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Journal of Sound and Vibration 286 (2005) 601–614

JOURNAL OF
SOUND AND
VIBRATION

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Reduction of acoustic radiation by impedance control with a perforated absorber system

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Received 27 January 2004; received in revised form 21 September 2004; accepted 13 October 2004

Available online 28 December 2004

Abstract

A model of sound radiation from an infinite plate with an absorptive facing is proposed and investigated theoretically from the viewpoint of acoustic power. Acoustic characteristics on the plate surface are represented by impedance derived from iso-absorption curves. A parametric study is carried out to clarify the effect of the impedance on the acoustic power. Results derived from this model show that acoustic radiation depends on change in impedance as well as the absorption coefficient, and there is a possibility of reducing the radiation from vibrating surface by introducing an appropriate impedance surface. In order to realize this effect, a model using a perforated board with a back cavity attached to the vibrating surface is proposed, in which the motion of the perforated board is made equal to that of the vibrating surface. To obtain fundamental data, a theoretical study is performed under a simplified condition, assuming an infinite plane piston. The calculated results are compared to experimental data measured by using an acoustic tube. The results, which are in good agreement in the reduction effect, show that this system can achieve the reduction of radiated sound power at arbitrary frequencies.

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1. Introduction

The acoustic radiation from plates has been studied theoretically as well as experimentally in many fields including room acoustics and structural acoustics. In order to attenuate the sound radiation, various structures have been proposed.

Experimental studies on the acoustic radiation from a single plate with an absorptive layer as a sound insulation construction were carried out by Mulholland [1] and Brown [2]. This structure was investigated theoretically from the viewpoint of the sound transmission by Takahashi [3], allowing for not only mass-law but also flexural vibrations. In the paper, the case that the vibrating surface itself was assumed to be given absorptive characteristics was also studied.

Structure-borne sound radiation from a double-leaf structure was discussed both theoretically and experimentally by Yairi [4]. It is shown that, although a double-leaf structure can attenuate the radiated sound power at mid and high frequencies, it is difficult to reduce that at low frequencies.

Many studies on the effect due to a perforated board have been performed. The effective boundary conditions for a perforated sandwich panel were proposed by Leppington [5] and the attenuation effect of a perforated sandwich panel excited by a line source was investigated theoretically from the viewpoint of acoustic radiation by Abrahams [6]. Takahashi [7] suggested the theoretical treatment of a perforated board and absorptive materials, in which the flexural vibrations of the plate were taken into account.

For the values of acoustic impedance which give the same absorption coefficient, there are innumerable combinations of the resistance term and the reactance term. The difference in the combination may strongly affect the acoustic radiation. In this paper, the acoustic characteristics of the plate surface are represented by impedance derived from iso-absorption curves, and a parametric study on the attenuation effect is carried out. In order to realize this effect, the model using a perforated board with a back cavity attached to the vibrating surface is proposed, in which the motion of the perforated board is made equal to that of the vibrating surface. A theoretical study is performed under simplified conditions, assuming an infinite plane piston so as to obtain the fundamental insight. The results are compared to experimental data measured by using an acoustic tube. A perforated board has the advantage of being able to change its impedance characteristics arbitrarily and extensively by tuning its parameters. In order to verify this superiority, a parametric study is also carried out.

2. Effect of impedance on the radiation from an absorptive facing

In this section, the effect of impedance on the radiation from an absorptive facing is clarified in the light of radiated acoustic power and the possibility of an absorptive facing for reduction of acoustic radiation is discussed. For this purpose, a simple and basic acoustic model is assumed.

Consider the acoustic radiation of the model as shown in Fig. 1, where the vibrating surface itself is assumed to be given any absorptive characteristics. The infinite plate located at $z = 0$, which has impedances Z_1 and Z_2 on both sides of the surface, respectively, is excited by a point force $q(r) = \delta(r)/2\pi r$, where $\delta(r)$ is the Dirac delta function. In this case, the equation of motion

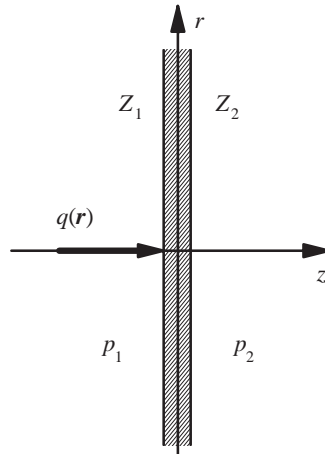


Fig. 1. Model of sound radiation from an infinite plate with absorptive facing.

for the axisymmetric displacement $w(r)$ can be written as

$$D\nabla^4 w(r) - \rho h \omega^2 w(r) = p_1(r) + \frac{\delta(r)}{2\pi r} - p_2(r), \tag{1}$$

where $\nabla^4 = [(\partial^2/\partial r^2) + (1/r)(\partial/\partial r)]^2$, and ω is the angular frequency, ρ and h are the density and the thickness of the plate, respectively. D is the flexural rigidity of the plate, defined as $D = [E(1 - i\eta)h^3/12(1 - \nu^2)]$, where i is the imaginary unit. E , η , and ν are Young's modulus, loss factor, and Poisson's ratio of the plate. The time factor $e^{-i\omega t}$ is suppressed throughout. The Hankel transform pair with respect to r and the wavenumber k are defined as

$$p_j(r) = \int_0^\infty P_j(k)J_0(kr)k \, dk, \quad P_j(k) = \int_0^\infty p_j(r)J_0(kr)r \, dr, \quad j = 1, 2, \tag{2}$$

$$w(r) = \int_0^\infty W(k)J_0(kr)k \, dk, \quad W(k) = \int_0^\infty w(r)J_0(kr)r \, dr, \tag{3}$$

where $J_0(kr)$ denotes the Bessel function of order zero. Taking the transform of Eq. (1) gives

$$W(k) = \frac{[2\pi\{P_1(k) - P_2(k)\} + 1]}{2\pi D(k^4 - k_b^4)}, \tag{4}$$

where $k_b = \rho h \omega^2 / D$.

With the Helmholtz–Kirchhoff integral formula, the sound pressures $p_j(r)$ can be expressed as

$$p_j(r) = 2 \iint_s G(r|r_0) \frac{\partial p_j(r_0)}{\partial n} \, ds, \quad j = 1, 2, \tag{5}$$

where G is the free space Green’s function given by

$$G(r|r_0) = \frac{e^{ik_0|r-r_0|}}{4\pi|r-r_0|}, \tag{6}$$

where $k_0 = \omega/c_0$ is the acoustic wavenumber, c_0 is the speed of sound, n is the outward normal to the surface and k_0 is the wavenumber in air. While in the case where the surface with the impedance $Z_j(j = 1, 2)$ is vibrating under the pressure $p_j(r_0)$, the following equation can be assumed [3]:

$$\frac{\partial p_j(r_0)}{\partial n} = -i\rho_0\omega \left\{ (-1)^{j-1}i\omega w(r_0) - \frac{p_j(r_0)}{Z_j} \right\}, \tag{7}$$

where ρ_0 is the density of air. Combining Eq. (7) with Eq. (5) yields

$$p_j(r) = \frac{1}{2\pi} \iint_s \left[(-1)^{j-1} \rho_0 \omega^2 w(r_0) + i\omega \rho_0 \frac{p_j(r_0)}{Z_j} \right] \frac{e^{ik_0|r-r_0|}}{|r-r_0|} ds, \quad j = 1, 2, \tag{8}$$

Taking the Hankel transform of Eq. (8) gives

$$P_j(k) = \frac{(-1)^{j-1}i\rho_0\omega^2}{\sqrt{k_0^2 - k^2 + \rho_0\omega/Z_j}} W(k). \tag{9}$$

One can obtain the solutions for unknown quantities $P_1(k)$, $P_2(k)$ and $W(k)$ by solving Eqs. (4) and (9) simultaneously, and then, substituting them into Eqs. (2) and (3) gives $p_1(r)$, $p_2(r)$ and $w(r)$. Radiated acoustic power P_w from the plate can be obtained with $p_2(r)$ and $w(r)$ by using the following equation:

$$P_w = \iint_s \frac{1}{2} \operatorname{Re} \left\{ p_2(r) \left(-i\omega w(r) - \frac{p_2(r)}{Z_2} \right)^* \right\} ds, \tag{10}$$

where the asterisk denotes conjugate.

Acoustic radiation from a vibrating surface may change due to its impedance, which is strongly related to its absorption coefficient. For example, in the case of normal incidence of a plane wave, the absorption coefficient α is expressed with the impedance ratio Z as follows:

$$\alpha = 1 - \left| \frac{1 - Z}{1 + Z} \right|^2 = \frac{4\operatorname{Re}\{Z\}}{[1 + \operatorname{Re}\{Z\}]^2 + \operatorname{Im}\{Z\}^2}. \tag{11}$$

Fig. 2 shows this relationship as the iso-absorption curves. These curves indicate that there are innumerable combinations of the resistance term and the reactance term for the values of impedance, which give the same absorption coefficient.

The calculated results of radiated acoustic power level (in dB re 10^{-12} W) for the case of a dense concrete plate of thickness 18 cm under a harmonic point excitation of 1 N force are shown in Fig. 3. In this calculation, the excitation-side or radiation-side surface itself is assumed to be the absorptive facing, which is given A to E combinations of impedance derived from Fig. 2. It is seen from Fig. 3 that the difference in the combination affects the acoustic power radiated from the vibrating surface.

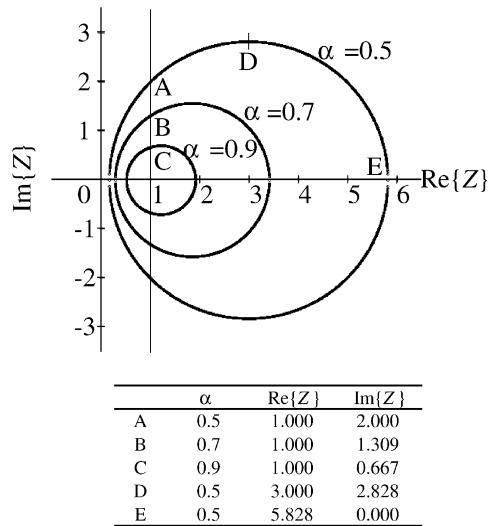


Fig. 2. Iso-absorption curves on the complex plane of impedance.

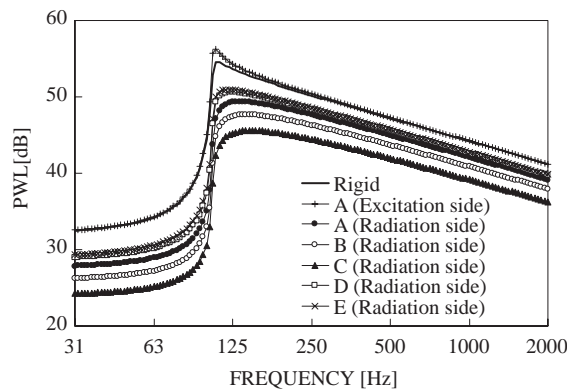


Fig. 3. Effect of absorptive facings on the acoustic power level radiated from the plate.

Let W_b be the total acoustic power from an absorptive surface and W_0 be that of the case without the absorption treatment; radiation reduction can be defined as [4]

$$RR = 10 \log_{10} \frac{W_0}{W_b}. \tag{12}$$

Results of radiation reduction (RR), which correspond to the difference of power level between a rigid surface and the absorptive surface, are shown in Fig. 4. The critical frequency of coincidence is 102 Hz. These results show that radiation-side absorptive facing can attenuate the radiation, especially at low frequencies below its critical frequency of coincidence, due to the increase in absorptive coefficient, and also due to decrease in the value of reactance term in the case of the

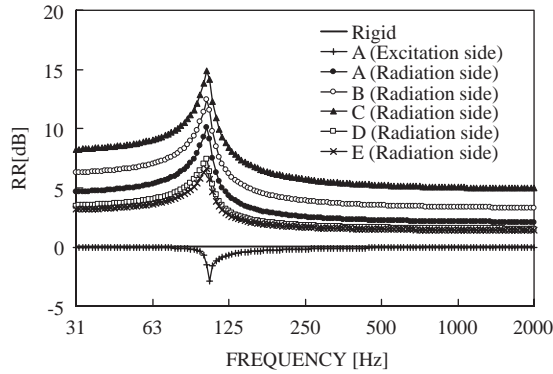


Fig. 4. Radiation reduction (RR) calculated from PWL.

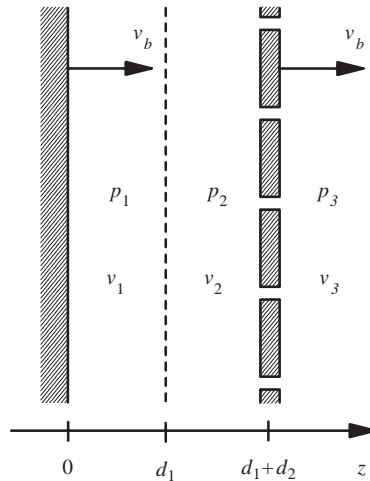


Fig. 5. Simple model of the perforated absorber system.

same values of absorption coefficient. However, it is seen that the excitation-side absorption facing causes a slight adverse effect around the coincidence frequency.

3. Realization of an absorptive facing

In this paper, absorber systems using a perforated board are proposed, in which one of the ways to realize the effect of the absorptive facing discussed in the preceding section is introduced. A perforated board with a back cavity filled with air and/or porous materials is located at the radiation side of a vibrating surface. However, in order to reproduce the absorptive facing, the radiation surface itself must have the absorptive effect and must have the same motion as the vibrating surface. Thus, the perforated board must be linked by stiffeners so as to be given the

same vibration. This condition is needed to satisfy the assumption of the absorptive facing, and the use of perforated board system makes it possible to realize this condition.

In order to discuss the validity of the realization and the fundamental character of this absorber system, unnecessary effects should be taken away. Therefore, the simplest model in a one-dimensional piston motion shown in Fig. 5 is considered. An infinite perforated board with a back cavity filled with air and/or porous materials is located at $z = d_1 + d_2$, which is in the radiation side of a rigid plate at $z = 0$. The perforated board is forced to be given the same vibration velocity as that of the rigid plate. Under this condition, the acoustic power radiated from the perforated board is calculated when the rigid plate is vibrating. The back cavity within $0 < z < d_1 + d_2$ is divided into two layers at $z = d_1$. The rigid plate side and the perforated board side are expressed by subscripts 1 and 2, respectively. The radiation side is also expressed by a subscript 3. Then, the sound pressures and the particle velocities in regions 1, 2 are written with region numbers $j = 1, 2$ as follows:

$$p_j(z) = p_j^+ e^{-\gamma_j z} + p_j^- e^{\gamma_j z}, \tag{13}$$

$$v_j(z) = \frac{1}{Z_j} (p_j^+ e^{-\gamma_j z} - p_j^- e^{\gamma_j z}), \tag{14}$$

where p_j^\pm are unknown coefficients, γ_j and Z_j are the propagation constant and the characteristic impedance of the layer. The sound pressure and the particle velocity in the region 3 are also written as

$$p_3(z) = p_3^+ e^{ik_0 z}, \tag{15}$$

$$v_3(z) = \frac{p_3^+ e^{ik_0 z}}{\rho_0 c_0}. \tag{16}$$

Acoustic coupling for a perforated board has been discussed [7]. Shown schematically in Fig. 6 is a cross-sectional view of the perforated board that is vibrating with a velocity v_b under any acoustic loading with a pressure difference Δp . Considering the acoustic wave lengths of relative low

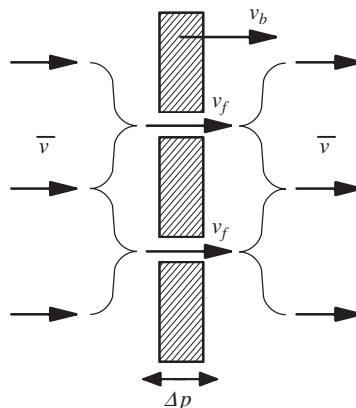


Fig. 6. Analytical model of a perforated board.

frequencies, interaction between the plate and surrounding air can be introduced in a spatially mean sense. The continuity of the volume velocity gives the following equation regarding the mean particle velocity \bar{v} of surrounding air in the vicinity of the perforated board,

$$\bar{v} = v_b + (v_f - v_b)\sigma, \quad (17)$$

where v_f is the spatially averaged particle velocity in the hole, and σ is the ratio of perforation. Let z_0 be the acoustic impedance of the hole, which is represented with its resistance term z_{resist} and reactance term z_{react} ,

$$z_0 = z_{\text{resist}} + z_{\text{react}}. \quad (18)$$

In this case, the viscous force at the air–solid interface in the hole depends on relative velocity $v_f - v_b$ and the inertial force depends only on v_f . Then, pressure difference Δp can be written as

$$\Delta p = z_{\text{resist}}(v_f - v_b) + z_{\text{react}}v_f. \quad (19)$$

Thus, combining Eq. (19) with Eq. (17) yields

$$\bar{v} = \zeta v_b + \frac{\Delta p}{z_0/\sigma}, \quad (20)$$

where $\zeta = 1 - (z_{\text{react}}/z_0)\sigma$. Allowing for Eq. (20), the boundary conditions in Fig. 5 are written as

$$v_1(0) = v_b, \quad (21)$$

$$p_1(d_1) = p_2(d_1), \quad (22)$$

$$v_1(d_1) = v_2(d_1), \quad (23)$$

$$v_2(d_1 + d_2) = v_3(d_1 + d_2) = \zeta v_b + \frac{p_2(d_1 + d_2) - p_3(d_1 + d_2)}{z_0/\sigma}. \quad (24)$$

One can obtain the solutions of five unknown quantities by solving these simultaneous equations.

In order to validate the realization of the attenuation effect discussed in the preceding section, calculated results of this model (Case 1) are compared with that of the absorptive facing model (Case 2), in which the impedance is calculated from a sound-absorbing model shown in Fig. 5 for normal incidence of a plane wave. In addition, another case is also considered as a reference (Case 3), in which the perforated board is located having no mechanical constraint. In this case, v_b in Eq. (24) is replaced with unknown quantity v_p , with an additional condition

$$p_2(d_1 + d_2) - p_3(d_1 + d_2) = -i\rho_p h_p \omega v_p, \quad (25)$$

where ρ_p and h_p are the density and thickness of the perforated board. This case results in simultaneous equations with six unknown quantities.

Acoustic impedance of the aperture is given by Maa's approximation formulas [8],

$$z_{\text{resist}} = \frac{8\eta_0 h_p}{(d_p/2)^2} \left(\sqrt{1 + \frac{X^2}{32}} + \frac{\sqrt{2}d_p X}{8h_p} \right), \quad (26)$$

$$z_{\text{react}} = -i\rho_0\omega h_p \left(1 + \frac{1}{\sqrt{9 + (X^2/2)}} + \frac{0.85d_p}{h_p} \right), \tag{27}$$

where $X = (d_p/2)\sqrt{\rho_0\omega/\eta_0}$ and d_p is the diameter of the aperture, η_0 is the viscosity coefficient of air. As parameters of a perforated board, density, thickness and perforation ratio are considered and these values are determined within the range where the approximate impedance of the aperture is in sufficient agreement with rigorous analytical impedance. The parameter Z_j and the propagation constant γ_j in Eqs. (13) and (14) are calculated from both the flow resistivity R_j and frequency f by the following Miki’s experimental equations [9]:

$$Z_j = \rho_0 c_0 \left[1 + 0.07 \left(\frac{R_j}{f} \right)^{0.632} + 0.107i \left(\frac{R_j}{f} \right)^{0.632} \right], \tag{28}$$

$$\gamma_j = k_0 \left[0.16 \left(\frac{R_j}{f} \right)^{0.618} - i \left\{ 1 + 0.109 \left(\frac{R_j}{f} \right)^{0.618} \right\} \right]. \tag{29}$$

Fig. 7 shows the calculated results of RR. Parameters of the perforated board are as follows: density is 600 kg/m³(plywood); thickness is 9 mm; diameter of the aperture is 6 mm; distance between apertures is 60 mm. The perforation ratio corresponding to these values is 0.79%. The thickness of a single layered air-back cavity is 150 mm. Case 1 is in good agreement with Case 2 at all frequencies except the resonance frequency 1150 Hz. Thus, it is shown that the effect due to an absorptive facing can be reproduced by assuming that the motion of the radiation surface is the same as that of the vibrating surface. It is also confirmed that, in Case 3, radiated acoustic power is increased at low frequencies although an attenuation effect can be obtained at mid and high frequencies.

Another example is shown in Fig. 8, in which the back cavity is double layered: the thickness of the rigid-surface-side layer filled with air is 100 mm; the thickness of the perforated-surface-side layer filled with a porous material is 50 mm; flow resistivity of the porous material is 10³ N s/m⁴. Other parameters are the same as in Fig. 7. Insertion of a porous material causes the alleviation of

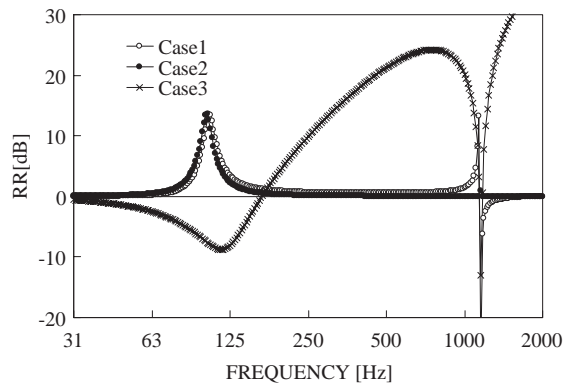


Fig. 7. RR by the absorber system with air-cavity.

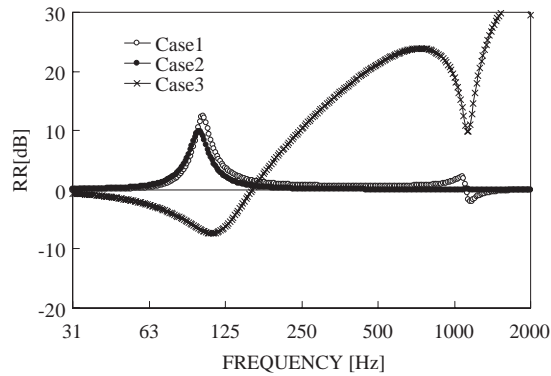


Fig. 8. RR by the absorber system with porous-cavity.

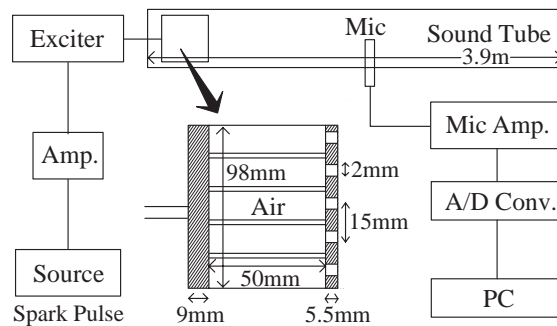


Fig. 9. Schematic diagram of the experimental setup.

the resonance effect, thus peak and dip at high frequencies are smoothed in both Cases 1 and 3. A slight disagreement between Cases 1 and 2 is seen at the peaks of the attenuation effects. Both effects of smoothing and the disagreements have a tendency to increase as the flow resistivity of a porous material increases.

4. Experimental study

In order to validate the possibility of realization discussed in preceding section, theoretical results are compared with experimental results. A schematic diagram of the apparatus is shown in Fig. 9. Sound tube was used as a substitute for piston vibration of an infinite plate. The perforated absorber sample was excited in the tube. The sample has a circular cross section of diameter 98 mm and is composed of a plywood of thickness 9 mm and a perforated board of thickness 5.5 mm. These plates are set in parallel with spacing 50 mm and linked tightly by steel bars so as to be given the same motion. Spark pulse was used as the sound source of excitation in order to eliminate the effect of reflection from the open end of the tube. Then, it becomes possible to

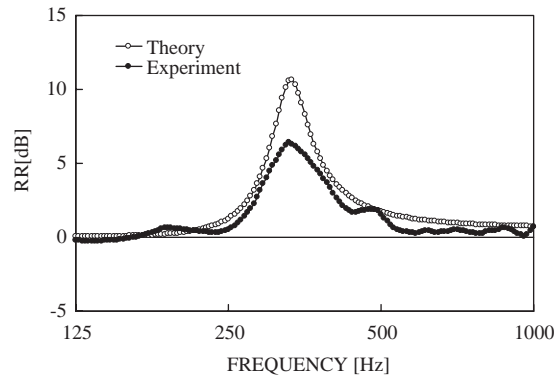


Fig. 10. Measured data of the RR in comparison with theoretical results.

extract the direct sound radiated from the sample because the reflected sound arrives at the microphone after the direct sound converges sufficiently.

In order to calculate the RR, the same measurement was carried out with another sample, in which the perforated board was replaced with the non-perforated one. These measurements were carried out four times, respectively, and the averaged values were used in the following procedure. Total acoustic power in a one-dimensional sound field is proportional to the squared sound pressure. The frequency domain of acoustic power was calculated from the Fourier transform of measured sound pressure waves and squared to obtain the power levels. The RR is the difference between them. Fig. 10 shows the experimental results in comparison with the theoretical results. There are somewhat differences between their peak values of RR. These deviations might be caused by roughness of apertures drilled into plywood board, insufficient support condition of the exciter, and so on. However, in general, the attenuation characteristics of theoretical results are in good agreement with the experiments.

5. Parametric study

This section is given to the discussion of the effect of the parameters on the radiation characteristics of a proposed absorber system from the viewpoint of theoretical results of RR. The following parameters are considered: ratio of perforation, thickness of a perforated board, thickness and flow resistivity of the single-layered back cavity.

Figs. 11 and 12 show the effect of perforation ratio and thickness of perforated board, respectively. The back cavity of thickness 15 cm is filled with air. It is seen from Fig. 11, where the thickness of the perforated board is 9 mm, that the maximal value of RR at low frequencies has a tendency to increase as the ratio of perforation increases, and also the reduction effects shift to higher frequencies. However, it is shown from Fig. 12, where the ratio of perforation is 2%, that they have the opposite tendency as thickness of the perforated board increases. Fig. 13 shows the effect of thickness of the air-back cavity. In this calculation, the ratio of perforation is 2% and thickness of the perforated board is 9 mm. It is seen from this figure that the maximal value at low frequencies increases as thickness of the cavity increases, and both the reduction effect and

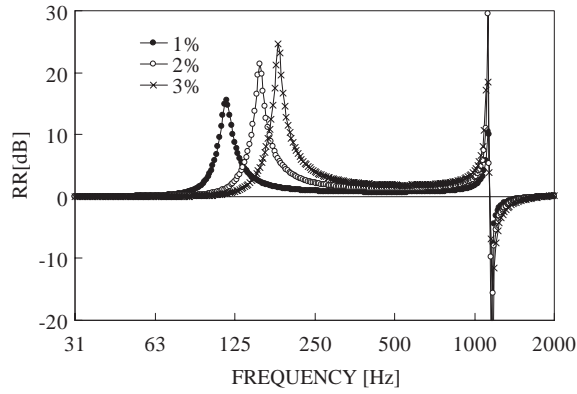


Fig. 11. Effect of perforation ratio on the RR.

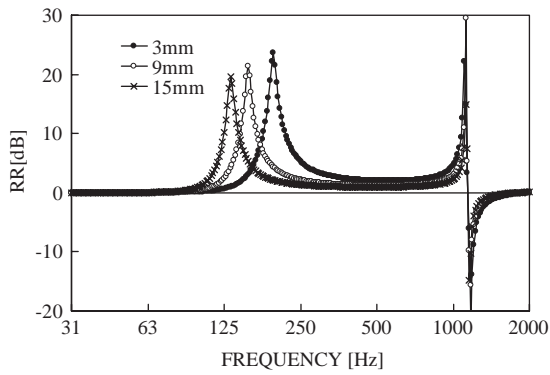


Fig. 12. Effect of perforated board thickness on the RR.

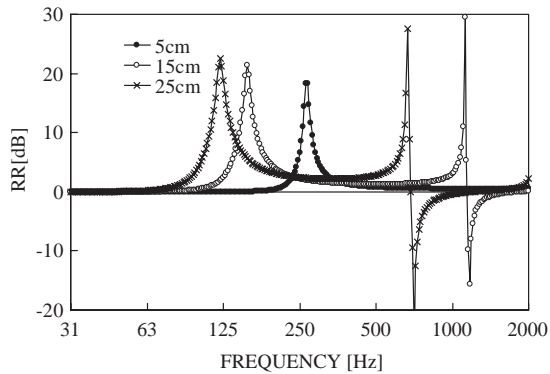


Fig. 13. Effect of air-back cavity thickness on the RR.

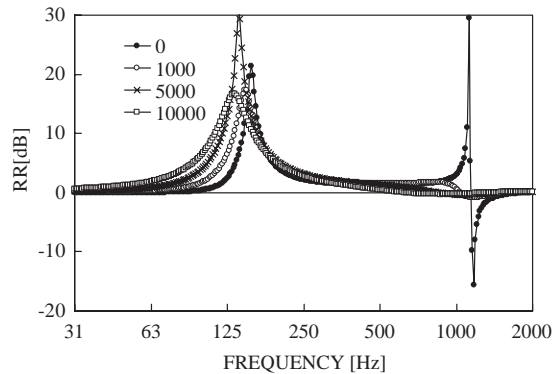


Fig. 14. Effect of flow resistivity on the RR.

resonance effect shift to lower frequencies. These tendencies may be desirable because it is difficult to reduce the radiation at low frequencies [4]. These results show that this absorber system can achieve the reduction of radiated sound power at arbitrary frequencies by tuning these parameters. However, in the case that the resonance effect due to the back cavity causes some problems, insertion of a porous material may be the effective solution as shown in Fig. 8. Fig. 14 shows the effect of flow resistivity of the porous material inserted in the back cavity. Peak and dip due to the resonance are smoothed as flow resistivity increases. The peak and dip are not outstanding in the case that flow resistivity is 10^3 N s/m^4 and above. It is also seen that there is the optimal value of flow resistivity in the light of the maximal value of RR at low frequencies. This non-monotonousness may be caused due to the balance between powers absorbed at the holes of the perforated board and the porous material. However, in order to explain the details of this behavior, further studies are necessary.

6. Conclusions

A model of sound radiation from an infinite plate with an absorptive facing was proposed and investigated theoretically from the viewpoint of acoustic power. Acoustic characteristics on the plate surface are represented by impedance derived from iso-absorption curves and their effect on the radiated power is clarified. Theoretical results show a possibility of reducing the acoustic radiation. In order to realize this effect, the simplified model in one-dimensional piston motion was proposed, in which a perforated board is used. The realization of this attenuation effect was achieved by assuming the same motion as the vibrating surface to the perforated board, and validated by the experimental study. Parametric studies were also performed and it is shown that the reduction effect is obtained at arbitrary frequencies by tuning parameters of the perforated board and the back cavity. Further studies regarding, for example, three-dimensional problems, transient characteristics, etc., will be necessary and carried out with the basis of the fundamental data obtained in this paper.

Acknowledgements

This research is supported by the Tostem Foundation for Construction Materials Industry Promotion.

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