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Short Communication

Buckling and vibration of sandwich beams with viscoelastic core under thermal environments

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Abstract

Sandwich beam having viscoelastic core is analyzed for its buckling and vibration behavior under thermal environments, using the finite element method. Variation of natural frequencies and loss factors with respect to temperature is investigated. The formulation is based on the displacement field proposed by Khatua and Cheung [International Journal for Numerical Methods in Engineering 6 (1973) 11–24]. The behavior of the beam considering both the temperature dependent and independent shear modulus of the core is studied. A parametric study is conducted with core thickness as parameter and its influence on buckling temperatures, natural frequencies and loss factors is studied. Two different core materials are considered for the analysis to find the influence of material on buckling and vibration behaviors.

1. Introduction

Sandwich structures are heavily used as sub-components in the construction of airplane, missile and spacecraft structures. Sandwich structures with viscoelastic cores are particularly useful in vibration damping over a wide range of frequencies. Khatua and Cheung [1] presented a finite element formulation for bending and vibration of multilayer beams and plates, with constrained cores. Sadasiva Rao and Nakra [2] carried out analysis of vibration of unsymmetrical sandwich

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beams and plates with viscoelastic cores. Rao [3] derived the complete set of equations of motion and boundary conditions governing the vibration of sandwich beams using energy approach. Sismore and Darvennes [4] have considered the effect of compression energy of the core on damping in addition to the conventional approach, which uses only shear deformation of the core for the estimation of damping. Recently, Banerjee [5] has used the dynamic stiffness method for free vibration analysis of three layer sandwich beams. Ha [6] developed a procedure for exact buckling of sandwich beams and framed structures subjected to arbitrary mechanical loading. The work of Lan et al. [7] presents the thermal buckling of bimodular sandwich beams having thick facings and moderately stiff cores. From the literature survey it is found that the study on buckling and vibration of sandwich beams with viscoelastic cores under thermal environment has not been carried out. Hence, the present study aims at finding the buckling temperatures and the changes in natural frequencies and loss factors due to thermally induced prestresses by making use of finite element method. The formulation uses the displacement field proposed by Khatua and Cheung [1]. The formulation is decoupled thermo-mechanical formulation where in layerwise temperatures are obtained by solving Fourier heat conduction equation and buckling and vibration behaviors are obtained by solving corresponding eigenvalue problems. The influence of temperature-dependent shear modulus, material on buckling and vibration behaviors is also attempted.

2. Finite element formulation

In carrying out buckling and free vibration analyses of the sandwich beam under thermal environments, the prerequisite is the evaluation of stress distribution in the structure under thermal load that governs the afore-mentioned behaviors. In the current work this is accomplished by calculating the layerwise temperature distribution, by solving steady-state two-dimensional Fourier heat conduction equation using 2D rectangular elements [8]. The finite element form of the Fourier heat conduction equation leads to the following elemental matrix equation:

$$[K_1^e]\{T^e\} + [K_2^e]\{T^e\} = \{P_1^e\} + \{P_2^e\}, \tag{1}$$

where $[K_1^e]$, $[K_2^e]$, $\{P_1^e\}$, $\{P_2^e\}$ and $\{T^e\}$ are elemental conduction matrix, convection matrix, load vector due to flux, load vector due to convection and vector of elemental nodal temperatures, respectively. The temperature field in the domain can be obtained by solving Eq. (1) in global sense.

Using the temperature distribution, prestresses, buckling behavior and vibration behavior of the sandwich beam are evaluated. The sandwich beam element is developed based on the displacement field proposed by Khatua and Cheung [1] (refer to Fig. 1).

3. Results and discussion

This section presents the frequency, damping and buckling behaviors of sandwich beam under thermal environment. Thermal stresses are estimated in the beam and buckling and vibration

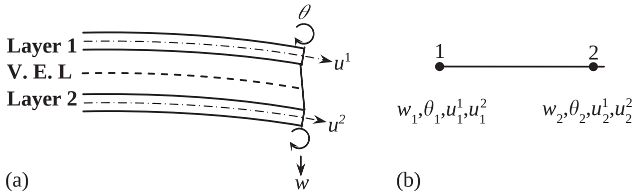

Fig. 1. Sandwich beam element.

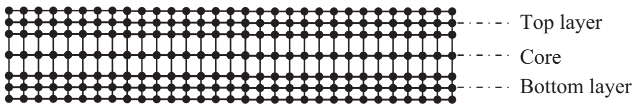

Fig. 2. Finite element mesh for layerwise temperature evaluation.

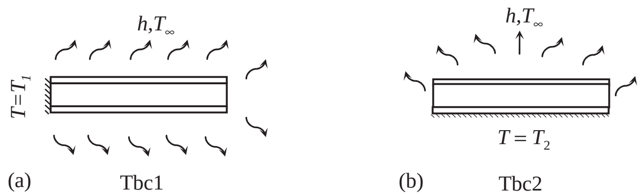

Fig. 3. Thermal boundary conditions: (a) Tbcl and (b) Tbc2.

analyses are carried out. A clamped beam is considered for the present study. The dimensions of the beam are given below.

Thickness of each stiff layer $= t_s = t^1 = t^2 = 3\,\text{mm}$ and length of the beam $l = 0.6\,\text{m}$.

3.1. Evaluation of thermal stresses

Thermal environments will influence the buckling and frequency behavior of a structure. This effect can be accounted for, by the estimation of thermally induced prestresses. Fig. 2 shows the finite element mesh for temperature evaluation. Each layer is meshed with two rectangular elements in thickness direction and thirty elements are used in the length direction. Two different thermal boundary conditions (Tbc1 and Tbc2) are considered for analysis as shown in Fig. 3. Temperatures on the middle line of each layer are extracted and these are considered to be approximate layer temperatures for prestress estimation.

The temperature distributions in the sandwich beam for the two different cases of thermal boundary conditions are shown in Fig. 4. The graphs are drawn for a specified temperature 140 °C for the given boundary condition. Fig. 4(a) shows temperature variation along the length of the beam with thermal boundary condition Tbc1. The temperature falls down from the specified temperature (140 °C) at left end following a second-order curve as indicated in Fig. 4(a). No appreciable variation of temperature in thickness direction of the beam can be observed with thermal boundary condition Tbc1. Fig. 4(b) show the variation of temperature across the

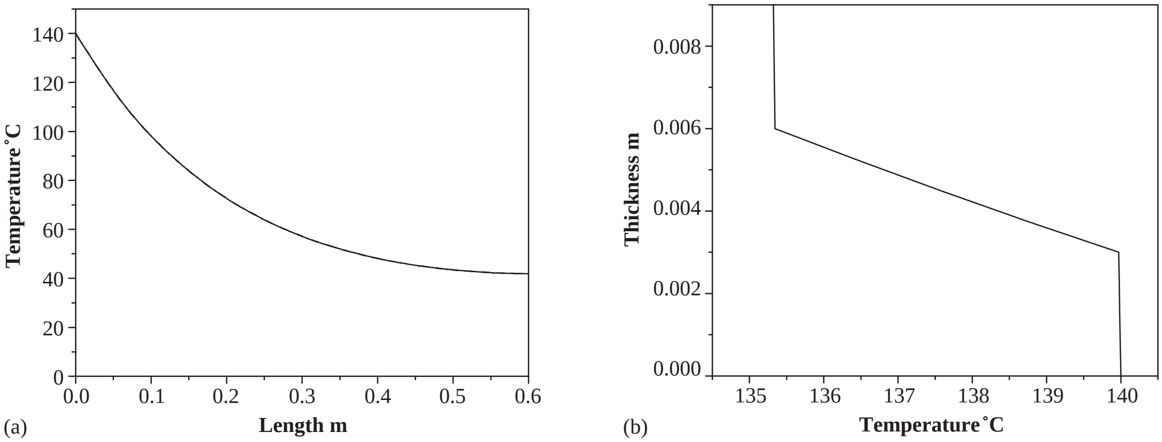

Fig. 4. (a) Temperature variation along the length of the beam for Tbc1 and (b) temperature variation across the thickness of the beam for Tbc2.

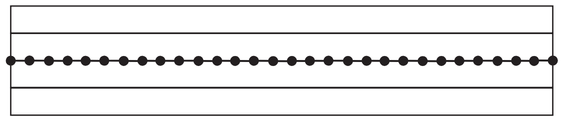

Fig. 5. Finite element mesh for structural analysis.

thickness of the beam with thermal boundary condition Tbc2. The temperature at any cross-section along the length of the beam remains almost constant in the bottom layer (140 °C), drops linearly in viscoelastic core and remains almost constant in the top layer. Temperature along the length of the beam is a constant with thermal boundary condition Tbc2.

Fig. 5 shows the finite element mesh for structural analysis. The beam is descretised by using 30 sandwich beam elements.

After evaluating layerwise temperatures prestresses in the beam can be evaluated.

3.2. Natural frequency analysis

The elementwise stress field $[\sigma^t]$, is used to formulate geometric stiffness matrix for each element given by

$$[K_g^e] = \int_x [B_g]^{\mathrm{T}}[\sigma^t][B_g]\,\mathrm{d}x. \tag{2}$$

The stress field at a specified temperature T is found for each element and using that global geometric stiffness matrix $[K_g^G]$ can be formulated. When the sandwich is under prestressed condition, the following eigenvalue problem has to be solved for obtaining natural frequencies:

$$\left[[K_R^G] + [K_g^G]\right]\{\phi\} - \omega^2[M^G]\{\phi\} = 0. \tag{3}$$

The modal loss factor η_j for jth mode $\{\phi_j\}$ can be found by modal strain energy method given by

$$\eta_j = \frac{\{\phi_j\}^{\mathrm{T}}[K_I^G]\{\phi_j\}}{\{\phi_j\}^{\mathrm{T}}([K_R^G]+[K_g^G])\{\phi_j\}}, \tag{4}$$

where $[K_R^G]$, $[K_I^G]$ are the real and imaginary parts of global stiffness matrix $[K^G]$, respectively. The shear modulus, loss factor values of two different viscoelastic core materials namely DYAD609 and EC2216 at 30 °C and at 100 Hz are 275.7 MPa, 0.4 and 620.5 MPa, 0.3 [9]. DYAD609 has relatively low shear modulus compared to EC2216. The temperature-dependent plots of G^{*c} and η for the above materials are given in Ref. [9]. A curve is fitted to the plots given in Ref. [9] such that values of G^{*c} and η can be obtained from it at any temperature in the operating range of temperatures. These curves are made use of for studying the buckling, frequency and loss factor variations with temperature when temperature-dependent properties of the core are used. Fig. 6(a) shows the variation of frequency with temperature when the core material is DYAD609. Fig. 6(b)

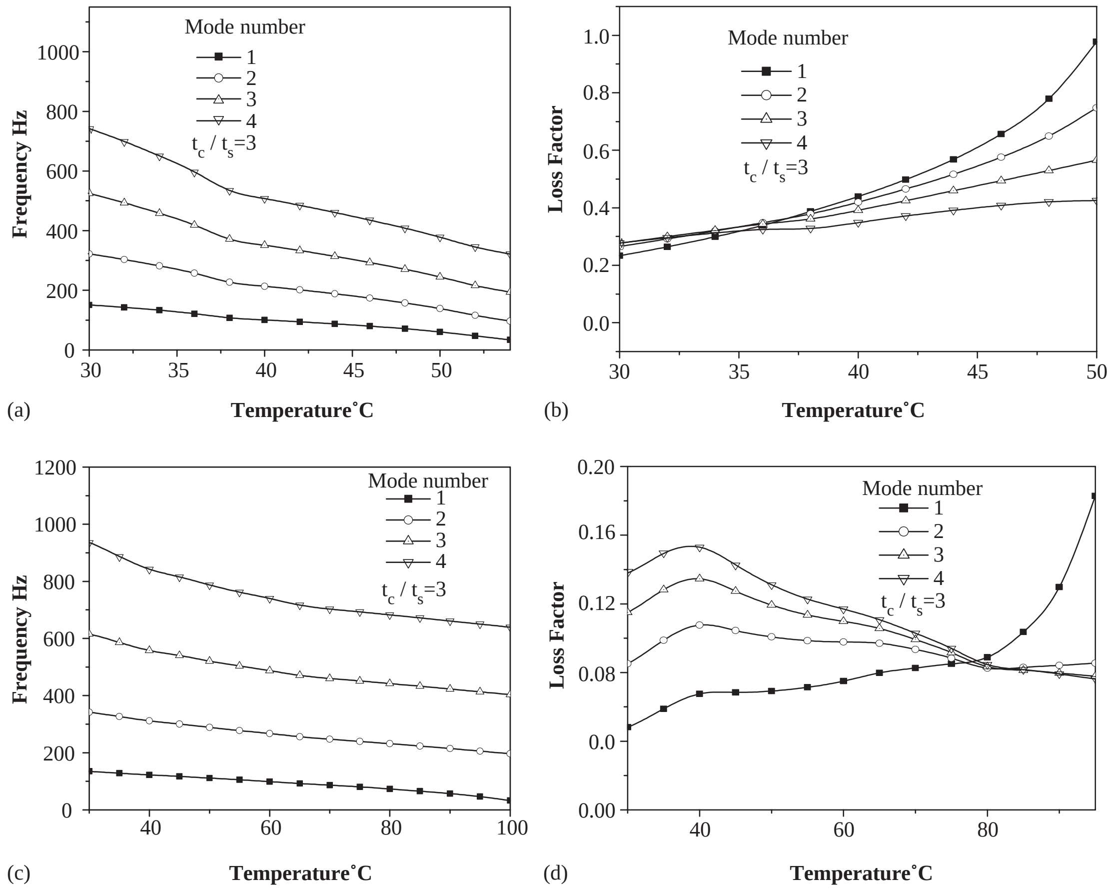

Fig. 6. Variation of frequency and loss factor with temperature for sandwich beam: Core (a), (b) DYAD609 and (c), (d) EC2216. *Note*: The thermal boundary condition used is Tbc2.

shows the variation of loss factor with temperature. Since the drop in the shear modulus of DYAD609 is very high with temperature, buckling temperatures are very low for DYAD609 loss factor reaches high value (0.95) near buckling temperature. Fig. 6(c) shows the variation of frequency with temperature when the core material is EC2216. Fig. 6(d) shows the variation of loss factor with temperature. First mode loss factor reaches high value (0.17) when temperature is nearing the buckling temperature.

3.3. Buckling analysis

The buckling temperature can be obtained by solving the following eigenvalue problem

$$[K_R^G] + \lambda[K_g^G] = 0. \tag{5}$$

The buckling temperature to be specified can be obtained depending upon temperature boundary condition as $T_b = \lambda T_{\text{ref}}$ where T_{ref} is the temperature at which $[K_g^G]$ is evaluated. When G^{*c} is assumed to be a function of temperature ($G^{*c} = f(T)$) the above procedure does not hold good. The following iteration procedure has to be carried out and the same has been carried out for a typical case.

1. Assume a particular temperature, and assume material properties at that temperature and evaluate λ.
2. Multiply λ with the reference temperature, which is supposed to be the buckling temperature corresponding to chosen temperature and corresponding material property.
3. Choose material properties corresponding to evaluated buckling temperature. Evaluate λ, in case λ is not 1 the procedure of above step is followed and iteration is carried out until $\lambda = 1$.

Instead of carrying out the above iterative procedure it is preferable to do the vibration analysis assuming different temperature for the beam and using the material properties corresponding to the temperature. The buckling temperature corresponds to temperature at which natural frequency becomes zero. In the present study for a typical case buckling temperature was found by both methods and they tallied very well. For parameter study the variation of frequency with temperature was studied and buckling temperature is evaluated. The results are cross verified by solving the buckling problem by using the buckling temperature and material properties corresponding to that temperature which leads to $\lambda = 1$.

Table 1 compares the buckling temperatures of the sandwich beam for each core material mentioned above for two different thermal boundary conditions. From Ref. [9] it is found that considering the temperature-dependent shear modulus DYAD609, it will fall drastically with temperature and becomes almost $\frac{1}{10}$ of its value (20.6e5 Pa) at room temperature for 20 °C temperature rise. The result is very low buckling temperatures (55 °C) as can be seen from Table 1. Shear modulus does not fall so drastically for EC2216 it only becomes only $\frac{1}{2}$ of its value (344.8e6 Pa) at room temperature for 20 °C temperature rise. Hence beam having EC2216 as core material has relatively higher buckling temperature. It is also found from the results when the shear modulus is higher the core thickness influences the buckling temperature much. In contrast if the shear modulus is low the core thickness does not influence buckling temperature much.

Table 1
Buckling temperatures

Description	Buckling temperature (°C)			
Core material	Tbc	$t_c/t_s = 1$	$t_c/t_s = 2$	$t_c/t_s = 3$
DYAD 606	1	96	103	109
	2	52	55	57
EC2216	1	—	—	—
	2	104	153	—

4. Conclusion

In the present study, for the first time in literature sandwich beam with viscoelastic core subjected to temperature is analyzed for its buckling temperature and variation of its frequency and damping with temperature. The following are the conclusions based on the present study.

1. The influence of material property with respect to temperature considerably affects the thermal buckling temperature and vibration behavior.
2. The results obtained for thermal buckling using iterative procedure correlates well with the temperature obtained for zero frequency of the beam accounting initial stresses.
3. The effect of core thickness on buckling temperature is more when the value of shear modulus of the core is high.
4. For the buckling temperature range considered in the present study η was increasing with temperature. In addition real stiffness of the system is falling with temperature. Hence in general the damping was increasing with temperature. It is felt that this conclusion may not hold good for the range of temperature where η decreases with temperature.

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