

Fig. 3. Photograph of testing model.

the middle and bottom of each bar (Fig. 2). The conductor line was replaced with steel chains connected to the three tower bars. Why this experiment pattern was chosen is mainly based on the following reasons. Firstly, the cross-section of most of the practical towers is square and the stiffness in two orthogonal directions are normally the same. The circular steel bar to replace the tower shelf can preferably simulate the dynamic performances of the tower during earthquakes in two orthogonal directions. Secondly, the attached mass along the model tower can match the mass distribution of the actual transmission towers. Thirdly, the steel chain to simulate the conductor linked to the tower with the hinge joint can embody the dynamic behaviors of the conductors well. Fourthly, the three model towers in the test are linked by two-span conductors with two layer lines for each span, which can perform the coupled system of the transmission towers and conductors.

Table 1
Seismic acceleration records

Site soil	No.	Earthquake	Event date	Magnitude	Station	PGA (gal)
Soft	1	San Fernando	Feb. 9, 1971	6.6	Port Hueneme	25.91
	2	San Fernando	Feb. 9, 1971	6.6	Univ.Avenue	56.36
	3	Tangshan	Nov. 15, 1976	7.1	Tianjing hosp.	104.18
Mid-hard	4	Imperial Valley	Jan. 23, 1951	5.6	El Centro	30.35
	5	Kern County	July 21, 1952	7.7	Taft	152.7
	6	Imperial Valley	May 18, 1940	6.7	El Centro	341.7
Hard	7	Landers	Jun. 28, 1992	7.5	Baker Fire	105.58
	8	Landers	Jun. 28, 1992	7.5	Fort Irwin	119.85
	9	Tangshan	1976.8.31	6.3	Qian'an	118.91

To investigate the accuracy of the above-proposed theoretical method, seismic responses of the model towers on different site conditions were obtained from the shaking-table experiments. Typical ground motion acceleration records were chosen as the seismic inputs listed in Table 1. In the course of test, the peak ground accelerations of these records were adjusted to 0.1 g on the scale for comparison.

The acceleration time-history responses of the testing model were recorded through channels 1-1 and 1-2. And their comparisons between theoretical and experimental results were done and typically illustrated with the out-of-plane results under the excitation of Tianjing hospital wave, El Centro wave and Qianan wave in Figs. 4, 5 and 6, in which the results in channel 1-1 were shown in the left figures, the channel 1-2 in the right figures, respectively.

Similarly, comparisons with in-plane results under the same seismic excitations were shown from Figs. 7 to 9.

Furthermore, theoretical and tested results of the peak accelerations of the coupled model system and their errors were listed in Tables 2 and 3, respectively.

It is noted from figures and tables of theoretical and test results that the maximum errors are 10.94% in channel 1-2 under out-of-plane vibration and 10.59% in channel 1-2 under in-plane vibration, respectively. In most cases, the out-of-plane results have higher accuracy than in-plane ones. Yet, these errors between theoretical and test results are acceptable in practical engineering area. Therefore, it can be concluded from the above results that the theoretical approaches presented here for the coupled system of transmission lines and their supporting towers are reasonable.

5. Simplified seismic calculation for coupled tower–line system

"Code for Seismic Design of Electrical Facilities" (GB 50260-96) [22] states that the modal combination response spectral method might be adopted for calculating the horizontal seismic force of large-span towers or steel towers with the height over 50 m, and the weights of lines and

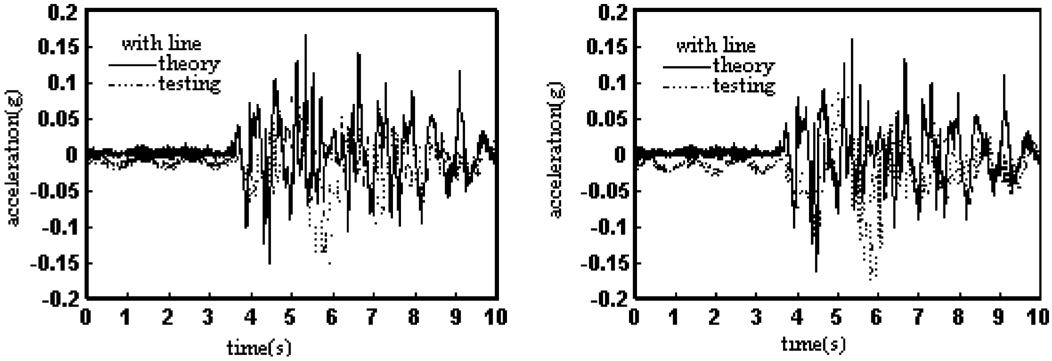

Fig. 4. Comparison under Tianjing hospital wave (out-of-plane).

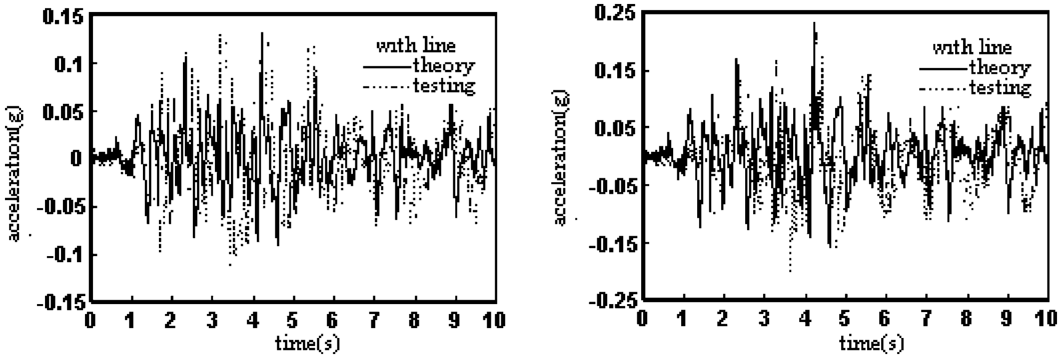

Fig. 5. Comparison under EL Centro wave (out-of-plane).

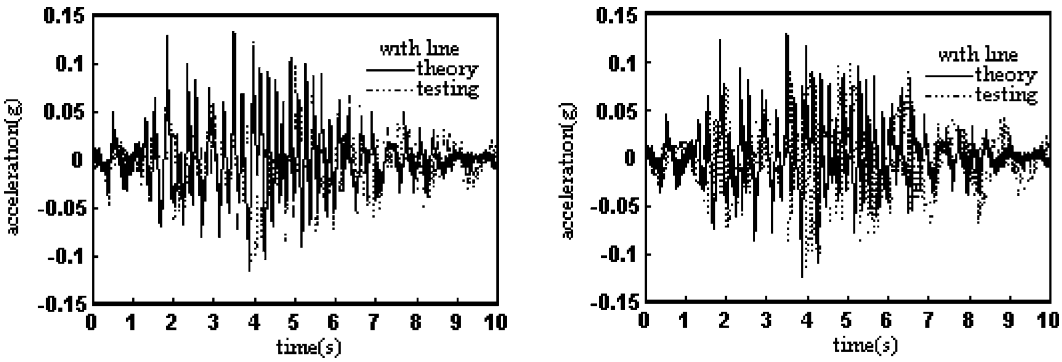

Fig. 6. Comparison under Qianan wave (out-of-plane).

lighting rods may not be considered in calculating the dynamic features of the towers. Thus, the seismic force from this response spectral approach is expressed by

$$F_{ji} = \zeta \alpha_j \gamma_j X_{ji} G_i \quad (i = 1, 2, \ldots, n; \ j = 1, 2, \ldots, m) \tag{15}$$

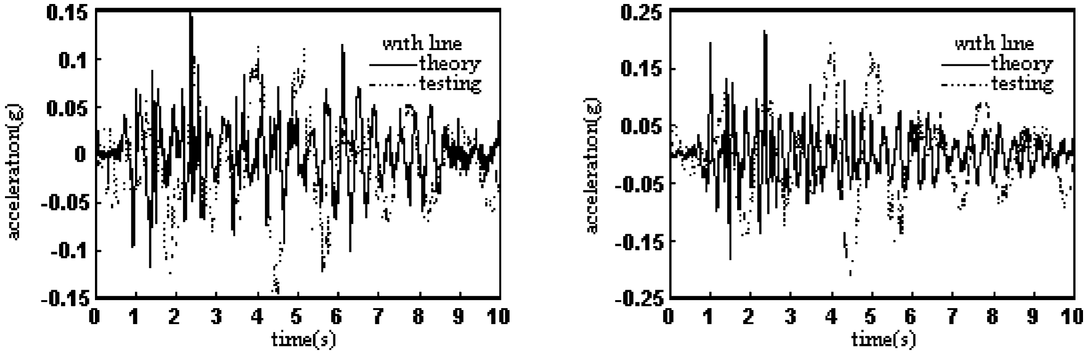

Fig. 7. Comparison under Tianjing hospital wave (in-plane).

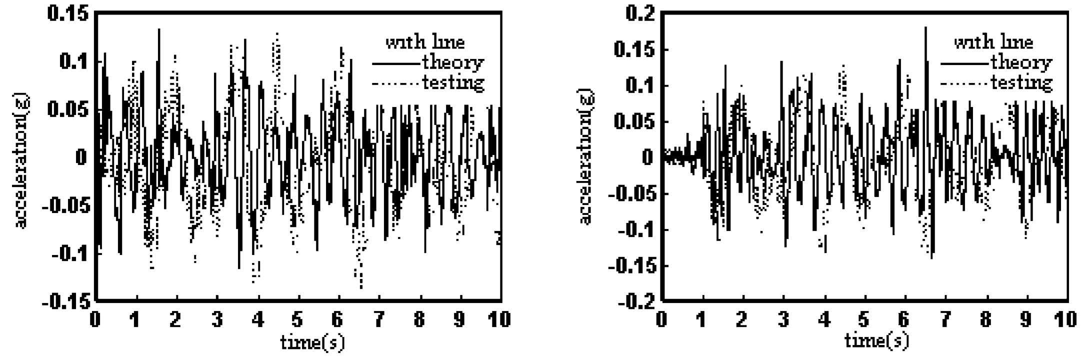

Fig. 8. Comparison under EL Centro wave (in-plane).

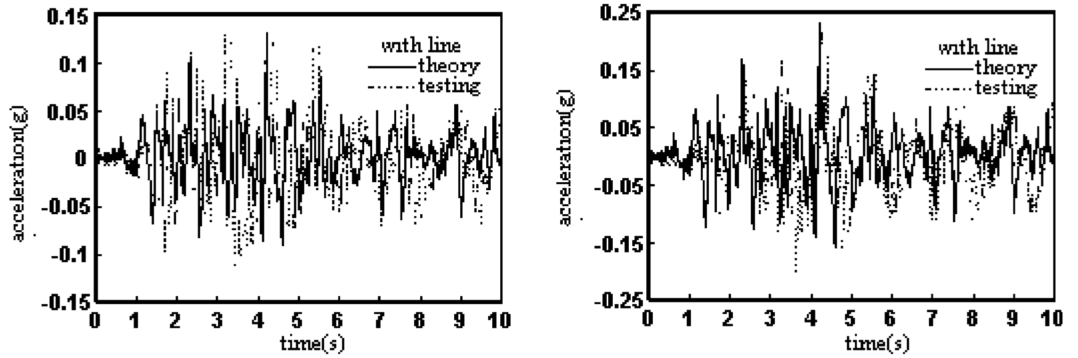

Fig. 9. Comparison under Qianan wave (in-plane).

where F_{ji} is the seismic force for the jth mode at the ith mass, ζ implies the structural factor, α_j denotes the seismic affecting factor in horizontal direction at the natural period of the jth mode, X_{ji} represents the relative displacement in horizontal direction of the jth mode at ith mass, G_i

Table 2
Comparisons between results of theory and test (out-of-plane)

Records	Tianjing hospital wave			El Centro wave			Qianan wave		
Results	Theory	Testing	Error (%)	Theory	Testing	Error (%)	Theory	Testing	Error (%)
Chann. 1-1	0.1489	0.1426	4.42	0.1322	0.1390	4.89	0.1312	0.1257	4.38
Chann. 1-2	0.2183	0.2115	3.22	0.1772	0.1752	1.14	0.1237	0.1115	10.94

Table 3
Comparisons between results of theory and test (in-plane)

Records	Tianjing hospital wave			El Centro wave			Qianan wave		
Results	Theory	Testing	Error (%)	Theory	Testing	Error (%)	Theory	Testing	Error (%)
Chann. 1-1	0.1624	0.1758	7.62	0.1300	0.1414	8.06	0.1312	0.1257	4.38
Chann. 1-2	0.1636	0.1789	8.55	0.2278	0.2139	6.50	0.1273	0.1151	10.59

means the gravity load of the ith mass including total permanent loads, fixing loads and the attached loads on the mass, and γ_j is the mode-participation factor of the jth mode given by

$$\gamma_j = \frac{\sum_{i=1}^{n} X_{ji} G_i}{\sum_{i=1}^{n} X_{ji}^2 G_i}. \tag{16}$$

It has been known from the above seismic response results of the coupled tower–line systems and Refs. [17–21] that the large-span transmission tower–line systems designed by the code method [22] may tend to be unsafe under the excitations of strong ground motions if the span is longer than the limitations given by Refs. [17,21]. Therefore, the effects on the system should not be neglected in the tower design. A simplified aseismic calculating method, i.e., by adding the attached mass Δm to G_i in Eq. (16) to consider the effects of conductors, is proposed here, in which Δm is given by

$$\Delta m = f(l_x) \times l_x \times q \tag{17}$$

where Δm is the attached mass of the tower after considering the effects of conductors (kg), l_x means the horizontal distance between two towers (m), q denotes the conductor mass per kilometer (kg/km) and $f(l_x)$ represents the attached mass factor determined by the following expression:

$$\text{out-of-plane}\ f(l_x) = \begin{cases} 0.17 + \dfrac{3l_x}{200l_0} & \text{soft} \\ 0.21 + \dfrac{l_x}{100l_0} & \text{mid-hard} \\ 0.35 + \dfrac{l_x}{20l_0} & \text{hard} \end{cases} \quad (\text{if } f(l_x) > 0.7,\ \text{then } f(l_x) = 0.7) \tag{18}$$

and

$$\text{in-plane } f(l_x) = 0.5 + \frac{l_x}{200 l_0} \text{ at all sites (if } f(l_x) > 1.0, \text{then } f(l_x) = 1.0). \tag{19}$$

To understand the accuracy of the presented approach, the seismic responses of the two kinds of transmission towers in Ref. [21] were given with two methods: the approach of the coupled tower–line model (integrated) in Section 2 and presented approach in Eqs. (18) and (19). The parameters of towers were listed in Table 4. In numerical computation, the number of lumped masses for each conductor line was simulated as 6. The comparisons of their results with the changes of line span from 100 to 1000 m in out-of-plane and from 50 to 500 m were shown typically with towers 1 and 2 in Figs. 10 and 11. It is noted from these figures that the results of two methods agree quite well and the maximum error is only about 6%. Hence, such small errors are acceptable in engineering.

Table 4
Main parameters of transmission towers

Tower No.	Height (m)	Tower mass (kg)	Line type	Lighting rod type	Line mass (kg/km)
Tower 1	74.6	33 427	LGJ-500/45	GJ-70	1688
Tower 2	90	40 430	LGJ-500/45	GJ-70	1688

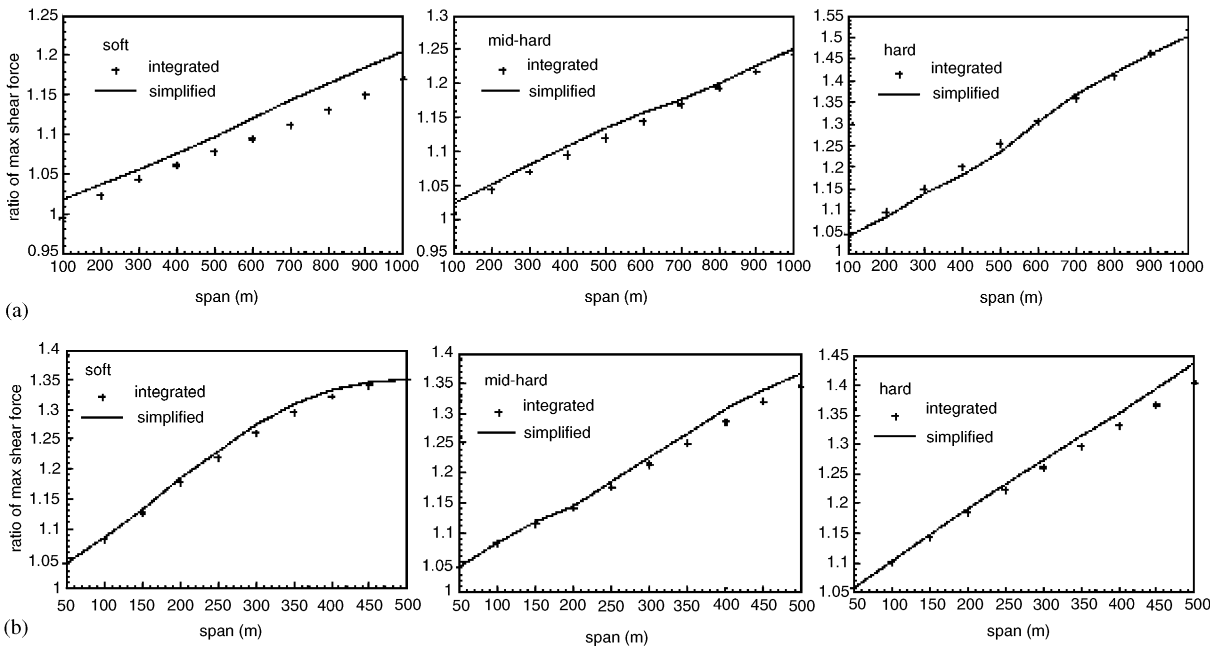

Fig. 10. Result comparisons between simplified and integrated model (out-of-plane). (a) Tower 1, (b) tower 2.

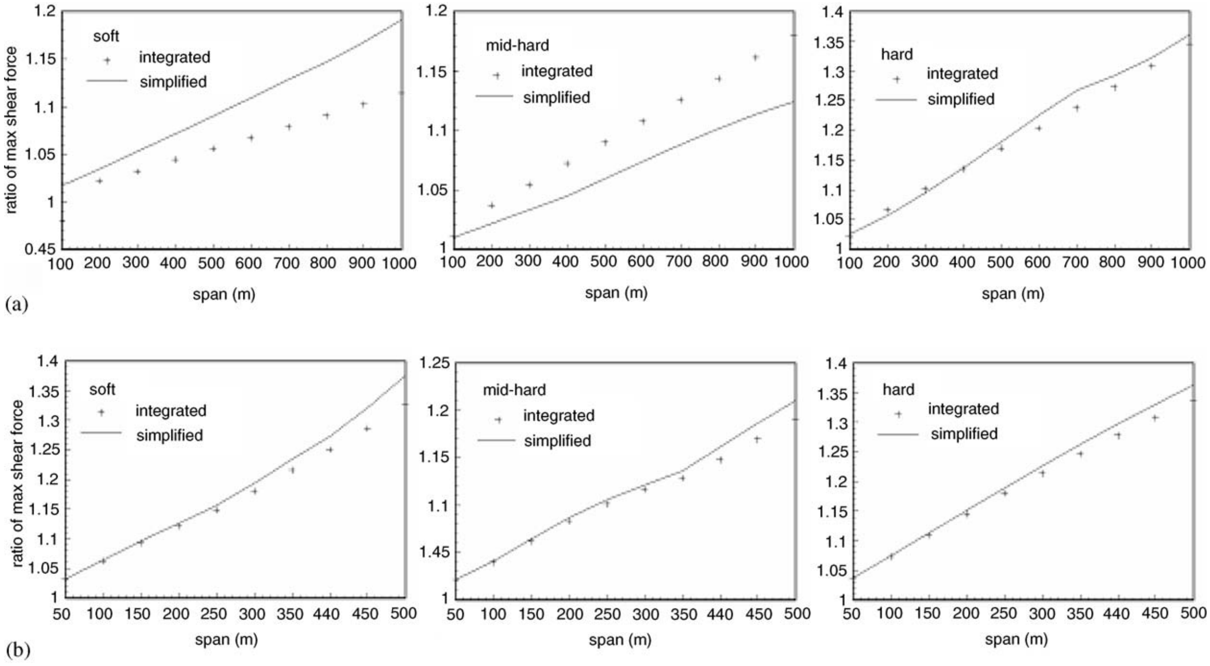

Fig. 11. Result comparisons between simplified and integrated model (in-plane). (a) Tower 1, (b) tower 2.

6. Conclusions

In this paper, the simplified models for coupled system of transmission lines and their supporting towers under the action of earthquake excitations are presented through the proofs of experiments and numerical analyses. The simplified analytical method can be applied to practical engineering projects. According to the experiments and numerical calculations, the following conclusions can be drawn as that the proposed theoretically computational models for analyzing the seismic responses of the coupled system of transmission towers and lines are reasonable. And the error analyses also indicate that the theoretical model for the out-of-plane vibration has higher accuracy than that under in-plane vibration.

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